The Role of Executive Functioning in Children's Arithmetic Calculation

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ABSTRACT

The cognitive underpinnings of arithmetic calculation in children are noted to involve working memory; however, other cognitive processes each related to arithmetic calculation and to working memory suggest that this relationship is more complex than previously stated. The central purpose of this study is to examine the role of executive functioning in arithmetic calculation in children. Results suggest two important findings. First, executive functioning emerged as a significant contributor to arithmetic calculation. Second, after controlling for reading, processing speed, short-term memory, and executive functioning, only visual-spatial working memory, and not verbal working memory contributed to arithmetic calculation. Results are discussed in terms of directions for future research on working memory in arithmetic calculation.

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CHAPTER ONE

INTRODUCTION

A major goal of school is the development of children's mathematical skills. All students, including those with disabilities and those at risk of school failure, need to acquire the knowledge and skills that will enable them to perform math-related problems that they encounter at school, in their daily lives, and in their future employment (Paglin & Rufolo, 1990; Rivera-Batiz, 1992; Stevenson, 1987). While it is viewed as a central responsibility of school, second only to the development of reading, little research has focussed on understanding the cognitive processes that contribute to the development of mathematical skills. Indeed, research in mathematics has lagged behind that of reading.

A recent development in the research on mathematics has been the attempt to combine research strategies from the fields of cognitive development, mathematical cognition, and learning disabilities (e.g., Geary, Hoard, & Hamson, 1999; Jordan & Montani, 1997). These studies follow children longitudinally and measure multiple competencies such as the development of early informal arithmetic skills (such as counting and problem-solving strategies) and more formal indicators of mathematical competence such as arithmetic calculation (Geary et al., 1999). Such studies investigate cognitive processes that are purported to support the development of mathematics and the acquisition of specific components of mathematics, such as knowledge of counting principles (Geary et al., 1999). As well, these studies examine children with mathematical disabilities and compare them to children who are normally achieving in mathematics. In many ways, this research design parallels earlier studies of reading acquisition in typically developing children and in children with reading difficulties. Such studies have proved critical to our understanding of reading disabilities. Regardless of the similarities between these fields of research, research on the cognitive underpinnings of mathematics is still in its infancy. While there is no consensus on the core cognitive processes that predict the development of mathematics, research has provided insight into the cognitive processes associated with specific areas of mathematics, for instance, arithmetic calculation.

The cognitive underpinnings of arithmetic calculation in children are noted to involve working memory (Adams & Hitch, 1997); however, other cognitive processes each related to arithmetic calculation and to working memory suggest that this relationship is more complex than previously stated (Berg, 2008b; Bull & Johnston, 1997; Case 1985). While some research has been directed at examining this relationship, with a focus upon processing speed and short-term memory (Berg, 2008b; Bull & Johnston, 1997), executive functioning has also surfaced as an area of potential importance. The purpose of this study is to examine the role of executive functioning in children's arithmetic calculation.

CHAPTER TWO

LITERATURE REVIEW

Working memory has received increased attention in the field of mathematics and has been hypothesized to be directly involved in arithmetic calculation for two reasons. First, numerous studies have reported that children with mathematical disabilities are impaired in several memory-based processes (Berg, 2008a; Bull & Johnston, 1997; Jordan & Montani, 1997; Swanson & Sachse-Lee, 2001). Second, the mental processes of arithmetic calculation (e.g., counting procedures and memory-based strategies) parallel the conceptual and operational characteristics of working memory (Case, 1985). Further, mental arithmetic performance and working memory functioning are facilitated by cognitive processes that interact with working memory (e.g., processing speed; Case, 1985). The following sections review literature related to the development of arithmetic calculation, describe the arithmetic difficulties and cogni `tive impairments of children with mathematical disabilities, and examine research on specific cognitive processes that are associated with arithmetic calculation.

Arithmetic Calculation

Arithmetic calculation is the practice of solving algorithmic equations. When first learning to solve simple addition problems (e.g., 5 + 3), children initially count both addends. These counting procedures are typically executed with the aid of fingers (i.e., the finger counting strategy) and progress to the verbal counting strategy (Geary, 2004; Siegler, 1987). The two most commonly used counting procedures are counting all and counting on (Fuson, 1982; Groen & Parkman, 1972). Counting all involves counting both addends starting from 1. The counting-on procedure typically involves stating the value of one addend and then counting the number equal to the value of the other addend, such as stating 2 and then counting 3, 4, 5, to solve 2 + 3. The development of procedural competencies is related in part to improvements in children's conceptual understanding of counting and is reflected in a gradual shift from the frequent use of the counting all to the counting on (e.g., Geary, Bow-Thomas, & Yao, 1992). This progression in counting strategies leads to the development of long-term memory representations of basic facts (Geary, 2004; Siegler, 1987; Siegler & Shrager, 1984). With direct retrieval, children state an answer that is associated in long-term memory with the problem presented, such as stating 8 when asked to solve 5 + 3.

The transition to memory-based processes results in arithmetic fluency (quick and accurate answers) to the solution of individual problems, which in turn, leads to a reduction in the cognitive demands required to perform more complex arithmetic calculations (Geary, 2004). Once formed, these representations support the use of memory-based problem-solving processes that support more complex arithmetic problems. One of the most common of these is decomposition. Decomposition involves reconstructing the answer based on the retrieval of a partial sum. For instance, the problem 5 + 7, might be solved by retrieving the answer to 5 + 5 (i.e., 10) and then adding 2 to this partial sum.

As the strategy mix matures, children solve problems more quickly because they use more efficient memory-based strategies and because, with practice, it takes less time to execute each strategy (Delaney, Reder, Staszewski, & Ritter, 1998; Geary, Bow-Thomas, Liu, & Siegler, 1996). This transition to memory-based processes results in the quick solution of individual problems and reduction of the working memory demands associated with solving these problems. The eventual automatic retrieval of basic facts and the accompanying reduction of the cognitive demands in turn appear to make the solving of more complex problems, in which the simple problems are embedded, less error prone (e.g., word problems, Geary & Widaman, 1992).

Learning Disabilities in Mathematics

Converging evidence reveals that 6-7% of the school-age population has a mathematical disability (Badian, 1983; Kosc, 1974; Rivera, 1997). Learning disabilities in mathematics have not been studied for as long as learning disabilities in reading and not as extensively. Nonetheless, research in the field of mathematics disabilities is increasing, particularly in the area of arithmetic calculation. The theoretical models and experimental methods used to study arithmetic calculation in children with mathematical disabilities have provided an important framework for guiding the study of the development of arithmetic calculation in normally achieving children. In particular, insights have been gained from research on the arithmetic difficulties and the cognitive processing impairments that are characteristic of children with mathematical disabilities.

Arithmetic Difficulties and Cognitive Impairments of Children with Mathematical Disabilities

Children with mathematics disabilities have difficulty performing basic arithmetic calculations such as addition, subtraction, multiplication, and division (e.g., Geary, 1990, 1993; Jordan & Montani, 1997). They struggle to develop fluency in basic math facts and have difficulty applying these basic math facts to more complex mathematical problems (e.g., Geary, Hamson, & Hoard, 2000). Geary (1993, 2004) delineated an inability to retrieve basic facts from long-term memory as a defining feature of mathematical disabilities. For instance, children with mathematical disabilities appear to be slower at developing fluency for arithmetic facts than their normally achieving peers (Geary, 1990, 1993; Jordan, Hanich, & Kaplan, 2003). In particular, children with mathematical disabilities compared to their normally achieving peers are typically less accurate (e.g., Geary, 1990, 2004; Ostad, 1998) and provide correct answers more slowly (e.g., Jordan & Montani, 1997). Further, when providing answers arithmetic problems, children with mathematical disabilities rely upon inefficient calculation strategies (e.g., finger counting) beyond the age when they would be expected to move to more efficient strategies such as direct retrieval from long-term memory (e.g., Geary, 1990; Geary & Brown, 1991). In view of the core difficulties children with mathematical disabilities experience in arithmetic calculation, memory functioning and memory-related processing abilities have become one focus as potential explanations for cognitive impairments that characterize children with mathematical disabilities (e.g., Berg, 2008a; Swanson & Sachse-Lee, 2001).

Three cognitive processes that appear to be characteristic impairments in children with mathematical disabilities are working memory, processing speed, and short-term memory (e.g., Berg, 2008a; Bull & Johnston, 1997). A large body of literature has examined working memory in children with mathematical disabilities and children normally achieving in mathematics (e.g., Berg, 2008a; Bull & Johnston, 1997; Hitch & McAuley, 1991; Siegel & Ryan, 1989; Swanson, 1994). Typically, these studies indicate that children with mathematical disabilities have poorer working memory than their normally achieving peers. However, working memory impairments have not been found consistently across research. Temple and Sherwood (2002) found that children with and without mathematical disabilities did not differ in working memory. Similarly, Berg

(2008a) reported that working memory impairments were not generalized across measures.

Further, these differences have been found in specific forms of working memory. For example, many studies have found that children with mathematical disabilities have lower verbal working memory than normally achieving children (Bull & Johnston, 1997; D'Amico & Guarnera, 2005; Hitch & McAuley, 1991; Passolunghi & Siegel, 2001; Swanson & Sachse-Lee, 2001). As well, studies have also reported that children with mathematical disabilities have visual-spatial working memory impairments (Berg, 2008a; Bull, Johnston, & Roy, 1999; D'Amico & Guarnera, 2005; McLean & Hitch, 1999). So prominent are the verbal and visual-spatial working memory impairments of children with mathematical disabilities, that current definitions include subtypes that refer specifically to these impairments (Geary, 1993; Robinson, Menchetti, & Torgesen, 2002; Rourke, 1993).

