

ARITHMETIC

PRIMARY - GRADE VI

A Teaching Guide

(REVISED EDITION)



Property of.....School

FOR THE USE OF THE TEACHER

Must be on hand for incoming teacher each school year.

Curriculum Division, Department of Education,
Halifax, Nova Scotia

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FOREWORD

On the authorization of the Curriculum Advisory Committee, the material contained in this booklet has been prepared for the Arithmetic Committee by Dr. J. P. McCarthy, Principal of the Nova Scotia Normal College.

In this edition the material has been revised as the result of a year's experimental use in the schools of the province.

Teachers may have additional suggestions that will improve the content of the course. Comments and suggestions, particularly with respect to teaching devices in arithmetic, will be welcomed. They should be sent to the Director of Curriculum and Research, Department of Education, Halifax.

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ARITHMETIC

PRIMARY TO GRADE VI

GENERAL STATEMENT

All people, children as well as adults, meet *quantitative* situations in their daily living. A pre-school child may know that there are two dogs or three cats in the backyard; that he has five cents and can buy an ice-cream cone; that he is four years old. He may be able to count by rote to ten, twenty, thirty or even a hundred. He may know nursery rhymes involving counting or quantity, e.g. "One, two, buckle my shoe" or "One, two, three, four, Mary at the cottage door." He may even know that "two and two are four" or "three and three are six." Of course, too, a child may come to school not able even to count by rote to ten although he is well aware of "quantity" in his daily life.

The point is that from early youth we meet situations involving number and quantity and we continue to do so all through life. The informal learning that takes place in the pre-school years usually arises from daily experiences that have *meaning* to the child. Although the *amount* of quantitative learning that children bring to school will differ from child to child, it usually is well learned because it came about in daily experiences using number and quantity. These quantitative experiences will continue as the child grows older.

The quantitative aspects of society and what we observe about the use of numbers by the pre-school child present two basic considerations which must underlie the program of studies in the common school grades Primary to Grade VI:

1. A knowledge of our number system and how to use and apply it must be part of the school program. Each child must be given thorough and systematic instruction leading to mastery of the fundamental skills.
2. The fundamental skills must be applied in situations which have meaning for the child. Pre-school number learnings show that this is necessary. The child learns about number because he sees some need for doing so.

Sometimes the social applications of arithmetic create situations which conflict with the logical organization of the subject. For example a situation may arise in which the child needs to know that $14+8=22$. He could *memorise* it or *count*, but if he is taught first that $4+8=12$, the other can and should follow when he learns the higher decade addition facts with bridging.

Three major principles must be followed in organizing course of study material in arithmetic:

1. *Arithmetic is a logical science in which there must be a sequential arrangement of topics.*
2. *Arithmetic is useful to children and to adults in their daily lives and has most meaning in its social applications.* Meaningful problems, therefore, should occupy an important place in the use of fundamental skills once such skills are mastered.

3. All new skills learned must be based on skills and understandings already mastered. This is a restatement of the principle that all learning proceeds from the known to the unknown. It is extremely important in its application to the learning of arithmetic.

It must be remembered that some topics do not follow in logical sequence. Where to place them is largely a matter of arbitrary judgment. For example the number of days in a week and the number of hours in a day may be learned as the need arises with no concern for logical placement in the study of seconds, minutes, hours, days, weeks, etc. The place of such topics in the work for any year must depend on the judgment of the teacher.

Content

In the past thirty years much has been learned about content of arithmetic programs, about methods of teaching arithmetic, and about the grade placement of various skills.

In years gone by it was customary to require children to do problems which people almost never meet or which were so far from reality that they had no real meaning. Involved problems may have a place in senior high school in order to strengthen a complete command of the intricacies of the number system. In the common school grades, however, the arithmetic taught should consist of the basic skills of addition, subtraction, multiplication and division of whole numbers with an *understanding* of them; simple fractions and how to use them; decimals and their uses; and an application of all of these to the kinds of problems which one meets in everyday situations. From the points of view of *computation* and *understanding*, the material for the common school grades should include only the kinds of problems one is likely to meet in daily living. Most of these problems involve numbers under one hundred.

Meaning and Method

It has been found to be unwise to plunge children into the manipulation of symbols before they know the meaning behind them. If children in the beginning are given a thorough readiness program using concrete, pictorial and semi-concrete materials, their mastery and understanding of the basic number facts is much greater. If children understand the decimal nature of the number system, that is, the use of the tens' place, the hundreds' place and so on, they are much better prepared to master addition and subtraction operations involving carrying and borrowing. The use of concrete materials, and the emphasis on understanding and meaning affect the grade placement of certain skills: use of number symbols, introduction of addition and subtraction facts, "carrying", "borrowing", addition and subtraction of fractions, percentage—in fact most skills—are introduced somewhat later in the course than has been the case in the past. When skills are deferred in order to place more emphasis on meaning, learning is much more effective.

Emphasis on readiness and meaning presupposes a teaching method different from simply presenting the process. The following simple examples illustrate the difference:

1 Miss A was teaching subtraction with borrowing. She placed this example on the blackboard:

$$\begin{array}{r} 52 \\ -34 \\ \hline \end{array}$$

Then she said, "We want to subtract 34 from 52. We can't take 4 from 2 so we borrow 1 from the 5. The 2 is now 12. Now 4 from 12 is 8. Put the 8 under the 4. Now we've taken 1 from the 5 leaving 4. 3 from 4 leaves 1. Write the 1 under the 3. The answer is 18." Then she went over it step by step with the children again. This was followed by similar problems for the children to work at their desks. Many of the children made errors. It was evident that most of the children did not understand the *meaning* of borrowing. Those who did the examples correctly had learned a mechanical process, but few of them understood it.

2. Miss B said, "We have 52 cents in our class fund. Yesterday we decided to buy some flower seeds for our window boxes. The seeds will cost 34 cents. How can we find out how much we have left?" She wrote the 52 with the 34 below it on the blackboard. One child said "34 and 10 is 44 and 10 is 54. That's 20. 54 is 2 more than 52. 20 is 2 more than 18." Another said "30 from 50 is 20. 4 is 2 more than 2. 2 from 20 is 18." A third said "If we had the money here we could take away the 34 cents and count what's left." Miss B produced 5 ten-cent pieces and 2 one-cent pieces. When the children saw that there were not enough one-cent pieces to take away, one child said "If we had ten pennies in place of one of the dimes we could do it." Miss B substituted 10 one-cent pieces for 1 ten-cent piece, the children took away the 34 cents and found that the remainder was 18 cents. Miss B then took a coin card or place value card and demonstrated what they had just done.

Tens	Ones	Tens	Ones
0 0 0 0 0	x x	0 0 0 0	x x x x x
-----	-----	-----	-----
			x x x x x

			x x

Then she said "Now with the numbers on the board we can use 'a trick' to get us the answer. We take 1 of the tens and change it to 10 ones. That gives us 4 tens and 12 ones. Think of the 2 as 12. Take away 4. That leaves 8. Think of the 5 tens as 4 tens. Take away 3 tens. That leaves 1 ten. Then the answer is 18." As she did this, she worked out the problem on the board. Several other examples were worked out with the coin card. Then the children were given several similar problems to do at their desks. All but the slowest *understood* and were able to get the correct answer.

While Miss B's lesson was probably not a perfect one, it was far superior to Miss A's in that it got at an understanding of "borrowing". Similarly all new skills should be taught not simply as mastery of a process but as seeing the *meaning* of and *mastering* the process, then using it in solving real problems. It is important also that concepts be introduced whenever possible through the medium of real problems using concrete materials. In the outlines for the various grades, each of the units in the grade should be introduced through real problems, whether or not the unit emphasizes

it, and once the skill is learned, it should be used in the solving of other real problems.

Problem Solving

One of the main objectives of the teaching of arithmetic in the common school grades is to have pupils acquire mastery of the fundamental operations with whole numbers, fractions, and decimals. It must be remembered, however, that the principal use of the fundamental operations is for the solution of problems. Hence problems which are real and within the experience of pupils must play an essential part in the arithmetic program throughout all the grades, from Primary up.

Every skill which is new should be introduced through the medium of a real problem so that pupils will appreciate the need for the skill, and when the mechanical skill has been acquired it should be put to use in the solution of real problems requiring the process. While this is necessary in the learning of a new skill, there is a danger, that problem solving itself may become mechanical if all the problems in a set require the same arithmetical process. Bright pupils, particularly, will quickly recognize that all the problems on a page can be solved by one process, and they may get correct answers without even reading the problem. Hence any sound arithmetic program must contain many sets of problems in which the pupil must decide for himself the operation to be performed. This will necessitate that he understand the problem, that he recognize the data given, that he know what has to be found, and that he decide the operation to use in order to secure the answer. Furthermore the pupil should be taught that he should not accept the answer he secures until he has checked the work.

The ability to solve problems is one that must be developed and nurtured with care. It requires the ability to reason which, on an immature level, is found in Primary Grade pupils and develops with the growth of maturity of the individual child.

In the early grades the problems should be real and of a very simple nature and should involve only one step up to the end of Grade III. Children in Grade IV should be able to do two-step problems. While problems should always be real, their difficulty should be gradually increased in keeping with the intellectual development of the pupil.

Drill

For some years the word *drill* has been in disrepute. The reason is that *drill* has been thought to replace *learning*. The chief results were to develop a distaste for number work and incompetence in arithmetic generally. Whether it is called *drill* or *practice* some means is needed to consolidate understandings and skills *once they are learned*. The pupil should *first learn* and then *practice* to make sure that he can perform the operation and apply it to real problems.

There is a need for what Morton* calls "telescoped reteaching." Children forget during the summer vacation and for various other reasons. The

*Morton, R. L., *Teaching Children Arithmetic*, New York. Silver Burdett Company, 1953, 566 pages (in Canada, W. J. Gage).

program which follows provides for this by reviewing at the beginning of each year the work covered in previous years. Such review should not be drill alone but a reteaching of the skills previously learned followed by plenty of practice. For example the forty-five easy addition facts are taught in Grade I, retaught in Grade II, reviewed and retaught at the beginning of Grade III. This is consistent with the way children's concepts and understandings develop. If children seem slow in grasping new concepts or old ones which are being retaught, *do not* resort to drill before meaning is grasped. *After* the material is understood and relearned plenty of practice should follow.

Testing and Diagnosis

It is important that the results of teaching should be evaluated from time to time. This is usually done by means of tests—either standardized or those constructed by the teacher. Such tests or examinations serve two major purposes: (1) to determine how the *objectives* of the program are being realized, (2) to give the teacher *information* about skills and understandings which various students have not learned so that reteaching may be done.

(1) One of the weaknesses of testing in arithmetic is that there is a tendency to emphasize only two of the objectives: computation and problem solving. Other objectives which should not be overlooked include an understanding of the number system, quantitative vocabulary, estimating results, "mental" or oral arithmetic, the use of short-cuts. One disadvantage of a standardized test is that norms for various grades may be set up on some items which are not included in the content for the grade. For example, if multiplication by a two-figure number is not included in the Grade III content but is included in computing the Grade III norm for the test, then the children cannot be expected to reach the norm of performance. Whether tests are standardized or prepared by the teacher, other means should be used to evaluate objectives which are not covered by the test. It is unwise, also, if children have been taught to work carefully and to check everything thoroughly, to place too much emphasis on speed. Allowance should be made for the slow but meticulous worker who may show up poorly on a test where speed influences the score. Brueckner, Morton, Spitzer and others all have excellent chapters on testing.* Teachers should read one or two of the references noted in the footnote below and in bibliography given on page 102.

(2) Either standard or teacher-made tests can serve one very important purpose: to determine weaknesses in a class or in individual students, as a basis for necessary reteaching. If the items on the test cover all the work done, the procedure is simply one of analyzing the items on the test to determine what things were not correct. If this is done for the class as a whole, the necessary reteaching can be done before proceeding to material

*Brueckner, L. J. and Grossnickle, F. E., *Making Arithmetic Meaningful*. Toronto. The John C. Winston Company, Limited, 1953. 570 pages Chapter II, "Evaluating Learning in Arithmetic."
Morton, R. L., *Teaching Children Arithmetic*. New York. Silver Burdett Company, 1953. 566 pages. Chapter 12, "Testing in Arithmetic" (in Canada: W. J. Gage).
National Society for the Study of Education: Fifteenth Yearbook, *The Teaching of Arithmetic*. University of Chicago Press, 1951. 305 pages. Chapter X.
Spitzer, H. F., *The Teaching of Arithmetic*. Boston. Houghton Mifflin Company, 1948. 397 pages. Chapter 12. "Testing in Arithmetic" (in Canada: Thomas Nelson and Sons Limited).

for which the unmastered skill or understanding is necessary. Such diagnosis followed by reteaching is well worth the time spent on it.

Teachers should not forget the importance of checking on day-to-day performance of pupils. Such daily checking is one of the most effective kinds of diagnosis.

Providing for Individual Differences

Children differ in the rate at which they can learn arithmetic just as they differ in learning to read or spell or do many other things in or out of school. How differences in ability to learn arithmetic may be looked after in the classroom is really a problem for the teacher of a particular class. Knowing the children in the class, the teacher should be the best judge of how to ensure that the most effective learning takes place.

Teachers should guard against the unsound assumption that the grouping used for reading instruction is the most effective way of providing for effective instruction in arithmetic. *This is not so.* Grouping may be used but it has to be a much more flexible and dynamic grouping than is used for reading instruction.

There are several considerations:

1. In a classroom there may be a "group" being retaught some skill which they have not learned or have forgotten. At the same time the bulk of the class are applying some newly learned skill or concept to simple problem situations. A few others will be able to apply the newly learned skill to much more difficult problems.

2. Guard against giving the brighter pupils just "more problems" to do unless you are sure that it will not turn them against arithmetic. Children who learn quickly and get all their work done fast should not be penalized by being given just "more work." There are many other purposeful and constructive things to which they could give their attention.

3. When a new skill or concept is taught, there are usually three things to be learned: (a) *understanding its meaning*, (b) *performing the operation* and (c) *applying it to solve a problem*. Often the slow learners in the class can do no more than to master the performance of the operation and apply it to simple problems. For example, in learning to subtract a two-figure number from a two-figure number with borrowing from the tens' place: (a) some children will *not* understand the meaning behind it but *may* learn how to do it. They may even have to use the crutch of putting a 1 beside the number in the ones' place—although this is to be discouraged; (b) most of the class will *understand* the process of changing one of the tens to ten ones, perform the operation successfully and apply it to simple problems; (c) a few will be able to go even farther by applying the operation to more difficult problems. It is with the slow pupils that much of the reteaching will be necessary. If the *meaning* behind operations is taught, then the logical nature of the subject matter, the necessity for maintenance of the fundamental operations and the desirability of "telescoped reteaching" provide, to a considerable degree, for individual differences in arithmetic. How they will be met in particular classrooms depends largely on the initiative of the teachers.

Some Materials for Teaching the Meaning of Numbers

1. A set of ten picture cards. Each card about 8" x 12". These can be oak tag, heavy cardboard, or wrapping paper, although the latter is not very durable. Each card has a picture or pictures showing numbers from one to ten. For example: 1 is a large house cut from a magazine; 2 is represented by two automobiles and so on. Old catalogues are useful in finding pictures for the larger numbers such as 8 toy trucks, 9 girls, 10 hats, etc.

2. A set of ten cards, same size as the above, with semi-concrete materials for each number. White oak tag with black construction paper mountings show up very well. For example: a large square for 1; 2 large circles for 2; 3 equilateral triangles; 4 diamonds; 5 small squares; 7 isosceles triangles, etc.

3. A large wall chart showing pictures and corresponding numbers from one through ten.

4. A set of ten cards, about 5" x 8", each with pictures of numbers from one through ten, to be used in class by teacher and pupils.

5. A similar set with semi-concrete materials, in bright colours, for class use. These show various groupings and help children learn to recognize groups of objects without counting.

6. A set of ten cards about 4" x 6" with numbers (one card for each number).

7. A similar set with number words printed in manuscript.

8. A set of cards about 4" x 6" showing groupings for addition. The pictures on these may be bird, flower, or animal seals obtainable at any stationery store. The pictures are arranged to show simple addition facts. The set includes one card for each addition fact taught in Primary.



(3+1)

9. A similar set with semi-concrete materials, mounted in the same way, showing addition facts. (These are mostly squares and rectangles cut from a paint colour chart.)

10. Strips made of oak tag each covered with a different coloured construction paper. Each strip is about 1½ inches wide. The "ten strip" is ten inches long, the "nine strip" is nine inches long, etc. Each strip is marked or scored, not cut, into 1-inch blocks, on down to the "one strip" which is one block 1 inch long. These are popular with the children because they are colourful and easy to handle. They are useful in teaching the value of numbers from one to ten, as well as teaching the addition facts and subtraction facts. Children soon see that if they lay the four strip on the five strip they need one more block to make five.

11. A vocabulary book. Construction paper of different colours used for the pages. On each page pictures are mounted, showing the idea of big, little; up, down; over, under; etc. Such pictures are not easily found and such a book can be a long-range project.

12. A classroom thermometer made of oak tag cut to resemble a large thermometer, marked 0, 10, 20, 30, 40, 50, etc. Holes are punched at either end and a length of red ribbon and equal length of string or white ribbon tied and threaded through. By pulling from the back the red ribbon can be made to go up or down as the temperature rises or falls. Daily reports are made by children.

13. Several boxes of coloured shapes (squares, triangles, circles, half-circles, diamonds, etc.) to be used by children at desks. These can be purchased but are easily made by cutting coloured construction paper into various shapes.

14. Small bead frames (purchased at toy counters). These should have ten beads in each row with each row a different colour. They can be constructed by making a wooden frame and stringing beads or buttons onto wires and fastening these to the wooden frame.

15. A flannel graph. Most stores have many posters of the easel type. They can be large or small. Cover the poster with two thicknesses of outing flannel, (light blue is a good colour to use). From magazines, colouring books, etc., cut animals, people, kites, *anything* and on the back of each one paste a small square of the outing flannel. To use the graph you place the cut-outs on it, pressing down slightly. They will cling there. With imagination the uses of this graph are unlimited. It can be used not only for number work but for reading, picture making, etc.

16. A coin card or place value card. Such a card has slits to hold one-cent pieces and ten-cent pieces. It can be used to show the meaning of two-figure numbers.

Tens' Place	Ones' Place
-----	-----
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It should be remembered that the use of concrete, pictorial and semi-concrete materials, number games and other devices for developing an understanding and command of the number system should not be confined to the Primary Grade. Such materials can and should be used in all grades.

THE PRIMARY GRADE

Preliminary Statement

Even before children enter school at the age of five or six they have experiences involving number and quantity, and have to be able to use numbers if they are to understand their environment fully. Number work to be real must be founded upon actual experiences of the pupil, and the knowledge gained by such experience must be given every opportunity to be put into practice. In the Primary Grade, the situations that give rise to these experiences should appear for the child to come naturally. Since out-of-school learning has been largely incidental, it seems reasonable to continue such experiences as a basis for further learning in the year of adjustment to the new social life of the school. The following list of situations with number possibilities indicates the ease with which teachers can find the means to carry on learning on a meaningful basis:

1. Playing number games.
2. Knowing one's own and other children's ages.
3. Learning dates of birthdays.
4. Counting pencils for each row; crayons for each row; scissors for each child; number of doors, windows, pictures on the walls, children in the room, desks and chairs in the room, children in a group, plants, lights, pieces of chalk, rubbers, overshoes, dots on dominoes, coat hangers, spools, sticks, beans, blocks, shells, clothes pegs, strokes of a clock, books on a shelf, marbles, ice-cream spoons, etc.
5. Learning number of days in the week.
6. Number of children present.
7. Learning telephone numbers.
8. Stating number of members in family.
9. Learning number of pounds one weighs.
10. Learning one's height.
11. Finding page numbers.
12. Reading house numbers.
13. Reading room numbers.
14. Reading the numbers on a clock (no Roman numerals).
15. Learning the meaning of a dozen rolls and a dozen eggs.
16. Reading numbers on rulers and yardsticks.
17. Learning the postage for letters.
18. Learning one's position in line (first, second, etc.)
19. Comparing the number of boys and the number of girls in the room.
20. Making price tags for a class store.

Though making use of these number situations when they arise naturally, teachers should also provide for number periods each day. Under the direction of a teacher who appreciates the value of number experiences as a background for the later systematic treatment of facts, process steps, and the development of judgment, this period may be invaluable to the child. To avoid mere incidental learning, which may be purely accidental, there is need for providing a modest program of number and quantity activities. Through both the incidental learning experiences and the

number periods, the child should learn counting, the reading and making of numbers, the use of money, how to tell time and the meaning of measurement. He may also learn some primary addition and subtraction facts although not receiving any formal drill on them. The number periods should provide experiences in counting, measuring, and quantitative ideas, *but not lessons to be memorized.*

There is a tendency, particularly on the part of the young teacher, to go too rapidly in introducing children to the meaning or concept of numbers, to the reading and making of numbers, to the idea of grouping, and to the understanding of the basis of the number system. Do not go too fast in doing these things. "Make haste slowly" should be the watchword in introducing children to number and quantity.

Objectives

The objectives for Primary Grade arithmetic are listed below—but not necessarily in the order in which they are to be taught. The teacher should have clearly in mind what each daily lesson is to contribute to the attainment of the objectives for the year. Only if this is done can the day-to-day work be considered worthwhile and meaningful. The objectives for the Primary Grade are as follows:

1. to know the meaning of, how to read, and how to make the numbers from 1 to 10
2. to enumerate to 30
3. to know the names of the days of the week, and the months of the year, the terms "yesterday" and "tomorrow"
4. to know the Canadian coins: one-cent piece, five-cent piece, and ten-cent piece
5. to know the liquid pint and the liquid quart
6. to know the meaning of one-half of a unit
7. to know the meaning of dozen
8. to have concrete, pictorial, and semi-concrete experiences with easy addition facts and the corresponding subtraction facts
9. to build a vocabulary as a basis for an understanding of arithmetic

Time Allotment

About thirty-five minutes a day should be devoted to number work in the Primary Grade. This 35-minute period should include the time the teacher works with the class and any seat-work that the children may be expected to do. The period should be broken up into at least two periods, and preferably three periods, during the day.

Grouping

Learning at any level can be successful only when the individual is ready for it. Readiness depends upon the child's mental growth, upon his accumulated experiences, and upon his attitude to the subject. The children must have acquired a sufficient level of mental maturity to handle abstractions and to see relationships. They should be grouped for instruction on the basis of the particular needs and activities of each child. These groups should be flexible and purposeful. In a class of thirty-five pupils

there should be three groups. *Do not assume that groups for instruction in number work will be the same as groups for reading instruction.*

Method

Every new learning should be presented through a problem. The problem should be one that has meaning for the children. Most problems should be presented in concrete form, and the child's experiences should be happy ones leading him to feel secure in his understanding of number meanings. When it is not possible to provide the children with enough concrete problems, the teacher should present problems which describe real situations. Vicarious experiences, too, give meaning to many number situations which the child cannot experience in real life. The teacher should set up imaginary situations that are full of number experiences both interesting and useful to the child. Through dramatic play or imaginative thinking, the child can learn to solve problems he encounters.

Every number experience must be presented first in a meaningful setting with concrete material. Picture representation should follow the use of concrete objects. The third stage should be through semi-concrete symbols, such as squares, dots, circles, diamonds, triangles, and stars, which will help the child to understand the concept. The fourth stage is the presentation of the concept in its abstract form. For example, the concept of 6 may be developed through (a) working with 6 clothes pegs and other objects, (b) seeing pictures of 6 chickens, (c) making 6 circles on the blackboard, and (d) making the figure "6".

Concepts of number, time, measurement, and space must become a natural part of the child's thinking. A specialized vocabulary necessary for the child to express these concepts meaningfully is important. As soon as written work is begun, the habit of working neatly and carefully should be encouraged. Well made figures and neat arrangement of the child's work will help him later to work accurately and quickly.

Content

The content of the work for the Primary Grade is outlined below by units. This plan is simply a guide to the teacher as to the order of presentation of the definite topics in the work. However, there are several topics in this outline which should not be used rigidly in any sequence of units. Such a unit for instance is "Unit 2" in which children are to be taught the names of the days of the week, etc. The information children should acquire from Unit 2 should be dealt with little by little from day to day throughout the year. The same is true of the meanings of what may be called the indefinite units in Unit 9.

The program for this grade contains a small number of topics involving number and quantity, but it is a full year's job for the average Primary Grade pupil. Proper instruction in this grade will require a great deal of "going over again" work that has been taught several times. Further suggestions regarding the teaching of the various topics will be found under each unit.

Unit 1: The meaning of, how to read and how to make the numbers from 1 to 10

The development of this unit should be carried on over a period of at least two months and more if necessary. Then there should be drill and review on it throughout the rest of the year. Emphasis should be put on the "meaning" of the number rather than on the recognition of the symbol. The meaning of a number should be developed through an abundance of concrete, picture and semi-concrete materials before the symbol is introduced. It must be remembered that *meaning comes before the symbol*.

In learning the meaning of the numbers from 1 to 10 children count. As a matter of fact counting is the first number activity and many children come to school with the ability to say the numbers from 1 to 10. Teachers should realize that the mere reciting of the number words or counting by "rote" is of no value except that it gives children the names of the numbers in order. Counting, if to be of any value, must be "rational". The child must point to, touch or move some object, saying the number for each as he proceeds, and stop when he comes to the last object.

When children have learned to count rationally they are ready to understand the meaning of a number. The most important thing to remember in teaching the meaning of, say, the number 6 is that the number 6 means a *group* of six individual things. It is the *group* of six that must be emphasized. This may be done by tying a string around six sticks; by grasping six coins; by drawing a circle around six stars drawn on the board—anything to emphasize the group idea.

Emphasis must be placed on the group idea of a number; otherwise the child might think that the last object counted represents the meaning of the number. For instance the child might think that the sixth coin named is the meaning of six whilst in reality it is only one of the coins in a group of six.

Not only is it essential that the meaning of the individual number be known but so also must the relationships of the numbers to each other. The child should know that 8 comes after 5, before 10 and between 7 and 9 without having to count from 1 to find out. Rational counting, the group idea of a number, and the relationships of the numbers to one another aid in developing the sequence of the numbers. The use of the ordinals, first to tenth, orally serves the same purpose.

Even if a child understands the group idea of six, his concept of "sixness" is not complete. He should be taught to see that within a group of 6 boys there may be groups of 4 boys and 2 boys, or 3 boys and 3 boys, or 5 boys and 1 boy. Similarly a doll carriage has 4 wheels, 2 on each side. Or again, a tricycle has 3 wheels, 1 at the front and 2 at the rear. Another illustration is that, of 5 children at a table, 3 may be on one side and 2 on the other side.

Recognizing groups within larger groups prepares the child for adding by groups, that is, it prepares him for learning the addition facts.

As the meaning of each number is understood, the child should learn to recognize and make the number symbol.

It cannot be emphasized too much that an abundance of materials must be used by the teacher in the classroom to introduce children to the meaning of numbers. Following are some suggestions of materials that should be found in every Primary Grade classroom.

Concrete Material—There is much concrete material in any classroom: the pupils themselves, chairs, desks, coats, boys, girls, books, pencils, and so on. In addition the teacher can easily provide such things as buttons, sticks, circular discs and blocks of wood. Children can bring in toy cars, dolls, building blocks, etc.

Pictorial Material—Each classroom should have a minimum of four sets of picture cards, with the same picture on each card, illustrating some of the following: birds, rabbits, dogs, horses, automobiles, flowers, fruits, toys, children, etc.

Semi-Concrete Material—A minimum of four sets of semi-concrete material, with the same material on each card in a set, such as: dots, circles, squares, triangles, lines, etc.

Several picture charts with groups of the same type depicting each of the numbers from 1 to 10.

A set of number cards with a card for each symbol, a card for each number word, and a card showing the number of things that each number represents.

Unit 2: The names of the days of the week, the months of the year, the date of the month, and the meaning of "yesterday" and "tomorrow"

The work of this unit should be done from day to day from the beginning of the term throughout the year. A few minutes each morning devoted to finding the name of the day, the month, etc., with the aid of a calendar (blank at the beginning of the month and built up day by day) will bring better results in the end than any short-time concentration on this unit. The names of the days of the week in order may be learned through the medium of rhymes and other devices.

Unit 3: The liquid quart and liquid pint and the relationship between the pint and quart

In developing this unit, pint and quart liquid measures must be available in the classroom. Emphasis should be placed on the recognition of these measures, what we buy in these quantities, and the discovery on the part of the pupil, through the use of the measures, of the relationship between them.

Unit 4: The addition concept and one of the subtraction concepts

The eight easy addition facts and the eight subtraction facts to be dealt with in the grade are shown below in units, arranged in order of difficulty.

$\begin{array}{r} 2 \ 4 \\ 2 \ -2 \\ \hline 4 \ 2 \end{array}$	$\begin{array}{r} 2 \ 1 \ 3 \ 3 \\ 1 \ 2 \ -1 \ -2 \\ \hline 3 \ 3 \ 2 \ 1 \end{array}$	$\begin{array}{r} 1 \ 2 \\ 1 \ -1 \\ \hline 2 \ 1 \end{array}$	$\begin{array}{r} 3 \ 1 \ 4 \ 4 \\ 1 \ 3 \ -1 \ -3 \\ \hline 4 \ 4 \ 3 \ 1 \end{array}$	$\begin{array}{r} 4 \ 1 \ 5 \ 5 \\ 1 \ 4 \ -1 \ -4 \\ \hline 5 \ 5 \ 4 \ 1 \end{array}$
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These facts should be taught in the concrete, pictorial, and semi-concrete stages only. They should not be developed in the abstract stage in this grade. It is recommended that the addition concepts be developed first, and after they have been dealt with, the "take-away" subtraction facts may be used together. This suggestion is made because confusion may arise if the

addition and subtraction concepts are introduced at the same time. Later there will be more insight if they are treated together. It should be pointed out from day to day that when we "put things together" we are adding; and that when we "take away" we are subtracting. The *take away idea of subtraction* is the only one dealt with in the Primary Grade.

Unit 5: The coins: the one-cent piece, the five-cent piece, and the ten-cent piece

Many children will be familiar with these coins by the time they begin school. However, the objective in this unit should be to have pupils recognize the coins, know what children's things they will buy, and know the relationships among them.

Reality should be introduced into this unit through the medium of a "store" which, in graded schools, may be set up in a convenient part of the room. The store may be "stocked" by the teacher and pupils bringing to the class empty cartons of things which children normally buy with the coins involved, such as candy bars (use the wrappers), pencils, apples, etc. Pictures of such things from magazines provide suitable substitutes. Children then can tell what they want to buy, tell how much it costs, and show the coins that will make the purchase.

Unit 6: The meaning of one-half of a unit

Even before children come to school they have need of the concept of "one-half" if they are to understand their environment. In real life, they divide apples, candy bars, etc., into halves, but children are often heard to speak of a "big half" and a "little half"; hence, the important ideas to get over in this unit are that there must be *two* parts, and the parts must be the same size.

Materials used in this unit should be things children actually need to divide into halves.

The unit does not include a lesson on the meaning of one-half of a group.

Unit 7: How to enumerate to 30

This unit will involve the learning of the names of the numbers from 11 to 30 in order. Much rote counting will be necessary to get the names of the numbers. However, the objective should be to have the child enumerate to 30. For example, there should be the requirement of the identification of a fixed number of children, such as 17, in the classroom's first three rows. Use many concrete, pictorial, and semi-concrete materials.

This unit does not require the pupils to learn to read or to make the numbers 11 to 30. However, some children will wish to learn how to read and make these numbers. When children are anxious to learn new materials, they will do so with enjoyment. Consequently, for those who express a wish to learn how to make numbers beyond 10, the following suggestions are given:

There are two ideas to emphasize here:

- (a) that 13, for instance, is made up of a group of 13 individual things;
- (b) that 13 is made up of 1 ten and 3 ones.

The place value of these two-figure numbers requires constant emphasis. Do not depend upon explaining the meaning of 13 by using only sticks. Coins are particularly useful in giving children the meaning of the figures in two-figure numbers. Coin-cards should be made in which slits for holding the coins are provided; for instance, a coin-card may have the number 13 on it and three slits above the 3 into which 3 one-cent pieces can be placed; and one slit above the 1 into which 1 ten-cent piece can be placed. Meaning is the important thing to keep in mind in developing the child's concept of the two-figure numbers.

Remember that the development of the meaning of the numbers beyond 10 should be attempted only with the most mature pupils in the Primary Grade. For the majority of the pupils this work will come in Grade I.

Unit 8: The meaning of "dozen"

Children need to know the meaning of "dozen" on account of its very frequent use in everyday activities.

It is well for teachers to remember that "dozen" and "12" are not synonymous terms. The term "dozen", meaning 12, is used only when referring to certain commodities; for instance, eggs, small fruits, products of the bake shop, and certain miscellaneous items like clothes pins, buttons, nuts and bolts, etc. It would be ridiculous to say that a man earns a dozen dollars a week.

In teaching this unit, therefore, the materials that are used to have the children discover how many things there are in a dozen should be things that are bought and sold by the dozen in everyday life. Easy materials to provide are cookies, clothes pins, fruits, etc. In any case, the materials should be real, and not drawings on the blackboard.

Unit 9: Building a number vocabulary

- (a) *To know the meaning of quantitative words* such as: many, few; big, little; fast, slow; large, small; long, short; thick, thin; tall, short; wide, narrow; near, far; heavy, light; many, most.
- (b) *To know the meaning of the terms we use for comparisons:* bigger, smaller; larger, smaller; more, less; longer, shorter; taller, shorter; nearer, farther; heavier, lighter; thicker, thinner; wider, narrower; quicker, slower; faster, slower; warmer, colder; higher, lower; softer, harder; more, most; fewer, fewest.

Although this is the unit placed last in the grade, it does not follow that the meaning of the terms referred to above must await development until this unit is reached. As a matter of fact, the meanings of these words should be developed from the very beginning of the year's work; for instance, in Unit 1, when dealing with the meaning of the numbers from 1 to 10, the teacher will certainly have to have the children understand the meaning of "more" and "less".

There is no compulsory vocabulary list of words to use in this unit. The objective is to develop a meaningful vocabulary to ensure an understanding of arithmetic. These are not words to be given as formal lessons, but the vocabulary is to be considered as if it were a living and growing organism planted in various environments of activity (games, songs, dances, etc.).

In teaching the meaning of the groups of words in this unit the teacher must bear in mind that definitions of the words should not be attempted. The proper procedure is to have the terms used correctly in a large variety of circumstances, always of course with a physical and visual aid. For instance, in teaching the meaning of "taller" and "shorter", the teacher should have two pupils, one of whom is taller than the other, stand before the class, and say to the class: "John is taller than Mary", "Mary is shorter than John". Have the children use such sentences to describe the difference in height of other boys and girls. In this way, meanings without definitions will be acquired. The other groups of words to be taught in this unit should be dealt with in the same manner.

GRADE I

Objectives

The objectives for Grade I arithmetic are given below. Do not, however, attempt to realize them with your pupils in the order in which they are listed. Each unit of the grade's program is related to a particular objective. As units are completed or carried on through the year, more and more of the objectives will be reached until, at the end of the program all of them should be realized.

1. To know the meaning of, how to read and how to make the numbers from 1 to 100.
2. To count by 2's to 20 and by 5's to 100.
3. To understand the meaning of addition.
4. To understand the meaning of subtraction.
5. To know the forty-five addition facts in which the sum does not exceed 10.
6. To know the forty-five subtraction facts corresponding to the addition facts in objective 5.
7. To be able to add correctly columns of not more than three addends in which the sum of the column does not exceed 10.
8. To know the ordinals *first* to *tenth*.
9. To know the Canadian coins: one-cent piece, five-cent piece, ten-cent piece.
10. To know the foot measure and how to use it.
11. To know the meaning of dozen.
12. To know the meaning of one-half of a unit.
13. To know the liquid quart and liquid pint.
14. To know the names of the days of the week, the months of the year, the date of the month, the terms "yesterday" and "tomorrow".
15. To build a vocabulary of quantitative and comparative words which occur in simple arithmetical situations.

Number Combinations for Grade I

These combinations are dealt with in the various units of the grade program. They are presented here in summary for the convenience of the teacher.

Group I

2	2	1	3	4
2	1	1	1	1
-	-	-	-	-

Group II

3	3	5	4	4
2	3	1	2	4
-	-	-	-	-

Group III

5	6	4	7	5
5	1	3	1	2
-	-	-	-	-

Group IV

8	6	5	9	7
1	2	3	1	2
-	-	-	-	-

Group V

6	8	5	7	6
3	2	4	3	4
-	-	-	-	-

Time Allotment

About 35-40 minutes a day should be devoted to number work in Grade I. This period should include the time the teacher works with the class and any seat-work that the child may be expected to do. The total time should preferably be broken up into at least two periods during the day.

Content

Unit 1: The meaning of, how to read and how to make the numbers from 1 to 10

Even though this unit has been developed in the Primary Grade, it should be done again in Grade I and developed with as much care as if it were being introduced for the first time. Children forget easily. A thorough development of this material again in Grade I will make for a better foundation on which to build number concepts. For the development of this unit teachers are referred to Unit 1 in the Primary Grade.

Unit 2: The names of the days of the week, the months of the year, the date of the month, and the meaning of "yesterday" and "tomorrow"

Proceed as in Unit 2 of the Primary Grade carrying on the activities throughout the year.

Unit 3: The liquid quart and liquid pint and the relationship between the pint and quart

Proceed as in Unit 3 of the Primary Grade. The material should be taught and reviewed from time to time throughout the year.

Unit 4: The number facts of Group I

The addition and subtraction facts to be dealt with in this unit are shown below in blocks called "Teaching Units" arranged in order of difficulty.

2	4
2	-2
-	-
4	2

2	1	3	3
1	2	-1	-2
-	-	-	-
3	3	2	1

1	2
1	-1
-	-
2	1

3	1	4	4
1	3	-1	-3
-	-	-	-
4	4	3	1

4	1	5	5
1	4	-1	-4
-	-	-	-
5	5	4	1

The facts of each teaching unit should first be taught, as in the Primary Grade, in the concrete, pictorial, and semi-concrete stages. After that is done the abstract stage may be introduced.

The general principle of method is to teach the addition and corresponding subtraction facts of each teaching unit before proceeding to the next teaching unit. However, it is suggested that all the addition facts of Group I be taught before doing the corresponding subtraction facts. Confusion is apt to arise in the beginning if addition and subtraction facts of Group I are introduced at the same time.

When the addition facts of Group I have been taught, the subtraction fact corresponding to each addition fact should be taught and they should be related to each other as shown in the blocks above.

With the number facts of Groups II, III, IV and V, both the addition and subtraction facts of each teaching unit should be taught before proceeding to the next teaching unit.

It should be noted that "teaching the addition and corresponding subtraction facts together" does not necessarily mean teaching them "in the same lesson". The addition facts of a teaching unit might be taught on Monday and reviewed on Tuesday; the subtraction facts could be taught in Tuesday's lesson and at the same time the addition and subtraction facts related as shown in the teaching unit.

When the addition and subtraction facts are being taught, the meaning of addition and the meaning of subtraction must be understood by the child if the facts are to have meaning and if he is to understand when numbers are to be added and when numbers are to be subtracted. Pupils must be reminded continuously and must be required to demonstrate and to explain that when they add, they "put things together" and when they subtract, they "take away". The "take away" idea of subtraction is the only one dealt with in Grade I.

After teaching the addition and subtraction facts from the concrete through to the abstract, complete mastery of them is essential. Mastery to secure automatic response can be had only through the use of properly motivated practice or drill. More practice should be given on the more difficult facts than on the easier ones.

A good procedure to attain mastery of the facts is:

- teach the facts as outlined earlier;
- drill the facts using straight drill, games and other devices aiming at automatic responses to the facts, real problems involving the facts;
- test on the facts; use the facts in oral problems;
- identify the facts that give difficulty;
- reteach and practice those that give difficulty;
- test again.

The procedure suggested above should be a continuous process throughout the term. The practice exercises and test items should be limited to not more than twelve facts with a lesser number earlier in the term.

Unit 5: The coins: the one-cent piece, the five-cent piece and the ten-cent piece

Proceed as in Unit 5 of the Primary Grade. The store idea is an excellent method of teaching much of the number work in the elementary grades.

Unit 6: The meaning of one-half a unit

Proceed as in Unit 6 of the Primary Grade. This should be emphasized from time to time throughout the year.

Unit 7: The number facts of Group II

Teach as in Unit 4 the teaching units shown below:

3 2 5 5 2 3 -2 -3 — — — — 5 5 3 2	3 6 3 -3 — — 6 3	5 1 6 6 1 5 -1 -5 — — — — 6 6 5 1	4 2 6 6 2 4 -2 -4 — — — — 6 6 4 2	4 8 4 -4 — — 8 4
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Follow the same procedure as in Unit 4 using concrete, pictorial and semi-concrete materials; drill on the bald facts, problems using the facts, and games using the facts until understanding, complete mastery and automatic response are secured.

Unit 8: Building a vocabulary of quantitative and comparative words

Refer to suggestions for words and procedure as in Unit 9 of the Primary Grade.

Although this unit is placed at this point the meanings of many of the words should be developed from the beginning of the year not only in arithmetic classes but in any situations which may arise. Do not teach definitions. Proceed as in the Primary Grade.

Unit 9: The meaning of, how to read and how to make the numbers from 1 to 100 by tens

- (a) The meaning of the numbers 10, 20, 30, etc., by tens to 100.
- (b) Counting by tens to 100.
- (c) Reading and making the numbers 10, 20, 30, etc., to 100.

Steps (a), (b) and (c) suggested above should not be taught separately but in combination. In teaching this unit concrete materials—such as ten-cent pieces, a number chart, etc., should be used.

The place value of the tens' digit must be emphasized.

The function of the zero is to hold the ones' place.

Unit 10: The meaning of, how to read and how to make the numbers from 11 to 19

These numbers are often rather difficult for children to understand.

Illustrate objectively the meaning of each number in terms of tens and ones. Use concrete, pictorial and semi-concrete materials. Emphasize

the place value of the digits. Show, for example, that while twelve means a group of twelve things, when the number 12 is written the 2 stands for two ones and the 1 means one ten. A coin card with slits into which one-cent pieces and ten-cent pieces may be placed is one objective device that may be used to help children understand the meanings of the numbers from 11 to 19.

Unit 11: The meaning of, how to read and how to make the numbers from 20 to 29

Following the procedures suggested in Unit 10 combined with a number chart. The numbers in this unit form the pattern for the numbers in the decades higher than twenty. For example 21 is 20 and 1; 22 is 20 and 2, and so on.

Unit 12: The meaning of, how to read and how to make the numbers from 30 to 100

Follow the procedures suggested in Units 10 and 11.

Unit 13: The meaning of dozen

Refer to and reteach Unit 8 of the Primary Grade.

Unit 14: The number facts of Group III

Teach as in Unit 4 the teaching units shown below:

5 10 5 -5 — — 10 5	6 1 7 7 1 6 -1 -6 — — — — 7 7 6 1	4 3 7 7 3 4 -3 -4 — — — — 7 7 4 3	7 1 8 8 1 7 -1 -7 — — — — 8 8 7 1	5 2 7 7 2 5 -2 -5 — — — — 7 7 5 2
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Unit 15: Counting by 5's to 100

Many children will have learned by rote to count by 5's to 100 from playing games like "Hide and Seek". This rote counting may be rationalized by having them count five-cent pieces saying as they touch each five-cent piece, "5, 10, 15, 20, etc." Piles of books containing five each may be counted in the same way. They will soon learn to appreciate that this is a short way of counting when there are groups of 5.

Unit 16: The foot as a measure of length

In developing this unit each pupil must have his own foot measure. The objective of the unit is to have the pupils recognize the foot, to know what it is used for, and how to use it. Pupils should be given opportunities of estimating such things as the length of the desk, the width of the door, etc., and checking their estimates by actual measurement. Such work on measurement with the foot measure should be carried out frequently throughout the rest of the term.

Unit 17: The number facts of Group IV

Teach as in Unit 4 the teaching units shown below:

$$\begin{array}{r} 8 \ 1 \ 9 \ 9 \\ 1 \ 8 \ -1 \ -8 \\ \hline 9 \ 9 \ 8 \ 1 \end{array}$$

$$\begin{array}{r} 6 \ 2 \ 8 \ 8 \\ 2 \ 6 \ -2 \ -6 \\ \hline 8 \ 8 \ 6 \ 2 \end{array}$$

$$\begin{array}{r} 5 \ 3 \ 8 \ 8 \\ 3 \ 5 \ -3 \ -5 \\ \hline 8 \ 8 \ 5 \ 3 \end{array}$$

$$\begin{array}{r} 9 \ 1 \ 10 \ 10 \\ 1 \ 9 \ -1 \ -9 \\ \hline 10 \ 10 \ 9 \ 1 \end{array}$$

$$\begin{array}{r} 7 \ 2 \ 9 \ 9 \\ 2 \ 7 \ -2 \ -7 \\ \hline 9 \ 9 \ 7 \ 2 \end{array}$$

Unit 18: Counting by 2's to 20

Counting by 2's to 20 is somewhat more difficult than counting by 5's to 100 and must be done with care. All things counted should be real things such as coins, books, pencils, buttons, beads, pupils in a row, pupils in aisles. There are many instances where counting by 2's is a useful and practical shortcut.

Unit 19: The number facts of Group V

Teach as in Unit 4:

$$\begin{array}{r} 6 \ 3 \ 9 \ 9 \\ 3 \ 6 \ -3 \ -6 \\ \hline 9 \ 9 \ 6 \ 3 \end{array}$$

$$\begin{array}{r} 8 \ 2 \ 10 \ 10 \\ 2 \ 8 \ -2 \ -8 \\ \hline 10 \ 10 \ 8 \ 2 \end{array}$$

$$\begin{array}{r} 5 \ 4 \ 9 \ 9 \\ 4 \ 5 \ -4 \ -5 \\ \hline 9 \ 9 \ 5 \ 4 \end{array}$$

$$\begin{array}{r} 7 \ 3 \ 10 \ 10 \\ 3 \ 7 \ -3 \ -7 \\ \hline 10 \ 10 \ 7 \ 3 \end{array}$$

$$\begin{array}{r} 6 \ 4 \ 10 \ 10 \\ 4 \ 6 \ -6 \ -4 \\ \hline 10 \ 10 \ 4 \ 6 \end{array}$$

Unit 20: The ordinals from first through tenth

The distinction between the meanings of the cardinal numbers and the meanings of the ordinal numbers must be made clear. For example while the cardinal number *six* means a group of six things the ordinal *sixth* refers to a single thing and to its particular place or position in a group. Many uses for ordinal numbers may be found in a classroom, e.g., the *third* aisle, the *seventh* book on the shelf, the *fourth* day of the week, etc.

Unit 21: Column addition with three addends, sum not to exceed 10

Mastery of the forty-five addition facts of Groups I-V is needed for success in column addition. Since column addition in this unit involves the addition of three one-place numbers several new difficulties arise that require careful attention:

- a wider attention span is involved in adding three numbers than is involved in adding the facts;
- the skill of adding an *unseen* number (the sum of the top two numbers) to a *seen* number (the bottom number of the column must be developed before beginning column addition;
- pupils must be taught to write the figures of the columns under each other in a straight line and to put the answer in its proper place at the bottom;
- have the pupil add from the top of the column down. When he is ready, he should be taught to check the answer by adding up.