Similar to literature highlighting working memory impairments in children with mathematical disabilities, several studies report that compared to their mathematically normally achieving peers, children with mathematical disabilities perform more poorly on a range of short-term memory tasks (Bull & Johnston, 1997; Siegel & Ryan, 1989). In a study investigating cognitive processing and mathematics in 7 year old children, Bull and Johnston (1997) found that children with mathematical disabilities had lower short-term memory spans. In a similar study with 9 to 12 year olds, Berg (2008a) found differences on short-term memory tasks for numerical and non-numerical information between children with and without mathematical disabilities.

A third cognitive area that seems to represent an impairment for children with mathematical disabilities is processing speed. Processing speed, defined as the maximum rate that cognitive functions can be performed (Kail, 1992), has been reported to represent a significant difference between children with and without severe learning disabilities (Kail, 1994; Martos, 1995). However, the majority of these studies in the area have focussed on reading difficulties and have focused on one aspect of processing speed, rapid automatized naming. Researchers have suggested the role of rapid automatized naming as a component in the double-deficit theory of reading disabilities (Wolf & Bowers, 1999). Nevertheless, literature in the field of mathematics has also suggested the importance of processing speed. Difficulty efficiently accessing and using information stored in long-term memory has been suggested to be a central characteristic of the challenge of children with mathematical disabilities to develop proficiency in arithmetic (e.g., Geary, 1990, 1993). This view has found some support. McLean and Hitch (1999) reported that children with mathematical disabilities were characterized by a difficulty accessing long-term memory. Using a missing items task that assessed children's performance on maintaining and manipulating remembered information, these researchers found that children with mathematical disabilities performed more poorly than their normally achieving peers.

Cognitive Predictors of Arithmetic Calculation

Compared to the substantial body of research focused on identifying the cognitive processes that undergird reading (e.g., phonological processing, Snow, Burns, & Griffin, 1998; Stanovich, 1982), little research has been directed at identifying the cognitive processes that are involved in arithmetic calculation. Current theoretical and empirical research suggests that working memory, a cognitive system that is in part responsible for the construction of information and the transfer of this information into long-term memory, is likely to play an important role in arithmetic calculation (Baddeley, 1986, 1996, 2001). While a growing body of evidence suggests that working memory is related to arithmetic calculation (e.g., Adams & Hitch, 1997; Geary et al., 2000); however, other cognitive processes each related to arithmetic calculation and to working memory suggest that this relationship is more complex than previously stated (Berg, 2008b; Bull & Johnston, 1997; Case 1985).

Working Memory

Working Memory is defined as a limited capacity processing resource involved in the preservation of information and simultaneous processing of the same or other information (e.g., Baddeley & Hitch, 1974; Just & Carpenter, 1992). Baddeley (1986) proposed a three component model of working memory. The central executive allows information to be manipulated in working memory and is thought to have direct access to long-term memory. It directs attention, coordinates procedures, and suppresses irrelevant information. The central executive integrates information from the other two components: the phonological loop and the visuo-spatial sketchpad. The phonological loop is responsible for the temporary storage of verbal information; items are held within the store through articulation. The visuo-spatial sketchpad is responsible for the storage of visual-spatial information over brief periods and plays a key role in the generation and manipulation of mental images. The central executive is viewed as responsible for coordinating activity within working memory but also devotes some of its resources to increasing the amount of information that can be held in the two subsystems (Baddeley & Logie, 1999).

Performance on measures of working memory predicts performance on a range of academic skills (Baddeley, Gathercole, & Papagno, 1998; Gyselinck, Ehrlich, Cornoldi, de Beni, & Dubois, 2000). Although much has been learned about the characteristics of the working memory system (Baddeley, 1986), its specific role in supporting arithmetic calculation is not well understood. As described above, there is evidence for working memory deficits in children with mathematical disabilities (e.g., Berg, 2008a; Bull & Johnston, 1997; Swanson & Sachse-Lee, 2001). More specifically, working memory occupies an important place in recent research on mathematical performance and is therefore a likely to contribute to arithmetic calculation.

Cognitive processes involved in performing arithmetic calculations have been argued to be associated with working memory system in that they require the storage of information while performing other mental operations. For instance, to solve 4 + 3 a child must concurrently retain two or more pieces of information (the phonological codes representing the numbers “4” and “3”) and then employ one or more procedures (e.g., counting) to combine the numbers to produce an answer. Alternatively, using carrying procedures involves maintaining parts of the problem while performing additional procedures. To solve 18 + 5, a child might retain the 3 from adding 8 + 5, while adding the 1 from the tens column of the 18 to the 1 from the tens column of the 13 produced from adding the 8 + 5.

While studies are limited in the area and conducted mostly with adults, findings suggest that working memory resources are required to solve problems that require several procedural steps (e.g., 125 + 97, Fürst & Hitch, 2000; Geary & Widaman, 1992; Logie, Gilhooly, & Wynn, 1994). In these problems, working memory appears to be related to retaining partial solutions during calculation and for carrying partial results across columns. Adams and Hitch (1997) explored this relationship in children aged 8 to 11 years using simple mental addition (8 + 1) and complex mental addition (231 + 16) problems. Results indicated that solving problems presented verbally was associated with high working memory, whereas solving problems presented visually was related to low working memory. As well, level of problem difficulty was associated with working memory contributions, with higher working memory spans corresponding with higher complexity problems.

Although research indicates a relationship between working memory and arithmetic calculation, the influence of other cognitive processes that inform working memory have received little attention. In particular, arithmetic calculation has been suggested to be related to two cognitive processes that are directly related to working memory: processing speed (Case, 1985) and short-term memory (Baddeley & Hitch, 1974).

Processing Speed and Short-term Memory

The speed that information is introduced into working memory and used within working memory has been viewed as important to the general functioning of working memory. For instance, Case (1985) suggested that processing speed plays a role in increasing the available short-term storage space. In investigations with children, Case and colleagues found a significant relationship between processing speed and storage capacity of working memory (Case, Kurland, & Goldberg, 1982). Specifically, they reported that faster counting speed was linked to higher counting spans. They explained that faster operations require less workspace, thus leaving more available for storage. The potential relationship among processing speed, short-term memory, and arithmetic calculation follows this rationale (Geary, 1990, 1993). For example, slow counting speed would increase the amount of time between a counted answer and a combined set of addends in a problem. This increase in time between paired associations (combined set of addends and an answer) increases the potential for decay on one or more of the pieces of information to be remembered, and thus, decreases the likelihood for encoding the paired association into long-term memory.

Bull and Johnston (1997) investigated the role of processing speed and short-term memory in the arithmetic calculation performance of 7 year old children. They found that when children were assessed on measures of short-term memory, long-term memory, processing speed (as measured by matching/crossing out speed and perceptual motor speed), and sequencing ability, the strongest predictor of arithmetic calculation was processing speed. This study, however, did not include measures of working memory. Swanson and Beebe-Frankenberger (2004) examined the contributions of working memory to arithmetic calculation in children in grades 1 to 3. Results suggest that processing speed contributed unique variance to each of these arithmetic areas in the presence of working memory, short-term memory, phonological processing, and age. In a similar study, Swanson (2006) examined the cognitive processes that contribute to arithmetic calculation in children with advanced mathematical skills in grades 1 to 3. In the presence of significant contributions of working memory, reading and age, processing speed did not emerge as a significant contributor.

Building on these studies, Berg (2008b) examined the relative contributions of processing speed, short-term memory, and working memory in children's arithmetic calculation. In a sample of children from grades 3 to 6, results corresponded with previous studies in younger children that have indicated a significant relationship between processing speed, short-term memory, and working memory and arithmetic calculation (Bull & Johnston, 1997; Swanson & Beebe-Frankenberger, 2004). A second notable finding of Berg's (2008b) research was that the contribution of working memory was not eliminated by the presence of processing speed (as measured by naming speed, articulation rate, and counting speed) and short-term memory. Although both of these latter processes are argued to be implicit to the working memory system (Case, 1985; Case et al., 1982) and associated with arithmetic calculation (Baddeley & Hitch, 1974; Swanson & Beebe-Frankenberger, 2004), the question arises as to what other processes account for the contribution of working memory to arithmetic calculation. Recent theoretical perspectives suggest that attentional resources within working memory (e.g., central executive functions related to switching and inhibition) are potential avenues for interpretation of Berg's (2008b) results and future studies (Baddeley, 2001).

Executive Functioning

William James, in his seminal *Principles of Psychology* (1890), remarked:

> Everyone knows what attention is. It is the taking possession by the mind, in clear and vivid form, of one out of what seem several simultaneously possible objects or trains of thought. Focalization, concentration, of consciousness are of its essence. It implies withdrawal from some things in order to deal effectively with others, and is a condition which has a real opposite in the confused, dazed, scatterbrained state which in French is called distraction, and Zerstreutheit in German. (403-404)

Since his initial statement, the study of attention (i.e., executive functioning) has become a significant area of focus within the field of cognition. Through a more general lens, executive functioning refers to the attentional control mechanisms that regulate general cognitive processes and various cognitive subprocesses (van der Sluis, de Jong, & van der Leij, 2004). More specifically, executive functioning refers to the cognitive processes required to plan and direct activities, task initiation and follow-through, sustained attention, performance monitoring, inhibition of impulses, and goal directed persistence (Baron, 2004). Executive functioning is often fractionated into a variety of distinct, though related processes. Baddeley (1996) distinguished four executive functions: inhibition, shifting, updating, and dual-task performance. Inhibition, or suppression of automatized responses in favour of more goal-appropriate actions, is most often associated with executive functioning. Shifting is the ability to switch attention between strategies or response sets. Shifting is characterized by the disengagement of an irrelevant task set or strategy and the subsequent activation of a more appropriate one. Updating is the encoding and evaluation of incoming information for relevance to a current task and subsequent revision of the information held in memory. Updating is characterized by the manipulation of the content of memory, information is not stored passively; rather, it is revised when new information is presented. Dual-task performance refers to the ability to coordinate performance on two tasks simultaneously. Performance on a baseline task deteriorates with the introduction of a secondary task that must be performed simultaneously (van der Sluis et al., 2004).

Little research has been directed at understanding the relationship between executive functioning and mathematics. Mathematics requires the ability to plan, organize, and self-monitor, several key components of executive functioning (Baron, 2004). In addition, impairments in executive functioning have been reported in children with learning disabilities. Denckla (1994) has demonstrated the connection between executive functioning and learning disabilities. Denckla stated that executive processing is linked to learning disabilities in ways similar to language or spatial abilities and is included under the definition as one of the psychological processes, particularly related to thinking and reasoning (Denckla, 1994).

The role of executive functioning in arithmetic calculation is likely to be related to the procedures that children have to use when engaged in mental arithmetic. For instance, when solving a problem (e.g., 5 + 8) the student must not only remember the numbers and the operation sign (i.e., storage), but also actively direct attention among these elements of the problem to engage in the strategy to solve the problem. For example, if the student uses a counting strategy, they must decide whether they will count from 1 to 13, from 5 to 13, or from 8 to 13. Any of these procedures requires the student to switch their attention from remembering the problem to solving the problem (thinking of an effective strategy), to focus their attention (actively remembering parts of the problem-numbers and operation), and to divide their attention (beginning to count from one of the numbers but remembering the other number as the sign for when to stop counting).

Several studies also demonstrated relations between arithmetic ability and performance on the Making Trails task, which measures shifting ability. McLean and Hitch (1999) presented children of high and low arithmetic ability with three versions of the Making Trails task, all of which required shifting. They found that children with mathematical disabilities were slower than children with normal arithmetic ability on all versions of the task. Bull and Scerif (2001) examined the ability to inhibit responses in children with mathematical disabilities. They administered Stroop tasks that required the naming of either color or quantity of stimuli under facilitating or interfering task conditions. The interference condition was assumed to provide an indication of the ability to inhibit automatized responses (i.e., the reading of words or digits) in favour of less automatized ones (i.e., the naming of ink color or number of digits). Only the naming of quantity in the interference condition appeared to be related to arithmetic ability.

An association between executive functioning and arithmetic has been suggested also in studies involving children with and without mathematical disabilities (e.g. Bull & Johnston, 1997; Bull, Johnston, & Roy, 1999; McLean & Hitch, 1999; Passolunghi & Siegel, 2001). Current, although limited, literature has been consistent in highlighting an impairment of executive functioning related to the central executive component within working memory in children with mathematical disabilities. These children typically perform worse than their same age peers on tasks such as the Wisconsin Sorting Card Tasks (Bull et al., 1999) and in memory tasks with a high executive demand such as Case, Kurland and Goldberg's (1982) counting span (Siegel & Ryan, 1989) or the digit span backward (Geary et al., 1999, 2000). Siegel and Ryan (1989), however, have shown that children with arithmetical difficulties showed impairment only in the central executive tasks that involved numerical information, while no differences between groups were found in the central executive tasks involving linguistic information, such as the Daneman and Carpenter Listening span task (1980). These results led Siegel and Ryan (1989) to argue that poor arithmetical performers did not have a general executive disorder, but rather that their failure in the central executive tasks derived from their difficulty in manipulating and maintaining numerical information. Furthermore, McLean and Hitch (1999) demonstrated that children with poor arithmetical skills showed impairment in central executive tasks that require a switch between numerical/linguistic retrieval strategies and are assumed to measure the ability to inhibit a pre-activated retrieval strategy.

Studies of children with mathematical disabilities suggest a second form of retrieval deficit, specifically, disruptions in the retrieval process due to difficulties in inhibiting the retrieval of irrelevant associations. This form of retrieval deficit was first noted by Barrouilet and colleagues (1997), based on the memory model of Conway and Engle (1994), and was recently confirmed by Geary et al. (2000). In this study, one of the arithmetic tasks required children to use only retrieval – the children were instructed not to use counting strategies – to solve simple addition problems (see also Jordan & Montani, 1997). Children with arithmetic difficulties, as well as children with reading difficulties, committed more retrieval errors than did their academically normal peers, even after controlling for intelligence. The most common of these errors was a counting-string associate of one of the addends. For instance, common retrieval errors for the problem 6 + 2 were 7 and 3, the numbers following 6 and 2 respectively, in the counting sequence.

The pattern in these more recent studies (e.g., Geary et al., 2000) and that of Barrouillet and colleagues (1997) is in keeping with Conway and Engle's (1994) position that individual differences in working memory and retrieval efficiency are related, in part, to the ability to inhibit irrelevant associations. In this model, the presentation of an arithmetic problem results in the activation of relevant information in working memory, including problem features; such as the addends in a simple addition problem and the information associated with these features. Problem solving is efficient when irrelevant associations are inhibited and prevented from entering working memory. Inefficient inhibition results in activation of irrelevant information, which functionally lowers working memory capacity. In this view, children might make calculation errors, in part, because they cannot inhibit irrelevant associations from entering working memory. Once in working memory, these associations either suppress or compete with the correct association for expression. This perspective suggests that the calculation difficulties of some children might originate from executive functions related to inhibitory mechanisms (Bull et al., 1999).

The Present Study

The relation between arithmetic calculation and working memory awaits consensus. One of the central issues is whether working memory in general or underlying cognitive processes are the central contributors to successful arithmetic calculation. Thus, investigations of arithmetic calculation are advantaged by the inclusion of potential mechanisms that are both related to arithmetic calculation and that inform the working memory system. The purpose of this study is to examine the role of executive functioning in children's arithmetic calculation. More specifically and in line with Berg's (2008b) findings, two questions will be investigated. First, does executive functioning contribute to arithmetic calculation? Second, does working memory contribute additional variance to arithmetic calculation after accounting for processing speed, short-term memory, and executive functioning?

Implications

This study will potentially make a valuable contribution to research by investigating the roles executive functioning in children's arithmetic calculation. A more complete understanding of the specific cognitive processes (e.g., executive functioning) that are involved in specific forms of mathematical performance (e.g., arithmetic calculation) will support investigations into other areas of arithmetic calculation (e.g., multiplication). And, as we begin to map better the development of children's proficiency in arithmetic calculation and the cognitive processes that facilitate proficiency, this knowledge can be used by classroom teachers and special education teachers to meet the needs of students normally achieving arithmetic calculation and students with learning difficulties in arithmetic. Uncovering which cognitive processes contribute to arithmetic calculation can guide the modelling of instructional approaches and curricular materials to correspond to these processes.

CHAPTER THREE

METHOD

Participants

Data for this study was part of a larger study previously conducted by Berg (2007). The purpose of the Berg study was twofold. The first purpose was to investigate the role of specific cognitive processes in children's arithmetic calculation. The second purpose was to examine the cognitive processing impairments of children with mathematical disabilities. The present study aligns with the first purpose.

A total of 90 children in Grades 2 and 3 from five schools in Atlantic Canada participated in this study. The socioeconomic status of individual children was not assessed; however, each of the schools that participated in the study was located in a predominately working-class middle-class neighbourhood. All children spoke English as their first language. No child had been identified as having English language difficulties that would have made it difficult for the child to complete any of the study activities. Children completed a series of tests that assessed children's academic achievement, fluid intelligence, naming speed, sequencing speed, short-term memory, working memory, executive functioning, and arithmetic calculation (written addition and mental addition). Only those tests that relate to the purpose of the present study were selected for analysis; these are described below.

Instruments

Children's performance on 5 test batteries was selected for analysis in the present study. The first battery focussed upon children's academic achievement in arithmetic and reading. The second battery measured children's processing speed. A third battery assessed children's short-term memory. The fourth battery measured children's working memory, and the fifth battery assessed children's executive functioning.

Academic Achievement

The Wide Range Achievement Test-Third Revision (WRAT3; Jastak & Jastak, 1993) was administered to measure children's arithmetic achievement and reading achievement. The arithmetic subtest focuses upon reading numbers, counting, mental arithmetic, and written calculation. The reading subtest focuses upon recognizing and naming letters, and pronouncing words. The WRAT3 has been used extensively to assess children's achievement in studies investigating arithmetic calculation (e.g., Berg, 2008; Mabbott & Bisanz, 2003; Wilson & Swanson, 2001). Raw scores and standard scores (M = 100, SD = 15) based on age-appropriate norms were calculated for each child. Cronbach's alpha for the WRAT3 arithmetic and reading subtests was .87 and .89 in the present study, respectively.

Naming Speed

Two tasks were administered to assess children's processing speed, specifically children's ability to name stimuli rapidly: rapid digit naming and rapid letter naming. Both tasks were taken from the Comprehensive Test of Phonological Processing (CTOPP, Wagner, Torgesen, & Rashotte, 1999).

Rapid Digit Naming

In the rapid digit naming task children were presented with a picture book containing two different pages, each containing four rows and nine columns of six randomly arranged single digit numbers. Children were instructed to start naming the numbers on the top row, from left to right, move to the next row, and so on, until all of the numbers had been named. A child's score was the total time to name the both arrays of numbers.

Rapid Letter Naming

In the rapid letter naming task children were presented with a picture book containing two different pages, each containing four rows and nine columns of six randomly arranged letters. Children were instructed to start naming the letters on the top row, from left to right, move to the next row, and so on, until all of the letters had been named. A child's score was the total time to name the both arrays of letters.