Various devices may be used for getting children ready for column addition as suggested in (b) above:

One common device is to ask the child the sum of 4 (which the teacher says) and 2 (which the teacher writes on the board).

The first lessons in column addition should be through the medium of objective material so that the pupils will know that the answer to the sum of the column is correct. Following this the teacher can teach the short way of getting the answer to the column. For instance in adding the column

$$\begin{array}{r} 2 \\ 1 \\ 4 \end{array}$$

starting at the top the teacher would ask "2 and 1 are how many?" "Remember the 3 and say 3 and 4 are 7. Write the 7 under the column."

Following are the 120 exercises in column addition with three addends using addition facts of Groups I-V. Much practice should be given using these exercises.

The 120 Exercises in Column Addition with Three Addends using Addition Facts of Groups I-V

1	1	1	2	1	1	3	1	1	4	1	1	5	1	1	6	1	1	7
1	1	2	1	1	3	1	1	4	1	1	5	1	1	6	1	1	7	1
1	2	1	1	3	1	1	4	1	1	5	1	1	6	1	1	7	1	1
-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
1	1	8	1	2	2	1	1	2	2	3	3	1	1	2	2	4	4	1
1	8	1	2	1	2	2	3	1	3	1	2	2	4	1	4	1	2	2
8	1	1	2	2	1	3	2	3	1	2	1	4	2	4	1	2	1	5
-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
1	2	2	5	5	1	1	2	2	6	6	1	1	2	2	7	7	1	3
5	1	5	1	2	2	6	1	6	1	2	2	7	1	7	1	2	3	1
2	5	1	2	1	6	2	6	1	2	1	7	2	7	1	2	1	3	3
-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-

Time Allotment

About 45-55 minutes a day should be devoted to number work in Grade II. This period should include the time the teacher works with the class and any seat-work that the child may be expected to do. The total time may be divided into two periods during the day if the teacher so desires.

Content

Unit 1: Review of Grade I work

As in outline for Grade I.

Unit 2: The meaning of two-figure numbers

The basis of our number system is the place value of the figures that make up the number. Since two-figure numbers are introduced here for the first time, great care must be taken to have pupils comprehend that in such a number as 36, the 6 means 6 ones and the 3 means 3 tens. The meaning of two-place numbers cannot be understood effectively unless their meaning is demonstrated objectively. Coins are a good medium through which meaning may be brought to two-figure numbers. For example in 36 cents the 6 stands for 6 one-cent pieces and the 3 stands for 3 ten-cent pieces; in other words, in the number 36 the 6 means 6 ones and the 3 means 3 tens.

Other sets of objective material must be used before a generalization is made. Sticks, books, blocks, beads, etc., should be used, putting in bundles ten individual items to represent the tens. Additional loose items (less than ten) represent the ones.

Unit 3: The addition of two-figure numbers of two addends with sums less than one hundred without carrying

This process should be introduced through the medium of real problems, and rationalized emphasizing ones and tens.

Be sure that the children are taught carefully to put the numbers under each other so that the ones are under the ones and the tens are under the tens. Emphasize, too, that in addition of two-place numbers we start on the right, i.e. we add the figures in the ones' column first and then the figures in the tens' column. In the sum of the numbers the ones' figure must be placed under the ones and the tens' figure under the tens.

In adding the numbers, *add down* and check by adding up.

Unit 4: The meaning of zero

Mathematically, zero means the absence of quantity. Actually the symbol 0 is not a number. Its function in our number system is that of a *place holder*. For instance, in the number 40 the zero is used to hold the ones' place or to fill the empty ones' place, since 40 means 4 tens and "no" or "not any" ones. In order that the 4 in 40 maintain its meaning the zero is written to hold the ones' place. Similarly, in the number 305 the 5 stands for 5 ones and the 3 stands for 3 hundreds. In order that the 3 and the 5

maintain their meaning, the 0 is written between them to hold the tens' place or to fill the empty tens' place since there are "no" or "not any" tens.

The function of zero as a place holder or to fill the empty space in a number should be emphasized from the beginning, although, to the young pupil, zero may be defined as meaning "not any", "none" or "nothing".

Unit 5: Addition with zero

In everyday affairs the simplest instance in which addition with zero is required is in the addition of two-figure numbers involving zeros, as in the examples:

$$\begin{array}{r} 42 \\ 30 \\ \hline \end{array} \quad \begin{array}{r} 20 \\ 35 \\ \hline \end{array} \quad \begin{array}{r} 50 \\ 20 \\ \hline \end{array}$$

The skill of adding two-figure numbers with zeros should be introduced through a real problem, such as:

"Jane had 45 cents and her father gave her 20 cents more. How many cents did she have then?"

The teacher should have the pupils suggest that to find how many cents Jane has now they would have to add 45 and 20. The teacher should write on the board:

$$\begin{array}{r} 45 \\ 20 \\ \hline \end{array}$$

By using ten-cent pieces and one-cent pieces, the pupils will be reminded that 4 ten-cent pieces and 5 one-cent pieces represent 45 cents; and that 2 ten-cent pieces represent 20 cents. Putting together or adding the coins gives 65 cents, which the pupils will know is the correct answer. The teacher should now write the answer 65, and the board work will appear as:

$$\begin{array}{r} 45 \\ 20 \\ \hline 65 \end{array}$$

Several other problems with coins should be worked out in a similar manner, and the results placed on the board. At this stage pupils should be reminded that the answers found by using the coins are correct, but the process of finding answers with the coins is slow and that it would be advantageous to find the answer without actually using the coins.

This should be done by reminding pupils that in the first problem the number 45 means 4 tens and 5 ones, and the number 20 means 2 tens and "not any" ones. To find the answer we add the ones and add the tens.

Tens	Ones
4	5
2	0
—	—
6	5

Starting on the right we add the ones. There are only 5 ones in the ones' column, so we write 5 in the ones' column in the answer. Now add the tens: "4 tens and 2 tens are 6 tens". Write the 6 in the tens' column in the answer.

The pupils will see that the answer secured in this way is correct because it is the same as the answer they got by actually putting the coins together.

The answers to the other problems previously worked out with the coins should be found in a similar manner embodying such examples as:

20	and	50
35		20
—		—

Lest the impression be left with pupils that addition of zeros is found only in problems involving money, answers should be developed to real problems in the addition of two-figure numbers involving zero using other objective materials such as pencils, books, etc., and hence to finding the answer by adding the ones and adding the tens.

When through use of objective materials and reference to ones and tens, the meaning of addition with zero is understood, the pupils are ready for the general principles of adding with zero, without thinking of coins, pencils, etc., or ones and tens. The teacher should lead the pupils to the discovery and expression of the following general principles:

- (1) When zero is added to a number the number does not change.

$$\begin{array}{r} 36 \\ 20 \\ \hline 56 \end{array}$$

- (2) When a number is added to zero the number does not change.

$$\begin{array}{r} 30 \\ 24 \\ \hline 54 \end{array}$$

- (3) When zero and zero are added the sum is zero.

$$\begin{array}{r} 40 \\ 30 \\ \hline 70 \end{array}$$

Unit 6: The meaning of, how to read and how to make the numbers from 100 to 1000 by hundreds

- (a) The meaning of the numbers 100, 200, 300, etc., to 1000.
 (b) Counting by 100's to 1000.
 (c) Reading and making the numbers 100, 200, 300, etc., to 1000.

The steps (a), (b) and (c) suggested above should be combined and not taught separately.

Unit 7: The meaning of one-half of a group

Review again one-half of a unit as in Unit 6, Grade I.

In teaching one-half of a group the concept must be developed that the group of objects is divided into two equal parts with the same number of objects in each part. This *does not* involve division by 2 because division by 2 has not been taught. Rather it involves dividing the group of objects by placing one object on one side, a second one on the other side, a third on the first side, etc., until the whole group is divided into two parts with the same number of objects in each part. This unit can be taught only through the use of many sets of objective materials which are real to the child and which would very frequently be a real problem for the child. It is a common problem for a child to divide such things as candy, marbles, coins, etc., equally with another child.

Unit 8: The meaning of, how to read and how to make the numbers from 100 to 1000

The place value of the figures in three-place numbers must be emphasized. For example in the number 374, pupils must understand that the 4 means 4 ones, the 7 means 7 tens, the 3 means 3 hundreds.

It may be difficult to represent objectively the quantity embraced by a number such as 374 but semi-concrete material prepared by the teacher may be presented to give meaning to such a number. Such a device as having the children see the number of pages that make a book or a portion of a book will help to give meaning to larger numbers.

Care should be taken at this time in the reading of whole numbers. For example the number 374 should be read: "three hundred seventy-four." Do not use "and" in reading three-figure numbers. "And", in the reading of numbers, has a special use which will be developed when pupils are taught to read mixed decimals.

Unit 9: The addition of three-figure numbers of two addends, with and without zeros and without carrying, sums not to exceed 999

Proceed as in Units 3 and 5.

Unit 10: The addition of two- and three-figure numbers with three and four addends with and without zeros, without carrying, sums not to exceed 999

Emphasize the placing of the ones under the ones, the tens under the tens and the hundreds under the hundreds.

Unit 11: Subtraction of two- and three-figure numbers without borrowing

Introduce this unit through real problems in which pupils will realize the meaning of and the need for subtraction. Emphasize that, when subtracting, the ones must be placed under the ones, the tens under the tens, etc.

In the Primary Grade and in Grade I, only the "how many left" idea of subtraction was developed. There are, however, two other subtraction ideas that must be developed in Grade II and beyond. In addition to the idea of subtracting to find how many are left, pupils must be taught that we subtract to find the "difference". Again they must learn that we subtract to find "how many more", "how much more", "how many fewer", "how many less", "how much taller", "how much shorter", "how much lighter", "how much heavier", etc. In problem exercises the words "left", "difference", "how much more", etc., may serve as key words to indicate subtraction although pupils must understand that the basic meaning of subtraction is the process of taking away part of a number leaving a smaller number except when zero is taken away.

Unit 12: The Canadian coins to fifty cents

Pupils should be taught to recognize the coins, to name things the coins will buy and to know the relationship between any two coins, for example, that one 25-cent piece will buy five ice cream cones while it will take five 5-cent pieces to buy five ice cream cones.

Unit 13: Subtraction with zero

In teaching subtraction with zero, follow the method suggested in unit 5 for addition with zero, using objective materials such as coins, crayons, sticks, etc., for meaning and for correct answers. This should be followed by discussion of subtraction of the ones and tens, thereby securing the same answer as found through the use of the objective materials. In the latter, in such an exercise as:

Tens	Ones
6	8
- 4	0
-	-
2	8

the procedure might be: Subtract the ones. There are not any ones to take away from 8 ones, so we write 8 under the ones. Then subtract the tens.

In such an exercise as $\begin{array}{r} 54 \\ -24 \\ \hline \end{array}$ the procedure might be: Subtract the ones.

When we take 4 ones from 4 ones there are no ones. Write zero in the ones' place. Subtract the tens.

The final step is to develop the general principles:

(1) When zero is subtracted from a number the number does not change.

$$\begin{array}{r} 63 \\ -40 \\ \hline 23 \end{array}$$

(2) Any number subtracted from itself leaves zero because nothing is left.

$$\begin{array}{r} 92 \\ -92 \\ \hline 40 \end{array}$$

(3) When zero is subtracted from zero the difference is zero.

$$\begin{array}{r} 70 \\ -30 \\ \hline 40 \end{array}$$

Unit 14: Subtraction of two- and three-figure numbers with zeros without borrowing

Proceed as in Unit 11.

Unit 15: The inch as a measure of length

Review the foot as in Unit 16, Grade I.

Children should be required to suggest that to measure the length of anything which is less than a foot a smaller unit of measure is needed. They should have practice in measuring lengths in inches with their foot measures, which are divided off into inches. Practice should also be given in estimating distances in inches and checking the estimates by accurate measurement. In the same way lengths like 2 feet 4 inches, 3 feet 5 inches, etc., should be estimated and measured.

Pupils should be required to discover for themselves how many inches there are in a foot.

Unit 16: The number facts of Group VI

Teach as in Unit 4, Grade I, the teaching units shown below:

$\begin{array}{r} 6 \ 12 \\ 6 \ -6 \\ \hline 12 \ 6 \end{array}$	$\begin{array}{r} 8 \ 3 \ 11 \ 11 \\ 3 \ 8 \ -3 \ -8 \\ \hline 11 \ 11 \ 8 \ 3 \end{array}$	$\begin{array}{r} 9 \ 2 \ 11 \ 11 \\ 2 \ 9 \ -2 \ -9 \\ \hline 11 \ 11 \ 9 \ 2 \end{array}$	$\begin{array}{r} 6 \ 5 \ 11 \ 11 \\ 5 \ 6 \ -5 \ -6 \\ \hline 11 \ 11 \ 6 \ 5 \end{array}$
$\begin{array}{r} 8 \ 4 \ 12 \ 12 \\ 4 \ 8 \ -4 \ -8 \\ \hline 12 \ 12 \ 8 \ 4 \end{array}$			

Unit 17: Telling time by the hour and half-hour

This topic may be introduced earlier than its position in this outline indicates. Some children may be able to tell time when they begin Grade II and many occasions will arise when a knowledge of how to tell time will be needed by the pupils. This is a topic which can be dealt with incidentally on many occasions throughout the year.

In teaching the telling of time by the hour and half-hour a real clock rather than blackboard drawings of clocks should be used. If no large clock is available in the classroom even an old alarm clock will serve the purpose. It is well to remember that people tell time from real clocks and watches rather than from drawings of clocks and watches.

Unit 18: The number facts of Group VII

$\begin{array}{r} 9 \ 3 \ 12 \ 12 \\ 3 \ 9 \ -3 \ -9 \\ \hline 12 \ 12 \ 9 \ 3 \end{array}$	$\begin{array}{r} 7 \ 4 \ 11 \ 11 \\ 4 \ 7 \ -4 \ -7 \\ \hline 11 \ 11 \ 7 \ 4 \end{array}$	$\begin{array}{r} 7 \ 5 \ 12 \ 12 \\ 5 \ 7 \ -5 \ -7 \\ \hline 12 \ 12 \ 7 \ 5 \end{array}$
$\begin{array}{r} 7 \ 14 \\ 7 \ -7 \\ \hline 14 \ 7 \end{array}$	$\begin{array}{r} 8 \ 16 \\ 8 \ -8 \\ \hline 16 \ 8 \end{array}$	

Unit 19: Addition by endings. To know the 135 higher decade facts, without bridging, sums not to exceed 39

The 135 higher decade addition facts (addition by endings) without bridging, whose sums do not exceed 39 are shown below in families:

$\begin{array}{r} 2 \ 12 \ 22 \ 32 \\ 2 \ 2 \ 2 \ 2 \\ \hline 4 \ 14 \ 24 \ 34 \end{array}$	$\begin{array}{r} 2 \ 12 \ 22 \ 32 \\ 1 \ 1 \ 1 \ 1 \\ \hline 3 \ 13 \ 23 \ 33 \end{array}$	$\begin{array}{r} 1 \ 11 \ 21 \ 31 \\ 2 \ 2 \ 2 \ 2 \\ \hline 3 \ 13 \ 23 \ 33 \end{array}$	$\begin{array}{r} 1 \ 11 \ 21 \ 31 \\ 1 \ 1 \ 1 \ 1 \\ \hline 2 \ 12 \ 22 \ 32 \end{array}$
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$\begin{array}{r} 3 \ 13 \ 23 \ 33 \\ 1 \ 1 \ 1 \ 1 \\ \hline 4 \ 14 \ 24 \ 34 \end{array}$	$\begin{array}{r} 1 \ 11 \ 21 \ 31 \\ 3 \ 3 \ 3 \ 3 \\ \hline 4 \ 14 \ 24 \ 34 \end{array}$	$\begin{array}{r} 4 \ 14 \ 24 \ 34 \\ 1 \ 1 \ 1 \ 1 \\ \hline 5 \ 15 \ 25 \ 35 \end{array}$	$\begin{array}{r} 1 \ 11 \ 21 \ 31 \\ 4 \ 4 \ 4 \ 4 \\ \hline 5 \ 15 \ 25 \ 35 \end{array}$
$\begin{array}{r} 3 \ 13 \ 23 \ 33 \\ 2 \ 2 \ 2 \ 2 \\ \hline 5 \ 15 \ 25 \ 35 \end{array}$	$\begin{array}{r} 2 \ 12 \ 22 \ 32 \\ 3 \ 3 \ 3 \ 3 \\ \hline 5 \ 15 \ 25 \ 35 \end{array}$	$\begin{array}{r} 3 \ 13 \ 23 \ 33 \\ 3 \ 23 \ 3 \ 3 \\ \hline 6 \ 16 \ 26 \ 36 \end{array}$	$\begin{array}{r} 5 \ 15 \ 25 \ 35 \\ 1 \ 1 \ 1 \ 1 \\ \hline 6 \ 16 \ 26 \ 36 \end{array}$
$\begin{array}{r} 1 \ 11 \ 21 \ 31 \\ 5 \ 5 \ 5 \ 5 \\ \hline 6 \ 16 \ 26 \ 36 \end{array}$	$\begin{array}{r} 4 \ 14 \ 24 \ 34 \\ 2 \ 2 \ 2 \ 2 \\ \hline 6 \ 16 \ 26 \ 36 \end{array}$	$\begin{array}{r} 2 \ 12 \ 22 \ 32 \\ 4 \ 4 \ 4 \ 4 \\ \hline 6 \ 16 \ 26 \ 36 \end{array}$	$\begin{array}{r} 4 \ 14 \ 24 \ 34 \\ 4 \ 4 \ 4 \ 4 \\ \hline 8 \ 18 \ 28 \ 38 \end{array}$
$\begin{array}{r} 6 \ 16 \ 26 \ 36 \\ 1 \ 1 \ 1 \ 1 \\ \hline 7 \ 17 \ 27 \ 37 \end{array}$	$\begin{array}{r} 1 \ 11 \ 21 \ 31 \\ 6 \ 6 \ 6 \ 6 \\ \hline 7 \ 17 \ 27 \ 37 \end{array}$	$\begin{array}{r} 4 \ 14 \ 24 \ 34 \\ 3 \ 3 \ 3 \ 3 \\ \hline 7 \ 17 \ 27 \ 37 \end{array}$	$\begin{array}{r} 3 \ 13 \ 23 \ 33 \\ 4 \ 4 \ 4 \ 4 \\ \hline 7 \ 17 \ 27 \ 37 \end{array}$
$\begin{array}{r} 7 \ 17 \ 27 \ 37 \\ 1 \ 1 \ 1 \ 1 \\ \hline 8 \ 18 \ 28 \ 38 \end{array}$	$\begin{array}{r} 1 \ 11 \ 21 \ 31 \\ 7 \ 7 \ 7 \ 7 \\ \hline 8 \ 18 \ 28 \ 38 \end{array}$	$\begin{array}{r} 5 \ 15 \ 25 \ 35 \\ 2 \ 2 \ 2 \ 2 \\ \hline 7 \ 17 \ 27 \ 37 \end{array}$	$\begin{array}{r} 2 \ 12 \ 22 \ 32 \\ 5 \ 5 \ 5 \ 5 \\ \hline 7 \ 17 \ 27 \ 37 \end{array}$
$\begin{array}{r} 8 \ 18 \ 28 \ 38 \\ 1 \ 1 \ 1 \ 1 \\ \hline 9 \ 19 \ 29 \ 39 \end{array}$	$\begin{array}{r} 1 \ 11 \ 21 \ 31 \\ 8 \ 8 \ 8 \ 8 \\ \hline 9 \ 19 \ 29 \ 39 \end{array}$	$\begin{array}{r} 6 \ 16 \ 26 \ 36 \\ 2 \ 2 \ 2 \ 2 \\ \hline 8 \ 18 \ 28 \ 38 \end{array}$	$\begin{array}{r} 2 \ 12 \ 22 \ 32 \\ 6 \ 6 \ 6 \ 6 \\ \hline 8 \ 18 \ 28 \ 38 \end{array}$
$\begin{array}{r} 5 \ 15 \ 25 \ 35 \\ 3 \ 3 \ 3 \ 3 \\ \hline 8 \ 18 \ 28 \ 38 \end{array}$	$\begin{array}{r} 3 \ 13 \ 23 \ 33 \\ 5 \ 5 \ 5 \ 5 \\ \hline 8 \ 18 \ 28 \ 38 \end{array}$	$\begin{array}{r} 7 \ 17 \ 27 \ 37 \\ 2 \ 2 \ 2 \ 2 \\ \hline 9 \ 19 \ 29 \ 39 \end{array}$	$\begin{array}{r} 2 \ 12 \ 22 \ 32 \\ 7 \ 7 \ 7 \ 7 \\ \hline 9 \ 19 \ 29 \ 39 \end{array}$
$\begin{array}{r} 6 \ 16 \ 26 \ 36 \\ 3 \ 3 \ 3 \ 3 \\ \hline 9 \ 19 \ 29 \ 39 \end{array}$	$\begin{array}{r} 3 \ 13 \ 23 \ 33 \\ 6 \ 6 \ 6 \ 6 \\ \hline 9 \ 19 \ 29 \ 39 \end{array}$	$\begin{array}{r} 5 \ 15 \ 25 \ 35 \\ 4 \ 4 \ 4 \ 4 \\ \hline 9 \ 19 \ 29 \ 39 \end{array}$	$\begin{array}{r} 4 \ 14 \ 24 \ 34 \\ 5 \ 5 \ 5 \ 5 \\ \hline 9 \ 19 \ 29 \ 39 \end{array}$
$\begin{array}{r} 0 \ 10 \ 20 \ 30 \\ 1 \ 1 \ 1 \ 1 \\ \hline 1 \ 11 \ 21 \ 31 \end{array}$	$\begin{array}{r} 0 \ 10 \ 20 \ 30 \\ 2 \ 2 \ 2 \ 2 \\ \hline 2 \ 12 \ 22 \ 32 \end{array}$	$\begin{array}{r} 0 \ 10 \ 20 \ 30 \\ 3 \ 3 \ 3 \ 3 \\ \hline 3 \ 13 \ 23 \ 33 \end{array}$	$\begin{array}{r} 0 \ 10 \ 20 \ 30 \\ 4 \ 4 \ 4 \ 4 \\ \hline 4 \ 14 \ 24 \ 34 \end{array}$

0	10	20	30
5	5	5	5
<hr/>			
5	15	25	35

0	10	20	30
6	6	6	6
<hr/>			
6	16	26	36

0	10	20	30
7	7	7	7
<hr/>			
7	17	27	37

0	10	20	30
8	8	8	8
<hr/>			
8	18	28	38

0	10	20	30
9	9	9	9
<hr/>			
9	19	29	39

The higher decade addition facts are not to be learned as separate facts but rather by families as shown in the blocks above. Let us look at the first

block in which the basic addition fact is $\begin{array}{r} 2 \\ - \\ 2 \end{array}$. Note that the ending of each

of the higher decade facts in the first block is the basic fact $\begin{array}{r} 2 \\ - \\ 4 \end{array}$ so that a

pupil knowing $\begin{array}{r} 2 \\ - \\ 4 \end{array}$ can say $\begin{array}{r} 22 \\ - \\ 24 \end{array}$, $\begin{array}{r} 32 \\ - \\ 34 \end{array}$ and $\begin{array}{r} 12 \\ - \\ 14 \end{array}$.

As a matter of fact it is better to leave the teens' higher decade fact

until the last because it breaks the rhythm $\begin{array}{r} 2 \\ - \\ 4 \end{array}$. The first block belongs to

the $\begin{array}{r} 2 \\ - \\ 4 \end{array}$ family. The other families are easily learned if the facts are known

well. Mastery of this unit depends entirely on a knowledge of the basic facts and a recognition of the families by endings.

The first family should be introduced with objective material starting

with $\begin{array}{r} 22 \\ - \\ 24 \end{array}$ and $\begin{array}{r} 32 \\ - \\ 34 \end{array}$.

The pupils should discover that the ending in each is the same as in

$\begin{array}{r} 2 \\ - \\ 4 \end{array}$. The pupils will then be able to give the answer to the higher decade

fact $\begin{array}{r} 12 \\ - \\ 14 \end{array}$.

The key to higher decade addition is to have pupils recognize the ending. If they know the ending they know the higher decade fact. Practice should be given not only in abstract form but with real problems.

Unit 20: The Roman Numerals to XII

Roman numerals still have some uses in our everyday affairs. They are used on some clock and watch faces, often to designate school grades, on individual volumes in sets of books, and to indicate dates on cornerstones of buildings.

Unit 21: Column addition with three and four addends to practice higher decade addition facts without bridging

(a) Practice sets with three addends in which the sum of the two top addends must exceed 9 and the sum of the three addends must not exceed 19. Following are examples that might be used:

8	7	3	7
3	7	9	5
6	3	4	6
<hr/>			
17	17	16	18

In constructing exercises for this unit the teacher must remember that the basic facts of Groups VIII and IX have not been taught and hence the only facts that can be used for the two top addends are the basic facts of Groups VI and VII. For teaching this unit the teacher is referred to Unit 21, Grade I.

(b) Practice sets with four addends in which the sum of the top two addends must exceed 9 and the sum of the four addends must not exceed 19. Examples of such exercises are:

6	8	7	2
5	8	4	9
4	1	3	2
2	2	4	3
<hr/>			
17	19	18	16

Use only basic facts of Group VI and VII for two top addends.

(c) Practice sets with four addends in which the sum of the top two addends does not exceed 9, the sum of the top three addends must exceed 9 and the sum of the four must not exceed 19. Examples of such exercises are:

5	2	2	4
3	7	5	4
3	2	5	8
7	5	7	1
<hr/>			
18	16	19	17

Note that the sum of the two top addends and the third addend is a basic fact of Group VI or VII.

Unit 22: The number facts of Group VIII

Teach as in Unit 4, Grade I the teaching units shown below:

$$\begin{array}{r} 8 \ 5 \ 13 \ 13 \\ 5 \ 8 \ -5 \ -8 \\ \hline 13 \ 13 \ 8 \ 5 \end{array}$$

$$\begin{array}{r} 9 \ 4 \ 13 \ 13 \\ 4 \ 9 \ -4 \ -9 \\ \hline 13 \ 13 \ 9 \ 4 \end{array}$$

$$\begin{array}{r} 8 \ 6 \ 14 \ 14 \\ 6 \ 8 \ -6 \ -8 \\ \hline 14 \ 14 \ 8 \ 6 \end{array}$$

$$\begin{array}{r} 9 \ 5 \ 14 \ 14 \\ 5 \ 9 \ -5 \ -9 \\ \hline 14 \ 14 \ 9 \ 5 \end{array}$$

$$\begin{array}{r} 7 \ 6 \ 13 \ 13 \\ 6 \ 7 \ -6 \ -7 \\ \hline 13 \ 13 \ 7 \ 6 \end{array}$$

Unit 23: Column addition with three and four addends as in Unit 21 using particularly the facts of Group VIII

See types of practice exercises in (a), (b) and (c) of Unit 21.

Real problems requiring the addition of three and four addends should be used.

Unit 24: The number facts of Group IX

Teach as in Unit 4, Grade I the teaching units shown below:

$$\begin{array}{r} 9 \ 18 \\ 9 \ -9 \\ \hline 18 \ 9 \end{array}$$

$$\begin{array}{r} 9 \ 6 \ 15 \ 15 \\ 6 \ 9 \ -6 \ -9 \\ \hline 15 \ 15 \ 9 \ 6 \end{array}$$

$$\begin{array}{r} 8 \ 7 \ 15 \ 15 \\ 7 \ 8 \ -7 \ -8 \\ \hline 15 \ 15 \ 8 \ 7 \end{array}$$

$$\begin{array}{r} 9 \ 7 \ 16 \ 16 \\ 7 \ 9 \ -7 \ -9 \\ \hline 16 \ 16 \ 9 \ 7 \end{array}$$

$$\begin{array}{r} 9 \ 8 \ 17 \ 17 \\ 8 \ 9 \ -8 \ -9 \\ \hline 17 \ 17 \ 9 \ 8 \end{array}$$

Unit 25: Column addition as in Units 21 and 23 using particularly the facts of Group IX

See types of practice exercises in (a), (b) and (c) of Unit 21.

Real problems requiring the addition of three and four addends should be used.

Unit 26: The meaning of one-fourth of a unit

Develop as in teaching the meaning of one-half of a unit using real problems and objective materials.

Unit 27: Adding two-figure numbers with two addends with carrying to the tens' place

This is a new and difficult skill for Grade II pupils. It must be understood that this skill is not to be taught in mechanical fashion at first. The proper procedure is to teach the skill so that the pupil will understand the reason for carrying before he is given exercises involving carrying.

Before beginning this unit the place values of two-figure numbers should be reviewed thoroughly.

The topic should be introduced through a real problem in which the child understands that to solve the problem he must learn a new skill. Since the meaning of 1's and 10's is most easily understood in our money system a concrete problem such as the following may be used: "John paid 35 cents for a ball and 27 cents for a toy car. How much did he spend?"

Write the 35 and the 27 on the board with the ones under the ones and the tens under the tens. Have 5 one-cent pieces and 3 ten-cent pieces to represent 35 cents. Place these on the desk or on a specially constructed card in which the coins may be inserted—the one-cent pieces to the right and the ten-cent pieces to the left. Have 7 one-cent pieces and 2 ten-cent pieces to represent 27 cents. Place these on the desk or card so that the ones are under the ones and the tens are under the tens. To find how many cents there are altogether we must put them together or "add."

Start on the right and add the ones: "5 ones and 7 ones are 12 ones." Have pupils suggest that 12 ones may be changed to 2 ones and 1 ten. Leave the 2 ones under the ones and carry the 1 ten up and add it to the tens, column saying "1 ten and 3 tens are 4 tens; 4 tens and 2 tens are 6 tens. Put the 6 tens (10-cent pieces) in the tens' column." So John spent 6 ten-cent pieces and 2 one-cent pieces or 62 cents.

Several problems of a like nature should be demonstrated objectively using coins. Lest the impression be given that carrying is used only in dealing with coins, additional problems should be used with other material—for instance using bundles of 10 sticks tied together to represent tens and additional loose sticks to represent ones. The ingenious teacher will think of other materials which may be used.

After the process of carrying is thoroughly understood by the pupils (and it may require several lessons to accomplish this) it should be pointed out that every time we wish to add numbers involving carrying the process would be too slow if we had to think of coins, bundles of sticks, books, etc. It should be suggested to them that we try to discover "a trick" which will allow us to get answers more quickly. If this work is properly developed and motivated some of the children, at least, will be able to suggest that to add two numbers involving carrying we start on the right and add the ones' column and put the right-hand figure of the sum of the ones' column under the ones and carry the left-hand figure of the sum to the tens' column and add it to the tens already there putting the sum of the tens' column under the tens.

For mastery of this skill much practice is required with exercises and in problem situations.

It is suggested that the "crutch" of placing the carry figure at the top of the tens' column be discouraged since this practice will be ultimately dropped and it is poor pedagogy to teach a child a skill which he will be required to unlearn later. The crutch is justified only in case of slower pupils who probably do not understand the reason for carrying and whose progress for the moment would be impeded if the crutch were not used.

Unit 28: Adding two-figure numbers with three addends, with and without zero with carrying to the tens' place

Develop as in Unit 27 using exercises and problems in which the figures in the ones' column correspond to exercises in Unit 21. If zero's are used pupils should be taught to skip the zero in adding.

Unit 29: Adding two-figure numbers with four addends, with and without zeros, with carrying to the tens' place

Develop as in Unit 27 using exercises and problems in which the figures in the ones' column correspond to exercises in Unit 21. If zeros are used pupils should be taught to skip the zero in adding.

Unit 30: Adding three-figure numbers with two and three addends, with and without zeros, with carrying to the tens' place only

Develop as in Unit 27 using exercises and problems with the figures in the ones' column corresponding to exercises in Unit 21 (a).

Unit 31: Building a vocabulary of quantitative words

A vocabulary of quantitative and comparative words has been developed since the beginning of the Primary Grade. See Unit 9, Primary Grade and Unit 8, Grade I. The vocabulary should be clarified and further developed throughout Grade II. Meaning should be emphasized rather than definitions. The arithmetical signs $+$ and $-$ may be said to form part of the required vocabulary.

GRADE III

Objectives

1. To know the meaning of, how to read and how to make the numbers from 1 to 10,000.
2. To know the common Canadian coins and bills; to know the sign for "cents" and the sign for "dollars".
3. To know the higher decade addition facts with and without bridging.
4. To be able to add two-figure numbers with carrying.
5. To be able to add three-figure numbers with carrying once or more than once.
6. To be able to add dollars and cents.
7. To be able to subtract two- and three-figure numbers with borrowing from the tens' place only, the hundreds' place only, and with double borrowing.
8. To be able to subtract dollars and cents.
9. To understand the meaning of multiplication.
10. To know the multiplication facts up to the "fives" and their reverses.
11. To know multiplication with zero.
12. To be able to multiply two- and three-figure numbers without carrying and with carrying once.
13. To be able to multiply dollars and cents.
14. To understand the meaning of division.
15. To know the division facts up to the "fives" and their reverses.
16. To be able to divide two- and three-figure numbers without carrying.
17. To know division with zero.
18. To be able to divide dollars and cents without carrying.
19. To know the easier common units of measurement: liquid measure, dry measure, length, and weight.
20. To be able to tell time and to read a calendar.
21. To acquire a further vocabulary of quantitative and comparative terms.
22. To know Roman numerals to XXXIX.
23. To know the meaning of fractions: one-half, one-quarter, one-third, one-fifth of a unit, of a group, or of a number.
24. To be able to do simple one-step problems.

Preliminary Statement

The first six weeks to two months of the Grade III year should be devoted to a systematic and thorough review of the arithmetic of Grades I and II. Teachers know that children who complete Grade II will have forgotten during the summer vacation a great deal of the arithmetic which they knew when school closed in June, hence the absolute necessity not of a cursory but of a thorough review of previous arithmetic skills. Furthermore, in many cases, reteaching of skills and systematic *drill* on the skills will be necessary before undertaking any new work for Grade III.

Any text prescribed for the work of Grade III should provide for the review work referred to above. However, the teacher should supplement the text with exercises and materials where it is felt students need more practice than is contained in the text. The text prescribed for Grade III at present provides the essential review work in its first seventy-five pages.

Time Allotment

About one hour a day should be devoted to number work in Grade III. This period should include the time the teacher works with the class and any seat-work that the child may be expected to do. The total time may be divided into two periods during the day if the teacher so desires.

Content

Unit 1: Review of the work of Grades I and II as in Grades I and II syllabus and as in the text

Unit 2: Reading and writing dollars and cents

Children should learn to read, write and to understand numbers representing dollars and cents. In Grade III such numbers should be limited to four-figure numbers. The \$ sign, called *the dollar sign* and the c. sign called *the cent sign* should be taught. Children should know that \$2.75 is read 2 dollars and 75 cents; that the figures after the point tell the number of cents. With a number such as \$3.00 they should know that the two 0's after the point tell that there are no cents. Pupils should know also that 75 cents may be written \$.75 or \$0.75 with the zero showing that there are no dollars.

Children should become familiar with the various denominations of coins and bills and know that there are 100 cents in a dollar.

Unit 3: Adding and subtracting dollars and cents

The addition of dollars and cents is best done through the medium of simple problems. Emphasize that the numbers be written with the decimal points under each other, the ones under the ones, the tens under the tens and the hundreds under the hundreds. A \$ sign goes at the top to the left and also to the left of the answer. The same procedures should be used in subtracting dollars and cents.

In adding cents where the sum is one dollar or more, emphasize that a decimal point is placed in the answer under the decimal points above it. Use the \$ sign at the top and before the answer. This gives the sum in dollars and cents.

$$\begin{array}{r} \$.22 \\ .31 \\ .43 \\ .20 \\ \hline \$1.16 \end{array}$$

Do not give addition problems involving dollars and cents beyond the kinds of column addition which the pupils have learned.

Unit 4: The higher decade addition facts without bridging with sums to 99

Higher decade addition consists of adding a one-figure number to a number of two or more figures—usually two figures.

When the sum of the higher decade fact is in the same decade as the two-figure number, it is a higher decade fact without bridging. The 135 such facts whose sums do not exceed 39 were taught in Unit 19, Grade II. The remaining groups of facts will be learned very rapidly once Unit 19 has been reviewed. They should continue to be taught by "families" with each "family" in Unit 19, Grade II extended as in the examples below. Emphasize that the ending of each higher decade fact is the same as the basic fact, and that the tens' figure in the sum is the same as the tens' figure in the two-figure addend.

2	12	22	32	42	52	62	72	82	92
2	2	2	2	2	2	2	2	2	2
—	—	—	—	—	—	—	—	—	—
4	14	24	34	44	54	64	74	84	94

4	14	24	34	44	54	64	74	84	94
3	3	3	3	3	3	3	3	3	3
—	—	—	—	—	—	—	—	—	—
7	17	27	37	47	57	67	77	87	97

3	13	23	33	43	53	63	73	83	93
5	5	5	5	5	5	5	5	5	5
—	—	—	—	—	—	—	—	—	—
8	18	28	38	48	58	68	78	88	98

Unit 5: The higher decade addition facts with bridging with sums less than 100

When the sum of the higher decade fact is in the next decade higher than the two-figure number, bridging is required:

$$\begin{array}{r} 17 \quad 34 \quad 66 \\ 8 \quad 9 \quad 5 \\ \hline 25 \quad 43 \quad 71 \end{array}$$

It might be said that we cross a bridge from one decade to the next higher decade.

The higher decade facts with bridging are much more difficult than those without bridging, hence they will require much more practice for mastery. Again the emphasis should be on recognizing that the ending of the higher decade fact is the same as the ending of the basic fact and that the tens' figure of the sum is one more than the tens' figure of the two-figure addend.

In presenting the facts which require bridging do not teach the process as one of carrying. In each decade there are forty-five facts which require bridging.

In the *tens'* decade, the table below shows to the right of the irregular line the forty-five facts which require bridging:

10	11	12	13	14	15	16	17	18	19
1	1	1	1	1	1	1	1	1	1
11	12	13	14	15	16	17	18	19	20
2	2	2	2	2	2	2	2	2	2
12	13	14	15	16	17	18	19	20	21
3	3	3	3	3	3	3	3	3	3
13	14	15	16	17	18	19	20	21	22
4	4	4	4	4	4	4	4	4	4
14	15	16	17	18	19	20	21	22	23
5	5	5	5	5	5	5	5	5	5
15	16	17	18	19	20	21	22	23	24
6	6	6	6	6	6	6	6	6	6
16	17	18	19	20	21	22	23	24	25
7	7	7	7	7	7	7	7	7	7
17	18	19	20	21	22	23	24	25	26
8	8	8	8	8	8	8	8	8	8
18	19	20	21	22	23	24	25	26	27
9	9	9	9	9	9	9	9	9	9
19	20	21	22	23	24	25	26	27	28

These may be taught in families, examples of which follow below:

4	14	24	34	44	54	64	74	84
9	9	9	9	9	9	9	9	9
13	23	33	43	53	63	73	83	93

8	18	28	38	48	58	68	78	88
4	4	4	4	4	4	4	4	4
12	22	32	42	52	62	72	82	92

5	15	25	35	45	55	65	75	85
7	7	7	7	7	7	7	7	7
12	22	32	42	52	62	72	82	92

18
4

Do not teach $\frac{18}{4}$ as "8 and 4 are 12, put 2 in the ones' column and carry 1; 1 and 1 are 2." As suggested above the higher decade facts should be taught as an extension of the basic fact, with emphasis on the ending and an increase of one in the tens' place of the sum.

Unit 6: The Roman numerals to XXXIX; how to tell time by the hour, half-hour and quarter-hour; how to read a calendar

In teaching Roman numerals and in telling time refer to Units 20 and 17 of the Grade II outline. Teaching how to read the calendar can best be done from day to day with a few minutes each morning devoted to reference to the day, the month, and the date.

Unit 7: Addition of two- and three-figure numbers with carrying to the tens' place only

The first lessons should be introduced with objective materials such as coins, sticks, etc., so that pupils may learn the *meaning* of carrying before acquiring the arithmetical skill of carrying. Refer to Units 27, 28, 29 and 30 of Grade II.

Unit 8: The addition of three-figure numbers with carrying to the hundreds' place only

Proceed as in Unit 7 above.

Unit 9: The meaning of four-figure numbers

For procedure refer to Units 2 and 8 of Grade II.

In developing the meaning of four-figure numbers emphasize the place values of the figures, the new place value being "thousands." It is not easy to find concrete or pictorial materials to demonstrate the thousands' place. It can be illustrated by using graph paper in which small squares represent one unit. The number 3472 is read "three thousand, four hundred, seventy-two." Do not use "and" in reading whole numbers.

Unit 10: Double carrying

Up to this point pupils have learned to carry to the tens' place only and to the hundreds' place only. In this unit these skills are combined in that there is carrying to both the tens' place and the hundreds' place. The skill to carry twice involves a wider attention span than carrying only once hence this unit must be taught with great care. Avoid using a crutch at first because it will have to be dropped later. For general procedure refer to Unit 27 of Grade II.

Unit 11: Adding two- and three-figure columns with carrying

Up to this point the carry number has always been 1. In this unit new skills are acquired as follows: (a) The carry number may be 1 or 2 or 3. (b) The columns may be uneven, in which case the blanks should be skipped. This skill should also be applied to column addition using dollars and cents.

Unit 12: Subtraction of two- and three-figure numbers with borrowing

- (a) Borrowing from the tens' place only.
- (b) Borrowing from the hundreds' place only.
- (c) Double borrowing.
- (d) Borrowing with zeros in the minuend.

The first lessons will involve the subtraction with borrowing of 2 two-figure numbers. In order that the meaning of borrowing may be understood this topic should be introduced through a real problem using objective materials such as coins, blocks, books, etc. At this point the meaning of borrowing is all-important. If pupils understand the meaning of borrowing, or what happens when they borrow, many of them will be able to discover the arithmetical rule for borrowing. Avoid the use of the "crutch" in teaching this topic.

Similar procedure should be followed in teaching borrowing from the hundreds' place only and in teaching double borrowing.

This skill should be applied to subtraction with borrowing involving dollars and cents.

Unit 13: The meaning of multiplication and the meaning of division

The meaning of multiplication and the meaning of division should be developed when the multiplication and division facts are being taught. These facts should be taught using concrete, pictorial, and semi-concrete material.

The pupils must see that multiplication is a short way of adding numbers whose addends are the same. For example "six 3's are 18" is a short way of adding $3+3+3+3+3+3=18$. This meaning may be enhanced

6
x3

by reading $\frac{6}{18}$ or $6 \times 3 = 18$, as "six three's are eighteen." This meaning

of the fact will be clearer than "six times three" or "six multiplied by three." These phrases may be used when we are sure the pupil understands the meaning of multiplication. Nothing is lost and much is gained by using, at all times, the first form suggested.

Although division is a short way of subtracting, its meaning is not as easy to demonstrate as is the case with the meaning of multiplication. Pupils should be taught that division is a way of finding how many times a number is contained in another number. For example, to find how many 3's in 21 we divide and find that 3 is contained in 21 seven times. The division

fact, $\frac{7}{3 \overline{)21}}$, should be read "3's in 21 are 7." Avoid at this time such phrases

as "3 divided into 21 gives 7", and "3 goes into 21 seven times." Reading the division fact "3's in 21 are 7", tends to emphasize the meaning of division.

Unit 14: The multiplication and division facts with 2

Multiplication facts with 2 and the corresponding division facts should be taught as a unit before proceeding to any other multiplication or division facts. This involves four sets of related facts:

(a) Multiplying by 2

1	2	3	4	5	6	7	8	9
x2	x2	x2	x2	x2	x2	x2	x2	x2
2	4	6	8	10	12	14	16	18

(b) Multiplying 2's which are the reverses of the facts under (a)

2	2	2	2	2	2	2	2	2
x1	x2	x3	x4	x5	x6	x7	x8	x9
2	4	6	8	10	12	14	16	18

(c) Dividing by 2

$\frac{1}{2 \overline{)2}}$	$\frac{2}{2 \overline{)4}}$	$\frac{3}{2 \overline{)6}}$	$\frac{4}{2 \overline{)8}}$	$\frac{5}{2 \overline{)10}}$	$\frac{6}{2 \overline{)12}}$	$\frac{7}{2 \overline{)14}}$	$\frac{8}{2 \overline{)16}}$	$\frac{9}{2 \overline{)18}}$
-----------------------------	-----------------------------	-----------------------------	-----------------------------	------------------------------	------------------------------	------------------------------	------------------------------	------------------------------

(d) Dividing when 2 is the quotient. These facts are the reverses of the facts under (c).

$\frac{2}{1/2}$	$\frac{2}{2/4}$	$\frac{2}{3/6}$	$\frac{2}{4/8}$	$\frac{2}{5/10}$	$\frac{2}{6/12}$	$\frac{2}{7/14}$	$\frac{2}{8/16}$	$\frac{2}{9/18}$
-----------------	-----------------	-----------------	-----------------	------------------	------------------	------------------	------------------	------------------

In the teaching of the multiplication and division facts by the use of concrete, pictorial and semi-concrete material the pupils should be required to discover the answers to the facts. After this is done the facts may be arranged in tables as shown above and practised for mastery. The close relationship between the direct and reverse multiplication facts and their relationship to the corresponding division facts should continuously be brought to the attention of pupils.

Unit 15: Multiplying and dividing two- and three-figure numbers without carrying using the facts of Unit 14

Unit 16: Finding one-half of a number

This unit ties in closely with Unit 14. The rule is developed: *to find one-half of a number, divide the number by 2.*

Unit 17: The meaning of dozen and half-dozen

Unit 18: The multiplication and division facts with 3

Proceed as in Unit 14.

Unit 19: Multiplication with 1

Develop two principles:

- (a) 1 multiplied by any number gives the number;
- (b) any number multiplied by 1 gives the number.

Unit 20: Multiplication with carrying

In multiplication with carrying, in Grade III, pupils are taught to carry only once either to the tens' place or to the hundreds' place. Carrying more than once is not taught until Grade IV.

Multiplication with carrying is similar to addition with carrying and it should be re-emphasized that multiplication is a short way of adding.

Unit 21: Multiplying dollars and cents

Proceed as in Unit 20.

The position of the decimal point in the multiplicand and product and the position of the dollar sign need to be emphasized.

Unit 22: Finding one-third of a number

Proceed as in Unit 16.

Unit 23: Division with 1

Develop the two principles:

- (a) any number divided by 1 gives the number;
- (b) any number divided by itself gives 1.

Unit 24: Dividing dollars and cents without carrying

The position of the decimal point in the dividend and quotient, and the position of the dollar sign need to be emphasized.

Unit 25: Measures of length

In this grade the measures of length taught are the inch, the foot and the yard. Use real measures to measure real things. Practice in estimating lengths and checking the estimates by exact measurement is an excellent way to have the measures become real and meaningful. Have pupils discover the relationships between the measures before they are memorized in tabular form.