Short-term Memory

Two tasks were administered to assess children's short-term memory: memory for digits and nonword repetition. Both tasks were taken from the CTOPP (Wagner et al., 1999).

Memory for Digits

In the memory for digits task children were presented with a series of recorded numbers ranging in length from 2 to 8 digits at a rate of 2 per second and then asked to repeat the numbers in the same order in which he or she heard them. The test was stopped when a child incorrectly recalled the digits in three consecutive sets. A child's score was the number of sets recalled correctly. In the current study, Cronbach's alpha for the memory for digits task measured .73.

Nonword Repetition

In the nonword repetition task children were presented with recorded nonwords that range in length from 3 to 15 sounds and then asked to repeat the made-up word exactly as he or she heard them. The test was stopped when a child incorrectly recalled the nonwords in three consecutive sets. A child's score was the number of sets recalled correctly. In the current study, Cronbach's alpha for the nonword repetition task measured .80.

Working Memory

Four tasks were administered to assess children's working memory: auditory digit sequence, semantic categorization, visual matrix, and mapping and directions. All tasks were taken from the Swanson-Cognitive Processing Test (Swanson, 1995).

Auditory Digit Sequencing

The auditory digit sequence task assessed a child's ability to recall numerical information contained within a short sentence. The researcher read aloud a sentence containing a street address, asked the child a process question, and then asked the child to recall part of the address. As an example of a single task administration, set 1 was: "Suppose somebody wanted to have you drive them to the library at 2-9 Maple Street." The researcher then asked the process question: "What is the name of the street?" If the child answered incorrectly, the task was stopped. If the child answered correctly they were then asked to state the number embedded within the address. If the child correctly recalled the address number the next sentence was administered. Sets ranged from 2 to 9 sentences. A child's score was the number of sets recalled correctly. In the current study, Cronbach's alpha for the auditory digit sequence task measured .58.

Semantic Categorization

The semantic categorization task assessed a child's ability to recall related words within prearranged groups. The researcher read aloud a set of words with a 2 second interval between words. Next the researcher asked the child a process question and then asked the child to recall each group name and each word within its respective group. As an example of a single task administration, set 1 was: "flower-rose-daisy". The researcher asked the process question "Which word 'rose' or 'violet' was presented?" If the child answered incorrectly, the task was stopped. If the child answered correctly they were then asked to recall the group and the words within that group. If the child responded correctly, the next word set was administered. Item set difficulty ranged from one group with two words to eight groups with three words in each group. A child's score was the number of sets recalled correctly. In the current study, Cronbach's alpha for the semantic categorization task measured .58.

Visual Matrix

The visual matrix task assessed a child's ability to recall dots arranged within a matrix. The researcher presented the child with a matrix containing a series of dots, gave the child 5 seconds to study the series of dots within the matrix, withdrew the matrix from the child, and then asked the child a process question. The process question was: "Are there any dots in the first column?" If the child answered incorrectly the task was stopped. If the child answered correctly the child was then asked to reproduce the dot arrangement onto a blank matrix of the same size. If the child correctly reproduced the original matrix the next matrix was administered. The items ranged in difficulty from a matrix of 4 squares with 2 dots to a matrix of 45 squares and 12 dots. A child's score was the number of matrices recalled correctly. In the current study, Cronbach's alpha for the visual matrix task measured .67.

Mapping and Directions

The mapping and directions task assessed a child's ability to recall a sequence of directions on a map that contained no visual symbol key. The researcher presented to each child a street map that contained a series of dots (signifying streetlights) connected by lines (signifying a route) and labelled with arrows (signifying directions). The child was given 10 seconds to study the map, the researcher withdrew the map, and then asked a process question ("Were there any stoplights in the first column?"). If the child answered the process question incorrectly the task was stopped. If the child answered correctly they were then asked to reproduce the lines, dots, and arrows on a blank map. If the child correctly drew all dots, lines, and arrows from the original map the next map was administered. The items range in difficulty from a map of 2 dots and 2 lines to a map of 20 dots and 20 lines. A child's score was the number of maps drawn correctly. In the current study, Cronbach's alpha for the mapping and directions task measured .59.

Executive Functioning

Eight tasks were administered to assess children's executive functioning. These tasks were adapted from similar tasks administered by van der Sluis et al. (2004). Four tasks assessed the switching process of executive functioning: making trails numbers, making trails letters, making trails coloured numbers, and making trails coloured letters.

Four tasks assessed the inhibition process of executive functioning: quantity naming, Stroop black words, quantity inhibition, and Stroop colour words.

Switching

Four tasks were administered to assess switching: making trails numbers, making trails letters, making trails coloured numbers, and coloured making trails letters. To assess switching the making trails numbers and the making trails letters tasks were used as control measures. That is, performance on the making trails coloured numbers task was compared with performance on the making trails numbers task and performance on the making trails coloured letters task was compared with performance on the making trails letters task.

Making Trails Numbers

In the making trails numbers task, children were presented with a series of 11 circles on a page, with each circle containing a number from 1 to 11. Children were instructed to draw lines as quickly as possible connecting each circle to the next in sequential order beginning at 1 and finishing at 11. Instructions to children emphasized the importance of first, to connect the circles in the correct order, and second, to connect the circles as quickly as possible. A child’s score was the total time to complete the sequence of numbers.

Making Trails Letters

In the making trails letters task, children were presented with a series of 11 circles on a page, with each circle containing a number from A to K. Children were instructed to draw lines as quickly as possible connecting each circle to the next in sequential order beginning at A and finishing at K. Instructions to children emphasized the importance of first, to connect the circles in the correct order, and second, to connect the circles as quickly as possible. A child's score was the total time to complete the sequence of letters.

Making Trails Coloured Numbers

In the making trails coloured numbers task, children were presented with a series of 22 circles on a page: 11 yellow circles contained the numbers 1 to 11 and 11 pink circles contained the numbers 1 to 11. Children were instructed to draw lines as quickly as possible connecting numbers in the yellow circles to the corresponding number in the pink circle then switch to the next number in the numeric sequence until the child reached the pink circle containing the number 11. Instructions to children emphasized the importance of first, to connect the circles in the correct order (yellow circle to pink circle in ascending numerical sequence), and second, to connect the circles as quickly as possible. A child's score was the total time to complete the sequence.

To identify specifically the child's switching ability for numbers, the time to complete the making trails numbers task was subtracted from the making trails coloured numbers task. Extra time needed to complete the making trails coloured numbers task, compared with the making trails numbers task, was attributed to the requirement to switch between two sets of number sequences. This value was used in all subsequent analysis.

Making Trails Coloured Letters

In the making trails coloured letters task, children were presented with a series of

22 circles on a page: 11 yellow circles contained the letters A to K and 11 pink circles contained the letters A to K. Children were instructed to draw lines as quickly as possible connecting letters in the yellow circles to the corresponding letter in the pink circle then switch to the next letter in the alphabetic sequence until the child reached the pink circle containing the letter K. Instructions to children emphasized the importance of first, to connect the circles in the correct order (yellow circle to pink circle in ascending alphabetic sequence), and second, to connect the circles as quickly as possible. A child's score was the total time to complete the sequence.

To identify specifically the child's switching ability for letters, the time to complete the making trails letters task was subtracted from the making trails coloured letters task. Extra time needed to complete the making trails coloured letters task, compared with the making trails letters task, was attributed to the requirement to switch between two sets of letters sequences. This value was used in all subsequent analysis.

Inhibition

Four tasks were administered to assess inhibition: quantity naming, Stroop black words, quantity inhibition, and Stroop colour words. To assess inhibition, the quantity naming and Stroop black words tasks were used as control measures. That is, performance on the quantity inhibition task was compared with performance on the quantity naming task and performance on the Stroop colour words task was compared with performance on the Stroop black words task.

Quantity Naming

In the quantity naming task, a series of triangles was presented in individual segments ranging from 1 to 4 triangles. Children were instructed to name the quantity of objects (e.g., triangles). For example, when the segment △△△ was presented, the correct response was ‘‘3.” Children were instructed to name correctly each quantity as quickly as possible. There were 8 quantity segments in each of 5 rows for a total of 40 segments. Children were instructed to begin by naming the quantity at the beginning (left side) of the top row, to continue to the end of the row, and start immediately at the next row after completing the previous row. Instructions to children emphasized the importance of first, to name correctly each quantity, and second, to name the quantity as quickly as possible. A child’s score was the total time to name the full array of quantities.

Quantity Inhibition

In the quantity inhibition task, the digits in each segment did not correspond to the number of digits in each segment. Children were instructed to name the quantity of digits in each segment as quickly as possible, and not to say the actual digits in each segment. For example, when the segment ‘‘222’’ was presented, the correct response was ‘‘3,” thus saying “two” or “two hundred twenty-two” was incorrect. There were 8 segments in each of 5 rows for a total of 40 segments. Children were instructed to begin by naming the number of digits in each segment at the beginning (left side) of the top row, to continue to the end of the row, and start immediately at the next row after completing the previous row. Instructions to children emphasized the importance of first, to name correctly the number of digits in each segment, and second, to name the number of digits as quickly as possible. A child’s score was the total time to name the full array of quantities.

To identify specifically the child's inhibition ability, the time to complete the quantity naming task was subtracted from the quantity inhibition task. Extra time needed to complete the quantity inhibition task, compared with the quantity naming task, was attributed to the requirement to inhibit one highly conditioned response (i.e., naming of digits) in favor of another response (i.e., naming the number of digits). This value was used in all subsequent analysis.

Stroop Black Words

In the Stroop black words task, a series of four words each referring to a specific colour (yellow, blue, red, and green) were presented in black ink in individual segments. Children were instructed to name (i.e., read) each word as quickly as possible. There were 8 word segments in each of 5 rows for a total of 40 segments. Children were instructed to begin by naming the word at the beginning (left side) of the top row, to continue to the end of the row, and start immediately at the next row after completing the previous row. Instructions to children emphasized the importance of first, to name correctly each word, and second, to name the word as quickly as possible. A child's score was the total time used to name the complete array of words.

Stroop Colour Words

In the Stroop colour words task, the words referring to a specific colour in each segment do not correspond to the colour in which the word was presented in each segment. Children were instructed to name the colour of the word as quickly as possible and not to say the actual word in each segment. There were 8 segments in each of 5 rows for a total of 40 segments. Children were instructed to begin by naming the colour of the word in each segment at the beginning (left side) of the top row, to continue to the end of the row, and start immediately at the next row after completing the previous row. Instructions to children emphasized the importance of first, to name correctly the colour of the word in each segment, and second, to name the colour of the word as quickly as possible. A child's score was the total time to name the full array of colours.

To identify the child's ability to inhibit, the time to complete the Stroop black word task was subtracted from the Stroop colour word task. Extra time needed to complete the Stroop colour word task, compared with the Stroop black word, was attributed to the requirement to inhibit one highly conditioned response (i.e., naming of words) in favor of another response (i.e., naming the colour the word is printed in). This value was used in all subsequent analysis.

Data Analysis

The central purpose of this study was to examine the relative contributions of processing speed, short-term memory, working memory, and executive functioning to arithmetic calculation in children. Two specific objectives informed subsequent analyses. First, does executive functioning contribute to arithmetic calculation? Second, in the presence of processing speed, short-term memory, and executive functioning, does working memory contribute unique variance to arithmetic calculation? To address these aims, specific associations among cognitive processing variables and arithmetic calculation were examined by fixed-order multiple regression analyses.

To determine whether the theoretical assumptions that processing speed, short-term memory, working memory and executive functioning measures reflect different though related constructs, a confirmatory factor analysis was conducted. This analysis is particularly important in light of literature differentiating between short-term and working memory. While agreement is increasing that short-term memory and working memory represent related, yet different constructs, literature in the area has not reached consensus (Engle, Tuholski, Laughlin, & Conway, 1999; Heitz, Unsworth, & Engle, 2005).

Following the confirmatory factor analysis, measures related to processing speed, short-term memory, working memory, and executive functioning were aggregated into composite scores by summing *z*-scores for each appropriate task based on the total sample. Using the composite z-scores for each cognitive construct, fixed-order multiple regression analyses were conducted to examine relative contributions of the cognitive constructs to arithmetic calculation. Because the order of entry influences the outcome of a regression equation, a series of models were created to examine the individual contribution of each cognitive domain, the cumulative variance accounted for by combined processes, and the unique variance accounted for by individual cognitive processing domains after accounting for the other cognitive domains.

CHAPTER FOUR

RESULTS

Descriptive Statistics

The means, standard deviations, and score ranges for all measures are reported in Table 1. Chronological age ranged from 86 to 110 months. Arithmetic calculation raw scores ranged from 16 to 34 and reading raw scores ranged from 18 to 39. Arithmetic calculation standard scores ranged from 66 to 123 and reading standard scores ranged from 69 to 129. Previous research in the area of arithmetic calculation has shown that chronological age and reading ability strongly relate to arithmetic ability (e.g., Berg, 2008b; Bull & Johnston, 1997). Furthermore, previous researchers in the area of arithmetic calculation have used chronological age and reading as control variables in their research design (e.g., Berg, 2008b). Given the wide range in chronological age and reading ability in the current study, these variables will be used as control variables in subsequent analysis.

Correlations

Correlations among the measures are reported in Table 2. Examination of the correlations among the cognitive tasks revealed that several significant relationships. Given the large range in chronological age among participants (86 to 110) and because of the significant correlation between chronological age and arithmetic calculation, $r(90) = .33$, $p < .01$, chronological age was used as a control variable in subsequent multiple regression analysis. Furthermore, there was regression analysis a significant correlation between reading and arithmetic calculation, $r(90) =.53$, $p < .01$, therefore, reading was used as control variable in the subsequent multiple regression analysis.

Table 1

Descriptive Statistics for Chronological Age, Academic Achievement, and Cognitive Processing Tasks

Variable	*M*	*SD*	Range (min-max)
Chronological Age (months)	96.47	5.47	86 - 110
Academic Achievement			
Arithmetic Calculation (Raw)	20.88	2.97	16 - 34
Arithmetic Calculation (Standard)	92.08	10.96	66 - 123
Reading (Raw)	27.66	4.84	18 - 39
Reading (Standard)	97.61	13.43	69 - 129
Control Tasks			
Making Trials Numbers	14.01	5.94	1.90 - 39.60
Making Trials Letters	13.95	5.22	6.70 - 33.90
Quantity Naming	37.70	13.09	20.90 - 126.80
Stroop Black Word Naming	25.03	7.28	14.70 - 66.30
Naming Speed			
Digits	42.97	11.35	26 - 90
Letters	45.69	11.72	27 - 100
Short-term Memory			
Memory for Digits	12.27	2.26	7 - 18
Nonword Repetition	7.79	3.19	2 - 14
Verbal Working Memory			

Auditory Digit Sequence	1.37	.83	0.0 - 3.0
Semantic Categorization	1.37	1.02	0.0 - 3.0
Visual-Spatial Working Memory			
Mapping and Directions	1.10	1.01	0.0 - 3.0
Visual Matrix	3.14	1.17	0.0 - 5.0
Switching			
Trails Numbers	29.80	17.46	8.70 - 97.91
Trails Letters	26.85	16.36	5.30 - 101.44
Inhibition			
Quantity	29.27	11.47	9.70 - 64.90
Stroop	60.92	21.39	17.39 - 128.0

Table 2

Intercorrelations among Chronological Age, Academic Achievement, and Cognitive Processing Tasks

	1	2	3	4	5	6	7	8	9
1. Chronological Age	-								
2. Arithmetic	.33**	-							
3. Reading	.13	.53**	-						
4. Digit Naming	-.03	-.39**	-.51**	-					
5. Letter Naming	.03	-.32**	-.50**	.84**	-				
6. Memory for Digits	.12	.31**	.23*	-.20**	-.11	-			
7. Nonword Repetition	.21*	.40**	.30**	-.28**	-.19	.61**	-		
8. Trails Digits	-.04	-.33**	-.18	.22*	.27*	-.18	-.30**	-	
9. Trails Letters	.06	-.24*	-.19	.22*	.34**	-.11	-.18	.68**	-
10. Quantity Inhibition	.00	-.44**	-.41**	.42**	.43**	-.24*	-.32**	.51**	.39**

Table 2 (continued)

	1	2	3	4	5	6	7	8	9
11. Stroop	-.16	-.40**	-.16	.24*	.24*	-.13	-.09	.22*	.08
12. Auditory Digit Sequence	.02	.42**	.43**	-.30**	-.30**	.42**	.52**	-.20	-.12
13. Semantic Categorization	.13	.30**	.33**	-.21*	-.26*	.23*	.35**	.34**	-.27
14. Visual Matrix	.16	.38**	.23*	-.12	-.11	.13	.07	.36**	-.35
15. Mapping and Directions	-.01	.22*	.08	-.07	-.09	-.01	-.01	-.20	-.12

Table 2 (continued)

	10	11	12	13	14	15
11. Stroop	.50**	-				
12. Auditory Digit Sequence	-.34**	-.18	-			
13. Semantic Categorization	-.35**	-.22*	.34**	-		
14. Visual Matrix	-.34**	-.36**	.15	.26*	-	
15. Mapping and Directions	-.16	-.18	.08	.25*	.36**	-

* $p < .05$ ** $p < .01$

Further inspection of the correlations among the other cognitive measures indicated that there are many shared relationships between the tasks. For instance, naming speed tasks were significantly related to executive functioning tasks and verbal working memory tasks. Also, short-term memory tasks and verbal working memory tasks were significantly related. Furthermore, it was found that the quantity inhibition task was significantly correlated to all other cognitive measures. However, visual-spatial working memory measures were not significantly related to the short-term memory tasks or the naming speed tasks. Other correlations that were not significant included the two tasks that were used to measure switching ability (trails numbers and letters) and reading. Overall, correlations revealed that many of the tasks shared significant relationships.