Unit 26: The multiplication and division facts with 4

Proceed as in Unit 14.

Unit 27: Multiplication with zero

This topic should be introduced through a real problem such as:

"If Jane learns to spell 20 new words in a month, how many does she learn in 3 months?"

20
The correct answer may be found by addition, thus 20
20
—
60

Pupils should be reminded that a short way to find the answer in addition when the addends are the same is by multiplication, hence

20
x3
—
60

Remembering that the multiplicand 20 means 2 tens and "not any" ones the correct answer may be found by following the procedure:

- (1) Multiply the *ones*. 20
- (2) There are *not any ones* in 20. x3
-
- (3) Write zero in ones' column. 60
- (4) Multiply the tens.

A similar procedure with several problems should lead to the development of the general principle: *Zero multiplied by any number is zero.* In this grade zero is found only in the multiplicand.

Unit 28: Finding one-fourth of a number

Proceed as in Unit 16.

Unit 29: Division with zero

This topic should be introduced through a real problem such as:

"Tommy mowed the lawn 3 times and earned 60 cents altogether. How much did he earn each time he mowed the lawn?"

To find the answer pupils should suggest that we divide 60 by 3.

The answer may be found by the following procedure:

$$\begin{array}{r} 20 \\ 3 \overline{)60} \end{array}$$

(1) Start at the left and think: "3's in six 10's are two 10's". Write the 2 in the tens' column above the 6.

(2) The zero in 60 means "not any ones." Write a 0 above the 0 to show there are not any 3's in zero.

So Tommy earned 20 cents each time he mowed the lawn.

A similar procedure with several problems should lead to the development of the general principle:

Zero divided by any number is zero.

Unit 30: Liquid Measures

Liquid measures: cup, pint, quart, and gallon are taught in this grade. Use real measures and have relationships between the measures discovered by the pupils.

Unit 31: Dry Measures

Dry measures taught are pint, quart, peck, and bushel. Use real measures and where feasible have pupils discover the relationships between measures, as well as the relationship between the liquid pint and quart and the dry pint and quart. The dry measures, the peck and bushel, are not commonly used to measure such things as vegetables and grains. These are usually bought by weight.

Unit 32: Measures of weight

The pound and ounce are taught in this grade. An appreciation of the meaning of these measures cannot be had by sight or size. It must be felt. Hence pupils should be given many opportunities of "feeling" things that weigh 1 lb., 5 lbs., 3 ounces, etc.

Unit 33: Multiplication and division with 5

Proceed as in Unit 14.

Unit 34: Finding one-fifth of a number

Proceed as in Unit 16.

Unit 35: Telling time by minutes

Telling time by the hour, half-hour, and quarter-hour has already been taught. In this unit pupils are taught to tell time to the nearest minute. They should learn the number of minutes in an hour and hours in a day as well as the meaning of the abbreviations a.m. and p.m.

Unit 36: Building a vocabulary

From the Primary Grade on pupils should have acquired (a) a body of quantitative words such as many, big, long, narrow, heavy, etc., (b) a body of words used for comparisons such as smaller, heavier, slower, hardest, fewest, etc., (c) some technical arithmetical terms and signs; namely, add, subtract, plus, minus, difference, sum, and the signs + and —.

In Grade III throughout the year as topics are developed, quantitative words and words used for comparisons should be reviewed, further developed and added to. The new technical words and signs introduced in this grade are: multiply, multiplier, multiplicand, product, divide, divisor, dividend, quotient, the signs \times , \div , and $\overline{)}$. Do not teach definitions. Aim at an understanding of the words and signs, and their uses.

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Déclaration de situation pour les prestations familiales et les aides au logement

2

Situation familiale

Vous vivez en couple

- Vous vivez en couple sans être marié ni pacsé depuis le | | | | |
- Vous êtes marié depuis le | | | | |
- Vous êtes pacsé depuis le | | | | |
- Vous avez repris la vie commune depuis le | | | | |

Vous vivez seul(e)

- Vous êtes célibataire | | | | |
- Vous êtes divorcé depuis le | | | | |
- Vous êtes séparé légalement (séparation sans divorce avec intervention du juge) depuis le | | | | |
- Vous êtes séparé sans intervention du juge ou avez rompu votre pacs depuis le | | | | |
- Vous êtes veuf ou veuve depuis le | | | | |

Parents séparés

• Si l'autre parent d'un de vos enfants est dans une ou plusieurs des situations suivantes, cochez la(les) case(s) correspondante(s) :

- il réside dans un des Etats de l'UE, EEE* (hors France) ou en Suisse
- il travaille pour un employeur dont le siège est situé dans l'un de ces pays
- il perçoit une pension (retraite, invalidité), ou une indemnisation (maladie, chômage) de l'un de ces pays

• Une pension alimentaire est-elle fixée et/ou versée pour vos enfants par l'autre parent ? oui non

* Cf liste des pays en page 4

Enfants à votre charge et autres personnes de votre foyer

Si besoin, vous pouvez déclarer d'autres enfants ou personnes sur papier libre à joindre à ce formulaire.

Nom et prénoms <small>(dans l'ordre de l'état civil)</small>	Date et lieu ou pays de naissance	Date d'arrivée au foyer et lien de parenté <small>(Fille, fils, nièce, neveu, enfant recueilli, parent, aucun lien...)</small>	Situation actuelle <small>(Scolarité, apprentissage, activité professionnelle...)</small>	Enfant résident à l'étranger	Enfant en résidence alternée**
1.	<input type="checkbox"/>	<input type="checkbox"/>
2.	<input type="checkbox"/>	<input type="checkbox"/>
3.	<input type="checkbox"/>	<input type="checkbox"/>
4.	<input type="checkbox"/>	<input type="checkbox"/>
5.	<input type="checkbox"/>	<input type="checkbox"/>

* Si l'enfant est salarié ou apprenti, précisez son dernier salaire net mensuel.

** En cas de séparation et de résidence alternée d'un ou plusieurs de vos enfants entre votre domicile et celui de l'autre parent, vous pouvez demander le partage des allocations familiales. Téléchargez le formulaire «Enfant(s) en résidence alternée - Déclaration et choix des parents» sur caf.fr ou msa.fr ou demandez-le à votre Caf ou MSA.

Si vous êtes âgé(e) de moins de 25 ans ou si votre conjoint(e), concubin(e) ou pacsé(e) est âgé(e) de moins de 25 ans

Si vous demandez des prestations, vos parents et/ou ceux de votre conjoint ne pourront plus bénéficier des prestations qu'ils perçoivent pour vous en tant qu'enfant à leur charge.

<p>L'un de vos parents perçoit-ils pour vous des prestations familiales, Apl, Rsa... ?</p> <p>Si oui, Nom du parent allocataire :</p> <p>Prénoms :</p> <p>Adresse :</p> <p>Code postal et commune :</p> <p>Organisme de prestations familiales :</p> <p>Numéro d'allocataire (s'il en possède un) :</p> <p>Numéro de sécurité sociale :</p>	<p>■ Parents de l'allocataire</p> <p><input type="checkbox"/> oui <input checked="" type="checkbox"/> non</p>	<p>■ Parents du conjoint, concubin ou pacsé</p> <p><input type="checkbox"/> oui <input checked="" type="checkbox"/> non</p>
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Emplacement réservé à la Caf

Date demande : 14/09/2016 FLORA GUITTON
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Mathematics Primary

**Implementation Draft
May 2013**

Website References

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Mathematics Primary, Implementation Draft

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Introduction

Background and Rationale

Mathematics curriculum is shaped by a vision that fosters the development of mathematically literate students who can extend and apply their learning and who are effective participants in society. It is essential that the mathematics curriculum reflect current research in mathematics instruction. To achieve this goal, Western and Northern Canadian Protocol's (WNCP) *The Common Curriculum Framework for K–9 Mathematics* (2006) has been adopted as the basis for the new mathematics curriculum in Nova Scotia.

The Common Curriculum Framework was developed by the seven ministries of education (Alberta, British Columbia, Manitoba, Northwest Territories, Nunavut, Saskatchewan, and Yukon Territory) in collaboration with teachers, administrators, parents, business representatives, post-secondary educators, and others. The framework identifies beliefs about mathematics, general and specific student outcomes, and performance indicators agreed upon by the seven jurisdictions. The outcomes and performance indicators have been adapted for Nova Scotia. This document is based on both national and international research by the WNCP and the National Council of Teachers of Mathematics (NCTM).

There is an emphasis in the Nova Scotia curriculum on particular key concepts at each grade that will result in greater depth of understanding and, ultimately, stronger student achievement. There is also a greater emphasis on number sense and operations concepts in the early grades to ensure students develop a solid foundation in numeracy.

Purpose

This document provides sets of outcomes and performance indicators to be used as a mandated common base for defining mathematics curriculum expectations. This common base should result in consistent student outcomes in mathematics within the province of Nova Scotia. It should also enable easier transfer for students moving within the province or from any jurisdiction that has adopted the WNCP framework. This document is intended to clearly communicate to all education partners across the province the high expectations for students' mathematical learning.

Program Design and Components

Assessment

Ongoing assessment for learning is essential to effective teaching and learning. Research has shown that assessment for learning (formative assessment) practices produce significant and often substantial learning gains, close achievement gaps, and build students' ability to learn new skills (Black & Wiliam 1998; OECD 2006). Student involvement in assessment promotes learning. Timely and effective teacher feedback and student self-assessment allow students to reflect on and articulate their understanding of mathematical concepts and ideas.

Assessment in the classroom includes

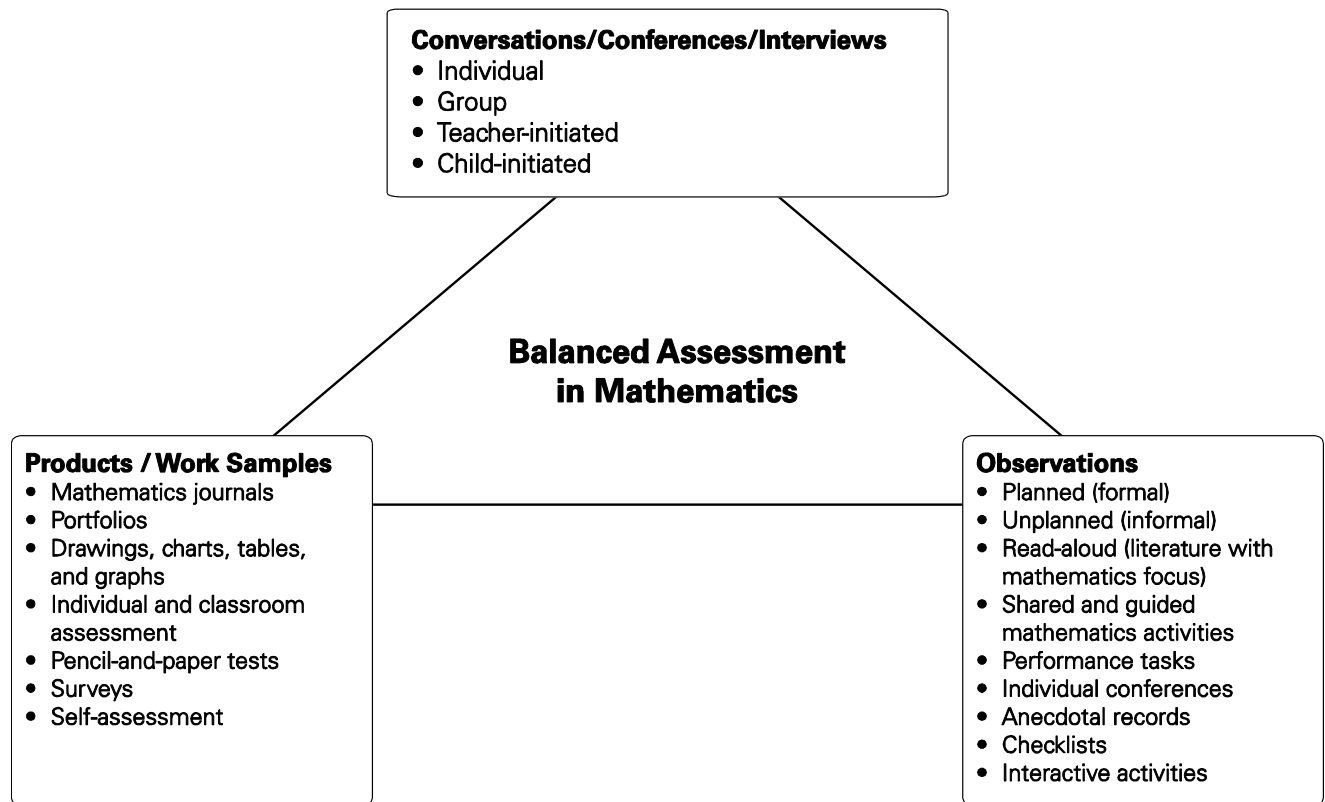
- providing clear goals, targets, and learning outcomes
- using exemplars, rubrics, and models to help clarify outcomes and identify important features of the work
- monitoring progress towards outcomes and providing feedback as necessary
- encouraging self-assessment
- fostering a classroom environment where conversations about learning take place, where students can check their thinking and performance and develop a deeper understanding of their learning

(Davies 2000)

Assessment for learning practices act as the scaffolding for learning, which only then can be measured through assessment of learning (summative assessment). Assessment of learning tracks student progress, informs instructional programming, and aids in decision making. Both forms of assessment are necessary to guide teaching, stimulate learning, and produce achievement gains.

Assessment of student learning should

- align with curriculum outcomes
- clearly define criteria for success
- make explicit the expectations for students' performance
- use a wide variety of assessment strategies and tools
- yield useful information to inform instruction



Time to Learn for Mathematics

The Time to Learn Strategy *Guidelines for Instructional Time: Grades Primary–6* includes time for mathematics instruction in the “Required Each Day” section. In order to support a constructivist approach to teaching through problem solving, it is highly recommended that the 45 minutes required daily in grades primary–2 and the 60 minutes required daily for grades 3–6 mathematics instruction be provided in an uninterrupted block of time.

Time to Learn guidelines can be found at

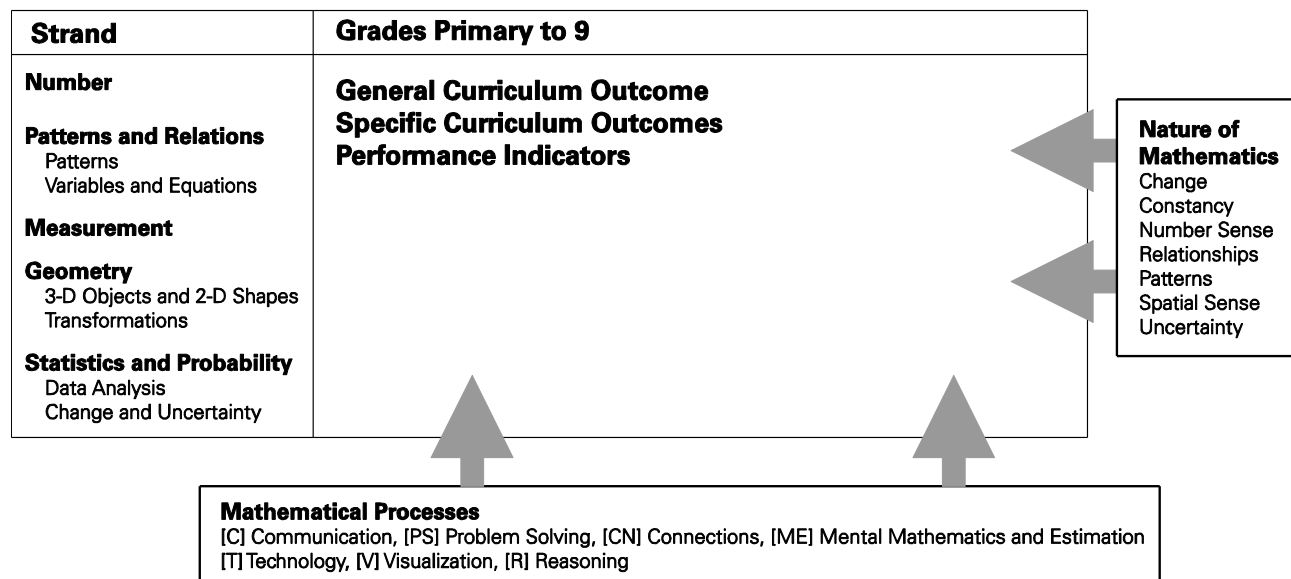
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Outcomes

Conceptual Framework for Mathematics Primary–9

The chart below provides an overview of how mathematical processes and the nature of mathematics influence learning outcomes.



(Adapted with permission from Western and Northern Canadian Protocol, *The Common Curriculum Framework for K–9 Mathematics*, p. 5. All rights reserved.)

Structure of the Mathematics Curriculum

Strands

The learning outcomes in the Nova Scotia Framework are organized into five strands across grades primary to 9.

- Number (N)
- Patterns and Relations (PR)
- Measurement (M)
- Geometry (G)
- Statistics and Probability (SP)

General Curriculum Outcomes (GCO)

Some strands are further subdivided into sub-strands. There is one general outcome (GCO) per sub-strand. GCOs are overarching statements about what students are expected to learn in each strand/sub-strand. The general curriculum outcome for each strand/sub-strand is the same throughout the grades.

NUMBER (N)

GCO: Students will be expected to demonstrate number sense.

PATTERNS AND RELATIONS (PR)

Patterns

GCO: Students will be expected to use patterns to describe the world and solve problems.

Variables and Equations

GCO: Students will be expected to represent algebraic expressions in multiple ways.

MEASUREMENT (M)

GCO: Students will be expected to use direct and indirect measure to solve problems.

GEOMETRY (G)

3-D Objects and 2-D Shapes

GCO: Students will be expected to describe the characteristics of 3-D objects and 2-D shapes and analyze the relationships among them.

Transformations

GCO: Students will be expected to describe and analyze position and motion of objects and shapes.

STATISTICS AND PROBABILITY (SP)

Data Analysis

GCO: Students will be expected to collect, display, and analyze data to solve problems.

Chance and Uncertainty

GCO: Students will be expected to use experimental or theoretical probabilities to represent and solve problems involving uncertainty.

Specific Curriculum Outcomes (SCOs) and Performance Indicators

Specific curriculum outcomes (SCOs) are statements that identify the specific conceptual understanding, related skills, and knowledge students are expected to attain by the end of a given grade.

Performance indicators are statements that identify specific expectations of the depth, breadth, and expectations for the outcome. Teachers use these statements to determine whether students have achieved the corresponding specific curriculum outcome.

Process Standards Key

[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

NUMBER (N)

- N01** Students will be expected to say the number sequence by
- 1s, from 1 to 20
 - 1s, starting anywhere from 1 to 10 and from 10 to 1 [C, CN, V]

Performance Indicators

- N01.01 Recite the number sequence from 1 to 20 and from 10 to 1.
 N01.02 Name the number that comes after a given number, 1 to 9.
 N01.03 Name the number that comes before a given number, 2 to 10.
 N01.04 Recite number names from a given number to a stated number (forward 1 to 10, backward 10 to 1) using visual aids.

- N02** Students will be expected to recognize, at a glance, and name the quantity represented by familiar arrangements of 1 to 5 objects or dots. [C, CN, ME, V]

Performance Indicators

- N02.01 Look briefly at a given familiar arrangement of 1 to 5 objects or dots and identify the number represented without counting.
 N02.02 Identify the number represented by a given dot arrangement on a five-frame.

- N03** Students will be expected to relate a numeral, 1 to 10, to its respective quantity. [CN, R, V]

Performance Indicators

- N03.01 Name the number for a given set of objects.
 N03.02 Match numerals with their given pictorial representations.
 N03.03 Hold up the appropriate number of fingers for a given numeral.
 N03.04 Construct a set of objects corresponding to a given numeral.
 N03.05 Record the numeral that represents the quantity of a given set of objects.

N04 Students will be expected to represent and describe numbers 2 to 10 in two parts, concretely and pictorially. [C, CN, ME, R, V]

Performance Indicators

N04.01 Show a given number as two parts (using fingers, counters, or other objects) and name the number of objects in each part.

N04.02 Show a given number as two parts, using pictures, and name the number of objects in each part.

N05 Students will be expected to compare quantities, 1 to 10, using one-to-one correspondence. [C, CN, V]

Performance Indicators

N05.01 Construct a set to show more than, fewer than, or as many as a given set.

N05.02 Compare two given sets through direct comparison and describe the sets using words, such as more, fewer, as many as, or the same number as.

N06 Students will be expected to demonstrate an understanding of counting to 10. [C, CN, ME, PS, R, V]

Performance Indicators

N06.01 Answer the question, “How many are in the set?” using the last number counted in a set.

N06.02 In a fixed arrangement, starting in different locations, show that the count of the number of objects in a set does not change.

N06.03 Count the number of objects in a given set, rearrange the objects, predict the new count, and recount to verify the prediction.

PATTERNS AND RELATIONS (PR)

PR01 Students will be expected to demonstrate an understanding of repeating patterns (two or three elements) by identifying, reproducing, extending, and creating patterns using manipulatives, sounds, and actions. [C, CN, PS, V]

Performance Indicators

PR01.01 Distinguish between repeating patterns and non-repeating sequences in a given set by identifying the part that repeats.

PR01.02 Copy a given repeating pattern and describe the pattern.

PR01.03 Extend a variety of given repeating patterns to two more repetitions.

PR01.04 Create a repeating pattern using manipulatives, musical instruments, or actions and describe the pattern.

PR01.05 Identify and describe a repeating pattern in the classroom, the school, and outdoors.

MEASUREMENT (M)

M01 Students will be expected to use direct comparison to compare two objects based on a single attribute, such as length, mass, volume, and capacity. [C, CN, PS, R, V]

Performance Indicators

M01.01 Compare the length of two given objects and explain the comparison using words such as **shorter, longer, taller, or almost the same.**

M01.02 Compare the mass of two given objects and explain the comparison using words such as **lighter, heavier, or almost the same.**

M01.03 Compare the capacity of two given objects and explain the comparison using words such as **holds less, holds more, or holds almost the same.**

M01.04 Compare the volume of two given objects and explain the comparison using words such as **bigger, smaller, or almost the same.**

GEOMETRY (G)

G01 Students will be expected to sort 3-D objects using a single attribute. [C, CN, PS, R, V]

Performance Indicators

G01.01 Sort a given set of familiar 3-D objects using a single attribute, such as size or shape, and explain the sorting rule.

G01.02 Explain the sorting rule used to sort a pre-sorted set.

G02 Students will be expected to build and describe 3-D objects. [CN, PS, V]

Performance Indicators

G02.01 Create a representation of a given 3-D object using building blocks and compare the representation to the original 3-D object.

G02.01 Describe a given 3-D object using words such as big, little, round, like a box, or like a can.

Mathematical Processes

There are critical components that students must encounter in a mathematics program in order to achieve the goals of mathematics education and encourage lifelong learning in mathematics.

Students are expected to

- communicate in order to learn and express their understanding of mathematics (Communication [C])
- develop and apply new mathematical knowledge through problem solving (Problem Solving [PS])
- connect mathematical ideas to other concepts in mathematics, to everyday experiences, and to other disciplines (Connections [CN])
- demonstrate fluency with mental mathematics and estimation (Mental Mathematics and Estimation [ME])
- select and use technologies as tools for learning and solving problems (Technology [T])

- develop visualization skills to assist in processing information, making connections, and solving problems (Visualization [V])
- develop mathematical reasoning (Reasoning [R])

The Nova Scotia curriculum incorporates these seven interrelated mathematical processes that are intended to permeate teaching and learning. The key to these process standards is presented in a box, as shown below, with each specific curriculum outcome within the strands.

Process Standards Key

[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

Communication [C]

Students need opportunities to read about, represent, view, write about, listen to, and discuss mathematical ideas. These opportunities allow students to create links between their own language and ideas, and the formal language and symbols of mathematics. Communication is important in clarifying, reinforcing, and modifying ideas, knowledge, attitudes, and beliefs about mathematics. Students should be encouraged to use a variety of forms of communication while learning mathematics.

Students also need to communicate their learning using mathematical terminology. Communication can help students make connections between and among the different representational modes—contextual, concrete, pictorial, linguistic/verbal, and symbolic—of mathematical ideas. Students must communicate *daily* about their mathematics learning. This enables them to reflect, to validate, and to clarify their thinking and provides teachers with insight into students’ interpretations of mathematical meanings and ideas.

Problem Solving [PS]

Learning through problem solving should be the focus of mathematics at all grade levels. When students encounter new situations and respond to questions of the type, How would you ...? or How could you ...? the problem-solving approach is being modelled. Students develop their own problem-solving strategies by being open to listening, discussing, and trying different strategies.

In order for an activity to be problem-solving based, it must ask students to determine a way to get from what is known to what is sought. If students have already been given ways to solve the problem, it is not a problem, but practice. A true problem requires students to use prior learning in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement, perseverance, and collaboration.

Problem solving is also a powerful teaching tool that fosters multiple, creative, and innovative solutions. Creating an environment where students openly look for and engage in finding a variety of strategies for solving problems empowers students to explore alternatives and develops confident, cognitive, mathematical risk takers.

When students are exposed to a wide variety of problems in all areas of mathematics, they explore various methods for solving and verifying problems. In addition, they are challenged to find multiple solutions for problems and to create their own problem.

Connections [CN]

Contextualization and making connections to the experiences of learners are powerful processes in developing mathematical understanding. When mathematical ideas are connected to one another or to real-world phenomena, students can begin to view mathematics as useful, relevant, and integrated. Learning mathematics within contexts and making connections relevant to learners can validate past experiences and increase student willingness to participate and be actively engaged. The brain is constantly looking for and making connections.

“Because the learner is constantly searching for connections on many levels, educators need to *orchestrate the experiences* from which learners extract understanding. ... *Brain research establishes and confirms that multiple complex and concrete experiences are essential for meaningful learning and teaching.*” (Caine and Caine 1991, p. 5).

Mathematics should be viewed as an integrated whole rather than as the study of separate strands or units. Connections must also be made between and among the different representational modes—contextual, concrete, pictorial, linguistic/verbal, and symbolic. The process of making connections, in turn, facilitates learning. Concepts and skills should also be connected to everyday situations and other curricular areas. For example, when developing literacy skills students learn to make text-to-world, text-to-text, and text-to-self connections. Students can also make connections to make mathematics come alive through math-to-world, math-to-math, and math-to-self connections.

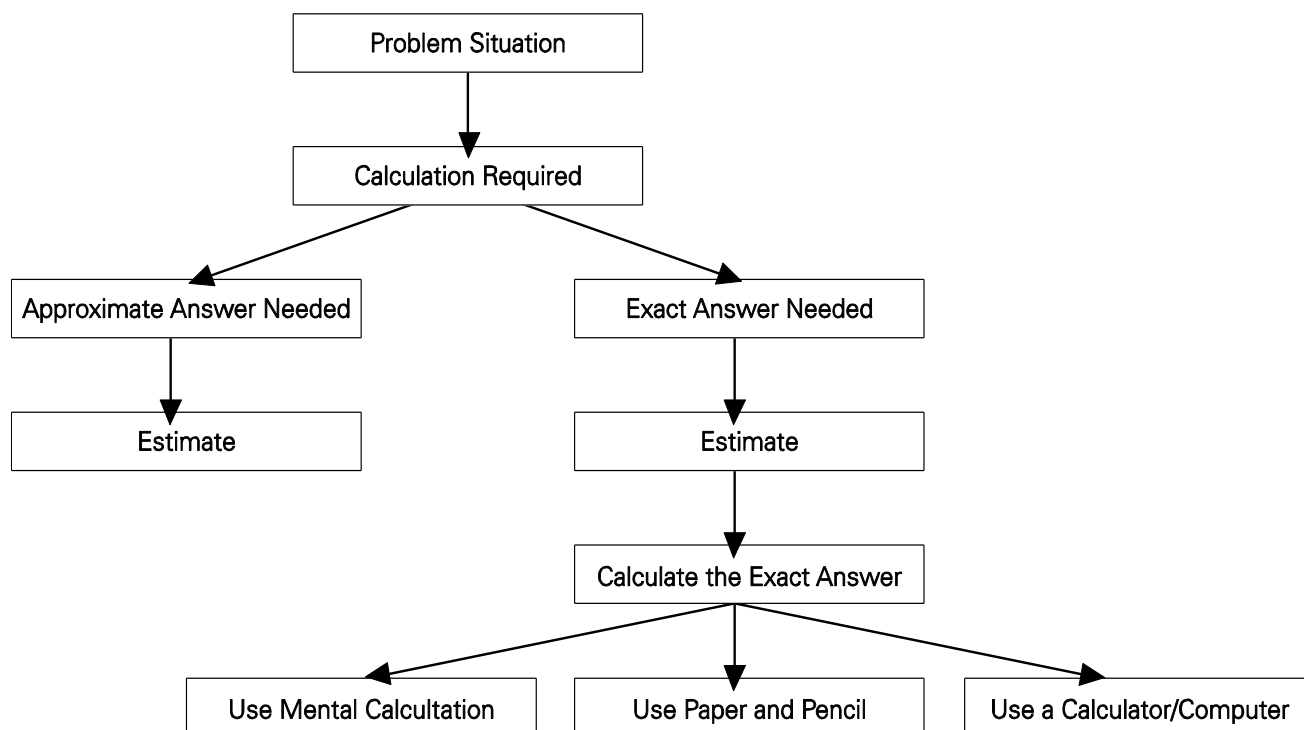
Mental Mathematics and Estimation [ME]

Mental mathematics is a combination of cognitive strategies that enhance flexible thinking and number sense. It is calculating mentally without the use of external aids. Mental mathematics enables students to determine answers without paper and pencil. It improves computational fluency by developing efficiency, accuracy, and flexibility. “Even more important than performing computational procedures or using calculators is the greater facility that students need—more than ever before—with estimation and mental math.” (National Council of Teachers of Mathematics, May 2005).

Students proficient with mental mathematics “become liberated from calculator dependence, build confidence in doing mathematics, become more flexible thinkers, and are more able to use multiple approaches to problem solving.” (Rubenstein 2001) Mental mathematics “provides a cornerstone for all estimation processes, offering a variety of alternative algorithms and nonstandard techniques for finding answers.” (Hope 1988, v)

Estimation is a strategy for determining approximate values or quantities, usually by referring to benchmarks or using referents, or for determining the reasonableness of calculated values. Students need to know how, when, and what strategy to use when estimating. Estimation is used to make mathematical judgments and develop useful, efficient strategies for dealing with situations in daily life.

Students need to develop both mental mathematics and estimation skills through context and not in isolation so they are able to apply them to solve problems. Whenever a problem requires a calculation, students should follow the decision-making process as illustrated below.



The skill of estimation requires a sound knowledge of mental mathematics. Both are necessary to many everyday experiences, and students should be provided with frequent opportunities to practise these skills.

Technology [T]

Technology can be effectively used to contribute to and support the learning of a wide range of mathematical outcomes and enables students to explore and create patterns, examine relationships, test conjectures, and solve problems.

Technology can be used to

- explore and demonstrate mathematical relationships and patterns
- organize and display data
- extrapolate and interpolate
- assist with calculation procedures as part of solving problems
- decrease the time spent on computations when other mathematical learning is the focus
- reinforce the learning of basic facts and test properties
- develop personal procedures for mathematical operations
- create geometric displays
- simulate situations
- develop number sense

The use of calculators is recommended to enhance problem solving, to encourage discovery of number patterns, and to reinforce conceptual development and numerical relationships. They do not, however, replace the development of number concepts and skills. Carefully chosen computer software can provide interesting problem-solving situations and applications.

Technology contributes to a learning environment in which the growing curiosity of students can lead to rich mathematical discoveries at all grade levels. While technology can be used in grades primary to 3 to enrich learning, it is expected that students will achieve all outcomes without the use of technology.

Visualization [V]

Visualization “involves thinking in pictures and images, and the ability to perceive, transform and recreate different aspects of the visual-spatial world.” (Armstrong 1999). The use of visualization in the study of mathematics provides students with opportunities to understand mathematical concepts and make connections among them. Visual images and visual reasoning are important components of number, spatial, and measurement sense. Number visualization occurs when students create mental representations of numbers. These mental images are needed to develop concepts and understand procedures. Images and explanations help students clarify their understanding of mathematical ideas in all strands.

Being able to create, interpret, and describe a visual representation is part of spatial sense and spatial reasoning. Spatial visualization and reasoning enable students to describe the relationships among and between 3-D objects and 2-D shapes.

Measurement visualization goes beyond the acquisition of specific measurement skills. Measurement sense includes the ability to determine when to measure, when to estimate and to know several estimation strategies. (Shaw and Cliatt 1989)

Visualization is fostered through the use of concrete materials, technology, and a variety of visual representations.

Reasoning [R]

Mathematical reasoning helps students think logically and make sense of mathematics. Students need to develop confidence in their abilities to reason and justify their mathematical thinking. High-order questions challenge students to think and develop a sense of wonder about mathematics. Mathematical experiences in and out of the classroom provide opportunities for inductive and deductive reasoning. Inductive reasoning occurs when students explore and record results, analyze observations, make generalizations from patterns, and test these generalizations. Deductive reasoning occurs when students reach new conclusions based upon what is already known or assumed to be true.

Mathematics reasoning involves informal thinking, conjecturing, and validating—these help students understand that mathematics makes sense. Students are encouraged to justify, in a variety of ways, their solutions, thinking processes, and hypotheses. In fact, good reasoning is as important as finding correct answers.

Nature of Mathematics

Mathematics is one way of trying to understand, interpret, and describe our world. There are a number of components that define the nature of mathematics, and these are woven throughout this document. These components include change, constancy, number sense, relationships, patterns, spatial sense, and uncertainty.

Change

It is important for students to understand that mathematics is dynamic and not static. As a result, recognizing change is a key component in understanding and developing mathematics. Within mathematics, students encounter conditions of change and are required to search for explanations of that change. To make predictions, students need to describe and quantify their observations, look for patterns, and describe those quantities that remain fixed and those that change. For example, the sequence 4, 6, 8, 10, 12, ... can be described as

- skip counting by 2s, starting from 4
- an arithmetic sequence, with first term 4 and a common difference of 2
- a linear function with a discrete domain

(Steen 1990, 184).

Constancy

Different aspects of constancy are described by the terms **stability**, **conservation**, **equilibrium**, **steady state**, and **symmetry** (AAAS–Benchmarks 1993, 270). Many important properties in mathematics and science relate to properties that do not change when outside conditions change. Some problems in mathematics require students to focus on properties that remain constant. The recognition of constancy enables students to solve problems. Examples of constancy include the following:

- The area of a rectangular region is the same regardless of the methods used to determine the solution.
- The sum of the interior angles of any triangle is 180° .
- The theoretical probability of flipping a coin and getting heads is 0.5.

Number Sense

Number sense, which can be thought of as intuition about numbers, is the most important foundation of numeracy (British Columbia Ministry of Education 2000, 146). A true sense of number goes well beyond the skills of simply counting, memorizing facts, and the situational rote use of algorithms. Number sense develops when students connect numbers to real-life experiences and use benchmarks and referents. This results in students who are computationally fluent, flexible with numbers, and have intuition about numbers. The evolving number sense typically comes as a by-product of learning rather than through direct instruction. However, number sense can be developed by providing rich mathematical tasks that allow students to make connections.

Relationships

Mathematics is used to describe and explain relationships. As part of the study of mathematics, students look for relationships among numbers, sets, shapes, objects, and concepts. The search for possible relationships involves the collection and analysis of data, and describing relationships visually, symbolically, orally, or in written form.

Patterns

Mathematics is about recognizing, describing, and working with numerical and non-numerical patterns. Patterns exist in all strands, and it is important that connections are made among strands. Working with patterns enables students to make connections within and beyond mathematics. These skills contribute to students' interaction with an understanding of their environment. Patterns may be represented in concrete, visual, or symbolic form. Students should develop fluency in moving from one representation to another. Students must learn to recognize, extend, create, and use mathematical patterns. Patterns allow students to make predictions and justify their reasoning when solving problems. Learning to work with patterns in the early grades helps develop students' algebraic thinking, which is foundational for working with more abstract mathematics in higher grades.

Spatial Sense

Spatial sense involves visualization, mental imagery, and spatial reasoning. These skills are central to the understanding of mathematics. Spatial sense enables students to reason and interpret among and between 3-D and 2-D representations and identify relationships to mathematical strands. Spatial sense is developed through a variety of experiences and interactions within the environment. The development of spatial sense enables students to solve problems involving 3-D objects and 2-D shapes. Spatial sense offers a way to interpret and reflect on the physical environment and its 3-D or 2-D representations. Some problems involve attaching numerals and appropriate units (measurement) to dimensions of objects. Spatial sense allows students to make predictions about the results of changing these dimensions. For example,

- knowing the dimensions of an object enables students to communicate about the object and create representations
- the volume of a rectangular solid can be calculated from given dimensions
- doubling the length of the side of a square increases the area by a factor of four

Uncertainty

In mathematics, interpretations of data and the predictions made from data may lack certainty. Events and experiments generate statistical data that can be used to make predictions. It is important to recognize that these predictions (interpolations and extrapolations) are based upon patterns that have a degree of uncertainty. The quality of the interpretation is directly related to the quality of the data. An awareness of uncertainty allows students to assess the reliability of data and data interpretation. Chance addresses the predictability of the occurrence of an outcome. As students develop their understanding of probability, the language of mathematics becomes more specific and describes the degree of uncertainty more accurately.

Curriculum Document Format

This guide presents the mathematics curriculum so that a teacher may readily view the scope of the outcomes that students are expected to achieve during that year. Teachers are encouraged, however, to examine what comes before and what follows after, to better understand how students' learning at a particular grade level is part of a bigger picture of concept and skill development.

The order of presentation in no way assumes or prescribes a preferred order of presentation in the classroom, but simply lays out the specific curriculum outcomes in relation to the overarching general curriculum outcomes (GCOs).

The footer of the document shows the name of the course, and the strand name is presented in the header. When a specific curriculum outcome (SCO) is introduced, it is followed by the mathematical processes and performance indicators for that outcome. A scope and sequence is then provided, which relates the SCO to previous and next grade SCOs. Also for each SCO, there is background information, assessment strategies, suggested instructional strategies, suggested models and manipulatives, mathematical language, and a section for resources and notes. For each section, the guiding questions should be used to help with unit and lesson preparation.

SCO**Mathematical Processes**

[C] Communication [PS] Problem Solving [CN] Connections
 [ME] Mental Mathematics and Estimation
 [T] Technology [V] Visualization [R] Reasoning

Performance Indicators

Describes observable indicators of whether students have achieved the specific outcome.

Scope and Sequence

Previous grade or course SCOs	Current grade SCO	Following grade or course SCOs
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Background

Describes the “big ideas” to be learned and how they relate to work in previous grade and work in subsequent courses.

Additional Information

A reference to Appendix A, which contains further elaborations for the performance indicators.

Assessment, Teaching, and Learning**Assessment Strategies****Guiding Questions**

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Sample tasks that can be used to determine students' prior knowledge.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Some suggestions for specific activities and questions that can be used for both instruction and assessment

FOLLOW-UP ON ASSESSMENT**Guiding Questions**

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Correlations to related resources.

Planning for Instruction**Guiding Questions**

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcome and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Suggested strategies for planning daily lessons.

SUGGESTED LEARNING TASKS

Suggestions for general approaches and strategies suggested for teaching this outcome.

Guiding Questions

- How can the scope and sequence be used to determine what prior knowledge needs to be activated prior to beginning new instruction?

SUGGESTED MODELS AND MANIPULATIVES**MATHEMATICAL LANGUAGE**

Teacher and student mathematical language associated with the respective outcome.

Resources/Notes

Contexts for Learning and Teaching

Beliefs about Students and Mathematics Learning

“Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge.” (National Council of Teachers of Mathematics 2000, 20).

The Nova Scotia mathematics curriculum is based upon several key assumptions or beliefs about mathematics learning that have grown out of research and practice. These beliefs include the following:

- Mathematics learning is an active and constructive process.
- Learning is most effective when standards of expectation are made clear with ongoing assessment and feedback.
- Learners are individuals who bring a wide range of prior knowledge and experiences and who learn via various styles and at different rates.
- Learning is most likely to occur when placed in meaningful contexts and in an environment that supports exploration, risk taking, and critical thinking and that nurtures positive attitudes and sustained effort.

Students are curious, active learners with individual interests, abilities, and needs. They come to classrooms with varying knowledge, life experiences, and backgrounds. A key component in successfully developing numeracy is making connections to these backgrounds and experiences.

Students develop a variety of mathematical ideas before they enter school. Children make sense of their environment through observations and interactions at home and in the community. Mathematics learning is embedded in everyday activities, such as playing, reading, storytelling, and helping around the home. Such activities can contribute to the development of number and spatial sense in children. Curiosity about mathematics is fostered when children are engaged in activities such as comparing quantities, searching for patterns, sorting objects, ordering objects, creating designs, building with blocks, and talking about these activities. Positive early experiences in mathematics are as critical to child development as are early literacy experiences.

Students learn by attaching meaning to what they do and need to construct their own meaning of mathematics. This meaning is best constructed when learners encounter mathematical experiences that proceed from the simple to the complex and from the concrete to the abstract. The use of models and a variety of pedagogical approaches can address the diversity of learning styles and developmental stages of students and enhance the formation of sound, transferable, mathematical concepts. At all levels, students benefit from working with and translating through a variety of materials, tools, and contexts when constructing meaning about new mathematical ideas. Meaningful discussions can provide essential links among concrete, pictorial, contextual, and symbolic representations of mathematics.

The learning environment should value and respect all students’ experiences and ways of thinking, so that learners are comfortable taking intellectual risks, asking questions, and posing conjectures. Students need to explore problem-solving situations in order to develop personal strategies and become mathematically literate. Learners must realize that it is acceptable to solve problems in different ways and that solutions may vary.

Goals for Mathematics Education

The main goals of mathematics education are to prepare students to

- use mathematics confidently to solve problems
- communicate and reason mathematically
- appreciate and value mathematics
- make connections between mathematics and its applications
- become mathematically literate adults, using mathematics to contribute to society

Students who have met these goals will

- gain understanding and appreciation of the contributions of mathematics as a science, a philosophy, and an art
- exhibit a positive attitude toward mathematics
- engage and persevere in mathematical tasks and projects
- contribute to mathematical discussions
- take risks in performing mathematical tasks
- exhibit curiosity about mathematics and situations involving mathematics

Opportunities for Success

A positive attitude has a profound effect on learning. Environments that create a sense of belonging, encourage risk taking, and provide opportunities for student success help develop and maintain positive attitudes and self-confidence. Students with positive attitudes toward learning mathematics are likely to be motivated and prepared to learn, participate willingly in classroom activities, persist in challenging situations, and engage in reflective practices.

To experience success, students must be taught to set achievable goals or assess their progress as they work toward these goals. Striving toward success and becoming autonomous and responsible learners are ongoing, reflective processes that involve revisiting the setting and assessing of personal goals.

Engaging All Learners

“No matter how engagement is defined or which dimension is considered, research confirms this truism of education: *The more engaged you are, the more you will learn.*” (Hume 2011, 6)

Student engagement is at the core of learning. This is critical for teachers to take into account when planning and implementing instruction. Effective instruction engages, embraces, and supports all learners through a range of learning experiences. This curriculum is designed to provide learning opportunities that reflect culturally proficient instructional and assessment practices and are equitable, accessible, and inclusive of the multiple facets of diversity represented in today’s classrooms.

Engagement in learning occurs when students are provided with opportunities to become more invested in their learning. When teachers know their students as individual learners and as individual people, their students are more likely to be motivated to learn, participate in classroom activities, persist in challenging situations, and engage in reflective practices. Students often become more engaged when teachers demonstrate a genuine belief in each student’s potential to learn.

SUPPORTIVE LEARNING ENVIRONMENTS

A supportive and positive learning environment has a profound effect on students' learning. In classrooms where students feel a sense of belonging, are encouraged to actively participate, are challenged without being frustrated, and feel safe and supported to take risks with their learning, students are more likely to experience success. It is realized that not all students will progress at the same pace or be equally positioned in terms of their prior knowledge of and skill with particular concepts and outcomes. Teachers provide all students with equitable access to learning by integrating a variety of instructional approaches and assessment activities that consider all learners and align with the following key principles:

- Instruction must be flexible and offer multiple means of representation.
- Students must have opportunities to express their knowledge and understanding in multiple ways.
- Teachers must provide options for students to engage in learning through multiple ways.

Teachers who know their students well become aware of individual learning differences and infuse this understanding into planned instructional and assessment decisions. They organize learning experiences to accommodate the many ways in which students learn, create meaning, and demonstrate their knowledge and understanding. Teachers use a variety of effective teaching approaches that may include

- providing all students with equitable access to appropriate learning strategies, resources, and technology
- offering a range of ways students can access their prior knowledge to connect with new concepts
- scaffolding instruction and assignments so that individual or groups of students are supported as needed throughout the process of learning
- verbalizing their thinking to model comprehension strategies and new learning
- balancing individual, small-group, and whole-class approaches to learning activities
- involving students in the co-creation of criteria for assessment and evaluation
- providing students with choice in how they demonstrate their understanding according to learning styles and preferences, building on individual strengths, and including a range of difficulty and challenge
- providing frequent and meaningful feedback to students throughout their learning experiences

LEARNING STYLES AND PREFERENCES

The ways in which students make sense of, receive, and process information, demonstrate learning, and interact with peers and their environment both indicate and shape learning preferences, which may vary widely from student to student. Learning preferences are influenced also by the learning context and purpose and by the type and form of information presented or requested. Most students tend to favour one learning style and may have greater success if instruction is designed to provide for multiple learning styles, thus creating more opportunities for all students to access learning. The three most commonly referenced learning styles are

- auditory (such as listening to teacher-presented lessons or discussing with peers)
- kinesthetic (such as using manipulatives or recording print or graphic/visual text)
- visual (such as interpreting information with text and graphics or viewing videos)

While students can be expected to work using all modalities, it is recognized that one or some of these modalities may be more natural to individual students than the others.

A GENDER-INCLUSIVE CURRICULUM

It is important that the curriculum respects the experiences and values of all students and that learning resources and instructional practices are not gender-biased. Teachers promote gender equity and inclusion in their classrooms when they

- articulate equally high expectations for all students
- provide equal opportunity for input and response from all students
- model gender-fair language and respectful listening in their interactions with students

VALUING DIVERSITY: TEACHING WITH CULTURAL PROFICIENCY

Teachers understand that students represent diverse life and cultural experiences, with individual students bringing different prior knowledge to their learning. Therefore, teachers build upon their knowledge of their students as individuals and respond by using a variety of culturally-proficient instruction and assessment strategies. “Instruction that is embedded in socially meaningful contexts, and tasks that are meaningful and relevant to the lives of students, will engage students in high-level problem-solving and reasoning and enhance students’ engagement (Frankenstein 1995; Gutstein 2003; Ladson-Billings 1997; Tate 1995).” (Herzig 2005)

STUDENTS WITH LANGUAGE, COMMUNICATION, AND LEARNING CHALLENGES

Today’s classrooms include students who have diverse backgrounds, abilities, levels of development, and learning challenges. By observing and interacting with students as they work on assigned activities, teachers can identify areas where students may need additional support to achieve their learning goals. Teachers can then respond with a range of effective instructional strategies. Students who have English as an Additional Language (EAL) may require curriculum outcomes at different levels, or temporary individualized outcomes, particularly in language-based subject areas, while they become more proficient in their English language skills. For students who are experiencing difficulties, it is important that teachers distinguish between students for whom curriculum content is challenging and students for whom language-based issues are at the root of apparent academic difficulties.

STUDENTS WHO DEMONSTRATE GIFTED AND TALENTED BEHAVIOURS

Some students are academically gifted and talented with specific skill sets or in specific subject areas. Most students who are gifted and talented thrive when challenged by problem-centred, inquiry-based learning and open-ended activities. Teachers may challenge students who are gifted and talented by adjusting the breadth, the depth, and/or the pace of instruction. Learning experiences may be enriched by providing greater choice among activities and offering a range of resources that require increased cognitive demand and higher-level thinking at different levels of complexity and abstraction. For additional information, refer to *Gifted Education and Talent Development* (Nova Scotia Department of Education 2010).

Connections across the Curriculum

The teacher should take advantage of the various opportunities available to integrate mathematics and other subjects. This integration not only serves to show students how mathematics is used in daily life, but it helps strengthen the students’ understanding of mathematical concepts and provides them with opportunities to practise mathematical skills. There are many possibilities for integrating mathematics in health education, literacy, music, physical education, science, social studies, and visual arts.

Number (N)

GCO: Students will be expected to demonstrate number sense.

Specific Curriculum Outcomes

Process Standards Key

[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

- N01** Students will be expected to say the number sequence by
- 1s, from 1 to 20
 - 1s, starting anywhere from 1 to 10 and from 10 to 1 [C, CN, V]
- N02** Students will be expected to recognize, at a glance, and name the quantity represented by familiar arrangements of 1 to 5 objects or dots. [C, CN, ME, V]
- N03** Students will be expected to relate a numeral, 1 to 10, to its respective quantity. [CN, R, V]
- N04** Students will be expected to represent and describe numbers 2 to 10 in two parts, concretely and pictorially. [C, CN, ME, R, V]
- N05** Students will be expected to compare quantities, 1 to 10, using one-to-one correspondence. [C, CN, V]
- N06** Students will be expected to demonstrate an understanding of counting to 10. [C, CN, ME, PS, R, V]

SCO N01 Students will be expected to say the number sequence by

- 1s from 1 to 20
- 1s, starting anywhere from 1 to 10 and from 10 to 1

[C, CN, V]

[C] Communication

[PS] Problem Solving

[CN] Connections

[ME] Mental Mathematics and Estimation

[T] Technology

[V] Visualization

[R] Reasoning

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- N01.01** Recite the number sequence from 1 to 20 and from 10 to 1.
- N01.02** Name the number that comes after a given number, 1 to 9.
- N01.03** Name the number that comes before a given number, 2 to 10.
- N01.04** Recite number names from a given number to a stated number (forward 1 to 10, backward 10 to 1) using visual aids.

Scope and Sequence

Mathematics Primary	Mathematics 1
<p>N01 Students will be expected to say the number sequence by</p> <ul style="list-style-type: none"> ▪ 1s from 1 to 20 ▪ 1s, starting anywhere from 1 to 10 and from 10 to 1 	<p>N01 Students will be expected to say the number sequence by</p> <ul style="list-style-type: none"> ▪ 1s, forward and backward between any two given numbers, 0 to 100 ▪ 2s to 20, forward starting at 0 ▪ 5s to 100, forward starting at 0, using a hundred chart or a number line ▪ 10s to 100, forward starting at 0, using a hundred chart or a number line

Background

Meaningful counting involves an understanding of the six principles of counting. These principles are addressed in various outcomes (N01, N03, and N06). The principle in bold face is the one addressed in this outcome.

1. One number is said for each item in the group and is counted once and only once. (one-to-one correspondence)
2. **Counting begins with the number 1, and there is a set number sequence. (stable order)**
3. The quantity in the set is the last number said. (cardinality)
4. The starting point and order of counting the objects does not affect the quantity. (order irrelevance)
5. The arrangement or types of objects does not affect the count. (conservation)
6. It does not matter what is being counted, the resulting count will always be the same. (abstraction)

Stable Order Principle: In everyday use there are two meanings of “to count.” One meaning is to say the numbers in order (outcome N01 addresses this meaning), which addresses the stable order principle. As early as two years of age, children can repeat words such as one, two, and three; however, children most likely do not understand the quantity represented by those numbers. Understanding quantity is the second meaning of “to count” (outcomes N02, N03, and N06 address this second meaning).

Learning the number names is an important prerequisite for counting items in a set. “Before there can be any meaningful counting, students must be able to recite the sequence beginning 1, 2, 3, 4, 5, etc.” (Small 2009, 84) There is a difference between being able to recite the number words (1, 2, 3, ...) and understanding how counting is used to describe a set. The counting sequence itself is a rote procedure; however, “The meaning attached to counting is the key conceptual idea on which all other number concepts are developed.” (Van de Walle and Lovin 2006, 39).

This outcome should be addressed throughout the year, allowing for its different aspects to be addressed in appropriate developmental stages.

1. Saying the number sequence from 1 to 10 can be addressed first since this is an important prerequisite for counting items in a set. As such, this counting sequence should be integrated with SCO N03 and SCO N06 and expanded as students’ fluency of number names develops.
2. Naming the number that comes after or before a given number, and counting forward and backward from any number can be addressed once students are more comfortable saying the number sequence from 1 to 10.
3. Saying the sequence of numbers from 11 to 20 can be addressed once students are very comfortable with numbers from 1 to 10. (**Note:** In primary, students are expected only to recite number names from 11 to 20 and are not expected to develop quantity sense for these numbers.)

Additional information

See Appendix A: Additional Information.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students’ prior knowledge.

- Ask students to start at 1 and recite the number sequence forward as far as they can.
- Ask students to recite the number sequence backwards from 10 to 1.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students to start at 1 and recite the number sequence forward as far as they can.
- Ask students to say the number sequence backward from 10 to 1.
- Observe whether or not students
 - recognize situations where they recite the number sequence
 - need to start from the beginning when saying the number sequence
 - correct one another as they say the number sequence together
- Say a number sequence (1 to 20 or from 10 to 1) incorrectly and ask students to identify your error and correct it.
- Begin reciting the numbers from 1 to 20 or from 10 to 1, but omit some numbers. Ask students to tell you the numbers you omitted.
- Ask students to begin saying the number sequence at a given number and continue up to 10 (e.g., ask students to begin at 4, and continue by saying, 5, 6, 7, 8, 9, and 10).
- Ask students to recite backwards starting at a given number (10 and less).
- Ask students to tell you what number comes after a given number and before a given number.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets K–2 (Bauman 2011)

- No Checkpoint for this outcome.

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- During the course of daily tasks, ensure students have frequent opportunities to say the number sequence, forwards and backwards. For example,
 - count backwards while taking items off a table or putting items away

- count down to special days
- count while skipping, hopping, bouncing a ball, or taking part in other physical activities
- count on while determining the total on a pair of number cubes
- Use children’s literature, such as counting songs, finger plays, and rhymes, to assist students in learning the number sequence.

SUGGESTED LEARNING TASKS

- **Echo counting:** The teacher recites the number sequence, and the students echo the teacher.
- **Orchestra counting:** Put students into groups. The first group begins reciting the number sequence forward from 1. When the teacher points to another group, they continue from the last number stated.
- Begin saying the number sequence from a given starting point less than 10, and ask students to continue up to the number 10. For example, say “3, 4, 5, ...” This activity can be repeated by reversing the number sequence. For example, “5, 4, 3, ...”
- Use poems, songs, and stories that have forward and backward counting sequences. Ask students to predict which number comes next as you read the poem or story.
- **I say... You say...:** The teacher says a number and the class or individual student responds with the number that comes after or before the stated number.
- Make up riddles for numbers that come before and after a given number less than 10. For example, “I am three. What comes before me?” “I am four. What is one more?”
- Count some items with the students. Cover the starting quantity of items. Have the students count on as you add more items (up to 10).

SUGGESTED MODELS AND MANIPULATIVES

- collections of classroom objects
- counters
- fingers
- students

MATHEMATICAL LANGUAGE

Teacher	Student (oral language)
<ul style="list-style-type: none"> ▪ backward, forward ▪ counting numbers: one to twenty ▪ position words: after, before, count forward, count on, count back, next 	<ul style="list-style-type: none"> ▪ backward, forward ▪ counting numbers: one to twenty ▪ position words: after, before, count forward, count back, count on, next

Resources/Notes

Print Resources

- *Making Math Meaningful to Canadian Students, K–8* (Small 2009), pp. 18–19, 83–84
- *Making Math Meaningful to Canadian Students, K–8, Second Edition* (Small 2013), pp. 17–20, 139–140
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 39–40

Videos

- *Teaching Number: 0 to 9* (14:47 min) (ORIGO Education 2010) (**Note:** Zero is not addressed in Mathematics Primary.)
- *Teaching Number: Counting* (10:49 min.) (ORIGO Education 2010)

Notes

SCO N02 Students will be expected to recognize, at a glance, and name the quantity represented by familiar arrangements of 1 to 5 objects or dots.

[C, CN, ME, V]

[C] Communication

[PS] Problem Solving

[CN] Connections

[ME] Mental Mathematics and Estimation

[T] Technology

[V] Visualization

[R] Reasoning

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N02.01 Look briefly at a given familiar arrangement of 1 to 5 objects or dots and identify the number represented without counting.

N02.02 Identify the number represented by a given dot arrangement on a five-frame.

Scope and Sequence

Mathematics Primary	Mathematics 1
N02 Students will be expected to recognize, at a glance, and name the quantity represented by familiar arrangements of 1 to 5 objects or dots.	N02 Students will be expected to recognize, at a glance, and name the quantity represented by familiar arrangements of 1 to 10 objects or dots.

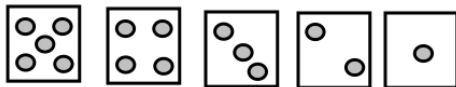
Background

Cardinality refers to the quantity of objects in a set. The cardinality can be obtained by counting (SCO N03) or by subitizing. This outcome addresses **subitizing**. Subitizing is the ability to recognize, at a glance, a quantity of objects, without one-to-one counting. For example, the ability to immediately recognize the number of dots on a standard die when it is rolled, or to immediately recognize a number on a five-frame when it is presented briefly.

Research indicates that young students are able to subitize a quantity equal to their age; for example, a three-year old can subitize one to three objects. As such, most students subitize quantity before they learn to determine quantity by one-to-one counting. Students in primary should come to realize that subitizing and counting are two ways of determining how many are in a set of objects and not think that one-to-one counting is the only legitimate way to find how many.

Subitizing small arrangements of objects helps students with counting on, and composing and decomposing (partitioning) numbers. Initially, for some arrangements, some students may count the objects or dots, but by the end of primary, they must be able to recognize arrangements up to five without counting. To avoid the misconception that an arrangement can only represent a specific quantity if it is arranged in a certain way, it is very important to vary the position of the objects, dots, or pictures.

Number Cube Arrangements



Examples of other familiar arrangements:



The two dot plates show 3.

The two dot plates show 5.

Additional Information

See Appendix A: Additional Information.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Use dot cards for 1, 2, and 3. Flash the cards in random order. After flashing each card, ask the students to tell you how many dots they saw.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Use your fingers to represent a quantity. Ask students to tell how many fingers you are holding up.
- Give each student five cards with the numerals 1 to 5 on them. Uncover a series of dots on the overhead projector or interactive whiteboard and have students hold up the card with the correct corresponding numeral on it. This gives you a quick way of identifying who is able to subitize. Students may also respond orally.
- Hold up a card with an arrangement of dots between 1 and 5 and ask, How many? Students may answer orally. Later in the year, they may write the corresponding numeral on a piece of paper and hold it up for you to see.
- Show three pie plate examples that display familiar arrangements of dots (two of the plates should display the same amount, but have different arrangements and the third should display a different amount). Ask students to identify which plate shows the different amount (without counting).
- Using coloured counters, ask students to show you three different arrangements that represent a particular number between 1 and 5.

- Ask students to roll a number cube with dots and to tell you the number rolled without counting the dots.
- Using individual five-frames, ask students to show you a number that you announce to the class. Walk around and see if they have the correct number of corresponding dots (counters).
- Display for three seconds a number represented in a five-frame. Ask students to tell how many there are.
- Ask students to arrange 5 counters in a way that will make it easy to tell that there are 5.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets K–2 (Bauman 2011)

- Kindergarten Checkpoint 2, Task 2, pp. 30–31
- Kindergarten Checkpoint 4, pp. 47–48 (Line Masters 2.5 and 2.6)

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Provide opportunities for students to represent quantities in various arrangements.
- Encourage students to display numbers whenever possible, either with materials or by enacting the number physically (e.g., showing fingers, clapping).
- Expect students to explain, verbally, how they know how many are in a set.
- Use children’s literature that shows a variety of arrangements for numbers.

SUGGESTED LEARNING TASKS

- Place counters on the glass of the overhead projector and cover them. Have students look at the screen. Uncover the counters for a few seconds only. Ask students to tell you how many counters they saw.
- Show a dot card for three seconds. Ask students to make the dot pattern they saw using counters.
- Show students a set of dot cards that all show the same number except for one card. Ask students to tell you which dot card does not belong in the set.
- Play a game of “concentration.” Place pairs of dot cards face down in an array. Students take turns turning over two cards at a time, and if they match, they keep the pair. If the cards do not match, then the student returns the cards.
- Use dot cards or similar cards that show familiar arrangements of dots for numbers. Show students two cards at a time and ask students to tell you which card has more/less without counting.
- Use a five-frame, and ask students to identify the number of dots at a glance. Extend this activity to other models and arrangements, such as linking cubes.
- Display a five-frame, and after three seconds cover it. Ask students to place counters on an empty five-frame to copy what they saw. Repeat with different quantities.
- Invite students to explore arrangements of numbers. For example, students could create books in which each two-page spread shows a particular number of items, but in a different physical arrangement.
- Have students make a Rekenrek and complete subitizing activities with it.

SUGGESTED MODELS AND MANIPULATIVES

- | | |
|-------------|-----------------|
| ▪ counters | ▪ five-frames |
| ▪ dominoes | ▪ linking cubes |
| ▪ dot cards | ▪ number cubes |
| ▪ fingers | ▪ Rekenrek |

MATHEMATICAL LANGUAGE

Teacher	Student (oral language)
<ul style="list-style-type: none"> ▪ dots, dot cards, dot arrangements ▪ five-frames, counters ▪ How many? ▪ sets 	<ul style="list-style-type: none"> ▪ dots, dot cards ▪ five-frames, counters ▪ How many?

Resources/Notes

Print Resources

- *Making Math Meaningful to Canadian Students, K–8* (Small 2009), pp. 89–91
- *Making Math Meaningful to Canadian Students, K–8, Second Edition* (Small 2013), pp. 145–148
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 43–44

Video

- *Developing Sight Recognition of Quantity* (12:03 min.) (ORIGO Education 2010) (Subitizing)

Notes

SCO N03 Students will be expected to relate a numeral, 1 to 10, to its respective quantity.

[CN, R, V]

[C] Communication

[PS] Problem Solving

[CN] Connections

[ME] Mental Mathematics and Estimation

[T] Technology

[V] Visualization

[R] Reasoning

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- N03.01** Name the number for a given set of objects.
- N03.02** Match numerals with their given pictorial representations.
- N03.03** Hold up the appropriate number of fingers for a given numeral.
- N03.04** Construct a set of objects corresponding to a given numeral.
- N03.05** Record the numeral that represents the quantity of a given set of objects.

Scope and Sequence

<p>Mathematics Primary</p> <p>N03 Students will be expected to relate a numeral, 1 to 10, to its respective quantity.</p>	<p>Mathematics 1</p> <p>N03 Students will be expected to demonstrate an understanding of counting to 20 by</p> <ul style="list-style-type: none"> ▪ indicating that the last number said identifies “how many” ▪ showing that any set has only one count ▪ using the counting-on strategy
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Background

Meaningful counting involves an understanding of the six principles of counting. These principles are addressed in various outcomes (N01, N02, N03, and N06). The principles in bold face are the ones addressed in this outcome.

- 1. One number is said for each item in the group and is counted once and only once. (one-to-one correspondence)**
2. Counting begins with the number 1, and there is a set number sequence. (stable order)
- 3. The quantity in the set is the last number said. (cardinality)**
4. The starting point and order of counting the objects does not affect the quantity. (order irrelevance)
5. The arrangement or types of objects does not affect the count. (conservation)
- 6. It does not matter what is being counted, the resulting count will always be the same. (abstraction)**

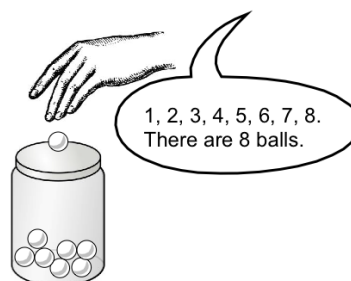
This outcome is connected to outcomes N02 and N06. All three outcomes relate to the quantity that results from one-to-one counting or from subitizing the objects in a set. As such, it is suggested that many lessons integrate these three outcomes. Outcomes N03 and N06 highlight a very critical counting principle (cardinality)—the last number said is the number (quantity) in the set, not the name of the last object touched.

Before students count sets of a certain size, they should be fluent with the number sequence involved (the stable order principle addressed in SCO N01) so they can concentrate on the counting process rather than on trying to recall the number names. Set size should increase throughout the year as student knowledge of number names increases; for example, set size in quantity development can start

with up to 3 objects once the numbers 1, 2, and 3 can be said by students fluently. Increase the set size as student development dictates.

One-to-One Correspondence: When students are counting, watch to see if they are touching objects while they count them and saying the appropriate numbers when they touch them. Encourage students to touch or move the objects as they count as this deepens their awareness of the one-to-one principle of counting.

Cardinality: Some students may very successfully perform a counting act of 8 objects, but when asked to show 8, pick up the last counter, thinking that this counter they touched when they said “eight” is 8, not the set of 8 counters. Help students develop this principle by engaging them in activities where objects are placed in a container as they are counted and directing students’ attention to the number of objects in the container after the counting act.



Abstraction Principle: Students should understand that the size of the objects does not influence the number that represents the quantity in a set; that is, the objects themselves do not have to be uniform in size and shape, and two sets may both represent the same number even if the objects in one set are larger than those in the other. For example, students can count 6 bears, 6 toy cars, and 6 chairs. The three sets are equivalent because they all represent 6.



As well, many, if not most, of the sets of objects from 1 to 5 should eventually be identified at a glance (SCO N02). By the end of the year, one-to-one counting should be used for sets from 6 to 10 and for those sets from 3 to 5 that may not be in readily identified arrangements.

Additional Information

See Appendix A: Additional Information.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Show students a set of 1 to 3 objects. Ask, How many objects are there? Observe whether the students are able to count the set of objects and name the number for the given set of objects. Ask students to hold up fingers to match the number of objects on the table.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following sample tasks (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Observe and note the way in which students count
 - Do they start at 1 and say the number words in the correct sequence?
 - Do they realize the last number said is the quantity of objects they have counted?
 - Do they touch each object as they count?
 - Do they set items aside as they count them?
 - Do they show confidence in their count or feel the need to check?
 - Do they check their counting in the same order as the first count or in a different order?
- Ask students, How many fingers am I holding up? or ask students to hold up a given number of finger(s) that represent a given numeral being displayed.
- Give each student a five-frame or a ten-frame. Announce a number to the class and ask students to show you that number on their five-frames or ten-frames. Observe whether they have the correct number of corresponding dots.
- Ask students to choose a numeral between 1 and 10 and create, and later draw, a set of objects that corresponds to that numeral. Repeat for several different numbers.
- Provide a variety of interesting photographs involving multiple people, animals, or objects. Ask students to tell how many are in the photographs and to record the corresponding numeral.
- Show students a numeral and ask them to clap that many times. For variation, clap and ask the students to point to the numeral that indicates the number of claps.
- Show a set of objects, dot cards, and five-frames or ten-frames. Ask students to name the number of objects shown and to record the numeral that matches the display.
- Fill paper bags with 10 different sized counters or objects (e.g., cubes, bear counters, erasers, crayons, large paper clips, etc). In turn, students grab one handful of objects from each bag, count them, and record the number of items on a blank sheet of paper or index card.
- Give students sets of numeral cards from 1 to 10. Ask them to match the 10 numerals with corresponding sets of objects. Show sets of up to 10 items and ask them to hold up the corresponding numeral card. You can also hold up dot cards with the numbers from 1 to 10.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets K–2 (Bauman 2011)

- Kindergarten Checkpoint 1, p. 21
- Kindergarten Checkpoint 2, Task 1, pp. 30–31 (Line Master 2.1)
- Kindergarten Checkpoint 15, pp. 139–140

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Use a variety of physical materials that students can manipulate and group as they count. Number activities should use concrete objects prior to using printed images.
- Encourage students to use familiar and interesting objects to represent and model numbers whenever possible. Also, encourage students to enact numbers physically (e.g., showing fingers, clapping).
- Expect students to explain, verbally, how they know how many are in a set.
- Invite children to create sets of a given size using a variety of objects.
- Work on writing numerals could begin by having students trace over numerals with their fingers. Later, students could make numerals in the air, before experimenting freely with writing numerals on unlined paper using markers, crayons, and pencils. By the end of Mathematics Primary, students should be introduced to writing numerals on lined paper.

SUGGESTED LEARNING TASKS

- Play a variety of games that use number cubes and counting.
- Ask students to hold up the number of fingers that represent a given numeral.
- Give each student a ten-frame and 10 counters. On the ten-frame, have students show you different numbers from one to ten. You might say the number or write the numeral on the board and then have students display the number in the ten-frame. Repeat with different examples.
- Provide students with cards on which the numerals 1 to 10 are written. Make number arrangements on an overhead, using ten-frames and counters. Ask the students to select and show the numeral card that represents the arrangement. This activity can also be done with pairs of students.

- Ask students to create sets of objects that represent a given numeral. For example, ask students to show sets of 7 in a variety of ways using different objects.
- Play a concentration game with pairs of cards that show numerals and matching pictorial representations of counters, five-frames, ten-frames, dot cards, or dominoes.
- Sort the numerals in terms of characteristics; for example, those with rounded parts (like 8, 0), those with only straight parts (like 1, 7), and those with both (like 5, 2).
- Play “rabbit ears.” Students place their right hand beside their right ear and their left hand beside their left ear. Ask them to use their fingers to show the number that you say. For example, a student may hold up 4 fingers on one hand and 3 fingers on the other hand.
- Present students with 6 cereal boxes in a stack and 6 cans of soup. Ask, Which stack has more? Why?
- Ask students to record numerals in the context of the activities above.

SUGGESTED MODELS AND MANIPULATIVES

- collections of objects
- counters
- dominoes
- dot cards
- five-frames and ten-frames
- number cubes

MATHEMATICAL LANGUAGE

Teacher	Student (oral language)
<ul style="list-style-type: none"> ▪ numeral ▪ one-to-one counting ▪ quantity ▪ set of objects 	<ul style="list-style-type: none"> ▪ numeral ▪ set of objects

Resources/Notes

Print Resources

- *Making Math Meaningful to Canadian Students, K–8* (Small 2009), pp. 88–91, pp. 93–94
- *Making Math Meaningful to Canadian Students, K–8, Second Edition* (Small 2013), pp. 144–148, 150–151
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 39–40

Videos

- *Teaching Number: 0 to 9* (14:47 min.) (ORIGO Education 2010) (**Note:** Zero is not addressed in Mathematics Primary.)
- *Teaching Number: Counting* (10:49 min.) (ORIGO Education 2010)
- *Using a Hands-On Approach to Represent Numbers to 10* (13:06 min.) (ORIGO Education 2010)

Notes

SCO N04 Students will be expected to represent and describe numbers 2 to 10 in two parts, concretely and pictorially.

[C, CN, ME, R, V]

[C] Communication

[PS] Problem Solving

[CN] Connections

[ME] Mental Mathematics and Estimation

[T] Technology

[V] Visualization

[R] Reasoning

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N04.01 Show a given number as two parts (using fingers, counters, or other objects) and name the number of objects in each part.

N04.02 Show a given number as two parts, using pictures, and name the number of objects in each part.

Scope and Sequence

Mathematics Primary	Mathematics 1
N04 Students will be expected to represent and describe numbers 2 to 10 in two parts, concretely and pictorially.	N04 Students will be expected to represent and partition numbers to 20.

Background

The ability to recognize that any number can be partitioned into two or more smaller numbers is an essential understanding in number development. Understanding the relationship between/among the parts and the whole is essential to developing a sense of numbers and an understanding of mathematical operations. It is important to understand that the action of partitioning a set of objects does not affect the count. (See counting principle 5 in SCO N06.) Students should be able to see small groups as part of a larger group. They should also be able to recognize how many objects are in each of the small groups. This helps students develop part-part-whole relationships.

It is important to note that this important understanding of part-part-whole relationships is only introduced in primary by having students use concrete materials and pictures to show the two parts of a number up to 10. This will make sense to students if these materials and pictures represent real situations. For example, they could model the number of possible combinations of boys and girls in a group of 8 students by using counters or cubes of two different colours, and describe the different combinations that make 8. Most students in primary, however, will likely not be convinced that the whole remains the same regardless of how it is partitioned. This will be a focus of partitioning activities in Mathematics 1 and Mathematics 2. Below three partitions of 8 are shown using linking blocks.



Counting a set of objects will not cause a child to focus on the fact that the set could be made up of two parts. This will require specific activities and questions that focus on a quantity in terms of its parts. Provide students with opportunities to demonstrate many ways of representing a number, both concretely and pictorially. Students need to talk about where and when they might use specific numbers and their parts. Representations may involve acting out situations, drawing, painting, or using concrete materials such as ten-frames. Students should then begin to use words to describe two parts of a number (five and five) and eventually record numerals to show the two parts (5 and 5). As with all

mathematical concepts, opportunities for students to generate multiple representations of the same idea are a critical component of developing a deeper understanding.

Additional Information

See Appendix A: Additional Information.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Show students dominoes representing numbers 2 to 4. Ask, How many dots are on each part of this domino? How many dots are there altogether?

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Hold up 3 fingers on one hand and 1 finger on the other hand. Ask students to name the parts by saying, "Four is three and one." Repeat using different combinations of fingers to represent 4.
- Shake and spill a handful of two different colours of counters (10 or less) on the overhead. Ask students to record (with pictures) how many of each colour there are and how many counters there are altogether. Eventually, they should also name the quantity in each part by recording the respective numeral beside or below each part of their drawing.
- Give students sticker dots of two different colours and ask them to make different dot plates of numbers, using the two colours (e.g., They might show 8 dots, with 5 red and 3 yellow or 2 red and 6 yellow). Ask them to share with the class the different ways they made the number.
- Provide students with a given number (10 or less) and ask them to make two-colour bars, using linking cubes (e.g., 5 blue and 3 red) to represent this number. Ask students to tell you about the number of cubes.
- Ask a student to count out 6 blocks/counters into your hand. Shake them up in both hands and then open hands to display a "6" combination, (e.g., 4 in one hand, 2 in the other). Ask students how many you have altogether. Repeat using different combinations. Observe whether or not students need to count.

- Ask students to each get 10 linking cubes—5 red, 3 green, and 2 yellow. Students can work in pairs to solve the following riddles.
 - I have 3 red cubes and 3 green cubes. How many cubes do I have?
 - I have 2 yellow cubes and the same number of red cubes. How many cubes do I have?
 - I have 8 cubes. There are 5 red cubes and the rest are green. How many cubes are green?
 - I built a tower of 7 cubes. The cubes are red and green. What could my tower look like? Which towers are the same? Which towers are different?
- Ask students to explore multiple ways to partition a number. For example, students could be asked to create books in which each page shows a different way to show the given number as two parts, using pictures. Students could be asked to record the number of objects in each part.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets K–2 (Bauman 2011)

- Kindergarten Checkpoint 2, pp. 30–31
- Kindergarten Checkpoint 5, pp. 55–56 (Line Master 5.1)

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Represent numbers in a variety of part-part-whole relationships using a variety of physical materials.
- Encourage students to display numbers in parts, whenever possible, either with manipulatives or by enacting the number physically (e.g., showing fingers, clapping).
- Encourage students to explain how they know how many are in each part of a whole.

SUGGESTED LEARNING TASKS

- Ask students to shake and spill a handful of two-colour counters (10 or less) and record (with pictures or numerals) how many counters there are altogether and how many of each colour there are. Students should explore making different combinations for the given number by flipping the counters over or by repeating the shake and spill process.
- Provide students with a number of linking cubes (10 or less) and have them make a “train.” Ask students to show how many different ways they can break the “trains” of cubes into two parts.
- Ask students to separate a given number of toys (10 or less) into two groups. Ask students to describe how many are in each group and how many there are in total.
- Students might play a variety of games that involve identifying the parts and the whole, for example bowling (counting both the pins knocked down and the pins left standing) or throwing bean bags (counting how many land in the target box and how many did not).
- Ask students to select a number between 5 and 10. Then, ask them to select two dot cards that combine to make that number. Challenge them to see how many different combinations they can make for their chosen number. This may also be done with dominoes.
- Use part-part-whole mats and counters and show students a set of 5 counters. Ask students to count how many are in the set. Then, partition the counters into two parts, for example 2 and 3, and ask, How many objects are there? How do you know? Observe whether students must recount all of the objects or if they know that partitioning the objects has not changed the quantity. Repeat with a different number of objects.

SUGGESTED MODELS AND MANIPULATIVES

- dominoes
- dot cards
- five-frames and ten-frames
- linking cubes
- two-colour counters

MATHEMATICAL LANGUAGE

Teacher	Student (oral language)
<ul style="list-style-type: none"> ▪ familiar arrangements ▪ partition numbers ▪ parts, whole ▪ part-part-whole relationships 	<ul style="list-style-type: none"> ▪ parts, whole

Resources/Notes

Print Resources

- *Making Math Meaningful to Canadian Students, K–8* (Small 2009), pp. 90–91
- *Making Math Meaningful to Canadian Students, K–8, Second Edition* (Small 2013), pp. 147–148
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 37–41, pp. 48–54

Videos

- *Teaching Number: 0 to 9* (14:47 min.) (ORIGO Education 2010) (**Note:** Zero is not addressed in Mathematics Primary.)
- *Teaching Number: Counting* (10:49 min.) (ORIGO Education 2010)
- *Using a Hands-On Approach to Represent Numbers to 10* (13:06 min.) (ORIGO Education 2010)

Notes

SCO N05 Students will be expected to compare quantities, 1 to 10, using one-to-one correspondence. [C, CN, V]			
[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- N05.01** Construct a set to show more than, fewer than, or as many as a given set.
- N05.02** Compare two given sets through direct comparison and describe the sets using words, such as more, fewer, as many as, or the same number as.

Scope and Sequence

Mathematics Primary	Mathematics 1
N05 Students will be expected to compare quantities, 1 to 10, using one-to-one correspondence.	N05 Students will be expected to compare sets containing up to 20 objects to solve problems using referents and one-to-one correspondence.

Background

Comparing is something students do many times a day. Many students start primary with some understanding of comparing. Young students often have a great sense of fairness that is the basis of their comparisons. For example, if they have 2 cookies and someone else has 4 cookies, they are aware they have fewer cookies.

Students in primary are expected to explore the concept of quantity as it relates to countable objects and to compare these quantities. Therefore, the word **fewer** is used (e.g., there are fewer counters in this set than in that set). It is interesting to note that although the concept of fewer is logically equivalent to the concept of more, the word **fewer** proves to be more difficult for students than the word **more**. Students will become more familiar with the term **less than** to compare numbers in later years. They will learn that 5 is less than 7 because there are 2 fewer items in a set of 5 than in a set of 7. When talking about sets that have the same number of objects, encourage the use of the terms **the same number as**, and **as many as**.

Additional Information

See Appendix A: Additional Information.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Show students two dot cards representing numbers from 1 to 5. Ask, Which card has more dots? Which card has fewer dots?

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Give each student a blank plate. Hold up a dot plate and ask students to
 - make a set of dots that is the “same as” this set
 - make a set of dots with “more” dots than this set
 - make a set of dots with “fewer” dots than this set
- Give students dominoes and have them tell you which side of the domino shows more dots and which side shows fewer dots. For example, “Five dots is more than 3 dots *and* 3 dots is fewer than 5 dots.”
- Use two sets of objects of differing sizes (e.g., 4 large objects in one set and 5 small objects in the other set). Ask, Which has more? How do you know?
- Show pairs of cards with sets of objects 10 or fewer (e.g., two that show sets of 8, two that show sets of 9, two that show sets of 6). As you show each pair, ask students if the pairs match. Make sure that some of the pairs are not equivalent. Use large stickers or illustrations when showing these to the whole group.
- Provide representations of two different sets each of 10 or fewer. Ask students which set has more and which has fewer (e.g., one set has 9 and the other has 7).
- Hold up a dot card that has between 2 and 8 dots. Give each student a set of three cards labelled “more,” “fewer,” and “same.” Beside each card, ask students to make a collection of counters; one set that has more, one set that has fewer, and one set that has the same as the original card.
- Give students a tower of 4 cubes and another tower of 7 cubes. Ask, Which is closer to 10? and then ask, How do you know? You could also ask students to tell you which is closer to 5 and to explain their thinking. Allow students to use more linking cubes to assist in solving this problem.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets K–2 (Bauman 2011)

- Kindergarten Checkpoint 4, pp. 47–48 (Line Masters 2.5 and 2.6)
- Kindergarten Checkpoint 8, pp. 81–82
- Kindergarten Checkpoint 15, pp. 139–140

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Use materials that students can manipulate when exploring one-to-one correspondence. Avoid using drawings as students cannot move these to experiment with different strategies.
- Encourage students to work in pairs or small groups to facilitate discussion and shared thinking.
- Provide many opportunities for students to pose or answer questions such as
 - Who has fewer blocks? Who has more blocks?
 - Are there more boys or girls here today?
 - Is there a child here for every name tag?
 - Are there the same number of chairs and students?

SUGGESTED LEARNING TASKS

- Place 5 counters on a paper plate and ask students to make sets that have more counters than the given set of counters.
- Display two ten-frames, each representing a different number. Ask students, Which ten-frame has more counters? Which ten-frame has fewer counters?

- Ask students to reach into a bag of blocks, take some blocks in one hand, and place them on a paper plate. Then direct them to put fewer / more / the same number of blocks than this on another paper plate.
- Display 4 blocks on a piece of paper. Ask students to use counters to make a set that is the same.
- Have students work in pairs to play “Dot Challenge.” Provide a deck of dot cards (1 to 10 dots each) for each student. Each student turns up a card. The student who has the card with more dots gets both cards.
- Play the traditional game of dominoes where the students play their domino by matching it with one that has the same number of dots.
- Give each student a ten-frame and 10 counters. Have all students show you the number 9 with 5 counters in the top row and 4 in the bottom, filling from left to right. Now ask students to explain what they will do to display the number 6. Ask them, Will you remove or add counters to the ten-frame? Is 6 more than 9? How do you know?

SUGGESTED MODELS AND MANIPULATIVES

- counters
- dominoes
- dot cards
- linking cubes
- links
- ten-frames

MATHEMATICAL LANGUAGE

Teacher	Student (oral language)
<ul style="list-style-type: none"> ▪ compare quantities/sets ▪ more than, fewer than, as many as, or the same as ▪ one-to-one 	<ul style="list-style-type: none"> ▪ more than, fewer than, as many as, or the same as

Resources/Notes

Print Resources

- *Making Math Meaningful to Canadian Students, K–8* (Small 2009), pp. 87–88
- *Making Math Meaningful to Canadian Students, K–8, Second Edition* (Small 2013), pp. 144–145
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 37–38

Videos

- *Teaching Number: 0 to 9* (14:47 min.) (ORIGO Education 2010) (**Note:** Zero is not addressed in Mathematics Primary.)
- *Teaching Number: Counting* (10:49 min.) (ORIGO Education 2010)
- *Using a Hands-On Approach to Represent Numbers to 10* (13:06 min.) (ORIGO Education 2010)

Notes

SCO N06 Students will be expected to demonstrate an understanding of counting to 10.

[C, CN, ME, PS, R, V]

[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N06.01 Answer the question, “How many are in the set?” using the last number counted in a set.

N06.02 In a fixed arrangement, starting in different locations, show that the count of the number of objects in a set does not change.

N06.03 Count the number of objects in a given set, rearrange the objects, predict the new count, and recount to verify the prediction.

Scope and Sequence

<p>Mathematics Primary</p> <p>N06 Students will be expected to demonstrate an understanding of counting to 10.</p>	<p>Mathematics 1</p> <p>N03 Students will be expected to demonstrate an understanding of counting to 20 by</p> <ul style="list-style-type: none"> ▪ indicating the last number said identifies “how many” ▪ showing that any set has only one count ▪ using the counting-on strategy <p>N07 Students will be expected to demonstrate an understanding of conservation of number for up to 20 objects.</p>
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Background

Meaningful counting involves an understanding of six principles of counting. These principles are addressed in various outcomes (N01, N02, N03, and N06). The principles in bold face are the ones addressed in this outcome.

1. One number is said for each item in the group and is counted once and only once. (one-to-one correspondence)
2. Counting begins with the number 1, and there is a set number sequence. (stable order)
3. **The quantity in the set is the last number said. (cardinality)**
4. **The starting point and order of counting the objects does not affect the quantity. (order irrelevance)**
5. **The arrangement or types of objects does not affect the count. (conservation)**
6. It does not matter what is being counted, the resulting count will always be the same. (abstraction)

Stable order is specifically addressed in SCO N01; one-to-one correspondence, cardinality, and abstraction are addressed in SCO N03; and this outcome specifically addresses order irrelevance and conservation. Each principle should be assessed within their respective outcome, but counting should be addressed as a single concept connecting all three outcomes.

In this outcome, students start developing an understanding of order irrelevance and conservation; however, in Mathematics 1, the development of counting will continue with the expectation of fully understanding and applying all six principles.

Order Irrelevance: Order irrelevance is the understanding that regardless of how objects are counted (right to left, left to right, from the middle out), it does not affect the count. Many students in primary will not be convinced that a different starting point or a different order of counting the objects will result in the same counts.

Conservation: Conservation of number is the understanding that the number of objects remains the same when they are rearranged spatially. When students do not demonstrate conservation of number, they may believe that the number of objects can increase or decrease when they are pushed out or in. The development of this principle starts in Mathematics Primary and continues in Mathematics 1.

Students may correctly count a set; however, the counting is not meaningful until they have come to terms with these two principles.

Additional Information

See Appendix A: Additional Information.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Provide students with a set of 1–5 objects. Ask them to count the objects. Note how they are counting (see observation list in Assessment Tasks below).

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Observe and note the way in which students count.
 - Do they start at 1 and say the number words in the correct sequence?
 - Do they realize the last number said is the quantity of objects they have counted?
 - Do they show confidence in their count or feel the need to check?
 - Do they correctly predict the count if the objects they have just counted have been rearranged?
 - Do they correctly count objects if they begin at different starting points?
 - Do they check their counting in the same order as the first count or in a different order?

- Present students with a set of 8 objects. Ask them to count the objects. Ask them to show you 8. Note whether they point to the last object counted or whether they show the whole set of 8 objects.
- Ask students to count a set of objects that are close together. Then, push the objects out so they cover a larger area, and ask students to predict how many there are now.
- Ask students to take a handful of linking cubes and make a train with them. Ask students to count the cubes and tell how many cubes they have.
- Place 5 yellow cubes and 3 green cubes in a row. Ask students to count the cubes beginning with the yellow cubes. Ask them to predict how many they think they would get if they counted them again starting with the green cubes.
- Place a set of counters in a five- or ten-frame and ask students to count the objects from left to right. Ask students to predict how many they think they would get if they started counting at the right or in the middle.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets K–2 (Bauman 2011)

- Kindergarten Checkpoint 1, pp. 21–22
- Kindergarten Checkpoint 3, pp. 38–39

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Ask questions or make requests in everyday situations that encourage counting of up to 10 objects.
- Present a set of objects in a variety of arrangements (spread out, close together, in rows) and

encourage students to explore how the count remains the same regardless of the physical arrangement of the objects in the set or the starting point for counting.

- Expect students to explain, verbally, how they know how many are in a set.

SUGGESTED LEARNING TASKS

- Provide students with a set of objects to count. Ask them to place the objects in a container as they count. Direct students’ attention to the number of objects in the container after counting has ended.
- Place a set of counters in a ten-frame. Ask students to count the objects from left to right. Ask students to predict how many they think they would get if they started at the right or in the middle. Have them count the same set of objects from the right or from the middle. Direct their attention to the fact that the count is the same whether they started at the left, at the right, or from the middle.
- Ask students to count a set of objects that are close together. Then, push the objects out so they cover a larger area, and ask students to predict how many there are now. Have them recount the objects. Direct their attention to the fact that they got the same count both times.
- Have students work in pairs. Give each pair of students a train or tower of linking cubes. Have them take turns starting at different ends of the trains or towers to count the linking cubes. Ask them whether they have the same number of cubes in the train or tower.
- Ask students to press a calculator key to show the number of windows in the room or the number of students wearing glasses. Have another student count the same objects, but in a different order to confirm the original count.
- Fill a small jar with a different item each day. Select a student to count the objects each day. Have the class confirm the number by counting aloud as the student points to each item. Be sure to vary the objects in the counting jar.
- Ask students to count the number of steps it takes to travel a specific distance, such as from the classroom door to the teacher’s desk.

SUGGESTED MODELS AND MANIPULATIVES

- calculator
- counters
- five-frames
- ten-frame
- linking cubes

MATHEMATICAL LANGUAGE

Teacher	Student (oral language)
<ul style="list-style-type: none"> ▪ count the number of objects in a given set ▪ order, left, right, middle ▪ set of objects ▪ the counting numbers, 1 to 9 ▪ the last number counted ▪ the same count 	<ul style="list-style-type: none"> ▪ order, left, right, middle ▪ set of objects ▪ the counting numbers, 1 to 9 ▪ the same count

Resources/Notes

Print Resources

- *Making Math Meaningful to Canadian Students, K–8* (Small 2009), pp. 84–86, 95
- *Making Math Meaningful to Canadian Students, K–8, Second Edition* (Small 2013), pp. 140–142, 151
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 39–40

Videos

- *Teaching Number: 0 to 9* (14:47 min.) (ORIGO Education 2010) (**Note:** Zero is not addressed in Mathematics Primary.)
- *Teaching Number: Counting* (10:49 min.) (ORIGO Education 2010)
- *Using a Hands-On Approach to Represent Numbers to 10* (13:06 min.) (ORIGO Education 2010)

Notes

Patterns and Relations (PR)

GCO: Students will be expected to use patterns to describe the world and solve problems.

GCO: Students will be expected to represent algebraic expressions in multiple ways.

Specific Curriculum Outcomes

Process Standards Key

[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

PR01 Students will be expected to demonstrate an understanding of repeating patterns (two or three elements) by identifying, reproducing, extending, and creating patterns using manipulatives, sounds, and actions. [C, CN, PS, V]

SCO PR01 Students will be expected to demonstrate an understanding of repeating patterns (two or three elements) by identifying, reproducing, extending, and creating patterns using manipulatives, sounds, and actions.

[C, CN, PS, V]

[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- PR01.01** Distinguish between repeating patterns and non-repeating sequences in a given set by identifying the part that repeats.
- PR01.02** Copy a given repeating pattern and describe the pattern.
- PR01.03** Extend a variety of given repeating patterns to two more repetitions.
- PR01.04** Create a repeating pattern using manipulatives, musical instruments, or actions and describe the pattern.
- PR01.05** Identify and describe a repeating pattern in the classroom, the school, and outdoors.

Scope and Sequence

<p>Mathematics Primary</p> <p>PR01 Students will be expected to demonstrate an understanding of repeating patterns (two or three elements) by identifying, reproducing, extending, and creating patterns using manipulatives, sounds, and actions.</p>	<p>Mathematics 1</p> <p>PR01 Students will be expected to demonstrate an understanding of repeating patterns (two to four elements) by describing, reproducing, extending, and creating patterns using manipulatives, diagrams, sounds, and actions.</p> <p>PR02 Students will be expected to translate repeating patterns from one representation to another.</p>
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Background

Patterns exist in many aspects of life and in multiple disciplines. Nowhere is this more apparent than in the field of mathematics where patterns exist in every strand of the discipline, and the discernment of these patterns is often the very essence of learning and doing mathematics. Working with patterns enables students to see relationships, to find connections, to make generalizations and predictions, not only within mathematics, but also with other disciplines and with the world in general.

Patterns are created using various attributes, such as colour, size, shape, sound, and texture. These attributes should be the focus of students' first visual patterning. It is important that the materials used in initial experiences focus on only one attribute. For example, for colour patterns, the size and shape of the material should remain constant with only colour differences.

Students in Mathematics Primary need to experience repeating patterns in a variety of different ways. They need both teacher-directed and independent activities. Teacher-directed activities should encourage students to analyze a variety of patterns. Independent activities provide students with the opportunity to explore, identify, reproduce, extend, and create patterns appropriate to their level of understanding. These pattern explorations should include patterns such as AB, AAB, ABB, and ABC.

When teaching the concept of patterns, it is essential to use a wide variety of manipulatives, and musical and tactile rhythms and beats.

Patterns may be represented concretely, pictorially, orally, or kinesthetically. Students will require many patterning experiences with concrete materials prior to recording patterns on paper and/or working from patterns of pictures. Students in primary need to be exposed to many forms of patterning, and should be able to copy, reproduce, and describe given patterns before being expected to create their own.

Repeating patterns include the continued extension of a sequence beyond what the student can actually see. The **core** of a repeating pattern is the shortest set of **elements** that repeat. A non-repeating pattern has no detectable core, therefore the identification of a core determines if a pattern is repeating or non-repeating.



It may be more difficult for students to identify patterns where the first and last elements of the core are the same, for example, ABA patterns.

Additional Information

See Appendix A: Additional Information.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Provide students with objects of two colours. Ask them to make a pattern with the objects. Note the type of pattern the students create.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students to build these pattern trains with linking cubes (e.g., RGRGRG, BYYBYBY, GRGRGR, BBWBBWBBW, or YRYRYR). Ask, Which patterns are the same and why?
- Show students a pattern sequence with an error and ask them what they would change to fix the pattern.
- Create an ABABAB pattern on the overhead projector with different shapes. Show it to students and then cover the pattern. Ask students to describe or draw what would come next. Have them explain how they know. This can also be done using other patterns such as, ABBABBABB or ABCABCABC.
- Show students a set of linking cubes arranged in a repeating pattern. Ask students to
 - add the next correct piece in the pattern
 - continue the pattern with additional pieces at least twice and explain the extension
 - copy the existing pattern using sounds, actions, shapes, etc.
 - identify the core of the pattern in the longer pattern
- Tell students, I made a pattern with 10 linking cubes and then it fell apart. All I have left are three together—RBG. Show the three cubes to students. Ask them to use cubes to build what they think my pattern might have looked like. After students have completed one pattern, ask, Might it have looked another way? Encourage students to show you other possibilities. It is most important that you ask students to explain their thinking.
- Ask students to describe a pattern that they see in the classroom.
- Ask students to build a pattern train with linking cubes. Ask them to make a pattern using sounds that is the same as the pattern they built with cubes.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets K–2 (Bauman 2011)

- Kindergarten Checkpoint 7, pp. 72–73

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?

- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Provide students with many experiences to sort objects such as paper clips, blocks, toys, beads, buttons, crayons, cubes, and counters. so they can increase their awareness of attributes.
- Ask students to communicate patterns in various ways (e.g., words, letters, actions).
- Provide opportunities for students to draw simple shape patterns. Computer software may be used to aid students in creating patterns.
- Encourage students to display concrete patterns whenever possible, either with manipulatives or by enacting the pattern physically.
- Encourage students to demonstrate oral patterns, including singing or clapping/stomping.
- Expect students to explain, verbally, how they know what comes next in a pattern.
- Extend experiences with patterns further by exploring familiar skipping-rope chants and performing the actions for songs. Students also benefit from hearing stories with repeated or predictable language and repeating the patterns.

SUGGESTED LEARNING TASKS

- Create a forward and backward pattern by having one student face forward and the next student face backward until all are included in the pattern around the room.
- Read a story that has a pattern, and have the students repeat the pattern once they recognize it.
- Ask students to reproduce and extend patterns that focus on
 - auditory:** clap, clap, clap, snap, clap, clap, clap, snap, clap, clap, clap, snap, ...
 - colour:** red block, blue block, red block, blue block, red block, blue block, ...
 - shape:** ☺ ♥ ☆ ☺ ♥ ☆ ☺ ♥ ☆
- Provide a pattern and ask the student to represent it with an action pattern. For example, “green, green, blue, green, green, blue, green, green, blue, ...” might be represented by “clap, clap, snap, clap, clap, snap, clap, clap, snap, ...”
- Show students a repeating pattern. Ask them to identify the elements in the pattern. Ask them to identify the core of the pattern.

SUGGESTED MODELS AND MANIPULATIVES

- | | |
|-----------------|------------------|
| ▪ beads | ▪ links |
| ▪ colour tiles | ▪ paper clips |
| ▪ counters | ▪ pattern blocks |
| ▪ crayons | ▪ toys |
| ▪ linking cubes | |

MATHEMATICAL LANGUAGE

Teacher	Student (oral language)
<ul style="list-style-type: none"> ▪ core ▪ elements ▪ identify, describe, reproduce, extend, and create ▪ patterns ▪ repeating, non-repeating patterns 	<ul style="list-style-type: none"> ▪ core ▪ elements ▪ patterns ▪ repeating, non-repeating patterns

Resources/Notes**Print Resources**

-
- *Making Math Meaningful to Canadian Students, K–8* (Small 2009), pp. 26–27, 568–573
 - *Making Math Meaningful to Canadian Students, K–8, Second Edition* (Small 2013), pp. 603–611
 - *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 276–279

Video

-
- *Using a Teaching Sequence for Repeating Patterns* (22:55 min.) (ORIGO Education 2010)

Notes

Measurement

GCO: Students will be expected to use direct and indirect measure to solve problems.

Specific Curriculum Outcomes

Process Standards Key

[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

M01 Students will be expected to use direct comparison to compare two objects based on a single attribute, such as length, mass, volume, and capacity. [C, CN, PS, R, V]

SCO M01 Students will be expected to use direct comparison to compare two objects based on a single attribute, such as length, mass, volume, and capacity.

[C, CN, PS, R, V]

[C] Communication

[PS] Problem Solving

[CN] Connections

[ME] Mental Mathematics and Estimation

[T] Technology

[V] Visualization

[R] Reasoning

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- M01.01** Compare the length of two given objects and explain the comparison using words such as **shorter, longer, taller, or almost the same.**
- M01.02** Compare the mass of two given objects and explain the comparison using words such as **lighter, heavier, or almost the same.**
- M01.03** Compare the capacity of two given objects and explain the comparison using words such as **holds less, holds more, or holds almost the same.**
- M01.04** Compare the volume of two given objects and explain the comparison using words such as **bigger, smaller, or almost the same.**

Scope and Sequence

Mathematics Primary	Mathematics 1
<p>M01 Students will be expected to use direct comparison to compare two objects based on a single attribute, such as length, mass, volume, and capacity.</p>	<p>M01 Students will be expected to demonstrate an understanding of measurement as a process of comparing by</p> <ul style="list-style-type: none"> ▪ identifying attributes that can be compared ▪ ordering objects ▪ making statements of comparison ▪ filling, covering, or matching

Background

Students should realize that the same object can have many measurable attributes. Specifically, in Mathematics Primary, students will explore dimensions of 3-D objects (length, width, and height), their masses, their volumes, and their capacities by comparing two objects directly. They will come to realize that they can use their sight to predict length, volume, and capacity, but will have to use their sense of touch to compare masses. Students compare two objects using measurable attributes such as length, mass, volume, and capacity.

It is important that students compare measurable attributes in meaningful contexts. For example, they might compare two pencils to determine which is longer, compare two students to determine who is taller, compare two books to determine which is heavier, compare two boxes to determine which is bigger, or compare two containers to determine which holds more.

Encourage students to engage in conversations using accurate language to identify which attributes (length, mass, volume, or capacity) they can use to make comparisons. These attributes are best recognized in students' everyday experiences. For example, Scott is taller than Susan or my book bag is heavier than yours.

In length, volume, and capacity activities be sure to place the two objects that are to be compared a distance apart and/or in different positions. Ask the students to compare two objects and observe if

students use a reliable strategy and are comparing the correct attribute. Students should make direct comparisons by looking at and handling the objects.

Additional Information

See Appendix A: Additional Information.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Provide students with pieces of string of different lengths. Ask them to tell which string is longer.
- Provide students with two bags containing objects of different masses. Ask them to tell you which bag is heavier.
- Provide students with two containers. Ask them which container holds less.
- Provide students with two empty boxes of different sizes. Ask them to tell you which box is bigger.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Give each student a piece of string and three pieces of paper that are labelled “shorter,” “same,” and “longer.” Ask students to find two objects that are longer than the string, two objects that are shorter than the string, and two objects that are almost the same length as the string. On the paper, students could draw pictures of the objects they found for each category.
- Give each student three pieces of paper labelled heavier, same, and lighter. Ask students to take off one of their shoes and then find two objects that are heavier than the shoe, two objects that are lighter than the shoe, and two objects that are almost the same mass as the shoe. On the paper, students could draw pictures of the objects they found for each category.
- Give each student a box and three pieces of paper labelled more, same, and less. Ask students to find two containers that would hold more than the box, two containers that would hold almost the same, and two containers that would hold less. On the paper, students could draw pictures of the objects they found for each category.
- Give each student two objects (e.g., crayon, paper clip, pencil, ruler, or eraser). Ask students to predict and then determine which item is longer or shorter. Ask students to explain their thinking.

- Give each student two objects (e.g., crayon, paper clip, large book, ruler, or stapler). Ask students to predict and then determine which item is lighter or heavier. Ask students to explain their thinking.
- Give each student two containers (e.g., one-litre milk container, empty tuna can, individual milk container, two-litre milk container, or thimble). Ask students to predict and then determine which item holds more or less. Ask students to explain their thinking.
- Have students tell you, for each of the following statements, if it is possible or impossible and explain their thinking.
 - A cat is heavier than my mom.
 - A bathtub holds less than a jug of milk.
 - My arm is longer than my foot.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets K–2 (Bauman 2011)

- Kindergarten Checkpoint 9, pp. 89–90 (length and mass only)
- Kindergarten Checkpoint 1, pp. 97–98

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Encourage students to develop intuitive notions for length, mass, capacity, and volume, using direct comparison.
- Ensure students first make estimates and then check their predictions.
- Expect students to explain how they determined their measurements.
- Emphasize the importance of a base line, as students compare the lengths of objects (i.e., comparing by starting both objects at the same place).

- Explore the transitive understanding of measurement with students (i.e., if A is larger than B, and B is larger than C, then A must be larger than C).
- Discuss with students situations in which direct measurement is difficult (e.g., comparing two large objects that cannot be directly compared).

SUGGESTED LEARNING TASKS

- Ask students to make meaningful measurements, including
 - comparing structures they have built-in terms of height and width
 - comparing their heights and arm lengths; finding objects in the class that are shorter, longer, or about the same length as their foot
 - comparing the capacities of different types of cups, spoons, pails, boxes, and bowls
 - comparing the masses of different rocks, fruits, or vegetables
 - comparing the volume of different boxes in order to arrange them from smallest to biggest
- Ask students to describe the steps, in order, that one would take to decide which of two objects is longer, heavier, bigger, or holds more.
- Create a sorting station at which students sort objects as longer, shorter, or about the same as a specified object at the station. This activity can be extended to explore mass, capacity, and volume.
- Ask students to estimate and then determine which of two toys is heavier by using a pan balance.
- Show students a length of string. Ask them if they think they are taller than the string without allowing them to stand beside it. After making a prediction, students should measure themselves against the string. They can then find items in the classroom that are longer and shorter than the string.

SUGGESTED MODELS AND MANIPULATIVES

- string
- various containers
- linking cubes
- pan balance
- various boxes

MATHEMATICAL LANGUAGE

Teacher	Student (oral language)
<ul style="list-style-type: none"> ▪ bigger, smaller, almost the same as ▪ compare ▪ holds less, holds more, holds almost the same as ▪ length, width, height, mass, volume, and capacity ▪ lighter, heavier, almost the same as ▪ shorter, longer, taller, almost the same as 	<ul style="list-style-type: none"> ▪ bigger, smaller, almost the same as ▪ compare ▪ holds less, holds more, holds almost the same as ▪ lighter, heavier, almost the same as ▪ shorter, longer, taller, almost the same as

Resources/Notes

Print Resources

- *Making Math Meaningful to Canadian Students, K–8* (Small 2009), pp. 22–23, 416–417, 421–423, 371, 430–431
- *Making Math Meaningful to Canadian Students, K–8, Second Edition* (Small 2013), pp. 411–416, 418–419, 461–463, 467–468, 476–477
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 227–242

Notes

Geometry

GCO: Students will be expected to describe the characteristics of 3-D objects and 2-D shapes and analyze the relationships among them.

Specific Curriculum Outcomes

Process Standards Key

[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

G01 Students will be expected to sort 3-D objects using a single attribute. [C, CN, PS, R, V]

G02 Students will be expected to build and describe 3-D objects. [CN, PS, V]

SCO G01 Students will be expected to sort 3-D objects using a single attribute.

[C, CN, PS, R, V]

[C] Communication

[PS] Problem Solving

[CN] Connections

[ME] Mental Mathematics and Estimation

[T] Technology

[V] Visualization

[R] Reasoning

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

G01.01 Sort a given set of familiar 3-D objects using a single attribute, such as size or shape, and explain the sorting rule.

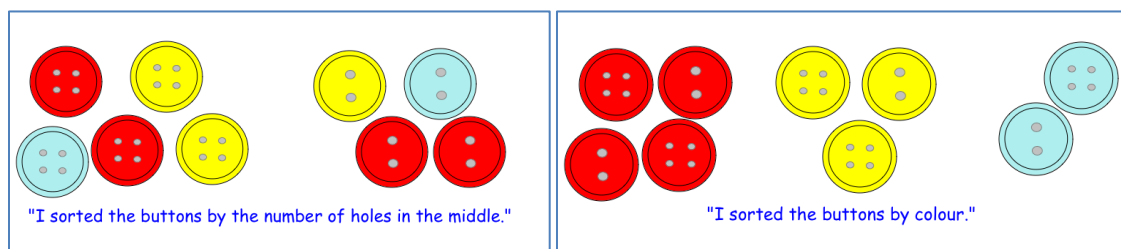
G01.02 Explain the sorting rule used to sort a pre-sorted set.

Scope and Sequence

Mathematics Primary	Mathematics 1
G01 Students will be expected to sort 3-D objects using a single attribute.	G01 Students will be expected to sort 3-D objects and 2-D shapes using one attribute and explain the sorting rule.

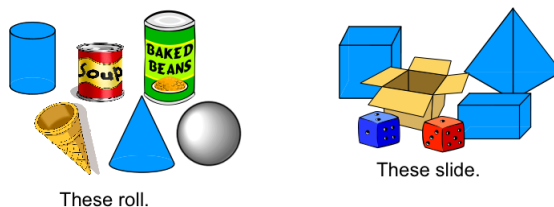
Background

Sorting objects into groups is a natural activity. If you watch preschoolers playing with blocks, buttons, or other objects, you will see them separating them into groups. For example, if they are playing with buttons, you would see the students putting all the white buttons in a group or all the buttons with four holes in another group. These early sorting experiences need to be fostered and extended in primary.



Before sorting 3-D objects, students can be introduced to the idea of sorting rules through activities involving sorting themselves into groups. For example, they could be sorted into groups: boys and girls; children with brown, blond, black, or other hair colours; left-handed and right-handed students; or students wearing sneakers and those wearing shoes. Choose a group of students who share something in common, such as all wearing sneakers, all wearing striped clothing, all having green eyes, or all wearing glasses. Once the selected students are standing in a group, the remaining classmates will identify the ways in which the group members are alike, which will result in the naming of the sorting rule you used. When the students have identified their rule, talk about other rules that might also apply. Ask students how they determined their sorting rule. It is important to provide students with opportunities to verbalize their sorting rule as this helps to build and solidify reasoning skills. It also enhances students' observations of multiple attributes.

Sorting activities should be planned using sets of 3-D objects found around the classroom or brought in from home specifically for sorting purposes, such as boxes, cans, jars, and other containers. Commercial sets of various 3-D wooden/plastic objects are available to provide students with other sorting opportunities, such as objects that come up to a point (pyramids and cones), objects that roll (cylinders and cones), and objects that do not roll (cubes and square/rectangular prisms). While there is no expectation that students will remember the names of the various shapes, these activities provide opportunities for them to hear the names as you use correct terminology.



As you work with students, model geometric terminology such as **circle**, **triangle**, **square**, **rectangle**, **sphere**, **cone**, **cylinder**, and **cube**; however, students are not expected to acquire this language in primary.

Additional Information

See Appendix A: Additional Information.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Provide students with a set of objects that can be sorted by one attribute. Ask them to sort them or to put them into groups. Ask them to explain their sorting rule.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

- Give students a series of 3-D shapes and ask them to sort the shapes into two groups, telling you how they decided to sort them. Make sure the objects are clearly related in at least two ways so that the students have some obvious choices (e.g., objects with rounded parts and objects with straight edges only, or very large objects and very small objects).
- Sort a set of objects into two groups. Ask students to explain your sorting rule. Ask a student to resort the objects. Ask students to explain the new sorting rule.
- Sort a set of objects into two groups leaving some objects out of the sort. Hold up one of the objects and ask students to identify where it belongs. Have students explain their thinking.
- Show students a group of objects that have been sorted. Ask them to describe the objects in each group.
- Ask a student to sort six other students into two groups. Ask the remaining members of the class to explain how the groups were sorted.
- Sort a set of eight students into two groups based on one attribute. Ask a student to identify the set to which he or she would belong and to explain his or her thinking.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets K–2 (Bauman 2011)

- Kindergarten Checkpoint 14, pp. 131–132 (Line Master 14.1)

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Use a variety of manipulatives and common objects for students to become familiar with the attributes of 3-D objects.
- Ask students to bring 3-D objects from home to share with the class. Have them tell one or two things that they find interesting about what they have brought from home.

SUGGESTED LEARNING TASKS

- Have students sort 3-D objects from around the room and block corner. Invite students to explain their sorting rule. Other questions could include
 - Explain why you put these objects together.
 - According to your (the) sort, where would ___ belong?
 - Which object does not belong to this set?
 - What other way could you sort these objects? Explain your sorting rule.
- Show students a set of 3D objects that you have sorted. Ask students to guess your sorting rule.
- Sort a set of objects into two groups leaving some objects out of the sort. Ask students to decide where the remaining objects belong and to justify their choice.

SUGGESTED MODELS AND MANIPULATIVES

- blocks
- modelling clay
- pattern blocks
- geometric solids
- collection of objects (boxes, cans, paper rolls, etc.)
- Polydrons

MATHEMATICAL LANGUAGE

Teacher	Student (oral language)
<ul style="list-style-type: none"> • 2-D shapes: triangles, squares, rectangles, circles • 3-D objects: pyramids, cones, prisms, cubes, cylinders • how they are alike / how they are different • smooth sides, sharp corners, ability to roll, ability to slide or ability to fit together, shapes that come up to a point • sorting groups of objects • sorting rule 	<ul style="list-style-type: none"> • 3-D objects • how they are alike / how they are different • smooth sides, sharp corners, ability to roll, ability to slide or ability to fit together, shapes that come up to a point • sorting groups of objects

Resources/Notes

Print Resources

- *Making Math Meaningful to Canadian Students, K–8* (Small 2009), pp. 19–21, 288–291
- *Making Math Meaningful to Canadian Students, K–8, Second Edition* (Small 2013), pp. 340–342, 344–347
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 191–195

Notes

SCO G02 Students will be expected to build and describe 3-D objects. [CN, PS, V]			
[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

G02.01 Create a representation of a given 3-D object using building blocks and compare the representation to the original 3-D object.

G02.01 Describe a given 3-D object using words such as big, little, round, like a box, or like a can.

Scope and Sequence

Mathematics Primary G02 Students will be expected to build and describe 3-D objects.	Mathematics 1 G02 Students will be expected to replicate composite 2-D shapes and 3-D objects. G03 Students will be expected to identify 2-D shapes in 3-D objects.
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Background

Students come to school having experience with 3-D objects. In fact, their most common experiences are those involving 3-D objects rather than 2-D shapes. They have likely played with building blocks, Lego, or other common children’s toys that involve construction of 3-D objects. Structuring meaningful contexts for students where they have opportunities to continue to explore, touch, manipulate, play, sort, and build with 3-D objects is important to developing spatial sense. Spatial sense involves visualization, mental imagery, and spatial reasoning. These skills are central to the understanding of mathematics.

Having a designated area of the classroom for building blocks and materials is important so students can regularly explore and experiment with 3-D objects. As students build, they begin to learn about the attributes of the various objects. It is through these investigations that students are able to learn the characteristics and properties of objects. Asking questions while the student is building with objects can provide valuable information regarding the student’s concept acquisition. Students’ exploration and development are enhanced by

- the teacher’s questions
- space and time to build and elaborate
- opportunities to discuss their creations
- time for free play or to make future additions to the structure

As you work with students, model geometric terminology such as **circle, triangle, square, rectangle, sphere, cone, cylinder, and cube**; however, students are not expected to acquire this language in primary. It is important for students to begin to understand some attributes of the various objects, such as big, little, round, like a box, like a can, or flat.

Additional Information

See Appendix A: Additional Information.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Show students a 3D object. Ask them to describe it for you. Note the language they use to describe it.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Show the class two 3-D objects. Ask students to describe in words or pictures the similarities and differences between the two shapes.
- Show the class a 3-D object built from linking cubes. Ask students to build an object like it with a different kind of block.
- Give each student enough modelling clay to build one or two objects. Ask students to build 3-D objects with the clay based on models you show (e.g., sphere, cylinder, and cube) and to describe how they are the same or different.
- Place four or five 3-D objects on a table and ask students to describe each one. For example, students might say, "This object looks like a can." "This one has points." "This one can roll." Cover the objects and remove one. Ask students to describe the missing object.
- Have students select shapes from the block corner to construct a model of a dog house (or other simple structure). When it is completed, encourage them to talk about their creations and to provide reasons for their designs (e.g., the kind/size of the dog, sleeping space, consideration of cold weather, size of model in relation to an actual dog house).

FOLLOW-UP ON ASSESSMENT**Guiding Questions**

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets K–2 (Bauman 2011)

- Kindergarten Checkpoint 13, pp. 122–123

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Use a variety of manipulatives and common objects for students to become familiar with the attributes of 3-D objects.
- Ask students to bring 3-D objects from home to share with the class. Have them tell one or two things that they find interesting about what they have brought from home.
- Have students identify the shapes needed to make a 3-D object.

SUGGESTED LEARNING TASKS

- Invite students to build imaginary buildings, monsters, or machines from 3-D objects such as boxes, cans, balls, paper cylinders, and cones. Ask them to describe their constructions.
- Have a student select, and hide from view, a wooden block. Ask him or her to describe the block to the class, one hint at a time, to see if they can determine which one was selected (or have the classmates guess by asking questions without using a 3-D object name).
- Ask one student to create a structure and ask his or her partner to create one that is quite different and to explain the differences using comparison words (big, small, tall, etc.). Also have students create a structure that is exactly the same as their partner.
- Use a bag or box with a cover for an “Object of the Day” activity. Place different objects inside the box or bag. Ask a student volunteer to reach into the box or bag and feel the object of the day

without seeing it and to describe it to the class using mathematics vocabulary (e.g., flat, round, points, curvy). Encourage students to predict the object that is in the bag based on the volunteer's description.

SUGGESTED MODELS AND MANIPULATIVES

- blocks
- modelling clay
- attribute blocks
- pattern blocks
- geometric solids
- collection of objects (boxes, cans, paper rolls, etc.)
- Polydrons

MATHEMATICAL LANGUAGE

Teacher	Student (oral language)
<ul style="list-style-type: none"> • 2-D shapes: circle, triangle, rectangle, square • 3-D object • 3-D objects: sphere, prism, pyramid, cone, cylinder, and cube • big, little, round, like a box, like a can, flat 	<ul style="list-style-type: none"> • 3-D objects • big, little, round, like a box, like a can, flat

Resources/Notes

Print Resources

- *Making Math Meaningful to Canadian Students, K–8* (Small 2009), pp. 19–21, 284–286
- *Making Math Meaningful to Canadian Students, K–8, Second Edition* (Small 2013), pp. 340–343
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 199–200

Notes

Appendices

Appendix A: Additional Information

Number (N)

SCO N01 Students will be expected to say the number sequence by			
<ul style="list-style-type: none"> ▪ 1s from 1 to 20 ▪ 1s, starting anywhere from 1 to 10 and from 10 to 1 			
[C, CN, V]			
[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- N01.01** Recite the number sequence from 1 to 20 and from 10 to 1.
- N01.02** Name the number that comes after a given number, 1 to 9.
- N01.03** Name the number that comes before a given number, 2 to 10.
- N01.04** Recite number names from a given number to a stated number (forward 1 to 10, backward 10 to 1) using visual aids.

Performance Indicator Background

N01.01 During the course of daily tasks, students should have frequent practice saying the number sequence from 1 to 10. The objective is to ensure that students know the counting sequences from 1 to 10 and from 10 to 1 so well that they can independently recite the number sequences correctly.

Later in the year, students should be expected to extend the counting sequence to 20. Learning the sequence of number names from 11 to 20 is challenging for young students. They simply have to remember that after ten comes eleven, twelve, thirteen, and so on. Knowing these number names is conventional knowledge; therefore, it will take many and frequent reinforcement activities to solidify and sustain these names.

N01.02 and N01.03 Spontaneously stating the number after or before a given number is an important step in the development of number sense in primary and is critical for performance indicator N01.04. Start by asking students to state the number after a given number (e.g., state a number and the students chant the number that comes after it). It could develop into leading a rhythmic chant: “I say 6, and you say __; I say 3, and you say __; I say 9, and you say __”; etc. This same approach can be applied to practice the number that comes before a given number.

N01.04 Students should also be able to recite the number names forward and backward between two given numbers. Starting and ending numbers should be in the range of 1 to 10. With frequent experiences (daily), students should become proficient at reciting forward and backward from any number, so much so that they are able to begin without any hesitation. If there is a delay, they may be counting internally from 1 or from 10 until they reach the starting number.

SCO N02 Students will be expected to recognize, at a glance, and name the quantity represented by familiar arrangements of 1 to 5 objects or dots.

[C, CN, ME, V]

[C] Communication

[PS] Problem Solving

[CN] Connections

[ME] Mental Mathematics and Estimation

[T] Technology

[V] Visualization

[R] Reasoning

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N02.01 Look briefly at a given familiar arrangement of 1 to 5 objects or dots and identify the number represented without counting.

N02.02 Identify the number represented by a given dot arrangement on a five-frame.

Performance Indicator Background

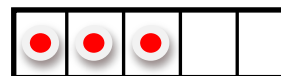
N02.01 Students should recognize that there are many ways to arrange a set of objects and that some arrangements are easier to subitize than others. Most students are familiar with using their fingers to represent a number; therefore, this is a good representation with which to start. When asking students to identify the number of fingers being displayed, use different combinations of fingers so that students do not believe that there is only one way to represent the number. For example, any two fingers represent the quantity of 2; however, many students may think that only the middle and index fingers represent 2 because these two fingers are the only ones that have been used when they have heard or said “two.” For these students, 2 may be seen as a shape rather than a quantity.

When students begin learning the arrangements of dots up to 5, they should start with the arrangements found on a standard die. Once students have had experience with this set of arrangements, other arrangements can be introduced. For most numbers, there are several common arrangements. These dot arrangements should be presented using paper plates, dot cards, as well as a standard die. It is important to note that as students are using a standard die they may be able to subitize this familiar arrangement for 6 but would not be evaluated on it.

N02.02 Students should be able to recognize, at a glance, any number up to 5 in a five-frame. A five-frame is a rectangle of five boxes with each box large enough to hold a counter. Five-frames focus on the relationship to five as an anchor for numbers. For example, this five-frame represents 3.

When using a five-frame, students should know that

- only one counter is permitted in each box of the five-frame
- counters are typically placed in the five-frame from the left to the right (the same way they read or write).



When subitizing, it is important to use the familiar (typical) arrangement; however, when students are creating their own representations of a number, other arrangements are acceptable. Once students have experience with five-frames and how each number is represented in a five-frame they will be ready to work on subitizing them. Display for three seconds a number represented in a five-frame. Ask students to sketch or use stickers to show on an empty five-frame what they saw. Repeat using other numbers. Ask students to tell how many there are.

SCO N03 Students will be expected to relate a numeral, 1 to 10, to its respective quantity. [CN, R, V]			
[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- N03.01** Name the number for a given set of objects.
- N03.02** Match numerals with their given pictorial representations.
- N03.03** Hold up the appropriate number of fingers for a given numeral.
- N03.04** Construct a set of objects corresponding to a given numeral.
- N03.05** Record the numeral that represents the quantity of a given set of objects.

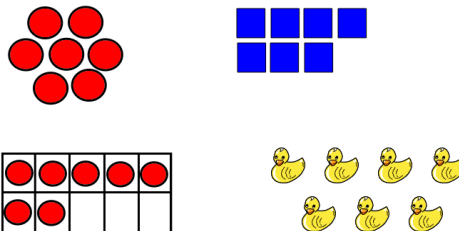
Performance Indicator Background

N03.01 When determining the number for a given set of objects, watch to see that students are using one-to-one correspondence. Listen to how the students are counting and verify that they start at 1 and say the number words in the correct sequence. Verify that students are realizing the last number said is the quantity of objects they have counted. Students should be able to demonstrate this on a variety of different sets of objects; therefore, a variety of materials should be used. Numbers from 1 to 10 should also be displayed in five- and ten-frames for students to name.

N03.02 Students should experience counting and identifying the quantity of concrete objects before counting images printed on paper. Eventually students start relating the numeral (the symbolic representation of a number) to the number of objects in a set, presented pictorially. For example, students can be given a set of numeral cards and asked to hold up the appropriate card that represents a picture of a set of objects.

N03.03 Students should be able to hold up the number of fingers that represent a given numeral being displayed. They should notice that different combinations of fingers can be used to represent the same numeral. For example, any six fingers can represent the numeral 6: 5 on one hand and 1 on the other, 4 on one hand and 2 on the other, or 3 on each hand.

N03.04 Once students are able to determine the number of objects in a set, they should create sets of objects for given numbers. Initially, a number should be presented orally for students to create a set of that quantity. Once students are able to match numerals with quantities, they should create sets of objects that represent any given numeral. For example, students can show 7 in a variety of ways, such as those below.



N03.05 This indicator is an extension of indicators N03.01 and N03.02 because students will be expected to not only name the number of objects and match it to its numeral, but also to record the appropriate numeral. Since it is important that students develop an efficient means of recording numerals, numeral writing should be taught as students are ready to record the appropriate numeral(s). Specific instruction and practise will be necessary. Integrating digit formation is an emergent writing opportunity that supports students in the development of writing skills. Initially, writing digits should not include the use of lined paper. Allow students to experiment freely on unlined paper using crayons and pencils or by using their fingers in a tray of sand. By the end of primary, students should be introduced to writing numerals on lined paper.

Observe students as they write numerals, both when copying from a model and when forming them from memory. While there are other acceptable ways to form numerals, students should be encouraged to start at the top when printing numerals. Patterns in the formation of numerals could be highlighted as well. For example, 2, 3, and 7 start at the top and go to the right; 5, 6, 8, and 9 start at the top and go to the left; 1 and 4 start at the top and go down. In these early years, it is not unusual for many students to reverse some of the digits as they write them. These students could be helped by getting them to compare the digits they write with ones you write, and by reminding them how to start making the digit. This practice should always be done in context.

SCO N04 Students will be expected to represent and describe numbers 2 to 10 in two parts, concretely and pictorially.

[C, CN, ME, R, V]

[C] Communication

[PS] Problem Solving

[CN] Connections

[ME] Mental Mathematics and Estimation

[T] Technology

[V] Visualization

[R] Reasoning

Performance Indicators

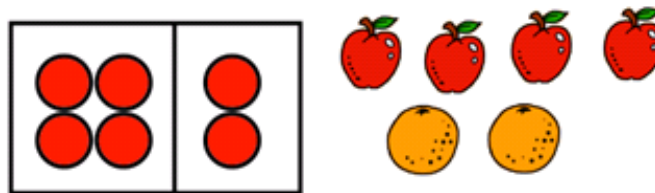
Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N04.01 Show a given number as two parts (using fingers, counters, or other objects) and name the number of objects in each part.

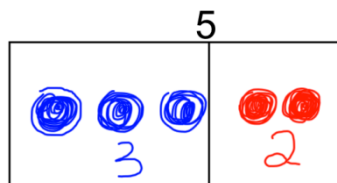
N04.02 Show a given number as two parts, using pictures, and name the number of objects in each part.

Performance Indicator Background

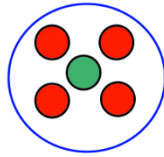
N04.01 To conceptualize a number being made up of two or more parts is the most important relationship that can be developed about numbers, and it will take time for students to internalize this idea. The number 4 is a good place to start. It can be represented by raising 3 fingers on one hand and 1 finger on the other hand, or by raising 4 fingers on one hand and none on the other, or by 2 fingers on each hand. A student should also be able to name the parts of 4 by saying, “four is three and one, four and none, or two and two.” A variety of representations and materials should be used to avoid the misconception that partitioning can only be represented using fingers. For example, below are various partitions of 6.



N04.02 While using objects to represent part-part-whole relationships is a good beginning, students should also draw how they represent numbers in two parts using pictures. Eventually, they should also name the quantity in each part by recording the respective numeral beside or below each part.



The main strategy that students in primary will use as they construct part-part-whole relationships is counting. However, subitizing activities can also reinforce part-part-whole thinking. For example, if the 5 pattern on a die is represented with 4 red dots and 1 green dot on a paper plate, the students are asked, How many dots do you see? How many are red? How many are green?



SCO N05 Students will be expected to compare quantities, 1 to 10, using one-to-one correspondence.

[C, CN, V]

[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N05.01 Construct a set to show more than, fewer than, or as many as a given set.

N05.02 Compare two given sets through direct comparison and describe the sets using words, such as more, fewer, as many as, or the same number as.

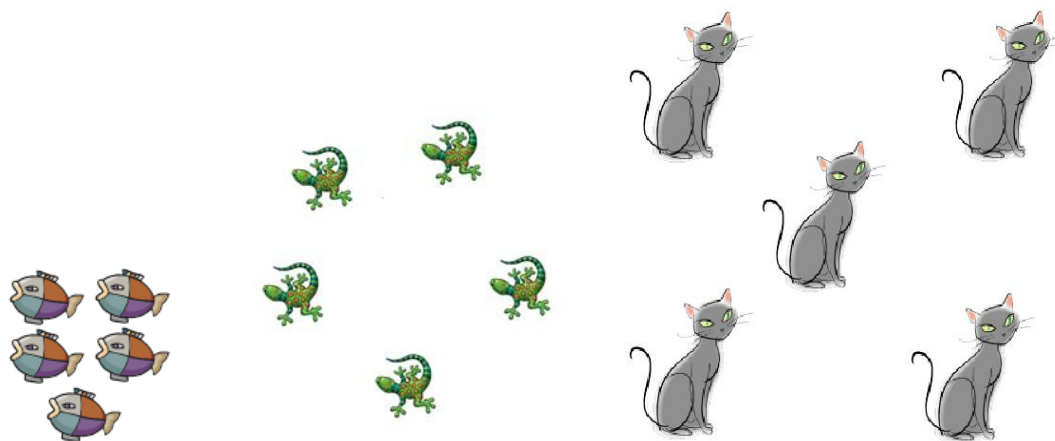
Performance Indicator Background

N05.01 Students should engage in activities in which they create sets that are more than, fewer than, and as many as a given set; and in which they create two sets that are more than, fewer than, or as many as one another. For example,

- place 5 counters on a paper plate and ask students to make sets that have more counters than this set
- ask students to reach into a bag of blocks, take some blocks in one hand, and place them on a paper plate, and then direct them to put fewer blocks than this on another paper plate
- display 4 blocks on a piece of paper (Ask students to use counters to make a set of the same size.)

N05.02 Given the concrete focus of students at this stage, comparing activities should frequently involve the use of concrete materials or pictorial representations of objects that can be matched. Pictorial representations should involve multiple cards, with one picture per card, so students will be able to manipulate them. When students are able to move the objects, they can experiment and start over again. Students should have experiences comparing and describing sets in which

- the items go together, such as forks and spoons
- the items are unrelated, such as glue containers and books
- the sets have the same quantity but take up a different amount of space, such as all these pictures of 5 objects



In primary, it is expected that students will make direct comparisons (one-to-one correspondence) when comparing sets of concrete objects. Students should realize that if the sets are very different in number and the objects are of similar size, one-to-one is not really needed. When there are similar-sized objects and the sets are close in number or when the objects are quite different in size using one-to-one is useful. Using five- and ten-frames is a great way to compare two sets. With practice, students will be able to quickly identify which set has more or fewer when displayed this way. Counting two sets of objects and making a comparison just by using those two counts requires more advanced thinking. It will take time for students to be convinced that 8 of something will always be more than 5 of the same thing, or 5 of something else. However, students will eventually be able to distinguish more, fewer, and the same numbers by counting, and will not always have to use one-to-one matching.

When comparing quantities, students may use strategies that involve subitizing, which means the student can recognize the number of objects without counting. This strategy becomes more challenging and less reliable as

- the number of objects increases
- the objects in the sets are of different sizes
- the spacing between the objects is different in the two sets

SCO N06 Students will be expected to demonstrate an understanding of counting to 10.

[C, CN, ME, PS, R, V]

[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- N06.01** Answer the question, “How many are in the set?” using the last number counted in a set.
- N06.02** In a fixed arrangement, starting in different locations, show that the count of the number of objects in a set does not change.
- N06.03** Count the number of objects in a given set, rearrange the objects, predict the new count, and recount to verify the prediction.

Performance Indicator Background

N06.01 Counting to find out how many objects are in a set is also addressed in SCO N03. It is important that students see the purpose in learning how to count and use it to perform common tasks. Ask questions or make requests in everyday situations that encourage counting of up to 10 objects. During these activities, observe whether students are spontaneously using counting as a strategy to complete these tasks; if not, demonstrate the use of counting or have a student share his or her strategy. This can be incorporated at any time during the day, not just during mathematics class.

N06.02 Have students explore the order irrelevance principle. For example, place a set of counters in a five- or ten-frame and ask the students to count the objects from left to right. After students have counted the number of objects in the set, explore the order irrelevance principle by asking the students to predict how many they think they would get if they started at the right or in the middle. (They may be unsure or predict a different number than the one they just counted. These responses indicate that order irrelevance is not yet part of their understanding.) Have them count the same set of objects from the right or from the middle, and focus their attention on the fact that they got the same count when they started at the left, at the right, or from the middle. (They may still believe that this is a coincidence!)

N06.03 Similarly, have students explore the conservation principle. For example, have students count a set of objects that are close together, then push the objects out so they cover a larger area, and ask them to predict how many there are now. (They may be unsure or predict a different number than the one they just counted. These responses indicate that conservation is not yet part of their understanding.) Have them recount the objects and focus their attention on the fact that they got the same count both times. (Again, they may still believe that this is a coincidence!)

Patterns and Relations (PR)

SCO PR01 Students will be expected to demonstrate an understanding of repeating patterns (two or three elements) by identifying, reproducing, extending, and creating patterns using manipulatives, sounds, and actions.

[C, CN, PS, V]

[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

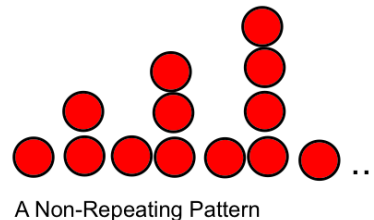
Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- PR01.01** Distinguish between repeating patterns and non-repeating sequences in a given set by identifying the part that repeats.
- PR01.02** Copy a given repeating pattern and describe the pattern.
- PR01.03** Extend a variety of given repeating patterns to two more repetitions.
- PR01.04** Create a repeating pattern using manipulatives, musical instruments, or actions and describe the pattern.
- PR01.05** Identify and describe a repeating pattern in the classroom, the school, and outdoors.

Performance Indicator Background

PR01.01 Repeating patterns include the continued extension of a sequence beyond what the student can actually see. The core of a repeating pattern is the shortest set of elements that repeats. A non-repeating pattern has no detectable core, therefore the identification of a core determines if a pattern is repeating or non-repeating.



It may be more difficult for students to identify patterns where the first and last elements of the core are the same, for example, ABA patterns.

PR01.02 Students are expected to reproduce and describe a pattern. For example, the AB pattern red, blue, red, blue, red, blue can be described as having a core of two different elements, red and blue that repeats and is, therefore, a two-element repeating pattern.



PR01.03 Whether students are working with simple or complex patterns, encourage them to extend their pattern two more repetitions with their chosen manipulative. Occasionally, students should be asked to extend the pattern as far as they can, as this will help them solidify their understanding that patterns go on and on. Repeating the core of a pattern three times is more likely to lead students to the anticipated result; however, there may be alternative extensions unless the pattern rule is described.

PR01.04 When creating their own patterns, students should work with concrete materials that allow them to explore and be flexible with their patterning creations. Such concrete patterns also enable students to revisit and extend patterning ideas. They should be able to describe the patterns they have created. Students enjoy the challenge of trying to determine the patterns in other students' work, so sharing their created patterns with other students is a very worthwhile activity. Students enjoy making patterns using their own bodies as they click their tongues, snap their fingers, pat their knees, or tap their toes to copy and create given patterns.

To help students understand the core of a pattern, provide students with two or more bingo daubers. Have them use the bingo daubers to create the core of a pattern on a small strip of paper and then tape that pattern to their desks. Then, ask them to create a pattern by iterating (repeating) the core on adding machine tape or long strips of paper.

PR01.05 Provide students with opportunities to see patterns that are in their environment; for example, in the classroom, outdoors, or on their clothes. Take advantage of opportunities that arise in the classroom to identify patterns. For example, line students up to go for physical education according to different patterns. For example, boy, girl, boy, girl, boy, girl, ... or short sleeve, long sleeve, short sleeve, long sleeve, short sleeve, long sleeve,

Measurement (M)

SCO M01 Students will be expected to use direct comparison to compare two objects based on a single attribute, such as length, mass, volume, and capacity.

[C, CN, PS, R, V]

[C] Communication

[PS] Problem Solving

[CN] Connections

[ME] Mental Mathematics and Estimation

[T] Technology

[V] Visualization

[R] Reasoning

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- M01.01** Compare the length of two given objects and explain the comparison using words such as **shorter, longer, taller, or almost the same.**
- M01.02** Compare the mass of two given objects and explain the comparison using words such as **lighter, heavier, or almost the same.**
- M01.03** Compare the capacity of two given objects and explain the comparison using words such as **holds less, holds more, or holds almost the same.**
- M01.04** Compare the volume of two given objects and explain the comparison using words such as **bigger, smaller, or almost the same.**

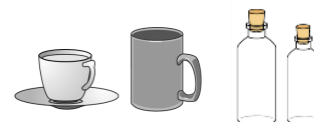
Performance Indicator Background

M01.01 Students generally find comparisons of length easier than comparisons of mass, volume, and capacity; therefore, length activities should be the first ones they encounter. It is also easier initially for students to concentrate on the attribute under discussion if the objects' other attributes are more alike. For example, if they are asked to compare two pencils to determine which is longer, it would be beneficial if those two pencils were both the same kind and colour so the only attribute that is different is length. With more experiences, students will be able to concentrate on the attribute being asked about and will not be distracted by other differences the objects may have. Students will line up the two pencils side by side to compare them directly and say the pencil on top is longer than the pencil on the bottom.



M01.02 Activities related to comparing masses should include different types of objects so comparisons must be made by lifting the objects rather than by visual means. For example, a piece of styrofoam may be bigger than a 1-L carton of milk and yet not as heavy. Students can determine which is heavier or lighter by picking them up before using a double pan balance.

M01.03 and M01.04 It is particularly important in volume and capacity comparisons that minimizing the number of differences is carefully considered. Students have difficulty taking into account the effect of differences in two dimensions on capacity and volume. Objects should initially have only one dimension difference. For example, 250-mL, 500-mL, and 1-L milk cartons have the same square footprint, so these could be used and cut to different heights to provide activities related to questions of which holds more or less. Students should determine which holds more by filling one container and pouring the contents into the other container. For volume, students should place one object on top of, or beside, the other to determine which object is larger or smaller.



Geometry (G)

SCO G01 Students will be expected to sort 3-D objects using a single attribute. [C, CN, PS, R, V]			
[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

Performance Indicators

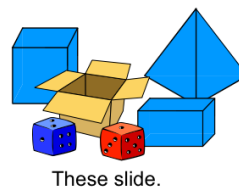
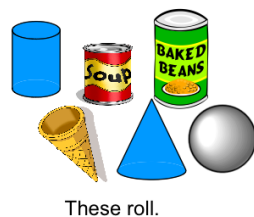
Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- G01.01** Sort a given set of familiar 3-D objects using a single attribute, such as size or shape, and explain the sorting rule.
- G01.02** Explain the sorting rule used to sort a pre-sorted set.

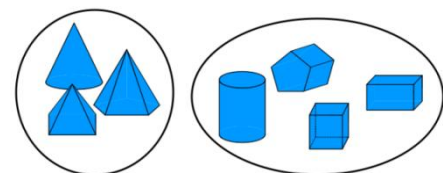
Performance Indicator Background

G01.01 In order to sort groups of objects, students need to be able to visually discriminate between objects and discuss how they are alike and how they are different. Therefore, students could be initially asked to examine one object looking for things such as flat surface, rounded surface, sharp corners, straight edges, ability to slide, ability to roll, and ability to fit together to make a bigger one of the same shape. Then students could be asked to compare just two objects looking for how they are alike and how they are different.

Sorting activities should be planned using sets of 3-D objects found around the classroom or brought in from home specifically for sorting purposes, such as boxes, cans, jars, and other containers. Commercial sets of various 3-D wooden/plastic objects are available to provide students with other sorting opportunities, such as objects that come up to a point (pyramids and cones), objects that roll (cylinders and cones), and objects that do not roll (cubes and square/rectangular prisms). While there is no expectation that students will remember the names of the various shapes, these activities provide opportunities for them to hear the names as you use correct terminology.



G01.02 As well as being able to sort 3-D objects with assigned rules or rules of their own, students should be able to determine the sorting rule someone else has used to sort sets of objects. For example, students could be presented with two sets of objects and asked to determine what rule was used to sort the shapes into these two groups. What's my sorting rule?



SCO G02 Students will be expected to build and describe 3-D objects.

[CN, PS, V]

[C] Communication

[PS] Problem Solving

[CN] Connections

[ME] Mental Mathematics and Estimation

[T] Technology

[V] Visualization

[R] Reasoning

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

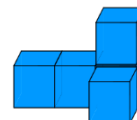
G02.01 Create a representation of a given 3-D object using building blocks and compare the representation to the original 3-D object.

G02.01 Describe a given 3-D object using words such as big, little, round, like a box, or like a can.

Performance Indicator Background

G02.01 Students should be given a 3-D object and asked to build one like it, and then asked, how is your creation the same as the given object? How is it different?

For example, show this shape built with cubes and ask students to build it with their blocks.



G02.02 As students are asked to describe objects, they have opportunities to explore geometric vocabulary. After they build 3-D objects, ask them questions such as, What words can you use to describe your creation? (round, flat, shaped like a box).

As you work with students, model geometric terminology such as circle, triangle, square, rectangle, sphere, cone, cylinder, and cube; however, students are not expected to acquire this language in primary. It is important for students to begin to understand some attributes of the various objects, such as big, little, round, like a box, like a can, or flat.

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Mathematics 1



CURRICULUM

Mathematics 1

**Implementation Draft
May 2013**

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Mathematics 1, Implementation Draft

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Introduction

Background and Rationale

Mathematics curriculum is shaped by a vision that fosters the development of mathematically literate students who can extend and apply their learning and who are effective participants in society. It is essential that the mathematics curriculum reflect current research in mathematics instruction. To achieve this goal, Western and Northern Canadian Protocol's (WNCP) *The Common Curriculum Framework for K–9 Mathematics* (2006) has been adopted as the basis for the new mathematics curriculum in Nova Scotia.

The Common Curriculum Framework was developed by the seven ministries of education (Alberta, British Columbia, Manitoba, Northwest Territories, Nunavut, Saskatchewan, and Yukon Territory) in collaboration with teachers, administrators, parents, business representatives, post-secondary educators, and others. The framework identifies beliefs about mathematics, general and specific student outcomes, and performance indicators agreed upon by the seven jurisdictions. The outcomes and performance indicators have been adapted for Nova Scotia. This document is based on both national and international research by the WNCP and the National Council of Teachers of Mathematics (NCTM).

There is an emphasis in the Nova Scotia curriculum on particular key concepts at each grade that will result in greater depth of understanding and, ultimately, stronger student achievement. There is also a greater emphasis on number sense and operations concepts in the early grades to ensure students develop a solid foundation in numeracy.

Purpose

This document provides sets of outcomes and performance indicators to be used as a mandated common base for defining mathematics curriculum expectations. This common base should result in consistent student outcomes in mathematics within the province of Nova Scotia. It should also enable easier transfer for students moving within the province or from any jurisdiction that has adopted the WNCP framework. This document is intended to clearly communicate to all education partners across the province the high expectations for students' mathematical learning.