Predictive models of arithmetic calculation

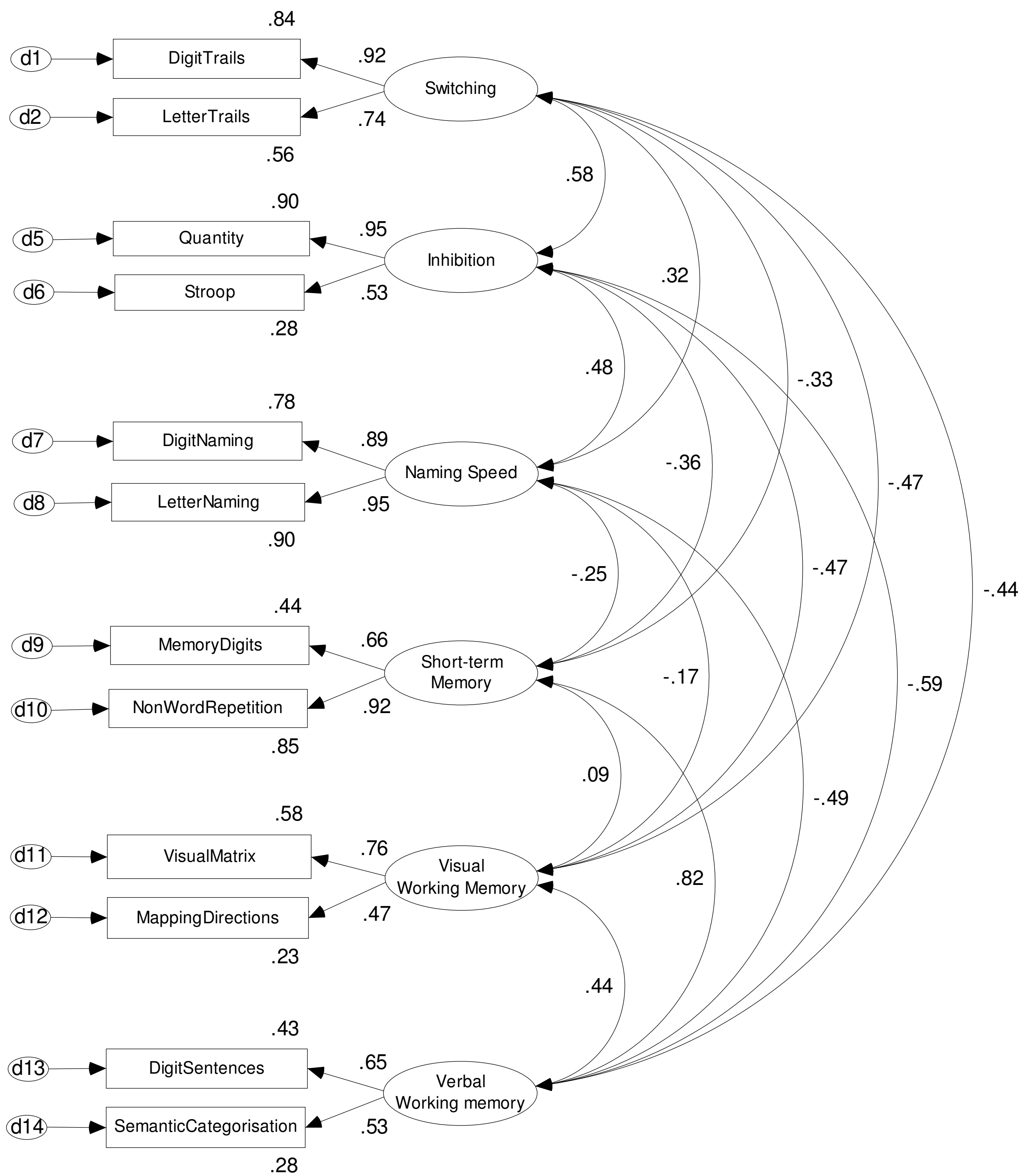
The purpose of this study was to examine the relative contributions of processing speed, short-term memory, executive functioning, and working memory to arithmetic calculation in children. Two specific objectives informed subsequent analyses. First, does executive functioning contribute to arithmetic calculation? Second, in the presence of processing speed, short-term memory, and executive functioning, does working memory contribute unique variance to arithmetic calculation? To address these aims, specific associations among cognitive processing variables and arithmetic calculation were examined by fixed-order multiple regression analyses.

To determine whether a priori assumptions that the cognitive measures used in the present study reflected different but related constructs, a confirmatory factor analysis was conducted. In light of literature differentiating between verbal working memory and visual-spatial working memory (e.g., Baddeley, 2001; Baddeley & Hitch, 1874), and because these working memory components were examined for individual and cumulative influence on arithmetic calculation in subsequent multiple regression models, each of these working memory components was examined as a separate latent construct. Processing speed was measured using two tasks: rapid digit naming and rapid letter naming. Short-term memory was measured by memory for digits and nonword repetition. Verbal working memory was measured by auditory digit sequencing and semantic categorization. Visual-spatial working memory was measured by mapping and directions and visual matrix. Executive functioning was also examined as separate latent constructs: inhibition and switching. Inhibition was measured using two tasks: quantity inhibition and the Stroop colour words task. Switching was measured using two tasks: making trials coloured numbers and making trails coloured letters.

Analysis supported a six-factor model representing the hypothesized latent constructs. Parameter estimates for the full model are presented in Figure 1. In figure 1, circles represent latent variables and rectangles represent observed variables. The chi-square test ($\chi^2 = 39.21$, $df = 39$, $p = .460$) suggested a good fit for the overall model. Additionally, the goodness-of-fit index (GFI) was .94, the root-mean-square error of approximation (RMSEA) was < .01, and the comparative fit index (CFI) was .99, indicating that the model was a good fit for the data. Results indicating the differentiation between short-term memory and working memory corresponded to studies using similar measures within child populations (e.g., Berg, 2008; Swanson & Beebe-Frankenberger, 2004). Further, the full factor model presented suggested that both working memory and executive functioning can be reliably separated into related but different factors: working memory into verbal and visual-spatial constructs, and executive functioning into switching and inhibition.

Figure 1

Parameter estimates for the Confirmatory Factor Analysis



Supported by the confirmatory factor analysis, measures related to processing speed, short-term memory, inhibition, switching, visual-spatial working memory, and verbal working memory were aggregated corresponding to theoretical assumptions into composite scores by summing *z* scores for each appropriate task based on the total sample. The correlations among composite scores are reported in Table 3. It was found that arithmetic calculation was related to all cognitive measures. However, correlations among the measures revealed that age was only related to arithmetic calculation. This suggested that the other cognitive measures are stable constructs for the age range. Further inspection revealed that inhibition and verbal working memory were significantly correlated with all other cognitive measures. Correlations that were not significant included switching and reading, which were measured to be unrelated constructs. Also, visual-spatial working memory was not related to reading, processing speed, and short-term memory. Given that reading has such a strong correlation to arithmetic ($r = .53$) and age has such a strong correlation to arithmetic ($r = .33$), this suggests the need to control for these variables in the subsequent analysis.

Using the composite *z* scores for each cognitive construct, fixed-order multiple regression analyses were conducted to examine relative contributions of the cognitive constructs to arithmetic calculation. To address the two objectives of the present study, a series of models was created that considered the individual contributions of each cognitive domain, the cumulative variance accounted for by combined processes, and the unique variance accounted for by individual cognitive domains. A total of five models were created and are reported in Table 4.

Table 3

Intercorrelations among Chronological Age, Academic Achievement Measures, and Cognitive Processing Composite Scales

	1	2	3	4	5	6	7	8	9
1. Chronological Age	-								
2. Arithmetic Calculation	.33*	-							
3. Reading	.13	.53**	-						
4. Processing Speed	-.00	-.38**	-.52**	-					
5. Short-term Memory	.18	.40**	.30**	-.23*	-				
6. Switching	.01	-.31**	-.21	.30**	-.23*	-			
7. Inhibition	-.10	-.49**	-.33**	.39**	-.25*	.38**	-		
8. Verbal Working Memory	.09	.44**	.47**	-.34**	.51**	-.31**	-.38**	-	
9. Visual-spatial Working Memory	.09	.36**	.18	-.12	.06	-.33**	-.37**	.27**	-

* $p < .05$. ** $p < .01$.

Table 4

Summary of Multiple Regression Analyses: Predictive Models of Arithmetic Calculation

Order of entry in equation	R^2	R^2 Change	*df*	*F* Ratio	*p*
			Model 1		
1. Chronological Age	.11	-	88	11.02	.001
2. Speed	.25	.14	87	16.33	<.001
2. STM	.23	.12	87	13.09	<.001
2. Switching	.21	.10	87	10.82	.001
2. Inhibition	.32	.21	87	26.56	<.001
2. VSWM	.22	.11	87	12.55	.001
2. VBLWM	.28	.17	87	19.85	<.001
2. Reading	.36	.24	87	32.99	<.001
			Model 2		
1. Chronological Age	.11	-	88	11.02	.001
2. Reading	.36	.24	87	32.99	<.001
3. Speed	.37	.02	86	2.49	.119
3. STM	.40	.04	86	6.27	.014
3. Switching	.40	.05	86	6.65	.012
3. Inhibition	.45	.10	86	15.50	<.001
3. VSWM	.42	.06	86	9.41	.003
3. VBLWM	.40	.04	86	5.75	.019
			Model 3		

1. Chronological Age	.11	-	88	11.02	.001
2. Reading	.36	.24	87	32.99	<.001
3. STM	.40	.04	86	6.27	.014
4. Switching	.43	.03	85	4.75	.032
4. Inhibition	.48	.08	85	13.17	<.001
4. VBLWM	.41	.02	85	2.12	.149
4. VSWM	.46	.06	85	10.16	.002
		Model 4			
1. Chronological Age	.11	-	88	11.02	.001
2. Reading	.36	.24	87	32.99	<.001
3. STM	.40	.04	86	6.27	.014
4. Switching	.43	.03	85	4.75	.032
5. Inhibition	.49	.06	84	9.40	.003
5. VSWM	.48	.04	84	6.94	.010
		Model 5			
1. Chronological Age	.11	-	88	11.02	.001
2. Reading	.36	.24	87	32.99	<.001
3. STM	.40	.04	86	6.27	.014
4. Inhibition	.48	.08	85	13.17	<.001
5. Switching	.49	.01	84	1.39	.241
5. VSWM	.51	.03	84	4.77	.032

Note. Speed = rapid naming, STM = short-term memory, VBLWM = verbal working memory, and VSWM = visual-spatial working memory.

Models

Model 1 reports the individual contributions of each cognitive domain and reading to arithmetic calculation after accounting for the influence of chronological age. Chronological age contributed 11% of the variance. Two findings are of particular importance. First, each cognitive domain contributed significant individual variance to arithmetic calculation after accounting for the influence of chronological age. Second, after controlling for chronological age, reading was the strongest contributor to arithmetic calculation, accounting for an additional 24% of the variance. In light of these results, follow-up models were constructed to examine the interaction among cognitive domains and reading on arithmetic calculation.