Program Design and Components

Assessment

Ongoing assessment for learning is essential to effective teaching and learning. Research has shown that assessment for learning (formative assessment) practices produce significant and often substantial learning gains, close achievement gaps, and build students' ability to learn new skills (Black & Wiliam 1998; OECD 2006). Student involvement in assessment promotes learning. Timely and effective teacher feedback and student self-assessment allow students to reflect on and articulate their understanding of mathematical concepts and ideas.

Assessment in the classroom includes

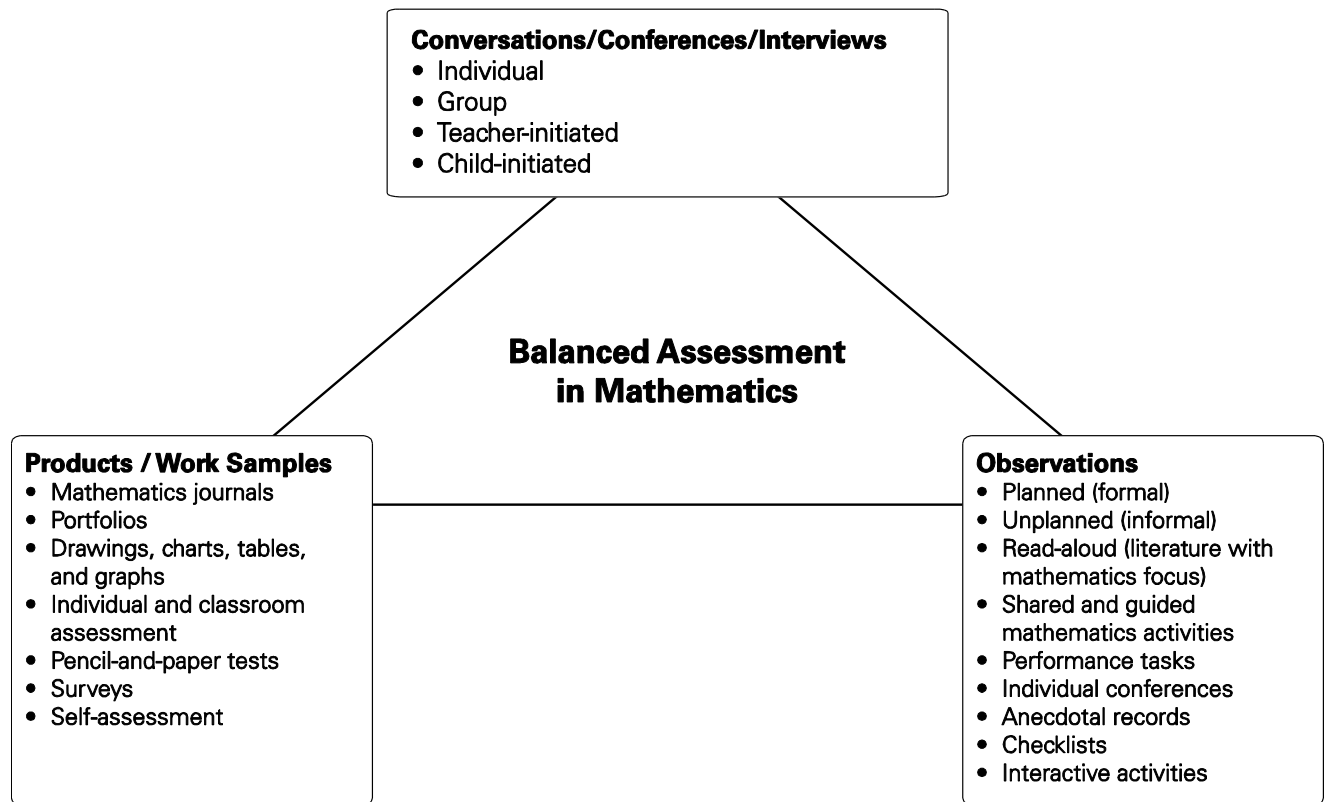
- providing clear goals, targets, and learning outcomes
- using exemplars, rubrics, and models to help clarify outcomes and identify important features of the work
- monitoring progress towards outcomes and providing feedback as necessary
- encouraging self-assessment
- fostering a classroom environment where conversations about learning take place, where students can check their thinking and performance and develop a deeper understanding of their learning

(Davies 2000)

Assessment for learning practices act as the scaffolding for learning, which only then can be measured through assessment of learning (summative assessment). Assessment of learning tracks student progress, informs instructional programming, and aids in decision making. Both forms of assessment are necessary to guide teaching, stimulate learning, and produce achievement gains.

Assessment of student learning should

- align with curriculum outcomes
- clearly define criteria for success
- make explicit the expectations for students' performance
- use a wide variety of assessment strategies and tools
- yield useful information to inform instruction



Time to Learn for Mathematics

The Time to Learn Strategy *Guidelines for Instructional Time: Grades Primary–6* includes time for mathematics instruction in the “Required Each Day” section. In order to support a constructivist approach to teaching through problem solving, it is highly recommended that the 45 minutes required daily in grades primary–2 and the 60 minutes required daily for grades 3–6 mathematics instruction be provided in an uninterrupted block of time.

Time to Learn guidelines can be found at

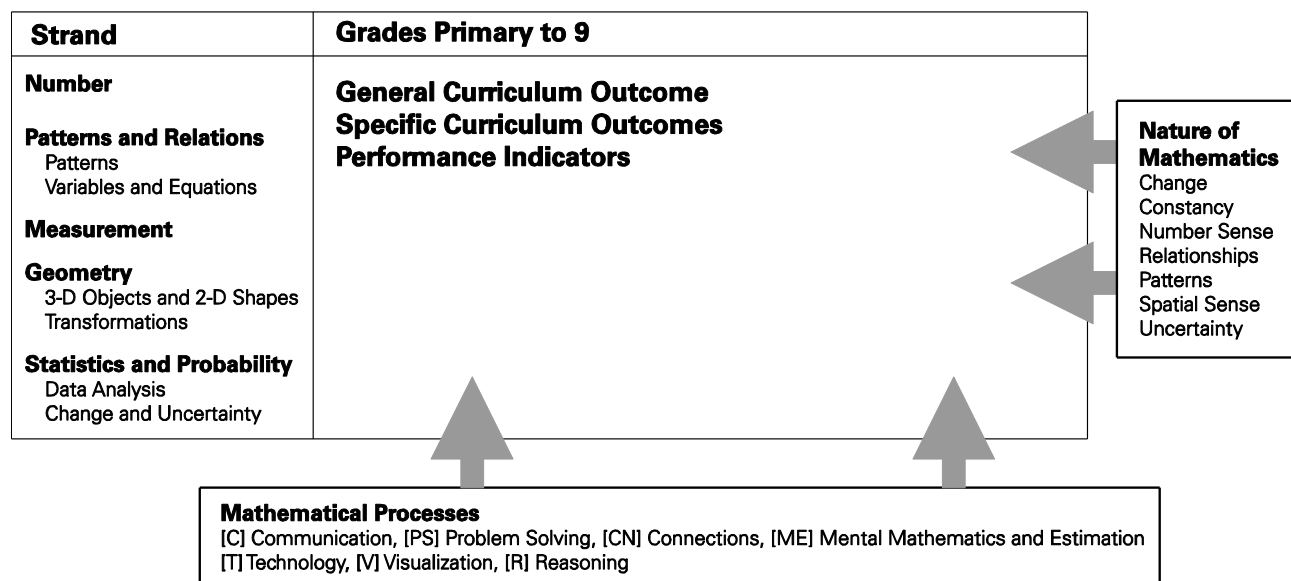
www.ednet.ns.ca/files/ps-policies/semestering.pdf

www.ednet.ns.ca/files/ps-policies/instructional_time_guidelines_p-6.pdf

Outcomes

Conceptual Framework for Mathematics Primary–9

The chart below provides an overview of how mathematical processes and the nature of mathematics influence learning outcomes.



(Adapted with permission from Western and Northern Canadian Protocol, *The Common Curriculum Framework for K–9 Mathematics*, p. 5. All rights reserved.)

Structure of the Mathematics Curriculum

Strands

The learning outcomes in the Nova Scotia Framework are organized into five strands across grades primary to 9.

- Number (N)
- Patterns and Relations (PR)
- Measurement (M)
- Geometry (G)
- Statistics and Probability (SP)

General Curriculum Outcomes (GCO)

Some strands are further subdivided into sub-strands. There is one general outcome (GCO) per sub-strand. GCOs are overarching statements about what students are expected to learn in each strand/sub-strand. The general curriculum outcome for each strand/sub-strand is the same throughout the grades.

NUMBER (N)

GCO: Students will be expected to demonstrate number sense.

PATTERNS AND RELATIONS (PR)

Patterns

GCO: Students will be expected to use patterns to describe the world and solve problems.

Variables and Equations

GCO: Students will be expected to represent algebraic expressions in multiple ways.

MEASUREMENT (M)

GCO: Students will be expected to use direct and indirect measure to solve problems.

GEOMETRY (G)

3-D Objects and 2-D Shapes

GCO: Students will be expected to describe the characteristics of 3-D objects and 2-D shapes and analyze the relationships among them.

Transformations

GCO: Students will be expected to describe and analyze position and motion of objects and shapes.

STATISTICS AND PROBABILITY (SP)

Data Analysis

GCO: Students will be expected to collect, display, and analyze data to solve problems.

Chance and Uncertainty

GCO: Students will be expected to use experimental or theoretical probabilities to represent and solve problems involving uncertainty.

Specific Curriculum Outcomes (SCOs) and Performance Indicators

Specific curriculum outcomes (SCOs) are statements that identify the specific conceptual understanding, related skills, and knowledge students are expected to attain by the end of a given grade.

Performance indicators are statements that identify specific expectations of the depth, breadth, and expectations for the outcome. Teachers use these statements to determine whether students have achieved the corresponding specific curriculum outcome.

Process Standards Key

[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

NUMBER (N)

- N01** Students will be expected to say the number sequence by
- 1s, forward and backward between any two given numbers, 0 to 100
 - 2s to 20, forward starting at 0
 - 5s to 100, forward starting at 0, using a hundred chart or a number line
 - 10s to 100, forward starting at 0, using a hundred chart or a number line [C, CN, V, ME]

Performance Indicators

- N01.01 Recite forward by 1s the number sequence between two given numbers, 0 to 100.
 N01.02 Recite backward by 1s the number sequence between two given numbers, 0 to 100.
 N01.03 Record a given numeral, 0 to 100, presented orally.
 N01.04 Read a given presented numeral, 0 to 100.
 N01.05 Skip count by 2s to 20 starting at 0.
 N01.06 Skip count by 5s to 100 starting at 0, using a hundred chart or a number line.
 N01.07 Skip count forward by 10s to 100 starting at 0, using a hundred chart or a number line.
 N01.08 Identify and correct errors and omissions in a given number sequence.

- N02** Students will be expected to recognize, at a glance, and name the quantity represented by familiar arrangements of 1 to 10 objects or dots. [C, CN, ME, V]

Performance Indicators

- N02.01 Look briefly at a given familiar arrangement of objects or dots and identify the number represented without counting.
 N02.02 Identify the number represented by a given arrangement of counters or dots on a ten-frame.

- N03** Students will be expected to demonstrate an understanding of counting to 20 by
- indicating that the last number said identifies “how many”
 - showing that any set has only one count
 - using the counting-on strategy [C, CN, ME, R, V]

Performance Indicators

- N03.01 Answer the question, How many are in the set? using the last number counted in a given set.
 N03.02 Identify and correct counting errors in a given counting sequence.
 N03.03 Show that the count of the number of objects in a given set does not change regardless of the order in which the objects are counted.

- N03.04 Record the number of objects in a set using the numeral symbol.
- N03.05 Determine the total number of objects in a given set, starting from a known quantity and counting on.

N04 Students will be expected to represent and partition numbers to 20. [C, CN, V]

Performance Indicators

- N04.01 Represent a given number up to 20 using a variety of manipulatives, including ten-frames and created materials.
- N04.02 Model a given number up to 20 using a variety of pictorial representations.
- N04.03 Find examples of a given number in the environment.
- N04.04 Place given numerals on a number line with benchmarks 0, 5, 10, 15, and 20.
- N04.05 Partition any given quantity up to 20 into two parts and identify the number of objects in each part.
- N04.06 Model a given number using two different objects.

N05 Students will be expected to compare sets containing up to 20 objects to solve problems using referents and one-to-one correspondence. [C, CN, ME, PS, R, V]

Performance Indicators

- N05.01 Build a set that has more, fewer, or as many objects as a given set, up to 20 objects.
- N05.02 Build several sets of different objects that have the same given number of objects in the set.
- N05.03 Compare two given sets using one-to-one correspondence and describe them using comparative words, such as more, fewer, or as many.
- N05.04 Compare a set to a given referent using comparative language.
- N05.05 Solve, using pictures and words, given story problems that involve the comparison of two quantities.

N06 Students will be expected to estimate quantities to 20 by using referents. [C, ME, PS, R, V]

Performance Indicators

- N06.01 Estimate a given quantity by comparing it to a given referent (known quantity).
- N06.02 Select an estimate for a given quantity by choosing between at least two possible choices and explain the choice.

N07 Students will be expected to demonstrate an understanding of conservation of number for up to 20 objects. [C, R, V]

Performance Indicators

- N07.01 Explain why for a given number of counters, no matter how they are arranged, the total number of counters does not change.
- N07.02 Group a set of given counters in more than one way.
- N07.03 Explain why for a given number of counters, no matter how they are grouped, the total number of counters does not change.

N08 Students will be expected to identify the number, up to 20, that is one more, two more, one less, and two less than a given number. [C, CN, ME, R, V]

Performance Indicators

N08.01 Name the number that is one more, two more, one less, or two less than a given number, up to 20.

N08.02 Represent a number on ten-frames that is one more, two more, one less, or two less than a given number.

N09 Students will be expected to demonstrate an understanding of the addition of two single-digit numbers and the corresponding subtraction, concretely, pictorially, and symbolically in join, separate, equalize/compare, and part-part-whole situations. [C, CN, ME, PS, R, V]

Performance Indicators

N09.01 Act out story problems that are presented orally or through shared reading.

N09.02 Model story problems with manipulatives or pictures, find and share solutions using counting strategies, and record number sentences that represent how they thought about the problems.

N09.03 Create story problems that connect to student experiences.

N09.04 Create story problems for given number sentences.

N10 Students will be expected to use and describe strategies to determine sums and differences using manipulatives and visual aids. Strategies include

- counting on or counting back
- one more or one less
- making ten
- doubles
- near doubles

Performance Indicators

N10.01 Use and describe a personal strategy to determine a sum.

N10.02 Use and describe a personal strategy to determine a difference.

N10.03 Use and describe how two different strategies can be used to determine a sum or difference.

PATTERNS AND RELATIONS (PR)

PR01 Students will be expected to demonstrate an understanding of repeating patterns (two to four elements) by describing, reproducing, extending, and creating patterns using manipulatives, diagrams, sounds, and actions. [C, PS, R, V]

Performance Indicators

PR01.01 Describe a given repeating pattern containing two to four elements in its core.

PR01.02 Identify errors in a given repeating pattern.

PR01.03 Identify the missing element(s) in a given repeating pattern.

PR01.04 Create and describe a repeating pattern using a variety of manipulatives, musical instruments and actions.

PR01.05 Reproduce and extend a given repeating pattern using manipulatives, diagrams, sounds and actions.

PR01.06 Identify and describe a repeating pattern in the environment (e.g., classroom, outdoors) using everyday language.

PR01.07 Identify repeating events (e.g., days of the week, birthdays, seasons).

PR02 Students will be expected to translate repeating patterns from one representation to another. [C, R, V]

Performance Indicators

PR02.01 Represent a given repeating pattern using another mode, e.g., actions to sound; colour to shape; ABC, ABC, ABC to blue, yellow, green, blue, yellow, green, blue, yellow, green,...

PR02.02 Describe a given repeating pattern using a letter code, e.g., ABC, ABC,ABC ...

PR03 Students will be expected to describe equality as a balance and inequality as an imbalance, concretely and pictorially (0 to 20). [C, CN, R, V]

Performance Indicators

PR03.01 Construct two equal sets using the same objects (same shape and mass) and demonstrate their equality of number using a balance scale.

PR03.02 Construct two unequal sets using the same objects (same shape and mass) and demonstrate their inequality of number using a balance scale.

PR03.03 Determine if two given concrete sets are equal or unequal and explain the process used.

PR04 Students will be expected to record equalities using the equal symbol. [C, CN, PS, V]

Performance Indicators

PR04.01 Represent a given pictorial or concrete equality in symbolic form.

PR04.02 Represent a given equality using manipulatives or pictures.

PR04.03 Provide examples of equalities where the given sum or difference is on either the left or right side of the equal symbol (=).

PR04.04 Record different representations of the same quantity (0 to 20) as equalities.

MEASUREMENT (M)

M01 Students will be expected to demonstrate an understanding of measurement as a process of comparing by

- identifying attributes that can be compared
- ordering objects
- making statements of comparison
- filling, covering, or matching [C, CN, PS, R, V]

Performance Indicators

M01.01 Identify common attributes, such as length, mass, volume, capacity, and area that could be used to compare a given set of two objects.

M01.02 Compare and order two given objects and identify the attributes used to compare.

M01.03 Predict which object in a set is longest/shortest, determine by matching and explain the reasoning.

- M01.04 Predict which object in a set is heaviest/lightest, determine by comparing and explain the reasoning.
- M01.05 Predict which object in a set is largest/smallest, determine by comparing and explain the reasoning.
- M01.06 Predict which object in a set holds the most/least, determine by filling and explain the reasoning.
- M01.07 Predict which figure in a set has the greatest/least area, determine by covering and explain the reasoning.

GEOMETRY (G)

- G01** Students will be expected to sort 3-D objects and 2-D shapes using one attribute and explain the sorting rule. [C, CN, R, V]

Performance Indicators

- G01.01 Sort a given set of familiar 3-D objects or 2-D shapes using a given sorting rule.
- G01.02 Sort a given set of familiar 3-D objects using a single attribute determined by the student and explain the sorting rule.
- G01.03 Sort a given set of 2-D shapes using a single attribute determined by the student and explain the sorting rule.
- G01.04 Determine the difference between two given pre-sorted sets of familiar 3-D objects or 2-D shapes and explain a possible sorting rule used to sort them.

- G02** Students will be expected to replicate composite 2-D shapes and 3-D objects. [CN, PS, V]

Performance Indicators

- G02.01 Select 2-D shapes from a given set of 2-D shapes to reproduce a given composite 2-D shape.
- G02.02 Select 3-D objects from a given set of 3-D objects to reproduce a given composite 3-D object.
- G02.03 Predict and select the 2-D shapes used to produce a composite 2-D shape, and verify by deconstructing the composite shape.
- G02.04 Predict and select the 3-D objects used to produce a composite 3-D object, and verify by deconstructing the composite object.

- G03** Students will be expected to identify 2-D shapes in 3-D objects. [C, CN, V]

Performance Indicators

- G03.01 Identify the shape of the faces of a 3-D object
- G03.02 Identify 3-D objects in the environment that have faces that are a given 2-D shape.

Mathematical Processes

There are critical components that students must encounter in a mathematics program in order to achieve the goals of mathematics education and encourage lifelong learning in mathematics.

Students are expected to

- communicate in order to learn and express their understanding of mathematics (Communication [C])
- develop and apply new mathematical knowledge through problem solving (Problem Solving [PS])
- connect mathematical ideas to other concepts in mathematics, to everyday experiences, and to other disciplines (Connections [CN])
- demonstrate fluency with mental mathematics and estimation (Mental Mathematics and Estimation [ME])
- select and use technologies as tools for learning and solving problems (Technology [T])
- develop visualization skills to assist in processing information, making connections, and solving problems (Visualization [V])
- develop mathematical reasoning (Reasoning [R])

The Nova Scotia curriculum incorporates these seven interrelated mathematical processes that are intended to permeate teaching and learning. The key to these process standards is presented in a box, as shown below, with each specific curriculum outcome within the strands.

Process Standards Key

[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

Communication [C]

Students need opportunities to read about, represent, view, write about, listen to, and discuss mathematical ideas. These opportunities allow students to create links between their own language and ideas, and the formal language and symbols of mathematics. Communication is important in clarifying, reinforcing, and modifying ideas, knowledge, attitudes, and beliefs about mathematics. Students should be encouraged to use a variety of forms of communication while learning mathematics.

Students also need to communicate their learning using mathematical terminology. Communication can help students make connections between and among the different representational modes—contextual, concrete, pictorial, linguistic/verbal, and symbolic—of mathematical ideas. Students must communicate *daily* about their mathematics learning. This enables them to reflect, to validate, and to clarify their thinking and provides teachers with insight into students' interpretations of mathematical meanings and ideas.

Problem Solving [PS]

Learning through problem solving should be the focus of mathematics at all grade levels. When students encounter new situations and respond to questions of the type, How would you ...? or How could you ...? the problem-solving approach is being modelled. Students develop their own problem-solving strategies by being open to listening, discussing, and trying different strategies.

In order for an activity to be problem-solving based, it must ask students to determine a way to get from what is known to what is sought. If students have already been given ways to solve the problem, it is not a problem, but practice. A true problem requires students to use prior learning in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement, perseverance, and collaboration.

Problem solving is also a powerful teaching tool that fosters multiple, creative, and innovative solutions. Creating an environment where students openly look for and engage in finding a variety of strategies for solving problems empowers students to explore alternatives and develops confident, cognitive, mathematical risk takers.

When students are exposed to a wide variety of problems in all areas of mathematics, they explore various methods for solving and verifying problems. In addition, they are challenged to find multiple solutions for problems and to create their own problem.

Connections [CN]

Contextualization and making connections to the experiences of learners are powerful processes in developing mathematical understanding. When mathematical ideas are connected to one another or to real-world phenomena, students can begin to view mathematics as useful, relevant, and integrated. Learning mathematics within contexts and making connections relevant to learners can validate past experiences and increase student willingness to participate and be actively engaged. The brain is constantly looking for and making connections.

“Because the learner is constantly searching for connections on many levels, educators need to orchestrate the experiences from which learners extract understanding. ... Brain research establishes and confirms that multiple complex and concrete experiences are essential for meaningful learning and teaching.” (Caine and Caine 1991, 5).

Mathematics should be viewed as an integrated whole rather than as the study of separate strands or units. Connections must also be made between and among the different representational modes—contextual, concrete, pictorial, linguistic/verbal, and symbolic. The process of making connections, in turn, facilitates learning. Concepts and skills should also be connected to everyday situations and other curricular areas. For example, when developing literacy skills students learn to make text-to-world, text-to-text, and text-to-self connections. Students can also make connections to make mathematics come alive through math-to-world, math-to-math, and math-to-self connections.

Mental Mathematics and Estimation [ME]

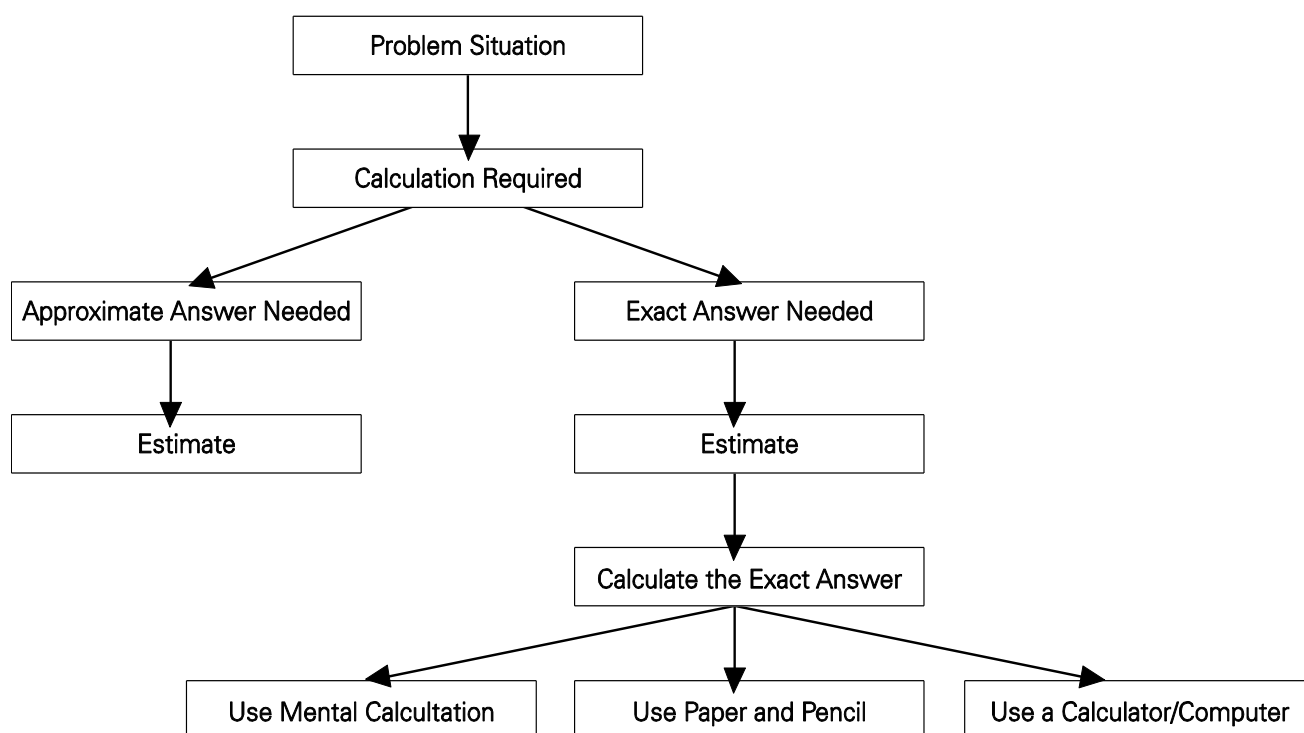
Mental mathematics is a combination of cognitive strategies that enhance flexible thinking and number sense. It is calculating mentally without the use of external aids. Mental mathematics enables students to determine answers without paper and pencil. It improves computational fluency by developing efficiency, accuracy, and flexibility. “Even more important than performing computational procedures or using calculators is the greater facility that students need—more than ever before—with estimation and mental math.” (National Council of Teachers of Mathematics, May 2005).

Students proficient with mental mathematics “become liberated from calculator dependence, build confidence in doing mathematics, become more flexible thinkers, and are more able to use multiple approaches to problem solving.” (Rubenstein 2001) Mental mathematics “provides a cornerstone for all

estimation processes, offering a variety of alternative algorithms and nonstandard techniques for finding answers.” (Hope 1988, v)

Estimation is a strategy for determining approximate values or quantities, usually by referring to benchmarks or using referents, or for determining the reasonableness of calculated values. Students need to know how, when, and what strategy to use when estimating. Estimation is used to make mathematical judgments and develop useful, efficient strategies for dealing with situations in daily life.

Students need to develop both mental mathematics and estimation skills through context and not in isolation so they are able to apply them to solve problems. Whenever a problem requires a calculation, students should follow the decision-making process as illustrated below.



The skill of estimation requires a sound knowledge of mental mathematics. Both are necessary to many everyday experiences, and students should be provided with frequent opportunities to practise these skills.

Technology [T]

Technology can be effectively used to contribute to and support the learning of a wide range of mathematical outcomes and enables students to explore and create patterns, examine relationships, test conjectures, and solve problems.

Technology can be used to

- explore and demonstrate mathematical relationships and patterns
- organize and display data
- extrapolate and interpolate
- assist with calculation procedures as part of solving problems
- decrease the time spent on computations when other mathematical learning is the focus

- reinforce the learning of basic facts and test properties
- develop personal procedures for mathematical operations
- create geometric displays
- simulate situations
- develop number sense

The use of calculators is recommended to enhance problem solving, to encourage discovery of number patterns, and to reinforce conceptual development and numerical relationships. They do not, however, replace the development of number concepts and skills. Carefully chosen computer software can provide interesting problem-solving situations and applications.

Technology contributes to a learning environment in which the growing curiosity of students can lead to rich mathematical discoveries at all grade levels. While technology can be used in grades primary to 3 to enrich learning, it is expected that students will achieve all outcomes without the use of technology.

Visualization [V]

Visualization “involves thinking in pictures and images, and the ability to perceive, transform and recreate different aspects of the visual-spatial world.” (Armstrong 1999). The use of visualization in the study of mathematics provides students with opportunities to understand mathematical concepts and make connections among them. Visual images and visual reasoning are important components of number, spatial, and measurement sense. Number visualization occurs when students create mental representations of numbers. These mental images are needed to develop concepts and understand procedures. Images and explanations help students clarify their understanding of mathematical ideas in all strands.

Being able to create, interpret, and describe a visual representation is part of spatial sense and spatial reasoning. Spatial visualization and reasoning enable students to describe the relationships among and between 3-D objects and 2-D shapes.

Measurement visualization goes beyond the acquisition of specific measurement skills. Measurement sense includes the ability to determine when to measure, when to estimate and to know several estimation strategies. (Shaw and Cliatt 1989)

Visualization is fostered through the use of concrete materials, technology, and a variety of visual representations.

Reasoning [R]

Mathematical reasoning helps students think logically and make sense of mathematics. Students need to develop confidence in their abilities to reason and justify their mathematical thinking. High-order questions challenge students to think and develop a sense of wonder about mathematics. Mathematical experiences in and out of the classroom provide opportunities for inductive and deductive reasoning. Inductive reasoning occurs when students explore and record results, analyze observations, make generalizations from patterns, and test these generalizations. Deductive reasoning occurs when students reach new conclusions based upon what is already known or assumed to be true.

Mathematics reasoning involves informal thinking, conjecturing, and validating—these help students understand that mathematics makes sense. Students are encouraged to justify, in a variety of ways, their solutions, thinking processes, and hypotheses. In fact, good reasoning is as important as finding correct answers.

Nature of Mathematics

Mathematics is one way of trying to understand, interpret, and describe our world. There are a number of components that define the nature of mathematics, and these are woven throughout this document. These components include change, constancy, number sense, relationships, patterns, spatial sense, and uncertainty.

Change

It is important for students to understand that mathematics is dynamic and not static. As a result, recognizing change is a key component in understanding and developing mathematics. Within mathematics, students encounter conditions of change and are required to search for explanations of that change. To make predictions, students need to describe and quantify their observations, look for patterns, and describe those quantities that remain fixed and those that change. For example, the sequence 4, 6, 8, 10, 12, ... can be described as

- skip counting by 2s, starting from 4
- an arithmetic sequence, with first term 4 and a common difference of 2
- a linear function with a discrete domain

(Steen 1990, 184).

Constancy

Different aspects of constancy are described by the terms **stability**, **conservation**, **equilibrium**, **steady state**, and **symmetry** (AAAS–Benchmarks 1993, 270). Many important properties in mathematics and science relate to properties that do not change when outside conditions change. Some problems in mathematics require students to focus on properties that remain constant. The recognition of constancy enables students to solve problems. Examples of constancy include the following:

- The area of a rectangular region is the same regardless of the methods used to determine the solution.
- The sum of the interior angles of any triangle is 180° .
- The theoretical probability of flipping a coin and getting heads is 0.5.

Number Sense

Number sense, which can be thought of as intuition about numbers, is the most important foundation of numeracy (British Columbia Ministry of Education 2000, 146). A true sense of number goes well beyond the skills of simply counting, memorizing facts, and the situational rote use of algorithms. Number sense develops when students connect numbers to real-life experiences and use benchmarks and referents. This results in students who are computationally fluent, flexible with numbers, and have intuition about numbers. The evolving number sense typically comes as a by-product of learning rather than through direct instruction. However, number sense can be developed by providing rich mathematical tasks that allow students to make connections.

Relationships

Mathematics is used to describe and explain relationships. As part of the study of mathematics, students look for relationships among numbers, sets, shapes, objects, and concepts. The search for possible relationships involves the collection and analysis of data, and describing relationships visually, symbolically, orally, or in written form.

Patterns

Mathematics is about recognizing, describing, and working with numerical and non-numerical patterns. Patterns exist in all strands, and it is important that connections are made among strands. Working with patterns enables students to make connections within and beyond mathematics. These skills contribute to students' interaction with an understanding of their environment. Patterns may be represented in concrete, visual, or symbolic form. Students should develop fluency in moving from one representation to another. Students must learn to recognize, extend, create, and use mathematical patterns. Patterns allow students to make predictions and justify their reasoning when solving problems. Learning to work with patterns in the early grades helps develop students' algebraic thinking, which is foundational for working with more abstract mathematics in higher grades.

Spatial Sense

Spatial sense involves visualization, mental imagery, and spatial reasoning. These skills are central to the understanding of mathematics. Spatial sense enables students to reason and interpret among and between 3-D and 2-D representations and identify relationships to mathematical strands. Spatial sense is developed through a variety of experiences and interactions within the environment. The development of spatial sense enables students to solve problems involving 3-D objects and 2-D shapes. Spatial sense offers a way to interpret and reflect on the physical environment and its 3-D or 2-D representations. Some problems involve attaching numerals and appropriate units (measurement) to dimensions of objects. Spatial sense allows students to make predictions about the results of changing these dimensions. For example,

- knowing the dimensions of an object enables students to communicate about the object and create representations
- the volume of a rectangular solid can be calculated from given dimensions
- doubling the length of the side of a square increases the area by a factor of four

Uncertainty

In mathematics, interpretations of data and the predictions made from data may lack certainty. Events and experiments generate statistical data that can be used to make predictions. It is important to recognize that these predictions (interpolations and extrapolations) are based upon patterns that have a degree of uncertainty. The quality of the interpretation is directly related to the quality of the data. An awareness of uncertainty allows students to assess the reliability of data and data interpretation. Chance addresses the predictability of the occurrence of an outcome. As students develop their understanding of probability, the language of mathematics becomes more specific and describes the degree of uncertainty more accurately.

Curriculum Document Format

This guide presents the mathematics curriculum so that a teacher may readily view the scope of the outcomes that students are expected to meet during that year. Teachers are encouraged, however, to examine what comes before and what follows after, to better understand how students' learning at a particular grade level is part of a bigger picture of concept and skill development.

The order of presentation in no way assumes or prescribes a preferred order of presentation in the classroom, but simply lays out the specific curriculum outcomes in relation to the overarching general curriculum outcomes (GCOs).

The footer of the document shows the name of the course, and the strand name is presented in the header. When a specific curriculum outcome (SCO) is introduced, it is followed by the mathematical processes and performance indicators for that outcome. A scope and sequence is then provided, which relates the SCO to previous and next grade SCOs. Also for each SCO, there is background information, assessment strategies, suggested instructional strategies, suggested models and manipulatives, mathematical language, and a section for resources and notes. For each section, the guiding questions should be used to help with unit and lesson preparation.

SCO		
Mathematical Processes		
[C] Communication	[PS] Problem Solving	[CN] Connections
[ME] Mental Mathematics and Estimation		
[T] Technology	[V] Visualization	[R] Reasoning
Performance Indicators		
Describes observable indicators of whether students have achieved the specific outcome.		
Scope and Sequence		
Previous grade or course SCOs	Current grade SCO	Following grade or course SCOs
Background		
Describes the “big ideas” to be learned and how they relate to work in previous grade and work in subsequent courses.		
Additional Information		
A reference to Appendix A, which contains further elaborations for the performance indicators.		
Assessment, Teaching, and Learning		
Assessment Strategies		
Guiding Questions		
<ul style="list-style-type: none"> What are the most appropriate methods and activities for assessing student learning? How will I align my assessment strategies with my teaching strategies? 		
ASSESSING PRIOR KNOWLEDGE		
Sample tasks that can be used to determine students' prior knowledge.		
WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS		
Some suggestions for specific activities and questions that can be used for both instruction and assessment		

FOLLOW-UP ON ASSESSMENT**Guiding Questions**

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Correlations to related resources.

Planning for Instruction**Guiding Questions**

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcome and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Suggested strategies for planning daily lessons.

SUGGESTED LEARNING TASKS

Suggestions for general approaches and strategies suggested for teaching this outcome.

Guiding Questions

- How can the scope and sequence be used to determine what prior knowledge needs to be activated prior to beginning new instruction?

SUGGESTED MODELS AND MANIPULATIVES**MATHEMATICAL LANGUAGE**

Teacher and student mathematical language associated with the respective outcome.

Resources/Notes

Contexts for Learning and Teaching

Beliefs about Students and Mathematics Learning

“Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge.” (National Council of Teachers of Mathematics 2000, 20).

The Nova Scotia mathematics curriculum is based upon several key assumptions or beliefs about mathematics learning that have grown out of research and practice. These beliefs include the following:

- Mathematics learning is an active and constructive process.
- Learning is most effective when standards of expectation are made clear with ongoing assessment and feedback.
- Learners are individuals who bring a wide range of prior knowledge and experiences and who learn via various styles and at different rates.
- Learning is most likely to occur when placed in meaningful contexts and in an environment that supports exploration, risk taking, and critical thinking and that nurtures positive attitudes and sustained effort.

Students are curious, active learners with individual interests, abilities, and needs. They come to classrooms with varying knowledge, life experiences, and backgrounds. A key component in successfully developing numeracy is making connections to these backgrounds and experiences.

Students develop a variety of mathematical ideas before they enter school. Children make sense of their environment through observations and interactions at home and in the community. Mathematics learning is embedded in everyday activities, such as playing, reading, storytelling, and helping around the home. Such activities can contribute to the development of number and spatial sense in children. Curiosity about mathematics is fostered when children are engaged in activities such as comparing quantities, searching for patterns, sorting objects, ordering objects, creating designs, building with blocks, and talking about these activities. Positive early experiences in mathematics are as critical to child development as are early literacy experiences.

Students learn by attaching meaning to what they do and need to construct their own meaning of mathematics. This meaning is best constructed when learners encounter mathematical experiences that proceed from the simple to the complex and from the concrete to the abstract. The use of models and a variety of pedagogical approaches can address the diversity of learning styles and developmental stages of students and enhance the formation of sound, transferable, mathematical concepts. At all levels, students benefit from working with and translating through a variety of materials, tools, and contexts when constructing meaning about new mathematical ideas. Meaningful discussions can provide essential links among concrete, pictorial, contextual, and symbolic representations of mathematics.

The learning environment should value and respect all students’ experiences and ways of thinking, so that learners are comfortable taking intellectual risks, asking questions, and posing conjectures. Students need to explore problem-solving situations in order to develop personal strategies and become mathematically literate. Learners must realize that it is acceptable to solve problems in different ways and that solutions may vary.

Goals for Mathematics Education

The main goals of mathematics education are to prepare students to

- use mathematics confidently to solve problems
- communicate and reason mathematically
- appreciate and value mathematics
- make connections between mathematics and its applications
- become mathematically literate adults, using mathematics to contribute to society

Students who have met these goals will

- gain understanding and appreciation of the contributions of mathematics as a science, a philosophy, and an art
- exhibit a positive attitude toward mathematics
- engage and persevere in mathematical tasks and projects
- contribute to mathematical discussions
- take risks in performing mathematical tasks
- exhibit curiosity about mathematics and situations involving mathematics

Opportunities for Success

A positive attitude has a profound effect on learning. Environments that create a sense of belonging, encourage risk taking, and provide opportunities for student success help develop and maintain positive attitudes and self-confidence. Students with positive attitudes toward learning mathematics are likely to be motivated and prepared to learn, participate willingly in classroom activities, persist in challenging situations, and engage in reflective practices.

To experience success, students must be taught to set achievable goals or assess their progress as they work toward these goals. Striving toward success and becoming autonomous and responsible learners are ongoing, reflective processes that involve revisiting the setting and assessing of personal goals.

Engaging All Learners

“No matter how engagement is defined or which dimension is considered, research confirms this truism of education: *The more engaged you are, the more you will learn.*” (Hume 2011, 6)

Student engagement is at the core of learning. This is critical for teachers to take into account when planning and implementing instruction. Effective instruction engages, embraces, and supports all learners through a range of learning experiences. This curriculum is designed to provide learning opportunities that reflect culturally proficient instructional and assessment practices and are equitable, accessible, and inclusive of the multiple facets of diversity represented in today’s classrooms.

Engagement in learning occurs when students are provided with opportunities to become more invested in their learning. When teachers know their students as individual learners and as individual people, their students are more likely to be motivated to learn, participate in classroom activities, persist in challenging situations, and engage in reflective practices. Students often become more engaged when teachers demonstrate a genuine belief in each student’s potential to learn.

SUPPORTIVE LEARNING ENVIRONMENTS

A supportive and positive learning environment has a profound effect on students' learning. In classrooms where students feel a sense of belonging, are encouraged to actively participate, are challenged without being frustrated, and feel safe and supported to take risks with their learning, students are more likely to experience success. It is realized that not all students will progress at the same pace or be equally positioned in terms of their prior knowledge of and skill with particular concepts and outcomes. Teachers provide all students with equitable access to learning by integrating a variety of instructional approaches and assessment activities that consider all learners and align with the following key principles:

- Instruction must be flexible and offer multiple means of representation.
- Students must have opportunities to express their knowledge and understanding in multiple ways.
- Teachers must provide options for students to engage in learning through multiple ways.

Teachers who know their students well become aware of individual learning differences and infuse this understanding into planned instructional and assessment decisions. They organize learning experiences to accommodate the many ways in which students learn, create meaning, and demonstrate their knowledge and understanding. Teachers use a variety of effective teaching approaches that may include

- providing all students with equitable access to appropriate learning strategies, resources, and technology
- offering a range of ways students can access their prior knowledge to connect with new concepts
- scaffolding instruction and assignments so that individual or groups of students are supported as needed throughout the process of learning
- verbalizing their thinking to model comprehension strategies and new learning
- balancing individual, small-group, and whole-class approaches to learning activities
- involving students in the co-creation of criteria for assessment and evaluation
- providing students with choice in how they demonstrate their understanding according to learning styles and preferences, building on individual strengths, and including a range of difficulty and challenge
- providing frequent and meaningful feedback to students throughout their learning experiences

LEARNING STYLES AND PREFERENCES

The ways in which students make sense of, receive, and process information, demonstrate learning, and interact with peers and their environment both indicate and shape learning preferences, which may vary widely from student to student. Learning preferences are influenced also by the learning context and purpose and by the type and form of information presented or requested. Most students tend to favour one learning style and may have greater success if instruction is designed to provide for multiple learning styles, thus creating more opportunities for all students to access learning. The three most commonly referenced learning styles are

- auditory (such as listening to teacher-presented lessons or discussing with peers)
- kinesthetic (such as using manipulatives or recording print or graphic/visual text)
- visual (such as interpreting information with text and graphics or viewing videos)

While students can be expected to work using all modalities, it is recognized that one or some of these modalities may be more natural to individual students than the others.

A GENDER-INCLUSIVE CURRICULUM

It is important that the curriculum respects the experiences and values of all students and that learning resources and instructional practices are not gender-biased. Teachers promote gender equity and inclusion in their classrooms when they

- articulate equally high expectations for all students
- provide equal opportunity for input and response from all students
- model gender-fair language and respectful listening in their interactions with students

VALUING DIVERSITY: TEACHING WITH CULTURAL PROFICIENCY

Teachers understand that students represent diverse life and cultural experiences, with individual students bringing different prior knowledge to their learning. Therefore, teachers build upon their knowledge of their students as individuals and respond by using a variety of culturally-proficient instruction and assessment strategies. “Instruction that is embedded in socially meaningful contexts, and tasks that are meaningful and relevant to the lives of students, will engage students in high-level problem-solving and reasoning and enhance students’ engagement (Frankenstein 1995; Gutstein 2003; Ladson-Billings 1997; Tate 1995).” (Herzig 2005)

STUDENTS WITH LANGUAGE, COMMUNICATION, AND LEARNING CHALLENGES

Today’s classrooms include students who have diverse backgrounds, abilities, levels of development, and learning challenges. By observing and interacting with students as they work on assigned activities, teachers can identify areas where students may need additional support to achieve their learning goals. Teachers can then respond with a range of effective instructional strategies. Students who have English as an Additional Language (EAL) may require curriculum outcomes at different levels, or temporary individualized outcomes, particularly in language-based subject areas, while they become more proficient in their English language skills. For students who are experiencing difficulties, it is important that teachers distinguish between students for whom curriculum content is challenging and students for whom language-based issues are at the root of apparent academic difficulties.

STUDENTS WHO DEMONSTRATE GIFTED AND TALENTED BEHAVIOURS

Some students are academically gifted and talented with specific skill sets or in specific subject areas. Most students who are gifted and talented thrive when challenged by problem-centred, inquiry-based learning and open-ended activities. Teachers may challenge students who are gifted and talented by adjusting the breadth, the depth, and/or the pace of instruction. Learning experiences may be enriched by providing greater choice among activities and offering a range of resources that require increased cognitive demand and higher-level thinking at different levels of complexity and abstraction. For additional information, refer to *Gifted Education and Talent Development* (Nova Scotia Department of Education 2010).

Connections across the Curriculum

The teacher should take advantage of the various opportunities available to integrate mathematics and other subjects. This integration not only serves to show students how mathematics is used in daily life, but it helps strengthen the students’ understanding of mathematical concepts and provides them with opportunities to practise mathematical skills. There are many possibilities for integrating mathematics in health education, literacy, music, physical education, science, social studies, and visual arts.

Number (N)

GCO: Students will be expected to demonstrate number sense.

Specific Curriculum Outcomes

Process Standards Key

[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

- N01** Students will be expected to say the number sequence by
- 1s, forward and backward between any two given numbers, 0 to 100
 - 2s to 20, forward starting at 0
 - 5s to 100, forward starting at 0, using a hundred chart or a number line
 - 10s to 100, forward starting at 0, using a hundred chart or a number line [C, CN, V, ME]
- N02** Students will be expected to recognize, at a glance, and name the quantity represented by familiar arrangements of 1 to 10 objects or dots. [C, CN, ME, V]
- N03** Students will be expected to demonstrate an understanding of counting to 20 by
- indicating that the last number said identifies “how many”
 - showing that any set has only one count
 - using the counting-on strategy [C, CN, ME, R, V]
- N04** Students will be expected to represent and partition numbers to 20. [C, CN, V]
- N05** Students will be expected to compare sets containing up to 20 objects to solve problems using referents and one-to-one correspondence. [C, CN, ME, PS, R, V]
- N06** Students will be expected to estimate quantities to 20 by using referents. [C, ME, PS, R, V]
- N07** Students will be expected to demonstrate an understanding of conservation of number for up to 20 objects. [C, R, V]
- N08** Students will be expected to identify the number, up to 20, that is one more, two more, one less, and two less than a given number. [C, CN, ME, R, V]
- N09** Students will be expected to demonstrate an understanding of the addition of two single-digit numbers and the corresponding subtraction, concretely, pictorially, and symbolically in join, separate, equalize/compare, and part-part-whole situations. [C, CN, ME, PS, R, V]
- N10** Students will be expected to use and describe strategies to determine sums and differences using manipulatives and visual aids. Strategies include
- counting on or counting back
 - one more or one less
 - making ten
 - doubles
 - near doubles

SCO N01 Students will be expected to say the number sequence by

- 1s, forward and backward between any two given numbers, 0 to 100
- 2s to 20, forward starting at 0
- 5s to 100, forward starting at 0, using a hundred chart or a number line
- 10s to 100, forward starting at 0, using a hundred chart or a number line

[C, CN, V, ME]

[C] Communication

[PS] Problem Solving

[CN] Connections

[ME] Mental Mathematics and Estimation

[T] Technology

[V] Visualization

[R] Reasoning

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- N01.01** Recite forward by 1s the number sequence between two given numbers, 0 to 100.
- N01.02** Recite backward by 1s the number sequence between two given numbers, 0 to 100.
- N01.03** Record a given numeral, 0 to 100, presented orally.
- N01.04** Read a given presented numeral, 0 to 100.
- N01.05** Skip count by 2s to 20 starting at 0.
- N01.06** Skip count by 5s to 100 starting at 0, using a hundred chart or a number line.
- N01.07** Skip count forward by 10s to 100 starting at 0, using a hundred chart or a number line.
- N01.08** Identify and correct errors and omissions in a given number sequence.

Scope and Sequence

Mathematics Primary	Mathematics 1	Mathematics 2
<p>N01 Students will be expected to say the number sequence by</p> <ul style="list-style-type: none"> ▪ 1s, from 1 to 20 ▪ 1s, starting anywhere from 1 to 10 and from 10 to 1 	<p>N01 Students will be expected to say the number sequence by</p> <ul style="list-style-type: none"> ▪ 1s, forward and backward between any two given numbers, 0 to 100 ▪ 2s to 20, forward starting at 0 ▪ 5s to 100, forward starting at 0 using a hundred chart or a number line ▪ 10s to 100, forward starting at 0, using a hundred chart or a number line 	<p>N01 Students will be expected to say the number sequence by</p> <ul style="list-style-type: none"> ▪ 1s, forward and backward, from any point to 200 ▪ 2s, forward and backward, starting from any point to 100 ▪ 5s and 10s, forward and backward, using starting points that are multiples of 5 and 10 respectively to 100 ▪ 10s, starting from any point to 100

Background

Meaningful counting involves an understanding of the six principles of counting. These principles are addressed in various outcomes (N01, N03, and N07). The principle in bold face is the one addressed in this outcome.

1. One number is said for each item in the group and is counted once and only once. (one-to-one correspondence)
- 2. Counting begins with the number 1, and there is a set number sequence. (stable order)**
3. The quantity in the set is the last number said. (cardinality)
4. The starting point and order of counting the objects does not affect the quantity. (order irrelevance)
5. The arrangement or types of objects does not affect the count. (conservation)
6. It does not matter what is being counted, the resulting count will always be the same. (abstraction)

Stable Order Principle: In everyday use there are two meanings of “to count.” One meaning is to say the numbers in order (outcome N01 addresses this meaning) which addresses the stable order principle. As early as two years of age, children can repeat words such as one, two, and three; however, children most likely do not understand the quantity represented by those numbers. Understanding quantity is the second meaning of “to count” (outcomes N03 and N07 address this second meaning). In primary, students learned the names of the numbers in the counting sequence to 20 and focused on meaningful counting for collections up to 10. In Mathematics 1, students will extend the number names in the counting sequence to 100 and will develop meaningful counting for collections up to 20.

Learning the number names is an important prerequisite for counting items in a set. “Before there can be any meaningful counting, students must be able to recite the sequence beginning 1, 2, 3, 4, 5, etc.” (Small 2009, 84) There is a difference between being able to recite the number words (one, two, three...) and understanding how counting is used to describe a set. The counting sequence itself is a rote procedure; however, “The *meaning* attached to counting is the key conceptual idea on which all other number concepts are developed.” (Van de Walle and Lovin 2006, 39).

Additional Information

See Appendix A: Additional Information.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students’ prior knowledge.

- Ask students to start at 1 and count forward to 20.
- Ask students to start at 20 and count backward.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students to say the number sequence forward by 1s starting at a given number less than 100.
- Ask students to say the number sequence backwards by 1s starting at a given number less than 100.
- Ask students to begin saying the number sequence by 1s at 13 and stop at 25.
- Ask students to begin saying the number sequence by 1s at 68 and stop at 49.

- Say a number sequence incorrectly and ask students to identify your error and correct it.
- Ask students to skip count by 2s (or 5s or 10s) starting at 0.
- Begin reciting a number sequence but omit some numbers. Ask students to tell you the numbers you omitted.
- Ask students to count by 2s (5s or 10s) as you clap. Have students tell you or record the final number when you finish clapping.
- Recite a number sequence with an error or a missing number. Ask students to correct the sequence.
- Have students “count-off” by 1s, 2s, 5s, and 10s. Observe whether students are able to follow the number sequence.
- Give each student a card with a numeral on it. Have students put themselves in order using the number cards. The cards can be used to show counting by 1s, 2s, 5s, and 10s.
- Ask students, If you count by twos, starting at 0, will you say 17? Why or why not?
- Provide a hundred chart. Tell the students, I counted from 10 to 50 and only said 5 numbers. What do you think I said?
- Tell students, I said, “10, 15, 20, 25,” when I was skip counting. By what number was I skip counting?

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets K–2 (Bauman 2011)

- Mathematics 1 Checkpoint 3, pp. 40–41

Planning for Instruction

Before introducing new material, consider ways to assess and build on students’ knowledge and skills.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of my students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Use songs, poems, finger plays, and children’s literature to support increasing and decreasing counting sequences.

- Use a hundred chart or hundred mat. Both are excellent tools to explore counting patterns. For example, when skip counting by 2s, students might put a counter on every other number, reading the number as the counter is placed on it. This can be extended to counting by 5s and 10s.
- Have students use a walk-on number line (on the floor) to explore counting and skip counting.

SUGGESTED LEARNING TASKS

- **Echo counting:** The teacher recites the number sequence and the students echo the teacher.
- **Orchestra counting:** Put students into groups. The first group begins reciting the number sequence forward from 1. When the teacher points to another group, they continue from the last number stated.
- Invite students to sing songs and to read poems that involve reciting number sequences backward and forward and skip counting.
- Invite students to use calculators to count. For example, as some students place cookies into a bag and count aloud, others may repeatedly add one on calculators to keep track of the count electronically.
- Ask students to count items that occur naturally in 2s (e.g., shoes, hands, eyes) or 5s (fingers, toes).
- Place number cards 1 to 100 in a container. Have a student select two cards. The students begin counting with the first number selected and stop when they get to the second number drawn.
- Use a horizontal number line or a large floor chart with numbers 1 to 100. Ask the class to skip count by 2s (or 5s or 10s) and ask a student volunteer to step on each number as it is said. Discuss the movements made by the student volunteer.
- Ask students to use the repeat function on the calculator to skip count to a target number. For example, if you start at 0 and want to end on 40, by which number(s) could you skip count? What if you started at a different point? What if you wanted to end at a different point?
- Ask the student to count aloud to 50 by 5s while using the “constant function” on the calculator. (Enter “5,” press “+ 5,” press “=” Each time the “=” is pressed, another 5 is added: 10, 15, 20, 25, ...)
- Have students work with a partner to create a number sequence with a missing number. Exchange sequence with another pair and identify the missing number.
- Ask students how many ways they can count to 30. Have students talk about their findings.

SUGGESTED MODELS AND MANIPULATIVES

- calculators
- counters
- hundred chart
- learning carpet
- linking cubes
- walk-on number line

MATHEMATICAL LANGUAGE

Teacher	Student (oral language)
<ul style="list-style-type: none"> ▪ counting numbers: one to one hundred ▪ forward, backward ▪ hundred chart, number lines ▪ numeral ▪ position words: between, count on, count back, next, after, before ▪ recite, skip count, number sequence 	<ul style="list-style-type: none"> ▪ counting numbers: one to one hundred ▪ forward, backward ▪ hundred chart, number lines ▪ numeral ▪ position words: between, count on, count back, next, after, before ▪ skip count, number sequence

Resources/Notes

Print Resources

- *Making Math Meaningful to Canadian Students, K–8* (Small 2009), pp. 83–87
- *Making Math Meaningful to Canadian Students, K–8, Second Edition* (Small 2013), pp. 139–144
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 37–41

Videos

- *An Introduction to Using Number Lines* (13:04 min.) (ORIGO Education 2010)
- *Analyzing Patterns (Skip Counting) on a Hundred Board* (27:16 min.) (ORIGO Education 2010)
- *Powerful Models to Help Struggling Students: Number Lines* (17:37 min.) (ORIGO Education 2010)
- *Teaching Number: 0 to 9* (14:47 min.) (ORIGO Education 2010)
- *Teaching Number: Counting* (10:49 min.) (ORIGO Education 2010)

Notes

SCO N02 Students will be expected to recognize, at a glance, and name the quantity represented by familiar arrangements of 1 to 10 objects or dots.

[C, CN, ME, V]

[C] Communication

[PS] Problem Solving

[CN] Connections

[ME] Mental Mathematics and Estimation

[T] Technology

[V] Visualization

[R] Reasoning

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N02.01 Look briefly at a given familiar arrangement of objects or dots and identify the number represented without counting.

N02.02 Identify the number represented by a given arrangement of counters or dots on a ten-frame.

Scope and Sequence

Mathematics Primary	Mathematics 1	Mathematics 2
N02 Students will be expected to recognize, at a glance, and name the quantity represented by familiar arrangements of 1 to 5 objects or dots.	N02 Students will be expected to recognize, at a glance, and name the quantity represented by familiar arrangements of 1 to 10 objects or dots.	—

Background

This outcome addresses **subitizing**. Subitizing is the ability to recognize, at a glance, a quantity of objects (cardinality) without counting, for example, the ability to immediately recognize the number of dots on a standard die when it is rolled or to immediately recognize a number on a five- or ten-frame when it is presented briefly. Students in Mathematics 1 should come to realize that subitizing and counting are two ways of determining how many are in a set of objects, and not think that counting is the only legitimate way to find how many. Students should recognize that there are many ways to arrange a set of objects, and that some arrangements are easier to subitize than others.

In primary, students subitized and named familiar arrangements of 1 to 5 dots/objects and identified the number represented by a given dot arrangement on a five-frame. In Mathematics 1, students should be able to recognize, without counting, configurations or spatial arrangements for numbers of items up to 10. Some students may subitize two parts of an arrangement and combine them. Having students share their subitizing strategies will benefit all students.

Additional Information

See Appendix A: Additional Information.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Flash five-frame representations for students. Ask students to indicate the number shown on each five-frame by selecting the appropriate numeral card from a set of cards.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Prepare a set of ten-frame cards. Show a card for two to three seconds. Ask students to respond by writing the numeral on their individual white board or paper that represents the ten-frame card shown. Ask students to describe what they saw (e.g., I saw 5 and 2 more, so I saw 7.).
- Use prepared dot arrangements for numbers 0 to 10. Hold up a dot plate for one to three seconds. Say, How many? How did you see it? Children might say, I saw 6; I saw 3 on one side and 3 on the other. Observe how quickly children can recognize the number of dots without counting.
- Ask students to draw an arrangement of counters that makes it easy to recognize 6.
- Ask students to arrange 8 counters (or another number) in a way that will make it easy to tell that there are 8.
- Ask students to make a sketch showing how they “see” 9 (or another number).
- Have a set of cards or objects in your pocket. At any time during the day, show students one of the cards or group of objects and ask them to tell you how many.
- Provide children with a set of counters. Flash a ten-frame card with dots for approximately three seconds. Have the students take the number of counters they think they would need to cover the dots displayed on the ten-frame. After students have made their sets of counters, place the card in front of one child who should then place his or her counters on the dots, while the other children count and check. Ask the child to explain how they identified the number represented on the ten-frame. Repeat this activity using other ten-frame cards with different representations of numbers to 10.
- Play “rabbit ears.” Students place his or her right hand beside his or her right ear and his or her left hand beside his or her left ear. Ask them to use their fingers to show the number that you say. For example, if you say 7, a student may hold up 4 fingers on one hand and 3 fingers on the other hand. This allows you to see who is counting by ones and who is able to subitize.

FOLLOW-UP ON ASSESSMENT**Guiding Questions**

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets K–2 (Bauman 2011)

- Mathematics 1 Checkpoint 2, Task 2, pp. 32–33 (Line Masters 2.5 and 2.6)
- Mathematics 2 Checkpoint 2, Task 2, pp. 34–35 (Line Masters 2.7 and 2.8)

Planning for Instruction

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of my students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Focus subitizing activities initially on arrangements of numbers from 1 to 5 and gradually increase to 10.
- Use “dot cards” and other models with easily recognizable configurations of numbers frequently so that students increase their familiarity with them. The level of difficulty can be adjusted by the arrangements used.
- Use less familiar arrangements often to promote greater automaticity in recognizing quantities.

SUGGESTED LEARNING TASKS

- Ask students to sort dot card arrangements into groups that display the same number.
- Arrange counters on an overhead projector or interactive whiteboard. Show the counters to the students for a few seconds, but not long enough for them to count the counters. Ask, What number was represented? (Repeat several times, using different configurations of the same number.) Ask, Which configuration was easiest to recognize? Why?
- Hold up a dot card for a few seconds. Ask, How many dots? What did you see first (all of the dots or a smaller group)? Include dot cards with both familiar and unfamiliar arrangements. Consider also using cards with two colours of dots.

- State a number or hold up a numeral card and ask students to find the corresponding dot card from a collection on their desks.
- Show students 5 counters arranged in an “L-shape” with equal sides. Ask them to find other numbers of counters that can be arranged to form “Ls”?
- Play a “concentration game.” (**Materials:** Two sets of dot cards, ten-frame cards, or both that show the same numbers.) Place 20 dot cards face down in a 5 x 4 array. Students take turns turning over two cards at a time trying to find a match. If the student finds a match, he or she keeps the pair. If he or she does not make a match, the cards are turned back over. Play continues until all pairs of dot cards are found.
- Students play in pairs. Players place their sets of ten-frame or dot cards face down in a pile in front of them. On a signal, both players turn over the top card and identify the number shown. The player who names the number first keeps the card.
- Play a favourite board game with your students using dot cards instead of a number cube.

SUGGESTED MODELS AND MANIPULATIVES

- collections of counters or objects
- dominoes
- dot cards
- five-frames and ten-frames
- number cubes
- playing cards

MATHEMATICAL LANGUAGE

Teacher	Student (oral language)
<ul style="list-style-type: none"> ▪ dots, dot cards, familiar dot arrangements ▪ five-frame or ten-frame, counters ▪ quantity of objects or dots ▪ sets 	<ul style="list-style-type: none"> ▪ dots ▪ five-frame or ten-frame, counters

Resources/Notes

Print Resources

- *Making Math Meaningful to Canadian Students, K–8* (Small 2009), pp. 89–90
- *Making Math Meaningful to Canadian Students, K–8, Second Edition* (Small 2013), pp. 145–148
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 43–44

Videos

- *Developing Sight Recognition of Quantity (Subitizing)* (12:03 min.) (ORIGO Education 2010)
- *Using a Hands-on Approach to Represent Numbers to 10* (13:06 min.) (ORIGO Education 2010)
- *Using Structured Patterns to Develop Number Combinations* (18:11 min.) (ORIGO Education 2010)

Notes

<p>SCO N03 Students will be expected to demonstrate an understanding of counting to 20 by</p> <ul style="list-style-type: none"> ▪ indicating that the last number said identifies “how many” ▪ showing that any set has only one count ▪ using the counting-on strategy <p>[C, CN, ME, R, V]</p>			
[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- N03.01** Answer the question, How many are in the set? using the last number counted in a given set.
- N03.02** Identify and correct counting errors in a given counting sequence.
- N03.03** Show that the count of the number of objects in a given set does not change regardless of the order in which the objects are counted.
- N03.04** Record the number of objects in a set using the numeral symbol.
- N03.05** Determine the total number of objects in a given set, starting from a known quantity and counting on.

Scope and Sequence

Mathematics Primary	Mathematics 1	Mathematics 2
<p>N03 Students will be expected to relate a numeral, 1 to 10, to its respective quantity.</p> <p>N06 Students will be expected to demonstrate an understanding of counting to 10.</p>	<p>N03 Students will be expected to demonstrate an understanding of counting to 20 by</p> <ul style="list-style-type: none"> ▪ indicating that the last number said identifies “how many” ▪ showing that any set has only one count ▪ using the counting-on strategy 	—

Background

Meaningful counting involves an understanding of the six principles of counting. These principles are addressed in various outcomes (N01 and N07). The principles in bold face are the ones addressed in this outcome.

1. **One number is said for each item in the group and is counted once and only once. (one-to-one correspondence)**
2. **Counting begins with the number 1, and there is a set number sequence. (stable order)**
3. **The quantity in the set is the last number said. (cardinality)**
4. **The starting point and order of counting the objects does not affect the quantity. (order irrelevance)**
5. The arrangement or types of objects does not affect the count. (conservation)
6. **It does not matter what is being counted, the resulting count will always be the same. (abstraction)**

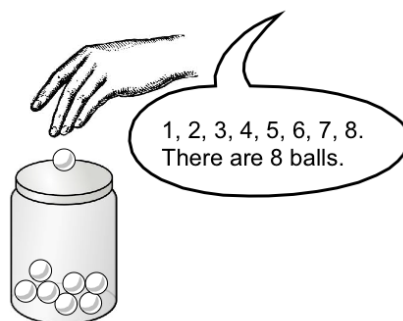
In primary, students were expected to successfully count sets up to 10 objects demonstrating the one-to-one correspondence, the stable order, the cardinality principles, and they explored the other principles. In Mathematics 1, the development of counting continues to sets up to 20 objects with the expectation of understanding all six principles. The conservation principle has its own outcome (N07) and should be addressed in conjunction with this outcome (N03).

Meaningful counting is the foundation on which all other number concepts are developed. For this reason, instruction on counting must emphasize quantity, numerals, and oral and written forms of numbers. It is also necessary to assess each student individually in order to determine his or her understanding of number, not only in the oral and written expression of numbers, but also in counting abilities and quantity sense.

Before students count sets of a certain size, they should be fluent with the number sequence involved, stable order principle (addressed in SCO N01), so they can concentrate on the counting process rather than trying to recall the number names.

One-to-One Correspondence: When students are counting, watch to see if they are touching objects while they count them and saying the appropriate numbers when they touch them. Encourage students to touch or move the objects as they count as this deepens their awareness of the one-to-one principle of counting.

Cardinality: Some students may successfully perform a counting act of 8 objects, but when asked to show 8, pick up the last counter, thinking that the counter they touched when they said “eight” is 8, not the set of 8 counters. Help students develop this principle by engaging them in activities where objects are placed in a container as they are counted and directing students’ attention to the number of objects in the container after the counting act.



Abstraction Principle: Students need to come to understand that the size of the objects does not influence the number that represents the quantity in a set; that is, the objects themselves do not have to be uniform in size and shape, and two sets may both represent the same number even if the objects in one set are larger than those in the other. For example, students can count 6 bears, 6 toy cars, and 6 chairs. The three sets are equivalent because they all represent 6.



Additional Information

See Appendix A: Additional Information.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Place a set of counters in a five-frame or ten-frame and ask students to count the objects from left to right. Ask students to predict how many they think they would get if they started counting at the right or in the middle.
- Ask students to select a numeral card (1 to 10) and to count out that number of counters. After the students count out the set, spread the counters out and ask students to tell you how many they think they have now.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Provide students with a number of objects. Ask them to count them and to tell you how many objects are in the set. After they have counted them once, rearrange the objects and ask them to tell you (without recounting) how many objects are there now. Observe children to determine their understanding of each of the principles underlying meaningful counting. Note the way in which students count:
 - Do they touch each object as they count?
 - Do they set items aside as they count them?
 - Do they show confidence in their count or feel the need to check?
 - Do they check their counting in the same order as the first count or in a different order?
- Ask students to count a number (1 to 20) of items in a photo. Observe how they count. Ask them to record, using numeral symbols, the number of items they counted.
- Ask the student to count out 16 blocks/counters onto the table. Rearrange them by moving them around the table and then display them in two parts to display a “16” combination (e.g., 9 in one group, 7 in the other). Ask the student how many you have altogether. Repeat using different combinations of 16. Observe the student's method of determining how many.

- Show two groups of objects. Hide one under a piece of paper labelled with its amount. Leave the other group showing. Ask the student how many objects there are in all.
- Ask the student to count out 6 counters. Once they have 6 counters, ask them to show you a total of 14 counters. Observe whether the student is able to count on from 6 or recounts starting at 1.
- Show students a number sequence with an error or a missing number. Ask students to correct the sequence.
- Place a large number line on the floor, positioning a child on the 8 and facing the higher numbers. Ask, Where would you be if you moved 4 spaces forward? (Ask additional questions, such as, Where would you be if you moved 3 spaces back?)

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets K–2 (Bauman 2011)

- Mathematics 1 Checkpoint 1, pp. 23–24
- Mathematics 1 Checkpoint 2, Task 1, pp. 32–35 (Line Masters 2.2, 2.3, and 2.4)
- Mathematics 1 Checkpoint 15, pp. 141–142
- Mathematics 2 Checkpoint 1, Tasks 1 and 3, pp. 25–26

Planning for Instruction

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of my students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Observe how the students count. Students who are successful counters have strategies to keep track of their count, such as touching and moving each object as it is counted.
- Encourage students to count items in natural situations that arise in the classroom.

- Play a variety of games that require counting. For example,
 - bowling (counting both the pins knocked down and the pins left standing)
 - board games (counting the number of spaces to be moved based on a spin)
 - throwing bean bags (counting how many land in the target box)

SUGGESTED LEARNING TASKS

- Request that students draw pictures of their favourite toys, and then ask them to count the number of toys in their pictures.
- Allow students to count the number of napkins, cups, plates, etc., that are on the table or are needed for snack time or for a special party.
- Tell students, I am thinking of something in the classroom of which there are exactly 5. What do you think it could be?
- Place 5 counters under a cup and tell the students that there are 5 underneath the cup. Show 3 more beside the cup. Ask, How many counters are there altogether? Students should count-on to determine how many are there.
- Use a walk-on number line. Invite a student to roll two number cubes. He or she chooses the value on one of the cubes to stand on that number on the number line and then moves along the number line by counting on the amount shown on the other number cube.
- Have students work in pairs. Place numeral cards face down. The first student draws a card making sure not to show it to his or her partner and reads the number shown. The partner records the numeral and makes a set of that size. The first student checks the written numeral and the set that was created. They then change roles.
- Provide students with a bag of counters. Tell them they must find out how many there are in total without counting by ones. Invite students to illustrate or demonstrate to the class how they counted and discuss which way of counting was the most efficient.

SUGGESTED MODELS AND MANIPULATIVES

- | | |
|--|---|
| <ul style="list-style-type: none"> ▪ coins, calculators ▪ collections of objects ▪ dot cards ▪ five-frames and ten-frames ▪ hundred chart | <ul style="list-style-type: none"> ▪ number cubes ▪ number lines ▪ open number lines ▪ walk-on number lines |
|--|---|

MATHEMATICAL LANGUAGE

Teacher	Student (oral language)
<ul style="list-style-type: none"> ▪ counting on ▪ left, right, middle ▪ numeral ▪ order, arrangements ▪ quantity of objects 	<ul style="list-style-type: none"> ▪ left, right, middle ▪ numeral ▪ quantity of objects

Resources/Notes

Print Resources

- *Making Math Meaningful to Canadian Students, K–8* (Small 2009), pp. 83–95
- *Making Math Meaningful to Canadian Students, K–8, Second Edition* (Small 2013), pp. 139–152
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006) pp. 39–47, 54–58

Videos

- *Teaching Number: Counting* (10:49 min.) (ORIGO Education 2010)
- *Using a Hands-on Approach to Represent Numbers to 10* (13:06 min.) (ORIGO Education 2010)

Notes

SCO N04 Students will be expected to represent and partition numbers to 20. [C, CN, V]			
[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- N04.01** Represent a given number up to 20 using a variety of manipulatives, including ten-frames and created materials.
- N04.02** Model a given number up to 20 using a variety of pictorial representations.
- N04.03** Find examples of a given number in the environment.
- N04.04** Place given numerals on a number line with benchmarks 0, 5, 10, 15, and 20.
- N04.05** Partition any given quantity up to 20 into two parts and identify the number of objects in each part.
- N04.06** Model a given number using two different objects.

Scope and Sequence

Mathematics Primary	Mathematics 1	Mathematics 2
N04 Students will be expected to represent and describe numbers 2 to 10 in two parts, concretely and pictorially.	N04 Students will be expected to represent and partition numbers to 20.	N04 Students will be expected to represent and partition numbers to 100.

Background

In order to fully develop deep meaning of and flexibility with numbers to 20, students need to experience multiple representations of those numbers. They should be given opportunities to model numbers concretely with a variety of materials, draw a variety of pictures, and identify a variety of real-world uses. **Note:** Verbal and symbolic representations are specifically addressed in SCO N01; however, these representations should be used in conjunction with this outcome.

The ability to recognize that any number can be partitioned into two or more smaller numbers is an essential understanding in number development. Understanding the relationship between/among the parts and the whole is essential to developing a sense of numbers and an understanding of mathematical operations. Students should be able to see small groups as part of a larger group. They should also be able to recognize how many objects are in each of the small groups. Partitioning numbers helps students develop part-part-whole thinking. (This is also seen in SCO N02.)

In primary, students have had experiences representing and describing numbers to 10, in one or two parts, concretely and pictorially. They have also been ordering quantities using objects, five-frames, ten-frames, or dot cards as well as relating a numeral, 1 to 10, to its respective quantity.

Additional Information

See Appendix A: Additional Information.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Present the following problem: I have 10 cubes in this bag. Some are blue and some are red. How many cubes might be blue and how many might be red? Can you tell another possible combination of blue and red cubes that might be in the bag?

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask children to show the number 15 in as many different ways as they can using manipulatives. This may also be done pictorially.
- Tell students, In my bowl, I have apples and bananas. There are 14 pieces of fruit altogether. How many apples are there? Draw a picture of the fruit. Are there other possibilities?
- Ask individual students, I was counting objects in our classroom. I counted exactly 18 of the same thing. What could I have been counting? Tell me why. What are some things I could not have been counting? Why could they not be the objects I was counting?
- Shake and spill a handful of two different colours of counters (20 or less) on the overhead. Ask students to record (with pictures) how many of each colour there are and how many counters there are altogether. Eventually, they should also name the quantity in each part by recording the respective numeral beside or below each part of their drawing.
- Provide students with a given number (20 or less) and ask them to make two-colour bars, using linking cubes to represent that number. Ask students to tell you about the number of cubes, describing each part and the whole. Ask them to find as many different two-colour bars as they can to represent their given number.
- Present students with a number line showing benchmarks 0, 5, 10, 15, and 20. Give them a number (1–20) and ask them to place the number on the number line and to explain how they know where it goes.
- Hold out a bar of linking cubes, a dot strip, a two-column strip, or a dot plate showing 6 or fewer. Say, “I wish I had six.” The students should respond with the part that is needed to make 6. Counting on can be used to check. The game can focus on a single whole, or the “I wish I had ...” number can change each time.
- Give the students a picture card and ask them to model or tell a number story about a part/whole relationship.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets K–2 (Bauman 2011)

- Mathematics 1 Checkpoint 5, pp. 57–58 (Line Master 5.2)
- Mathematics 2 Checkpoint 5, pp. 59–60 (Line Master 5.2)

Planning for Instruction

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of my students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Provide students with many opportunities to represent numbers in part-part-whole relationships using a variety of physical materials or pictorial representations and to explain how many are in each part of a whole.
- Provide students with counting activities in which sets of 11 to 19 items are counted. Students will develop number sense and will recognize that certain partitions, such as a group of 10 and 7 more, make it easier to determine the size of the set. This is an important **pre-place value** concept.

SUGGESTED LEARNING TASKS

- Tell students, There are 16 squirrels in a park that has one big tree and one small tree. One day, all the squirrels were in the trees. How many squirrels could be in the big tree and in the small tree? Draw two trees on the board and have construction paper squirrels to place in the trees. Change the position of the squirrels as students offer alternative answers.
- **Part-Part-Whole Mats Option 1:** Show students a set of 15 counters. Ask students to count how many are in the set. Place the 15 counters on the part-part-whole mats to show two parts, for example 12 and 3. Ask, How many objects are there? How do you know? Observe whether students

must recount all of the objects or if they know that partitioning the objects has not changed the quantity. Repeat with a different number of objects.

- **Part-Part-Whole Mats Option 2:** Ask students to count out 10 counters and place them on one side of a part-part-whole mat. Ask them to place 5 counters on the other side. Together, count all the counters by ones to determine the total number of counters. Then, identify each part and the whole. Say, 10 and 5 more is 15 and 15 is 10 and 5 more. Turn the mat around and say, 5 and 10 more is 15 and 15 is 5 and 10 more. Repeat with other numbers (11 to 20) without changing the 10 side of the mat.
- **Part-Part-Whole Mats Option 3:** Ask students to count out 11 counters beside a three-part mat. Have them place 5 counters on one side, 5 in the middle, and 1 on the other side. Together count all the counters by ones. Say, “5 and 5 and 1 is 11.” Turn the mat around and say, “1 and 5 and 5 is 11.” Repeat with other numbers on the three sections of the mat without changing the total of eleven counters.
- Ask students to find objects in the classroom that represent numbers from 1 to 20 (e.g., 12: there are 12 windows in the classroom).
- Ask students to explore multiple ways to partition a number. For example, students could be asked to create books in which each page shows a different way to represent a given number as two parts, using pictures.
- Keep a number line in the classroom showing benchmark numbers (0, 5, 10, 15, and 20). Prepare number cards from 0 to 20. Have one student each day select a number card and place it on the number line. Ask each student to explain his or her thinking about the number and its placement in relation to the benchmark numbers.

SUGGESTED MODELS AND MANIPULATIVES

- counters
- dominoes
- linking cubes
- part-part-whole mats
- various number lines

MATHEMATICAL LANGUAGE

Teacher	Student (oral language)
<ul style="list-style-type: none"> ▪ numeral ▪ partition numbers/quantities ▪ part-part-whole relationships ▪ parts or whole ▪ ten-frames, number lines 	<ul style="list-style-type: none"> ▪ numeral ▪ parts or whole ▪ ten-frames, number lines

Resources/Notes

Print Resources

- *Making Math Meaningful to Canadian Students, K–8* (Small 2009), pp. 83–95
- *Making Math Meaningful to Canadian Students, K–8, Second Edition* (Small 2013), pp. 139–152
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 47–58

Videos

- *Powerful Models to Help Struggling Students: Number Lines* (17:37 min.) (ORIGO Education 2010)
- *Teaching Number: Relative Position* (20:41 min.) (ORIGO Education 2010)
- *Teaching Place Value: Teen Numbers* (8:30 min.) (ORIGO Education 2010)
- *Using Structured Patterns to Develop Number Combinations* (18:11 min.) (ORIGO Education 2010)

Notes

SCO N05 Students will be expected to compare sets containing up to 20 objects to solve problems using referents and one-to-one correspondence.

[C, CN, ME, PS, R, V]

[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- N05.01** Build a set that has more, fewer, or as many objects as a given set, up to 20 objects.
- N05.02** Build several sets of different objects that have the same given number of objects in the set.
- N05.03** Compare two given sets using one-to-one correspondence and describe them using comparative words, such as more, fewer, or as many.
- N05.04** Compare a set to a given referent using comparative language.
- N05.05** Solve, using pictures and words, given story problems that involve the comparison of two quantities.

Scope and Sequence

Mathematics Primary	Mathematics 1	Mathematics 2
N05 Students will be expected to compare quantities, 1 to 10, using one-to-one correspondence.	N05 Students will be expected to compare sets containing up to 20 objects to solve problems using referents and one-to-one correspondence.	N05 Students will be expected to compare and order numbers up to 100.

Background

Comparing is something students do many times a day. Even before coming to school, they will have developed a sense of fairness when comparing sets of objects. In primary, students had experiences comparing quantities, 1 to 10, by constructing a set to show more than, fewer than, or as many as a given set and by direct comparison.

In Mathematics 1, these same comparisons will be expected for numbers to 20. Students will explore the concept of quantity as it relates to countable objects and to compare these quantities. Therefore, the word **fewer** is used; for example, there are fewer counters in this set than in that set. It is interesting to note that although the concept of fewer is logically equivalent to the concept of more, the word **fewer** proves to be more difficult for students than the word **more**. Students will become more familiar with the term **less than** to compare numbers in later years. They will learn that 5 is less than 7 because there are 2 fewer items in a set of 5 than in a set of 7. When talking about sets that have the same number of objects encourage the use of the terms **the same number as**, **as many as**, and **equal**. In Mathematics 1, students will be introduced to referents (benchmarks) and will solve story problems involving comparison situations. Some students may know one number is more than another simply because it is said later in the counting sequence.

Additional Information

See Appendix A: Additional Information.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Show students a set of 9 counters. Ask them to make a set that has as many counters as your set. Ask them to make a set that has fewer counters than yours. Ask them to make a set that has more counters than yours.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Identify a target number and give each student a set of three cards labelled “more,” “fewer,” and “same.” Show students a double ten-frame that represents a number between 11 and 20. Ask students to compare the double ten-frame display with the target number and to hold up the appropriate card to show whether the double ten-frame has more, fewer, or the same amount of counters as the target number. Later, ask them to tell how many more or fewer are shown.
- Provide representations of two different sets each with 20 or fewer. Ask students which set has more and which has fewer (e.g., one set has 19 and the other has 12) and to explain how they know.
- Hold up a dot card that has between 11 and 20 dots. Give each student a set of three cards labelled “more,” “fewer,” and “same.” Beside each card, ask students to make a collection of counters; one set that has more, one set that has fewer, and one set that has the same as the original card.
- Give students a tower of 14 cubes and another tower of 17 cubes. Ask, Which is closer to 10? and then ask, How do you know? Which is closer to 20? How do you know? You could also ask students to tell you which tower is closer to 15 and to explain their thinking. Allow students to use more linking cubes to assist in solving this problem.
- Give small groups of students interlocking cubes. Give a variety of directions that use the words **more**, **fewer**, and **the same as** and have students build towers. For example, Build a tower that is one more than 11; Build a tower that is two fewer than 9; Build a tower that is two more than 18. Challenge students by saying, “What number do you think is one fewer than 15? Let’s build a tower and see.”
- Present students with story problems that involve comparing two quantities and ask them to use manipulatives or pictures to solve the problems.
- Provide three sets of objects that are not equal and have students number them, in order, from least to greatest. Then, ask students to write comparative sentences about them.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets K–2 (Bauman 2011)

- Mathematics 1 Checkpoint 4, pp. 49–50 (Line Masters 2.5 and 2.6)
- Mathematics 1 Checkpoint 8, pp. 83–84
- Mathematics 1 Checkpoint 15, pp. 141–142
- Mathematics 2 Checkpoint 4, Tasks 1 and 3, pp. 51–51 (Line Masters 2.5, 2.6, and 4.1)
- Mathematics 2 Checkpoint 1, Task 2, pp. 25–26

Planning for Instruction

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of my students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Encourage students to work in pairs or small groups to facilitate discussion and share thinking.
- Provide many opportunities for students to pose or answer questions such as,
 - Who has fewer? Who has more?
 - Are there more boys or girls here today?
 - Are there the same number of chairs and students?
- Provide students with many opportunities to represent numbers concretely.
- Allow students to make purposeful links between concrete, pictorial, and symbolic representations of numbers.
- Expect students to verbally explain their answers about numbers.
- Use real-world contexts for comparison problems whenever possible.

SUGGESTED LEARNING TASKS

- Place 15 counters on a paper plate and ask students to make sets that have more, fewer or the same numbers of counters as the given set of counters.
- Display two double ten-frames, each representing a different number. Ask students, Which double ten-frame has more counters? Which double ten-frame has fewer? How many more? How many fewer?
- Have students work in pairs to play “Dot Challenge.” Provide a deck of dot cards (1 to 20 dots each) for each student. Each student turns up a card; the student whose card has more dots gets both cards. This can also be played so that the student whose card has fewer dots captures both cards.
- Present students with story problems that involve comparing two quantities and ask them to use manipulatives or pictures to solve them. For example, Mary had 6 cookies. Chantelle had 8 more cookies than Mary. How many cookies did Chantelle have?
- Students work in pairs. Place a set of dot cards (1–20) face down. Turn over the top card. Have students identify the number shown. Turn over a word card (more, fewer, the same as) to determine whether students will make a set that has more, fewer, or the same as the one shown on the original card.

SUGGESTED MODELS AND MANIPULATIVES

- counters
- dot cards
- linking cubes
- ten-frames
- various number lines

MATHEMATICAL LANGUAGE

Teacher	Student (oral language)
<ul style="list-style-type: none"> ▪ compare sets ▪ more than, greater than, fewer than, as many as, or the same as (equal), less than ▪ one-to-one ▪ size of objects ▪ ten-frames, number lines 	<ul style="list-style-type: none"> ▪ compare sets ▪ more than, greater than, fewer than, as many as, or the same as (equal) ▪ size of objects ▪ ten-frames, number lines

Resources/Notes

Print Resources

- *Making Math Meaningful to Canadian Students, K–8* (Small 2009), pp. 87–89
- *Making Math Meaningful to Canadian Students, K–8, Second Edition* (Small 2013), pp. 144–145
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 37–38, 54–58, 69

Notes

SCO N06 Students will be expected to estimate quantities to 20 by using referents.

[C, ME, PS, R, V]

[C] Communication

[PS] Problem Solving

[CN] Connections

[ME] Mental Mathematics and Estimation

[T] Technology

[V] Visualization

[R] Reasoning

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N06.01 Estimate a given quantity by comparing it to a given referent (known quantity).

N06.02 Select an estimate for a given quantity by choosing between at least two possible choices and explain the choice.

Scope and Sequence

Mathematics Primary	Mathematics 1	Mathematics 2
—	N06 Students will be expected to estimate quantities to 20 by using referents.	N06 Students will be expected to estimate quantities to 100 by using referents.

Background

The ability to estimate quantities, a key reasoning skill in mathematics, should develop with regular practice over the course of the year. Estimation also helps develop useful benchmarks for thinking about numbers. It is important for students to understand what makes a good estimate. All counting activities can be modified to include estimation. Students should estimate how many are in a set prior to finding the actual count. Prepare daily estimation tasks by presenting several objects and having students record their estimates. Be sure to have students share their strategies for arriving at their estimates. Sometime throughout the day count the objects and compare the estimates to the actual number.

Students should understand the difference between guessing and estimating; that is, estimating involves the use of strategies to find a carefully considered approximation rather than a number “pulled out of the air.” The two words should never be combined as “guesstimation” because it further confuses the terms. Many students are reluctant to give an estimate because they want to be “right.” They need to be convinced that there can be a number of different estimates that are acceptable for a given collection of objects.

After estimates are made, there should be discussion of reasonableness and accuracy of those estimates. Some students also find it easier to commit to an estimate if initial estimation activities involve selecting from two or more stated estimates. Estimation skills will improve over time with many activities and refinement.

Additional Information

See Appendix A: Additional Information.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Place 10 or fewer counters on the table. Ask students, About how many counters are on the table? Ask students to explain their thinking. Note whether the estimate is reasonable.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Fill a container with cubes. The container should hold almost 20 cubes. Show it to the student and then ask, How many cubes do you think are in the container? Have the student count the cubes. Then ask, Was your estimate too small, too large, or just right? How do you know?
- Provide three different sets of objects such as cubes, colour tiles, or unifix cubes. Ask students to take a handful of one of the objects and estimate the number of objects they have. They should then count to check. Ask, Was your estimate too small, too large, or just right? How do you know? Have students repeat the task with the other objects.
- Place between 10 and 20 counters on the overhead projector. Turn the projector on for five seconds and then turn it off. Ask students to write down their estimate. Turn the projector on and move 5 counters (referents) off to the side but still on the screen. Tell students, "Here are 5 counters. Do you want to adjust your estimate? If you want to adjust it, will it be more or less than your first estimate? Ask students to explain their decision.
- Show students a set of objects and ask them to choose between the two estimates you've provided. Ask them to explain their choice.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets K–2 (Bauman 2011)

- Mathematics 1 Checkpoint 2, Task 3, pp. 32–33 (Line Masters 2.7 and 2.8)

Planning for Instruction

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of my students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

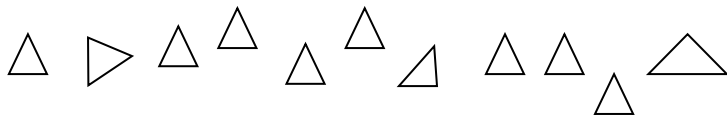
Consider the following strategies when planning daily lessons.

- Expect students to verbally explain their answers about numbers.
- Provide students with a variety of estimation activities. For small groups ask, Is it closer to 5 or 10? For large collections, one might be asking whether the group is closer to 20.
- Develop an understanding of the concept of “about” as it relates to estimation activities. Use varied language, such as
 - More or less than ___? Are there more or fewer than 15 counters on the overhead projector?
 - Closer to ___ or to ___? Do I have closer to 10 cubes or closer to 15 cubes in the clear glass?
 - Less than ___, between ___ and ___, or more than ___?

SUGGESTED LEARNING TASKS

- Find or create pictures of sets of objects. Show a picture and suggest two possible estimates. Ask students to select one of the two possible estimates and to justify their choice.

Estimate how many triangles are on this page—15 or 9?



- Provide students with a set of 16 objects without telling them the number of objects in the set. Tell them that three students estimated the number of objects and their estimates were 4, 13, and 20. Ask them to discuss each estimate and to decide if the estimate is too large, too small, or just right and to explain their thinking.
- Use three identical transparent containers. Fill one container. Use the other containers as referents. For example, put 16 blocks in the first container. Put 5 blocks in the second container and 10 in the third container. Have students use the referents to estimate the number of blocks in the full container. Ask students to explain their thinking as they use the referents to estimate.
- Provide students with a bag of cubes. Ask them to reach in and grab about 5, 10, or 15 cubes. Ask the students to count the cubes they took from the bag and to determine if they took too many, too few, or just the right number of cubes.

- Have students gather groups of classroom objects (between 5 and 20). Take digital pictures of the sets, and have students identify two possible estimates for the set. The pages can be compiled into a class estimation book.

SUGGESTED MODELS AND MANIPULATIVES

- cubes
- counters
- objects of various sizes

MATHEMATICAL LANGUAGE

Teacher	Student (oral language)
<ul style="list-style-type: none">▪ estimate quantities▪ referents, benchmarks	<ul style="list-style-type: none">▪ estimate

Resources/Notes

Print Resources

- *Making Math Meaningful to Canadian Students, K–8*, Second Edition (Small 2013), pp. 144–145
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 58–60

Notes

SCO N07 Students will be expected to demonstrate an understanding of conservation of number for up to 20 objects.

[C, R, V]

[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N07.01 Explain why for a given number of counters, no matter how they are arranged, the total number of counters does not change.

N07.02 Group a set of given counters in more than one way.

N07.03 Explain why for a given number of counters, no matter how they are grouped, the total number of counters does not change.

Scope and Sequence

Mathematics Primary	Mathematics 1	Mathematics 2
N06 Students will be expected to demonstrate an understanding of counting to 10.	N07 Students will be expected to demonstrate an understanding of conservation of number for up to 20 objects.	—

Background

Meaningful counting involves an understanding of the six principles of counting. These principles are addressed in various outcomes (N01, N03, and N07). The principle in bold face is the one addressed in this outcome.

1. One number is said for each item in the group and is counted once and only once. (one-to-one correspondence)
2. Counting begins with the number 1, and there is a set number sequence. (stable order)
3. The quantity in the set is the last number said. (cardinality)
4. The starting point and order of counting the objects does not affect the quantity. (order irrelevance)
- 5. The arrangement or types of objects does not affect the count. (conservation)**
6. It does not matter what is being counted, the resulting count will always be the same. (abstraction)

Of the six counting principles discussed in SCO N03, the conservation principle is the only one addressed by its own outcome. This is done to highlight its importance in number development, but should be addressed in conjunction with outcome N03. While it was explored in primary, it is often the last principle to be attained by students, so it is an essential part of counting expectations in Mathematics 1. Trusting that you will get the same count no matter where you start, or in which order, or how the objects are arranged is a very significant development in students' thinking. Many students in Mathematics 1 also believe they may get a different quantity if they count in a different way. It is critical that by the end of grade one, students are convinced they will get the same count whether they count by 1s, 2s, 5s, or 10s.

Conservation: Conservation of number is the understanding that the number of objects remains the same when they are rearranged spatially. When students do not demonstrate conservation of number, they may believe that the number of objects can increase or decrease when they are pushed out or in.

Additional Information

See Appendix A: Additional Information.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Provide students with a set of 10 objects. Ask them to count the objects. Rearrange the objects while students watch. Ask students to tell you how many objects are there now. Note whether students recount the objects or whether they know that the number of objects has not changed.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Observe and note the way in which students count.
 - Do they show confidence in their count or feel the need to check?
 - Do they correctly predict the count if the objects they have just counted has been rearranged?
 - Do they correctly predict the count if the objects they have just counted have been grouped to make counting easier (by 2s, 5s)?
- Provide a selection of buttons, linking cubes, or bread tags to represent any number less than 20. Ask students to count them and tell the number. Then, ask them to rearrange the objects to make counting easier and to predict the number of objects that are there before they count. Note whether students understand that the count has not changed even though the arrangement has.
- Give the children a set of counters and ask them to group the counters in different ways. They record their work with drawings of their various groupings. Ask them to explain why the number of counters is the same though the groupings are not.
- Explain why it is possible to have a number such as 13 described using two or more parts in more than one way.

- Ask a student to model a number in two or more parts. Ask students to represent that number in as many different ways they can. Ask them to explain why the arrangements are different but the number is the same.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets K–2 (Bauman 2011)

- Mathematics 1 Checkpoint 5, pp. 57–58 (Line Master 5.2)
- Mathematics 2 Checkpoint 5, pp. 59–60 (Line Master 5.2)

Planning for Instruction

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of my students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Provide students with one type of material, such as linking cubes. Explore how many different combinations for a particular number can be made using different arrangements or groupings of the cubes. Direct students' attention to the fact that the arrangements and groupings do not change the total number of cubes that they have.

SUGGESTED LEARNING TASKS

- Observe and note the way in which students count.
 - Do they show confidence in their count or feel the need to check?
 - Do they correctly predict the count if the objects they have just counted has been rearranged?
 - Do they correctly predict the count if the objects they have just counted have been grouped to make counting easier (by 2s, 5s)?

- Ask students to bring in a set of between 2 and 20 of their favourite things. Alternatively, you could provide pairs of students with sets of between 2 and 20 objects. Ask students to count their favourite things/objects and to record the number. Then ask students to place their favourite things/objects in various locations (on a desk, on a large table, in a cup, on a window ledge, etc.) and in various arrangements (close together, spread out, in a row, in a circle) around the classroom. After placing them in each new location and/or new arrangement, ask students to count the objects to confirm that the number of objects has remained the same. If a digital camera is available, students could take pictures of each of their locations/arrangements and could use the pictures to create “Conservation” books.
- Give each pair of students a set of 2–20 objects. Ask them to count their objects and to record the number of objects. Then ask them to work together to find different ways to group the objects (by 2s, by 5s, with groups and singles). After each new grouping, ask students to count their objects again to confirm that the number of objects remains the same. Students can record with drawings the various groupings they discover. When they have completed the task, ask students to explain why the number of objects is the same though the groupings are not.
- Provide each student with a set of paper plates or index cards and some dots. Have each student select a number (20 or less). Students will then make a set of dot cards/plates to represent the number chosen. The dot cards/plates should show various arrangements or groupings.
- Request up to 20 student volunteers. Play a game of “Simon Says” and count the group after each of Simon’s directions and direct students’ attention to the fact that the size of the group has not changed though Simon’s directions have arranged them in different ways. For example, Simon says, “Stand in a small circle,” or “Stand in a line at the front of the room,” or “Spread out in the hallway (or the gym),” or “Stand in pairs around the room.”