Model 2 examined the individual contributions of each cognitive domain after the influence of chronological age and reading was controlled. It was found that processing speed did not contribute to arithmetic calculation after controlling for chronological age and reading. Therefore, the tasks used to measure reading ability in the present study appeared to be capturing the processing speed variance in arithmetic calculation. In light of these results, follow-up models were constructed to examine the interaction among cognitive domains on arithmetic calculation after controlling for chronological age and reading. Because it failed to contribute significant variance to arithmetic calculation, processing speed was not included in subsequent models. However, short-term memory, switching, inhibition, verbal working memory, and visual-spatial working memory each contributed significant variance after controlling for age and reading. For instance, additional variance ranged from 4% for short-term memory to 10% for inhibition. Switching was found to contribute 5% additional variance to arithmetic calculation.

Furthermore, both visual-spatial working memory and verbal working memory contributed unique variance to arithmetic calculation, 6% and 4%, respectively.

Model 3 examined the individual contributions of each cognitive domain after the influence of chronological age, reading, and short-term memory was controlled. Again, switching and inhibition both surfaced as significant contributors to arithmetic calculation, accounting for an additional 3% and 8%, respectively. In this model, it was found that verbal working memory did not contribute additional variance to arithmetic calculation. As it failed to contribute significant variance to arithmetic calculation, verbal working memory was not included in the subsequent models. However, visual-spatial working memory was found to still contribute to arithmetic calculation, with an additional 6% of the variance.

Model 4 examined the individual contributions of each cognitive domain after the influence of chronological age, reading, short-term memory, and switching were controlled. In this model, it was found that inhibition and visual-spatial working memory contributed additional variance to arithmetic calculation, 6% and 4% respectively. That is, short-term memory and switching did not eliminate the contribution of visual-spatial working memory to arithmetic calculation.

Model 5 examined the individual contributions of each cognitive domain after the influence of chronological age, reading, short-term memory, and inhibition were controlled. In this model, it was found that switching did not contribute additional variance to arithmetic calculation. Therefore, while inhibition contributed unique variance after switching, switching did not contribute additional variance after inhibition. This suggested that the tasks used to measure inhibition (stroop and quantity naming) may be capturing the ability to switch, as measured by making trails numbers and letters.

However, visual-spatial working memory still contributed significant variance to arithmetic calculation (3%).

As reported in Model 5, a full regression model predicting arithmetic calculation suggested that age, reading, short-term memory, inhibition, and visual-spatial working memory accounted for 51% of the variance, $F(5, 84) = 17.35$, $p < .001$. Within the complete model, after controlling for chronological age and reading ability, visual-spatial working memory contributed significant variance (3%) to arithmetic calculation in the presence of short-term memory and inhibition.

Summary of predictive models of arithmetic calculation

The first purpose of this study was to investigate the contributions of executive functioning, specifically inhibition and switching, to arithmetic calculation. Indeed, inhibition and switching both emerged as significant contributors of variance to arithmetic calculation. Inhibition contributed unique variance to arithmetic calculation after controlling for chronological age, reading, short-term memory, and switching. Similarly, switching also contributed unique variance to arithmetic calculation after controlling for chronological age, reading, and short-term memory. However, after controlling for chronological age, reading, short-term memory, and inhibition, switching did not contribute additional variance to arithmetic calculation. This suggested that inhibition may be capturing the ability to switch in the current study.

The second purpose of this study was to investigate the contributions of working memory to arithmetic calculation in the presence of processing speed, short-term memory, and executive functioning. It was found that visual-spatial working memory emerged as a significant contributor to arithmetic calculation. However, the contribution of verbal working memory disappeared after taking into account short-term memory. As reported in Model 5, a full regression model predicting arithmetic calculation suggested that age, reading, short-term memory, inhibition, and visual-spatial working memory accounted for 51% of the variance. Within models 4 and 5, after controlling for chronological age and reading ability, and short-term memory, visual-spatial working memory contributed significant variance to arithmetic calculation in the presence of either switching or inhibition.

CHAPTER FIVE

DISCUSSION

The central purpose of this study was to examine the relative contributions of processing speed, short-term memory, working memory, and executive functioning to arithmetic calculation in children. Two specific objectives informed subsequent analyses. First, does executive functioning contribute to arithmetic calculation? Second, in the presence of processing speed, short-term memory, and executive functioning, does working memory contribute unique variance to arithmetic calculation? To address these aims, specific associations among cognitive processing variables and arithmetic calculation were examined by fixed-order multiple regression analyses. Each objective is discussed in turn.

Executive Functioning and Arithmetic

The first purpose of the current study was to address the contribution of executive functioning to arithmetic calculation. Results suggest four important findings. First, executive functioning did indeed contribute to arithmetic calculation. Second, the results indicated that the individual executive functioning components of inhibition and switching are significant contributors to arithmetic calculation. Third, both executive functions of inhibition and switching emerged as a significant variables contributing to arithmetic calculation after controlling for chronological age, reading, and short-term memory. Fourth, after controlling for chronological age, reading, short-term memory, and inhibition, the contribution of switching was not significant. Each finding is discussed in turn.

In the current study, a major finding was that executive functioning did indeed contribute to arithmetic calculation. This finding adds to the limited amount of research that has been directed at understanding the relationship between executive functioning and arithmetic. Arithmetic requires the ability to plan, organize, and self-monitor, several key components of executive functioning (Baron, 2004). An association between executive functioning and arithmetic has been suggested also in studies involving children with and without mathematical disabilities (e.g. Bull & Johnston, 1997; Bull, Johnston, & Roy, 1999; McLean & Hitch, 1999; Passolunghi & Siegel, 2001). Current, although limited, literature has been consistent in highlighting an impairment of executive functioning related to the central executive component within working memory in children with mathematical disabilities. The role of executive functioning in arithmetic calculation is likely to be related to the procedures that children have to use when engaged in mental arithmetic. For instance, when solving a problem (e.g., 5 + 8) the student must not only remember the numbers and the operation sign (i.e., storage), but also actively direct attention among these elements of the problem to engage in the strategy to solve the problem. The inclusion of executive functioning tasks specific to switching and inhibition in the present study was an attempt to provide a more clear understanding of these relationships.

Another important finding in the current research was that individual executive functioning processes were significant contributors to arithmetic calculation. Several studies have suggested relations between arithmetic performance and switching ability. McLean and Hitch (1999) found that children with mathematical disabilities were slower than children with normal arithmetic ability on making trails tasks. As well, Bull and Scerif (2001) examined the ability to inhibit responses in children with mathematical disabilities. They administered Stroop tasks that required the naming of either colour or quantity of stimuli under facilitating or interfering task conditions. Only the naming of quantity in the interference condition appeared to be related to arithmetic ability. The interference condition was assumed to provide an indication of the ability to inhibit automatized responses (i.e., the reading of words or digits) in favour of less automatized ones (i.e., the naming of ink colour or number of digits). The present results extend these findings by suggesting the role of switching and inhibition in general arithmetic calculation performance. Both switching and inhibition contributed to children's arithmetic calculation.

One or a combination of these executive functioning processes might be interacting during arithmetic calculations. For instance, when solving a problem (e.g., 5 + 8) the student must not only remember the numbers and the operation sign (i.e., storage), but also actively direct attention among these elements of the problem to engage in the strategy to solve the problem. For example, if the student uses a counting strategy, they must decide whether they will count from 1 to 13, from 5 to 13, or from 8 to 13. Any of these procedures requires the student to switch their attention from remembering the problem to solving the problem (thinking of an effective strategy), to focus their attention (actively remembering parts of the problem-numbers and operation), and to divide their attention (beginning to count from one of the numbers but remembering the other number as the sign for when to stop counting).

The next important finding in the current research is that both executive functions of inhibition and switching emerged as a significant variables contributing to arithmetic calculation after controlling for chronological age, reading, and short-term memory. This finding is consistent with previous research such as van der Sluis et al. (2004) and

Swanson and Sachse-Lee (2001). One of the findings in the Swanson and Sachse-Lee (2001) study was that domain-general working memory (i.e., executive functioning) contributed significantly to solving mathematical word problems. However, that the individual executive functions of inhibition and switching both contributed unique variance to arithmetic calculation in the current study represents a novel finding to the research for two reasons. First, Swanson and Sachse-Lee (2001) did not measure executive functioning directly. Rather, these researchers used statistical procedures to identify common variance shared by verbal working memory and visual-spatial working memory. This shared variance was assumed to be the executive processes of working memory. Second, Swanson and Sachse-Lee (2001) measured general executive functioning, as opposed to the specific executive functions (e.g., inhibition and switching). The executive processes that these researches identified were viewed as general processing abilities that were separate from the storage processes of working memory. The current study sought to extend this research and measured directly the specific executive functions of inhibition and switching.

Another interesting finding of the present study is that processing speed contributed significant variance to arithmetic calculation after controlling for chronological age, which corresponds with findings of Swanson and Sachse-Lee (2001). However, in the current study, processing speed (as measured by naming speed) did not contribute to arithmetic calculation after controlling for chronological age and reading. This finding is similar to Swanson's (2006) study where in the presence of significant contributions of working memory, reading, and age, processing speed did not emerge as a significant contributor. Although several studies have highlighted a relationship between processing speed and arithmetic calculation (Bull & Johnston, 1997; Swanson & Beebe-

Frankenberger, 2004), the absence of corresponding results might be due, in part, to task selection. The current study used rapid naming tasks to measure processing speed. Naming speed is often included under phonological processes (Wagner et al., 1999) and, thus, is considered as an indication of the speed with which phonological codes are retrieved from long-term memory (e.g. Torgesen, Wagner, Rashotte, Burgess, & Hecht, 1997). In general, the relationship between naming speed and arithmetic and reading and arithmetic may be related to the relationship between naming speed and reading.

Fourth, after controlling for chronological age, reading, short-term memory, and inhibition, the contribution of switching was not significant to arithmetic calculation. In the current study, inhibition seems to be a more important contributor to arithmetic calculation than switching. Model 4 revealed that when switching was entered into the regression model before inhibition, both contributed significant variance to arithmetic calculation. However, model 5 revealed that when inhibition was entered before switching in the regression models, switching no longer contributed to arithmetic calculation. In other words, inhibition cancelled out the contribution of switching, but switching did not cancel out the contribution of inhibition to arithmetic calculation. Therefore, inhibition seems to be more a more important contributor to arithmetic calculation than switching. It is possible that inhibition is utilizing the same process as switching and potentially another cognitive process such as updating.

The pattern in recent studies (e.g., Geary et al., 2000) and that of Barrouillet and colleagues (1997) is in keeping with Conway and Engle's (1994) position that individual differences in working memory and retrieval efficiency are related, in part, to the ability to inhibit irrelevant associations. In this model, the presentation of an arithmetic problem results in the activation of relevant information in working memory, including problem features; such as the addends in a simple addition problem and the information associated with these features. Arithmetic calculation is efficient when irrelevant associations are inhibited and prevented from entering working memory. Inefficient inhibition results in activation of irrelevant information, which lowers working memory capacity. In this view, children might make calculation errors, in part, because they cannot inhibit irrelevant associations from entering working memory. Research indicates that when solving arithmetic calculations problems that can be interpreted as having two meanings can be more difficult to solve. For instance, to solve 3 + 5 one must inhibit the answer 15 (the answer for 3 x 5) in favour of the correct answer 8 (De Rammelaere, Stuyven, & Vandierendonck, 1999). Once in working memory, these associations either suppress or compete with the correct association. This perspective suggests that the calculation difficulties of some children might originate from executive functions related to inhibitory mechanisms (Bull et al., 1999).

Working Memory and Arithmetic

The second purpose of the current study was to examine the unique variance working memory contributes to arithmetic calculation in the presence of processing speed, short-term memory, and executive functioning. Results suggest four important findings. First, in correspondence with previous studies, working memory did indeed contribute to arithmetic calculation (Berg, 2008b; Swanson & Sachse-Lee, 2001). Second, both individual working memory components, verbal and visual-spatial working memory, contributed unique variance to arithmetic calculation. Third, the contribution of verbal working memory to arithmetic calculation was not significant after controlling for chronological age, reading, and short-term memory. Fourth, the current results found that visual-spatial working memory was not eliminated by the presence of chronological age, reading, short-term memory, and inhibition. Each finding is discussed in turn.

Consistent with previous studies (i.e., Berg, 2008b, Swanson & Sachse-Lee, 2001), working memory did indeed contribute unique variance to arithmetic calculation in the current study. Working memory has been hypothesized to be directly involved in arithmetic calculation for two reasons. First, numerous studies have reported that children with mathematical disabilities are impaired in several memory-based processes (Berg, 2008a; Bull & Johnston, 1997; Jordan & Montani, 1997; Swanson & Sachse-Lee, 2001). Second, the mental processes of arithmetic calculation (e.g., counting procedures and memory-based strategies) parallel the conceptual and operational characteristics of working memory (Case, 1985). Further, arithmetic performance and working memory functioning are facilitated by cognitive processes that interact with working memory (e.g., processing speed; Case, 1985). While studies are limited in the area and conducted mostly with adults, findings suggest that working memory resources are required to solve problems that require several procedural steps (e.g., 125 + 97, Fürst & Hitch, 2000; Geary & Widaman, 1992; Logie, Gilhooly, & Wynn, 1994). In these problems, working memory appears to be related to retaining partial solutions during calculation and for carrying partial results across columns.

To further understand the role of working memory the current study separated each component of working memory, visual-spatial working memory and verbal working memory, to examine their individual relationship to arithmetic calculation. The current results revealed that both individual working memory components, verbal working memory and visual-spatial working memory, contributed individual variance to arithmetic calculation. Consistent with previous research (i.e., Adams and Hitch, 1997) individual components of working memory appear to contribute to arithmetic calculation. For instance, to solve 4 + 3 a child must concurrently retain two or more pieces of information (the phonological codes representing the numbers “4” and “3”) and then employ one or more procedures (e.g., counting) to combine the numbers to produce an answer. Adams and Hitch (1997) explored this relationship in children aged 8 to 11 years using simple mental addition (8 + 1) and complex mental addition (231 + 16) problems. Results indicated that solving problems presented verbally was associated with high working memory, whereas solving problems presented visually was related to low working memory. As well, level of problem difficulty was associated with working memory contributions, with higher working memory spans corresponding with more complex problems.

The current study also found that after controlling for chronological age, reading, and short-term memory, the contribution of verbal working memory to arithmetic calculation was eliminated. This is not consistent with previous research (e.g., Berg, 2008b; Swanson & Sachse-Lee, 2001). In Berg’s (2008b) study, results indicated that after controlling for all other variables, both verbal working memory and visual-spatial working memory each contributed significant variance to arithmetic calculation. A plausible explanation for this finding could be the tasks used in the current study. The short-term memory tasks used in the current study (memory for digits and nonword repetition) are both verbal tasks, therefore verbal working memory contributions may have been captured by the short-term memory tasks. Research would benefit from differentiating between visual-spatial short-term memory and verbal short-term memory measures. It could be possible that inclusion of a visual-spatial short-term memory task may have captured some of the visual-spatial working memory variance contributing to arithmetic calculation in the current study.

Finally, the current results found that visual-spatial working memory was not eliminated by the presence of chronological age, reading, short-term memory, and inhibition. This finding corresponds with previous research (e.g., Berg 2008b; Swanson & Sachse-Lee, 2001). For example, Berg (2008b) found that visual-spatial working memory emerged as a unique contributor in the presence of the other working memory components. A potential relationship between visual-spatial working memory and arithmetic calculation could be the procedures children use. Using carrying procedures involves maintaining parts of the problem while performing additional procedures (Heathcote, 1994). To solve 18 + 5, a child might retain the 3 from adding 8 + 5, while adding the 1 from the tens column of the 18 to the 1 from the tens column of the 13 produced from adding the 8 + 5 (Heathcote, 1994).

Further evidence of the role of working memory and related cognitive processes in arithmetic calculation are provided from the results of the current study (e.g., Adams & Hitch, 1997; Swanson & Beebe-Frankenberger, 2004). However, the specific role of the each of the working memory components remains unclear. Evidence from adult populations indicates that the verbal and visual-spatial components of working memory have specialized roles in solving arithmetic calculation – verbal working memory is involved in retaining parts of the solution, and visual-spatial working memory is involved in encoding the problem addends when the problem is presented visually (Logie et al., 1994). Further explanation for the contribution of visual-spatial working memory can be found in Logie et al. (1994). The present study assessed general arithmetic calculation performance on the WRAT3 which is presented visually. That is, visual-spatial working memory might have been used to encode the visual presentation of the arithmetic problems.

Limitations

A limitation of the present study was the tasks used to assess short-term memory. One of the current findings was that the contribution of verbal working memory to arithmetic calculation was eliminated in the presence of chronological age, reading, and short-term memory. Furthermore, the tasks used to measure short-term memory tasks were indeed both verbal tasks and therefore may have captured the verbal working memory measures. Therefore, inclusion of a visual-spatial short-term memory task may have potentially captured some of the visual-spatial working memory variance contributing to arithmetic calculation after controlling for chronological age, reading, short-term memory, and inhibition. Further research in the area of working memory would benefit from differentiating between visual-spatial and verbal short-term memory tasks in their research design.

Another limitation to the current study was that the executive functioning tasks only measured two specific forms of executive functioning; inhibition and switching. Inclusion of other executive functioning tasks, such as updating and dual-task performance task, would be beneficial to differential between executive functions and their individualized roles in arithmetic calculation.

In contrast to previous studies (Swanson & Beebe-frankenberger, 2004), the current study revealed that processing speed did not contribute to arithmetic calculation in the presence of chronological age and reading. Therefore, differentiating between processing speed and rapid naming tasks and the cognitive processes facilitating these tasks would be beneficial. For example, reading seems to have an important relationship to naming speed. However, other measures of processing speed may not be capturing the same cognitive processes as rapid naming.

Another limitation to the current study was that the socioeconomic status of the participants was not assessed. Although the schools in the current study were predominantly located in working-class middle-class communities, the population of these communities are quite diverse. Research has shown a potential connection between socioeconomic status and arithmetic calculation (Jordan, Hutenlocher, & Levine, 1992).

Implications

This study contributed to research by investigating the roles of executive functioning in children's arithmetic calculation. A more complete understanding of the specific cognitive processes (e.g., executive functioning) that are involved in specific forms of mathematical performance (e.g., arithmetic calculation) will support investigations into other areas of arithmetic calculation (e.g., multiplication). And, as we begin to map better the development of children's proficiency in arithmetic calculation and the cognitive processes that facilitate proficiency, this knowledge can be used by classroom teachers and special education teachers to meet the needs of students normally achieving arithmetic calculation and students with learning difficulties in arithmetic. Uncovering which cognitive processes contribute to arithmetic calculation can guide the modelling of instructional approaches and curricular materials to correspond to these processes.

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