SUGGESTED MODELS AND MANIPULATIVES

- counters
- dot cards
- double ten-frames
- linking cubes
- ten-frames

MATHEMATICAL LANGUAGE

Teacher	Student (oral language)
<ul style="list-style-type: none"> ▪ groups ▪ singles 	<ul style="list-style-type: none"> ▪ groups ▪ singles

Resources/Notes

Print Resources

- *Making Math Meaningful to Canadian Students, K–8* (Small 2009), pp. 83–86; 95
- *Making Math Meaningful to Canadian Students, K–8, Second Edition* (Small 2013), pp. 141–142, 151
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 47–52; 54–55

Notes

SCO N08 Students will be expected to identify the number, up to 20, that is one more, two more, one less, and two less than a given number.

[C, CN, ME, R, V]

[C] Communication

[PS] Problem Solving

[CN] Connections

[ME] Mental Mathematics and Estimation

[T] Technology

[V] Visualization

[R] Reasoning

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N08.01 Name the number, up to 20, that is one more, two more, one less, or two less than a given number.

N08.02 Represent a number on ten-frames that is one more, two more, one less, or two less than a given number.

Scope and Sequence

Mathematics Primary	Mathematics 1	Mathematics 2
—	N08 Students will be expected to identify the number, up to 20, that is one more, two more, one less, and two less than a given number.	<p>N08 Students will be expected to demonstrate and explain the effect of adding zero to or subtracting zero from any number.</p> <p>N10 Students will be expected to apply mental mathematics strategies to quickly recall basic addition facts to 18 and determine related subtraction facts.</p>

Background

Developing an understanding of the relationship between numbers is an important aspect of number sense. Students do not necessarily reflect on the connection between two numbers when they are counting; therefore, they should have experiences that direct their attention to these relationships. When talking about numbers, such as 6 and 8, students need to say that 8 is two more than 6, and 6 is two less than 8. Similarly, the relationship between numbers that differ by 1 can be explored. Identifying these relationships should build on students' experiences with counting on and counting back allowing them to deepen their understanding of numbers.

It is interesting to note that although the concept of fewer is logically equivalent to the concept of more, the word **fewer** proves to be more difficult for students than the word **more**. The words **fewer** and **less** are often used incorrectly. The word **fewer** should be used when referring to countable quantities; for example, there are fewer students in this class than the class next door. The word **less** is used when referring to measures and non-countable items; for example, there is less water in this bottle. However, when comparing numbers, the phrases, **less than** and **greater than**, should be used.

Additional Information

See Appendix A: Additional Information.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Show students a ten-frame representing the number 6. Ask students to identify the number shown. Then, ask them to select another ten-frame from the group on the table that has more (or fewer) than the ten-frame showing 6.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Give students dot cards and some counters. Ask them to create sets that are one more than, one less than, two more than, and two less than the dot cards and to name the sets that are created.
- Call out a number up to 20. Ask students to select a ten-frame card that shows the number that is one more, one less, two more or two less than the number you called out.
- Use ten-frames on the overhead to show numbers up to 20. After showing the ten-frame(s), ask students to write the number that is one more, one less, two more, or two less than the number shown.
- Create two groups of students—7 boys and 6 girls. Ask, What must be changed to make the number of girls equal to the number of boys? What must be changed to make the number of boys equal to the number of girls?
- Ask children to sort a collection of buttons by various criteria into two sets so that the sets are one more or one less than each other. Compare the size of the sets.
- Students should be able to create a set equal in number to a given set. Ask students to change their set to equal a number that is one or two more (less) than their current set. For example, change your set of 8 counters to show a set that is two more. Name the new set that you have created.
- Place 3 red counters and 3 blue counters in one group and 3 blue and 2 red in another, as follows: RRR BBB BBB RR. Ask, How do you know that there are more blue counters than red in the second group? How many more?

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets K–2 (Bauman 2011)

- Mathematics Primary Checkpoint 6, pp. 64–65
- Mathematics 1 Checkpoint 1, Task 1, pp. 23–24
- Mathematics 1 Checkpoint 4, Task 2, pp. 49–50 (Line Masters 2.5 and 2.6)

Planning for Instruction

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of my students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Ensure that students' initial exploration of numbers that are one more than, one less than, two more than, and two less than is done concretely using sets of objects.
- Give students frequent opportunities to transfer their thinking from one representation to another. For example, showing 6 and 7 with linking cubes and then displaying the same numbers with counters on a ten-frame.
- Ensure students are able to create a set that is one more / one less than a given set and two more / two less than a given set.

SUGGESTED LEARNING TASKS

- Show students a number of counters on an overhead projector or interactive whiteboard. Have the students count them. Have students close their eyes while you change the amount of counters by one or two. Ask the students to open their eyes and tell you how the group of counters has changed.
- Invite students to play "One More Than Dot Cover."
 - Create cards using symbols and dot patterns of numbers up to 8.
Rules:
 - > Take turns rolling a number cube.
 - > Cover any one square on the card that is one more (or two more, one less, or two less) than the top number on the number cube.
 - > The player who first covers three in a row is the winner.
 - > This game could be extended to numbers to 20 using two number cubes or by using dice with more sides and changing the student cards.

- Students work in pairs. Give each pair of students a set of numeral cards marked 1 to 20 and a set of word cards with the words “one more,” “one less,” “two less,” and “two more” written on them. The first student takes a number card and makes that number on a ten-frame. The other student takes a word card and makes the new number on a ten-frame. Students compare the ten-frames to verify that the two numbers are correct. They then record their work using words. For example, if the first student drew a “16” and the second student drew the “two more” word card, the students would record “18 is two more than 16.”
- Invite students to make up story problems to solve. For example, if I have 20 teeth and I lose a tooth, how many teeth will I have? If I have 18 teeth and one more tooth grows in, how many teeth will I have? I had 20 teeth, but I lost my 2 front teeth. How many teeth do I have now?
- Invite students to create their own story problems that involve one more, one less, two more, and two less situations.

SUGGESTED MODELS AND MANIPULATIVES

- dot cards
- five-frames
- number cubes
- Rekenrek
- ten-frames

MATHEMATICAL LANGUAGE

Teacher	Student (oral language)
<ul style="list-style-type: none"> ▪ count on, count back ▪ one more, two more, one less, two less, before, after ▪ ten-frames, double ten-frames, number lines 	<ul style="list-style-type: none"> ▪ count on, count back ▪ one more, two more, one less, two less, before, after ▪ ten-frames, double ten-frames, number lines

Resources/Notes

Print Resources

- *Making Math Meaningful to Canadian Students, K–8* (Small 2009), pp. 87–89
- *Making Math Meaningful to Canadian Students, K–8, Second Edition* (Small 2013), pp. 144–145
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 37–38, 44–45, 54–55

Videos

- *Using a Hands-on Approach to Develop Mental Strategies for Addition* (11:04 min.) (ORIGO Education 2010)
- *Using a Hands-on Approach to Develop Mental Strategies for Subtraction* (6:45 min.) (ORIGO Education 2010)

Notes

SCO N09 Students will be expected to demonstrate an understanding of the addition of two single-digit numbers and the corresponding subtraction, concretely, pictorially, and symbolically in join, separate, equalize/compare, and part-part-whole situations.

[C, CN, ME, PS, R, V]

[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- N09.01** Act out story problems that are presented orally or through shared reading.
- N09.02** Model story problems with manipulatives or pictures, find and share solutions using counting strategies, and record number sentences that represent how they thought about the problems.
- N09.03** Create story problems that connect to student experiences.
- N09.04** Create story problems for given number sentences.

Scope and Sequence

Mathematics Primary	Mathematics 1	Mathematics 2
—	N09 Students will be expected to demonstrate an understanding of the addition of two single-digit numbers and the corresponding subtraction, concretely, pictorially, and symbolically in join, separate, equalize/compare, and part-part-whole situations.	N09 Students will be expected to demonstrate an understanding of addition (limited to 1- and 2-digit numerals) with answers to 100 and the corresponding subtraction by <ul style="list-style-type: none"> ▪ using personal strategies for adding and subtracting with and without the support of manipulatives ▪ creating and solving problems that involve addition and subtraction ▪ explaining and demonstrating that the order in which numbers are added does not affect the sum ▪ explaining and demonstrating that the order in which numbers are subtracted matters when finding a difference

Background

As with many early concepts, the development of the meaning of addition and subtraction cannot be rushed. It is desirable to explore adding and subtracting situations in meaningful contexts. Students should have extensive investigative experiences in which they use a variety of concrete materials and pictures to model and compare contexts, before recording the number sentence that represents their thinking. It is important that problems be personalized, but students also need experience interpreting how addition and subtraction are portrayed in print.

When working with addition, include examples of

- active situations that involve the physical joining of sets (join problems) (For example, I had 4 pencils. My teacher gave me 3 more. How many do I have now?)
- static situations (part-part-whole) that involve the implied joining of sets that are not physically joined to form a whole (For example, there are 4 cars parked on one side of the road. There are 3 cars parked on the other side of the road. Altogether, how many cars are parked on the road?)

With subtraction, it is important to include examples of

- active situations that involve the physical separating of sets (separate problems) (For example, I had 8 pencils. I gave 4 of them to my friend. How many do I have left?)
- static situations (part-part-whole) that involve the implied separating of sets that are not physically joined to form a whole (For example, there are 7 red and green cars parked on the road. Four of them are red. How many cars are green? [In this situation, the whole remains the same, nothing is added or taken away, we are looking to find the two parts that make up the whole]).

Addition and subtraction problems can be categorized based on the kinds of relationships they represent. It is important that all of the following structures of problems be presented and that these are developed from students' experiences. These structures include

Join			Part-Part-Whole	Compare
Result Unknown	Change Unknown	Start Unknown	Whole Unknown	Difference Unknown
Pat has 8 marbles. Her brother gives her 4. How many does she have now? $8 + 4 = ?$	Pat has 8 marbles, but she would like to have 12. How many more does she need to get? $8 + ? = 12$ or $12 - 8 = ?$	Pat has some marbles. Her brother gave her 4 and now she has 12. How many did she have to start? $? + 4 = 12$ or $12 - 4 = ?$	Pat has 8 blue marbles and 4 green marbles. How many does she have in all? $8 + 4 = ?$	Pat has 8 blue marbles and 4 green marbles. How many more blue marbles does she have? $8 - 4 = ?$ or $4 + ? = 8$
Separate			Part-Part-Whole	Compare
Result Unknown	Change Unknown	Start Unknown	Part Unknown	Smaller or Larger Unknown
Pat has 12 marbles. She gives her brother 4 of them. How many does she have left? $12 - 4 = ?$	Pat has 12 marbles. She gives her brother some. Now she has 8. How many marbles did she give to her brother? $12 - ? = 8$ or $12 - 8 = ?$	Pat has some marbles. She gives her brother 4 of them. Now she has 8. How many marbles did she have to start? $? - 4 = 8$ or $8 + 4 = ?$	Pat has 12 marbles. Eight are blue and the rest are green. How many are green? $8 + ? = 12$ or $12 - 8 = ?$	Pat has 8 blue marbles and some green marbles. She has 4 more blue marbles than green ones. How many green marbles does she have? $8 - 4 = ?$ or $? + 4 = 8$

Initial work with addition and subtraction may focus on join and separate types of problems because students associate the actions in these problems with the operations. Eventually, however, students should experience all story problem structures. **Note:** Compare problems are addressed as part of SCO N05.

More information on each of these structures can be found in *Teaching Student-Centered Mathematics* by Van de Walle and Lovin (2006).

Symbols for addition, subtraction, and equals should be introduced slowly and should only be used as a way to record number sentences that represent the contexts that are modelled. (See outcome PR04.) Model the symbolic recording when students share solutions orally. This modelling can be done well before expecting students to begin recording number sentences. When working with the symbols for addition and subtraction, the sign (–) is called a minus and the sign (+) is called a plus. Students should be exposed to a variety of language phrases that correspond to these signs, depending upon the contexts. For addition phrases, such as $4 + 2$, these could include

- 4 add 2
- 4 and 2
- 4 plus 2
- 2 more than 4
- the sum of 4 and 2
- the total of 4 and 2

For subtraction phrases, such as $8 - 2$, these could include

- 8 subtract 2
- 8 take away 2
- 8 minus 2
- 2 less than 8
- the difference between 8 and 2

Some care should be taken with the equal sign (=). The equal sign means “is the same as” or “is equivalent to.” However, most children come to think of it as a symbol that tells you that the answer is coming up. A good idea is to often use the phrase “is the same as” in place of, or in conjunction with, equals as you read number sentences with students.

Students need to be able to explain how they got their answers demonstrating their use of addition and subtraction. By observing students at work, we can assess their understanding of how they solve addition and subtraction problems.

Additional Information

See Appendix A: Additional Information.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Give students counters and ask them to act out the story problem you will tell them. Tell students a simple join story problem using numbers less than 5, such as "I have 3 flowers and my friend gave me 2 more flowers. How many flowers do I have now?" Observe as students solve the problem.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Model this problem for a pair of students: I had 5 pennies and now I have 9. How many pennies did I earn? Ask the pair to make up a similar problem, using objects of their choice, and to model and describe it.
- Ask students to tell an addition/subtraction story involving 8 and 5 while manipulating a model.
- Provide the students with a given number of counters. Ask them to add/remove 3 or another number of counters and tell how many are now there. Ask them to represent this symbolically.
- Tell the student that Jake had 9 pencils and lost 3, while Martha had 7 pencils and lost 2. Ask, Who has more pencils left? Explain how you know.
- Tell the student that you had 9 marbles, but lost some. There are only 4 marbles left. Ask, How many did I lose? Show how you know.
- Present students with a number sentence. Ask them to create a story problem that matches the number sentence.
- Present students with a pictorial representation of a story problem. Ask them to tell a story that matches the picture.
- Provide students with manipulative materials and present them with each of the following number problems. Ask students to solve each problem and record the number sentence.
 - Chen has 8 cards. David has 4 more cards than Chen. How many cards does David have?
 - Brodie has 18 toys. Eight of his toys are cars and the rest are animals. How many animals does Brodie have?
 - Sophie had 12 apples. She gave some to her mother and now she has 8 apples. How many did she give to her mother?
 - Shona had 15 hairbands. Her dad gave her some more. Now she has 18 hairbands. How many did dad give her?
- Have students create their own word problems for the number family 7, 9, and 16 (numbers related by addition and subtraction). Ask them to write a problem that uses these numbers in addition and another problem that uses these numbers in subtraction.

FOLLOW-UP ON ASSESSMENT**Guiding Questions**

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets K–2 (Bauman 2011)

- Mathematics 1 Checkpoint 6, pp. 66–67 (Line Master 6.1)
- Mathematics 2 Checkpoint 5, Task 1, pp. 59–60 (Line Master 5.2)
- Mathematics 2 Checkpoint 6, pp. 68–69 (Line Masters 6.1 and 6.2)

Planning for Instruction

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of my students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Personalize word problems for students. Encourage students to create a variety of meaningful problems based on situations with which they are familiar. Use daily classroom experiences to tell story problems that represent addition and subtraction.
- Manipulate concrete materials to represent the context as you or students relate a solution to a word problem. Verbalize the process as you manipulate the model.
- Provide a wide variety of problem types and include all four problem structures.

SUGGESTED LEARNING TASKS

- Choose a book, or make up a story, that tells about an addition or a subtraction situation and ask a student to model the situation with counters as you read the book. Reread the story and use pictures and number sentences to record the actions. Ask students to make up other problems about the story.
- Model the telling of story problems. Ask students to orally tell story problems for other students to act out and answer. Story mats can be used for support.
- Use a story mat and some counters for each student. Present problem scenarios and ask students to act them out using manipulatives.
- Have students model addition and subtraction questions concretely using a Rekenrek.
- Pose story problems, such as Janet has 6 baseball cards. Mario gives her some hockey cards. She now has 13 sports cards. How many sports cards did Mario give her? Observe how students solve the problem. Students should be encouraged to share strategies and to explain their thinking.

- Ask students to make a drawing to model this and other structures of problems: Robert had some baseball cards. He gave his brother 2 of the cards. Robert now has 8 cards. How many cards did Robert have to start?
- Ask students to think of a situation in which someone might add. Ask students to think of a situation when they might subtract.
- Provide storyboards for students to use with manipulatives to create, model, and solve story problems. Storyboards can be created by drawing a simple scene, such as a fence, an ocean, or a tree, on a half-sheet of $8\frac{1}{2} \times 11$ " paper. As well, a piece of black construction paper can be used to represent outer space or nighttime, sandpaper for a beach, and blue paper for the sky. Many different problems can be created using the same storyboards. Students should share their story problems with others and record the corresponding number sentence for each of their problems.
- Present students with a number sentence. Ask students to create a story problem that matches the number sentence.

SUGGESTED MODELS AND MANIPULATIVES

- counters
- dominoes
- linking cubes
- number cubes
- Rekenrek
- ten-frames

MATHEMATICAL LANGUAGE

Teacher	Student (oral language)
<ul style="list-style-type: none"> ▪ add, subtract, ▪ count on, count back ▪ equal, is the same as ▪ minus, plus ▪ more than, less than ▪ number sentence ▪ story problem, situation ▪ sum, difference, total 	<ul style="list-style-type: none"> ▪ add, subtract, ▪ count on, count back ▪ equal, is the same as ▪ minus, plus ▪ more than, less than ▪ number sentence ▪ story problem, situation ▪ sum, difference, total

Resources/Notes

Print Resources

- *Making Math Meaningful to Canadian Students, K–8* (Small 2009), pp. 103–110
- *Making Math Meaningful to Canadian Students, K–8, Second Edition* (Small 2013), pp. 159–166
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 65–75

Videos

- *Using Language Stages to Develop Addition Concepts* (15:38 min.) (ORIGO Education 2010)
- *Using Language Stages to Develop Subtraction Concepts* (18:32 min.) (ORIGO Education 2010)
- *Using Static Problems to Relate Addition and Subtraction and Introduce Equality* (13:25 min.) (ORIGO Education 2010)
- *Using Static Problems to Relate Addition and Subtraction and Introduce Functions* (18:59 min.) (ORIGO Education 2010)

Notes

SCO N10 Students will be expected to use and describe strategies to determine sums and differences using manipulatives and visual aids. Strategies include			
<ul style="list-style-type: none"> ▪ counting on or counting back ▪ one more or one less ▪ making ten ▪ doubles ▪ near doubles 			
[C, CN, ME, PS, R, V]			
[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- N10.01** Use and describe a personal strategy to determine a sum.
N10.02 Use and describe a personal strategy to determine a difference.
N10.03 Use and describe how two different strategies can be used to determine a sum or difference.

Scope and Sequence

Mathematics Primary	Mathematics 1	Mathematics 2
—	<p>N010 Students will be expected to use and describe strategies to determine sums and differences using manipulatives and visual aids. Strategies include</p> <ul style="list-style-type: none"> ▪ counting on or counting back ▪ one more or one less ▪ making ten ▪ doubles ▪ near doubles 	<p>N010 Students will be expected to apply mental mathematics strategies to quickly recall basic addition facts to 18 and determine related subtraction facts.</p>

Background

Students' first strategy for addition and subtraction involved counting sets and recounting to find results. For example, for $6 + 7$, students used materials to count out a set of 6 and a set of 7, and then counted both sets together starting at 1 until they reached 13. Similarly, for $12 - 4$, students used materials to count out a set of 12, from this set of 12 they took away 4 one at a time, and then counted how many were left. This outcome addresses exploring alternative strategies to this "count-all" strategy. Students will need many rich experiences exploring these strategies concretely and pictorially.

Modelling specific examples of the strategies will be an important aspect of the development.

Strategy	Examples
Counting on	Find $6 + 3$, by making a set of 6 counters and a set of 3 counters, covering the 6 with your hand as you say, "6," and then pointing to each counter in the set of 3, as you say, "7, 8, 9."
Counting back	Find $8 - 2$ by making a row of 8 counters, removing 1 counter and placing it below as you say "7," and removing another counter and placing it below as you say "6." You stop because there are 2 counters below, and the 6 you said is how many counters were left in the row above.

Strategy	Examples
One more	Find $4 + 1$, $5 + 1$, and $6 + 1$ by building a set of cube towers for a series of consecutive numbers 4, 5, 6, and 7. Place the towers side by side, showing $4 + 1$ by placing 1 cube on the 4 tower to see that it becomes a 5 tower. Similarly show that $5 + 1$ becomes a 6 tower and $6 + 1$ becomes a 7 tower. Then generalize that adding 1 to a number results in the next number in the counting sequence.
One less	Find $7 - 1$, $6 - 1$, and $5 - 1$ by using the same set of towers as for one more, show that removing 1 cube from the 7 tower to see that it becomes a 6 tower, and so on. Then generalize that subtracting 1 from a number results in the number that comes before it in the counting sequence.
Making 10	Find $9 + 5$ by making 9 in one ten-frame with counters and 5 in another ten-frame, removing 1 counter from the 5 and putting it with the 9, creating 10 and 4, saying, "Nine plus five is the same as ten plus four, which is fourteen."
Doubles	Find $6 + 6$ by displaying a picture of an egg carton, which is known to contain 12 eggs (a dozen) and is in two rows of 6. Doubles should be developed using associations with things with which students are familiar.
Near doubles	Find $6 + 7$ by building a 6-cube tower and a 7-cube tower, placing them side by side, removing 1 cube from the 7 to create two 6-cube towers and a single cube, and as you point, saying, "Six and six are twelve and one more is thirteen."

In Mathematics 1, the strategies are the focus of the learning experiences; students are not expected to recall facts. In Mathematics 2, students will be applying these strategies mentally without using materials, so this foundational work in Mathematics 1 is critical. It is important that opportunities are provided for student discussion and sharing of a wide variety of strategies, including their own.

Additional Information

See Appendix A: Additional Information.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Ask students to tell you the number that
 - comes after 8
 - comes before 6
- Ask students to show a set of 2 counters and a set of 8 counters. Ask them to tell you how many counters there are in all.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students to explain how they solve each of the following computations using the strategies studied.

$$7 + 1 \quad 8 - 2 \quad 8 + 9 \quad 7 - 1 \quad 6 + 4 \quad 7 + 8 \quad 4 + 7 \quad 9 + 6$$

- Ask students to explain how they could solve each of the following computations using two different strategies.

$$8 + 9 \quad 9 + 6$$

- Ask students how they could use $6 - 4 = 2$ to figure out $6 - 3$. Students may use materials to model this.
- Have students work in pairs and answer the following question. If you did not know the answer to $9 + 6$, what are some strategies you can use to get the answer? Encourage students to come up with more than one strategy to find the sum. Students discuss their ideas with their partner and then present their ideas to the class.

FOLLOW-UP ON ASSESSMENT**Guiding Questions**

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets K–2 (Bauman 2011)

- Mathematics 1 Checkpoint 6, pp. 66–67 (Line Master 6.1)
- Mathematics 2 Checkpoint 6, pp. 68–69 (Line Masters 6.1 and 6.2)

Planning for Instruction

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of my students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Provide students with opportunities to develop their own strategies for determining a given sum or difference.
- Encourage students to use strategies for solving problems.
- Ask students to employ as many representations as possible for determining sums and differences, including physically acting out, drawing pictures, verbally explaining their ideas, using concrete materials and number lines, and writing number sentences.
- Encourage students to create their own word problems. They can write the problems or dictate them to a scribe.

SUGGESTED LEARNING TASKS

- Have students spill 10 two-coloured counters from a cup. Ask them to describe the addition and subtraction situations that are shown. For example, if 7 red and 3 yellow counters are spilled, students could state, “7 and 3 make 10; 3 and 7 make 10; 10 less 3 is 7; 10 less 7 is 3.”
- Have students show a variety of combinations of numbers that make 10 using ten-frames. Students can record their work with pictures, numbers, and words.
- Use picture cards of real-life contexts to illustrate doubles; for example, $5 + 5$ could be illustrated with two hands; $6 + 6$ could be illustrated with an egg carton showing a dozen eggs, etc.
- Prepare a set of addition and subtraction problem cards. Have students select a card and identify a strategy that could be used to find the answer. The cards can then be placed under that strategy heading.
- Show a ten-frame for a short time. Ask students to select the ten-frame from their set that is one more or one less, than the card you showed.
- Make missing part cards. Each card has a numeral for the whole and two dot sets with one set covered by a flap. Ask students how many are covered and write the number sentence.
- Ask students to build a linking cube train of 10 cubes with two colours in different ways.
- Have a group of approximately 12 students stand in front of the room. Ask the class how many students are at the front. Divide the group into two smaller groups. Ask the class how many students are at the front now and how do they know. Explore the different ways to partition the larger group.
- Play “I Wish I Had” with students. The teacher shows a number of counters and says, “I have 6 counters. I wish I had 10.” Students respond with “You need 4 more because $6 + 4 = 10$.”
- Play a game with students. Provide each student with a card on which a variety of number sentences are written. Each card should contain different number sentences. The teacher calls out a number, and each student covers any number sentence on his or her card that has the number as an answer. For example, if you call out 5, a student might cover $4 + 1$, $6 - 1$, or $10 - 5$.
- Request students to work in pairs and by joining fingers, demonstrate the doubles from 1 to 10. This also offers opportunity to reinforce skip counting by 2s.

SUGGESTED MODELS AND MANIPULATIVES

- coins
- counters
- dominoes
- dot cards
- hundred chart
- linking cubes
- number lines
- ten-frames

MATHEMATICAL LANGUAGE

Teacher	Student (oral language)
<ul style="list-style-type: none"> ▪ add, subtract ▪ counting on, counting back ▪ doubles, near doubles ▪ making ten ▪ one more, one less ▪ strategy ▪ sum, difference 	<ul style="list-style-type: none"> ▪ add, subtract ▪ counting on, counting back ▪ doubles, near doubles ▪ making ten ▪ one more, one less ▪ sum, difference

Resources/Notes

Print Resources

- *Making Math Meaningful to Canadian Students, K–8* (Small 2009), pp. 111–115
- *Making Math Meaningful to Canadian Students, K–8, Second Edition* (Small 2013), pp. 166–170
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 40–41, 44–45, 47, 56

Videos

- *An Introduction to Teaching Addition Number Facts* (15:51 min.) (ORIGO Education 2010)
- *Teaching the Think-Addition Subtraction Fact Strategy* (13:41 min.) (ORIGO Education 2010)
- *Using a Hands-on Approach to Develop Mental Strategies for Addition* (11:04 min.) (ORIGO Education 2010)
- *Using a Hands-on Approach to Develop Mental Strategies for Subtraction* (6:45 min.) (ORIGO Education 2010)
- *Using Static Problems to Relate Addition and Subtraction and Introduce Equality* (13:25 min.) (ORIGO Education 2010)
- *Using Static Problems to Relate Addition and Subtraction and Introduce Functions* (18:59 min.) (ORIGO Education 2010)

Notes

Patterns and Relations (PR)

GCO: Students will be expected to use patterns to describe the world and solve problems.

GCO: Students will be expected to represent algebraic expressions in multiple ways.

Specific Curriculum Outcomes

Process Standards Key

[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

- PR01** Students will be expected to demonstrate an understanding of repeating patterns (two to four elements) by describing, reproducing, extending, and creating patterns using manipulatives, diagrams, sounds, and actions. [C, PS, R, V]
- PR02** Students will be expected to translate repeating patterns from one representation to another. [C, R, V]
- PR03** Students will be expected to describe equality as a balance and inequality as an imbalance, concretely and pictorially (0 to 20). [C, CN, R, V]
- PR04** Students will be expected to record equalities using the equal symbol. [C, CN, PS, V]

SCO PR01 Students will be expected to demonstrate an understanding of repeating patterns (two to four elements) by describing, reproducing, extending, and creating patterns using manipulatives, diagrams, sounds, and actions.

[C, PS, R, V]

[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

PR01.01 Describe a given repeating pattern containing two to four elements in its core.

PR01.02 Identify errors in a given repeating pattern.

PR01.03 Identify the missing element(s) in a given repeating pattern.

PR01.04 Create and describe a repeating pattern using a variety of manipulatives, musical instruments and actions.

PR01.05 Reproduce and extend a given repeating pattern using manipulatives, diagrams, sounds and actions.

PR01.06 Identify and describe a repeating pattern in the environment (e.g., classroom, outdoors) using everyday language.

PR01.07 Identify repeating events (e.g., days of the week, seasons).

Scope and Sequence

Mathematics Primary	Mathematics 1	Mathematics 2
<p>PR01 Students will be expected to demonstrate an understanding of repeating patterns (two or three elements) by identifying, reproducing, extending, and creating patterns using manipulatives, sounds, and actions.</p>	<p>PR01 Students will be expected to demonstrate an understanding of repeating patterns (two to four elements) by describing, reproducing, extending, and creating patterns using manipulatives, diagrams, sounds, and actions.</p> <p>PR02 Students will be expected to translate repeating patterns from one representation to another.</p>	<p>PR01 Students will be expected to demonstrate an understanding of repeating patterns (three to five elements) by describing, extending, comparing, and creating patterns using manipulatives, diagrams, sounds, and actions.</p>

Background

The foundation of all mathematics learning is investigating patterns and their representations. This outcome focuses on repeating patterns and ways of representing these patterns. Not only do students need to recognize the pattern, but they must also be able to describe and extend the pattern. Patterns are explored in all the strands and are also developed through connections with other disciplines, such as science, social studies, English language arts, physical education, and music. In primary, students sorted objects by attributes; copied, extended, described, and created a repeating pattern with a core of two or three elements; and identified the core of a repeating pattern.

Repeating patterns have what mathematicians refer to as a “core” with a number of “elements.” The core of a pattern is the shortest set of elements that iterates (repeats). For example, the colour pattern red, yellow, green, red, yellow, green, red, yellow, green, ..., has a core of three different elements that are repeated. The pattern red, red, yellow, yellow, red, red, yellow, yellow, red, red, yellow, yellow ...,

has a core of four elements even though this core has repeated elements. If students are expected to identify, describe, extend, or reproduce a pattern, it is important that the core of the pattern be repeated at least three times. Students should have many opportunities to work with materials that have one attribute difference, such as cubes, coloured tiles, two-colour counters, collections of small items of one kind (bread tags of different colours), before using materials that have more than one visible attribute, such as attribute blocks or sets of small plastic animals.

While the patterns being explored in Mathematics 1 are repeating patterns, you should be aware that some students may create or describe other types of patterns, such as growing patterns. The detection of patterns should be an ongoing and natural process. Students need to experience both teacher-directed and independent patterning activities. Teacher-directed activities should encourage students to analyze a variety of visual, kinaesthetic, and auditory patterns. Independent activities provide students with the opportunity to explore, reproduce, extend, and create patterns appropriate to their level of understanding.

Additional Information

See Appendix A: Additional Information.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Make an AB pattern with cubes. Ask the students to copy, extend, describe, and identify the core of the pattern.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Show a pattern of cubes; for example, RGGRGGRGG (red, green, green, ...), and ask students to tell you what the pattern is. Then show a different pattern of cubes; for example, YBYBYBYB (yellow, yellow, blue, ...). Have students identify the new pattern. Then, ask students to tell you how the patterns are different and how they are the same.

- Tell students, “Mary has 6 green triangles and 3 orange squares.” Show students the pieces on the overhead. Ask, Can she make two different patterns? Ask students to draw two possible patterns that Mary could make and explain the patterns. Ask, What comes next in the pattern?
- Tell the students that you think there is a pattern to the days (Monday, Tuesday, ...) in a week. Ask students to explain the pattern.
- Have students look at a repeating visual pattern, or listen to a repeating sound pattern, that contains an error. Ask students to correct the error and explain how they know.
- Take students on a walk around the inside and outside of the school looking for patterns. Students can draw a pattern they found and can describe the pattern to a classmate.
- Show students a pattern and have them draw the missing part of the pattern.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets K–2 (Bauman 2011)

- Mathematics 1 Checkpoint 7, pp. 74–75

Planning for Instruction

Before introducing new material, consider ways to assess and build on students’ knowledge and skills.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of my students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Provide students with many opportunities to represent patterns concretely. As students create repeating patterns ask them questions such as
 - What comes next/before/after? How do you know?
 - Can you extend the pattern to the left? To the right?
 - Which part of the pattern repeats? What is the pattern core?
 - What are the elements in the core?
 - Can you make a new pattern with the same materials?

- What else could you use to make the same pattern?
- Are these patterns the same? How do you know?
- How is this pattern different from that pattern?
- Allow students to identify patterns in their daily lives. This can include repetitive songs and rhythmic chants that are based on repeating patterns. Songs and poems can be used to explore patterns.
- Provide a variety of examples of patterns to explore: rhythmic/sound patterns, action patterns, colour patterns, shape patterns, patterns of attributes, and patterns of size.
- Provide students with opportunities to create patterns and to identify the core of a pattern visually.
- Integrate patterns in physical education, music, visual arts, and other subject areas to provide a context.
- Expect students to describe and to explain their thinking about patterns.

SUGGESTED LEARNING TASKS

- Ask students to use pattern blocks or attribute blocks to construct a pattern with two to four elements in its core. Ask them to explain the pattern to another student.
- Ask students to create clapping patterns, or to use stickers or coloured counters to make visual patterns.
- Show students a pattern with an error or missing element in the pattern. Ask students to identify and correct the error or add the missing element.
- Show students a pattern that you have begun. Ask them to continue the pattern in two different ways.
- Give students pattern blocks and ask them to create an ABB pattern. Then ask students, What would this pattern sound like? Continue asking for other patterns, such as ABC, AAB, or ABBC.

SUGGESTED MODELS AND MANIPULATIVES

- | | |
|--|--|
| <ul style="list-style-type: none"> ▪ attribute blocks ▪ calendars ▪ dot cards ▪ geometric solids | <ul style="list-style-type: none"> ▪ linking cubes ▪ pattern blocks ▪ stamps and ink pads ▪ stickers |
|--|--|

MATHEMATICAL LANGUAGE

Teacher	Student (oral language)
<ul style="list-style-type: none"> ▪ core, element ▪ describe, reproduce, extend, create ▪ repeating pattern 	<ul style="list-style-type: none"> ▪ core, element ▪ describe, reproduce, extend, create ▪ repeating pattern

Resources/Notes

Print Resources

- *Making Math Meaningful to Canadian Students, K–8* (Small 2009), pp. 568–569, 571–573
- *Making Math Meaningful to Canadian Students, K–8, Second Edition* (Small 2013), pp. 605–611
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 276–280

Video

- *Using a Teaching Sequence for Repeating Patterns* (22:55 min.) (ORIGO Education 2010)

Notes

SCO PR02 Students will be expected to translate repeating patterns from one representation to another.

[C, R, V]

[C] Communication [PS] Problem Solving [CN] Connections [ME] Mental Mathematics and Estimation
[T] Technology [V] Visualization [R] Reasoning

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

PR02.01 Represent a given repeating pattern using another mode, e.g., actions to sound; colour to shape; ABC, ABC, ABC to blue, yellow, green, blue, yellow, green, blue, yellow, green,...

PR02.02 Describe a given repeating pattern using a letter code, e.g., ABC.

Scope and Sequence

Mathematics Primary	Mathematics 1	Mathematics 2
<p>PR01 Students will be expected to demonstrate an understanding of repeating patterns (two or three elements) by identifying, reproducing, extending, and creating patterns using manipulatives, sounds, and actions.</p>	<p>PR01 Students will be expected to demonstrate an understanding of repeating patterns (two to four elements) by describing, reproducing, extending, and creating patterns using manipulatives, diagrams, sounds, and actions.</p> <p>PR02 Students will be expected to translate repeating patterns from one representation to another.</p>	<p>PR01 Students will be expected to demonstrate an understanding of repeating patterns (three to five elements) by describing, extending, comparing, and creating patterns using manipulatives, diagrams, sounds and actions.</p>

Background

Students should recognize many different forms of the same pattern structure. They need to see that patterns constructed with different materials, or in other ways, have the same pattern structure. For example, all these repeating patterns have the same pattern structure:

- clap, clap, snap, clap, clap, snap, clap, clap, snap, ... (sound pattern)
- red, red, yellow, red, red, yellow, red, red, yellow, ... (colour pattern)
- square, square, triangle, square, square, triangle, square, square, triangle, ... (shape pattern)
- 4, 4, 1, 4, 4, 1, 4, 4, 1, ... (number pattern)

Additional Information

See Appendix A: Additional Information.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Show students an ABC pattern with cubes. Ask students to describe the pattern. Ask them to continue the pattern.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Present students with a pattern made with pattern blocks. Ask them to describe the pattern using a letter code and to make a sound pattern that could be described with the same letter code.
- Show a pattern to students. Provide a variety of materials for students and ask them to represent that pattern in two different ways.
- Show students a collection of visual repeating patterns. Ask them to identify the patterns that match and to explain why they match.
- Clap out a pattern. Ask students to represent that same pattern using pattern blocks.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets K–2 (Bauman 2011)

- No Checkpoint for this outcome.

Planning for Instruction

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of my students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Model how patterns can be translated from one medium to another, using objects, pictures, sounds, actions, or letters.
- Have students create their own patterns and translate them to a different medium; for example, concrete to action, concrete to pictorial, or concrete to auditory; action to pictorial, action to concrete, or action to auditory; pictorial to concrete, pictorial to action, or pictorial to letters.
- Expect students to explain their thinking about patterns and to describe how the patterns are the same.
- Provide students with opportunities to use letter codes to describe their patterns.

SUGGESTED LEARNING TASKS

- Give students pattern blocks and ask them to create an ABB pattern. Then ask students, What would this pattern sound like? Continue using other patterns, such as ABC, AAB, or ABBC.
- Give students a variety of patterns. Ask students to translate these patterns into other representations, such as letters, actions, manipulatives, or sounds.
- Have students create audiotapes of clapping patterns and have them use stickers to translate the pattern to a visual pattern.
- Provide students with a selection of patterns such as ABB, ABC, AAB, or ABBC from which to choose. Ask students to select the pattern they wish to create, and use materials to create it. When students have finished creating their patterns, invite them to look at the patterns that have been created by other students and to find all of the patterns that match the one they created.
- Prepare a set of pictorial patterns and their letter descriptions. Have students match the picture to the correct letter description.

SUGGESTED MODELS AND MANIPULATIVES

- | | |
|--------------------|-----------------------|
| ▪ calendars | ▪ pattern blocks |
| ▪ dot cards | ▪ Power Polygons |
| ▪ geometric solids | ▪ stamps and ink pads |
| ▪ linking cubes | ▪ stickers |

MATHEMATICAL LANGUAGE

Teacher	Student (oral language)
<ul style="list-style-type: none">▪ core, element▪ repeating pattern	<ul style="list-style-type: none">▪ core, element▪ repeating pattern

Resources/Notes

Print Resources

- *Making Math Meaningful to Canadian Students, K–8* (Small 2009), p. 573
- *Making Math Meaningful to Canadian Students, K–8, Second Edition* (Small 2013), p. 611
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 276–280

Video

- *Using a Teaching Sequence for Repeating Patterns* (22:55 min.) (ORIGO Education 2010)

Notes

SCO PR03 Students will be expected to describe equality as a balance and inequality as an imbalance, concretely and pictorially (0 to 20).

[C, CN, R, V]

[C] Communication

[PS] Problem Solving

[CN] Connections

[ME] Mental Mathematics and Estimation

[T] Technology

[V] Visualization

[R] Reasoning

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

PR03.01 Construct two equal sets using the same objects (same shape and mass) and demonstrate their equality of number using a balance scale.

PR03.02 Construct two unequal sets using the same objects (same shape and mass) and demonstrate their inequality of number using a balance scale.

PR03.03 Determine if two given concrete sets are equal or unequal and explain the process used.

Scope and Sequence

Mathematics Primary	Mathematics 1	Mathematics 2
—	PR03 Students will be expected to describe equality as a balance and inequality as an imbalance, concretely and pictorially (0 to 20).	PR03 Students will be expected to demonstrate and explain the meaning of equality and inequality by using manipulatives and diagrams (0 to 100).

Background

Balance activities form a basis for understanding equality. Working with balance scale problems involving concrete materials and pictures, helps students build foundations for further study in operations, algebra, and equation solving. Using concrete materials will help students see the equality or inequality relationship between the quantities on each pan of the balance scale. It is important that balances be reasonably calibrated and the items used be heavy enough that the differences in mass are visible.

Throughout balance activities, students should answer questions, such as

- What is the relationship between the two quantities if the scale is balanced? This provides opportunities to use the phrases “is the same as” and “is equal to.”
- What is the relationship between the quantities if the left balance pan is lower than the right pan? This provides opportunities to use the phrases “is more than” and “is not equal to,” and to discuss the “is fewer than” relationship between the right and left quantities on the pans.
- What is the relationship between the quantities if the left balance pan is higher than the right pan? This provides opportunities to use the phrases “is fewer than” and “is not equal to,” and to discuss the “is more than” relationship between the right and left quantities on the pans.

After students have had many concrete experiences interpreting the relationships between quantities on two pans of a balance scale, they could be asked to interpret a variety of balance scale pictures.

Additional Information

See Appendix A: Additional Information.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Give students a set of between 5 and 10 counters. Ask them to make a set that is the same.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Working in partners, one student puts cubes in a paper bag and places the bag on a pan balance. The partner predicts the number of cubes in the bag. (He or she may change the prediction as the scale begins to balance). The partner then begins to add cubes to the other side of the balance to verify the prediction. Once the scale is balanced, ask, How many cubes do you think are in the bag? How do you know? The partners count and compare the number of cubes on both sides.
- Provide students with two bags of counters and ask them to determine if the sets are equal or unequal and to explain how they know.
- Show students a pan balance that has 5 cubes on the left side and 7 cubes on the right side. Ask if the two sets are equal or not equal. Ask them to predict how many more cubes need to be added to the left side to make the scale balance and to make the sets equal.
- Ask students to use cubes and a pan balance to prove that 8 is equal to $7 + 1$.
- Ask students to use cubes and a pan balance to prove that 8 is not equal to $5 + 4$.
- Provide students with two sets of cubes and ask them to tell if the two sets are equal or not equal using a pan balance.
- Ask students to create two sets of cubes that are equal. Ask them to use a pan balance to prove that they are equal. Then ask them to change the sets so that they are not equal.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets K–2 (Bauman 2011)

- Mathematics 2 Checkpoint 8, pp. 85–86

Planning for Instruction

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of my students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Provide students with a variety of experiences so they come to understand that the equal sign represents a relation, not an operation. Use of the words “the same as” for the equal sign will help them see the relation.
- Use balance activities to assist with the development of an understanding of equality. Use concrete materials so students can examine how a balance operates like the seesaw on the playground.
- Provide students with many opportunities to represent number sentences concretely and to use language to describe those number sentences.
- Expect students to verbally explain their answers about equalities and inequalities.

SUGGESTED LEARNING TASKS

- Use a pan balance. Explain how the scale works. When the scale is balanced the sets are equal. Have students use objects of equal weight, such as unifix cubes, to practise making sets that are equal and sets that are not equal. Students should be encouraged to use oral language to describe the sets and to explain why they are or are not equal.
- Ask students to work in pairs. Ask them to use linking cubes to make trains to show as many different ways as possible to make a number from 1 to 20. Ask them to use the pan balance to prove that all of the trains for a given number are equal.

- Show students a pan balance that has some cubes on the left side and a different number of cubes on the right side. Ask students to tell if the two sets are equal or not equal and to explain how they know.
- Ask students to create two sets of cubes that are equal. Ask them to use a pan balance to prove that they are equal. Then, ask them to change the sets so that they are not equal.
- Provide students with number sentences. Ask them to use pan balances and cubes to show whether the sentences are true or not. For example, $4 = 1 + 3$; $3 + 2 = 5$; $6 + 1 = 5 + 2$.
- Ask students to work in pairs. Provide each pair of students with a set of word cards that say “Equal” and “Not Equal.” The first student draws a word card, but does not let his or her partner see the card. The card tells whether the first student will create two sets of objects that are equal or not equal. For example, if the student drew a not equal card, he or she might make a set of 5 and a set of 7. The second student looks at the sets of objects that have been created and determines whether the card said equal or not equal by stating “five is not equal to seven.” The students verify the statement by placing the sets on pan balances.

SUGGESTED MODELS AND MANIPULATIVES

- dot cards
- linking cubes
- pan balance

MATHEMATICAL LANGUAGE

Teacher	Student (oral language)
<ul style="list-style-type: none"> ▪ compare ▪ left, right, balance, imbalance ▪ more than, fewer than, same as/equal, equality/inequality, equal sign, not equal to/not same as ▪ one-to-one ▪ quantity 	<ul style="list-style-type: none"> ▪ compare ▪ left, right ▪ more than, fewer than, same as/equal, equal sign, not equal to/not same as ▪ quantity

Resources/Notes

Print Resources

- *Making Math Meaningful to Canadian Students, K–8* (Small 2009), pp. 585–586
- *Making Math Meaningful to Canadian Students, K–8, Second Edition* (Small 2013), pp. 626–627
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 299–302

Notes

SCO PR04 Students will be expected to record equalities using the equal symbol.			
[C, CN, PS, V]			
[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- PR04.01** Represent a given pictorial or concrete equality in symbolic form.
- PR04.02** Represent a given equality using manipulatives or pictures.
- PR04.03** Provide examples of equalities where the given sum or difference is on either the left or right side of the equal symbol (=).
- PR04.04** Record different representations of the same quantity (0 to 20) as equalities.

Scope and Sequence

Mathematics Primary	Mathematics 1	Mathematics 2
—	PR04 Students will be expected to record equalities using the equal symbol.	PR04 Students will be expected to record equalities and inequalities, symbolically, using the equal symbol or the not equal symbol.

Background

This outcome could be addressed at the same time as the equality aspect of outcome PR03. Balance activities form a basis for understanding equality. Throughout balance activities, students should answer the question, What is the relationship between the two quantities if the scale is balanced? This provides opportunities to use “is the same as” and also to use the equal sign (=) in the related number sentences.

Students need to understand that the equal sign (=) is a relationship, not an operation. It means “the same as”; that is, what is on one side of the equal sign is the same as what is on the other side. Often, when students only have experience using the equal sign in addition and subtraction, they mistakenly believe the sign (=) means “is the answer to” and, therefore, do not accept as possible number sentences such as $8 = 5 + 3$, $6 = 6$, and $4 + 5 = 2 + 7$. However, through balance activities, students see that such number sentences express equality relationships between quantities on the two sides of a balance scale, or in a one-to-one correspondence. As such, these numbers sentences are not only possible, but are necessary to express the relationships.

Note: Extensive research in children’s development of the meaning of equality suggests that

- while you may be tempted to use an equal sign (=) in concrete and picture models, it is strongly recommended that you use, and encourage students to use, the equal sign (=) only with other symbols, such as in number sentences and open sentences
- when you read a number sentence, it is recommended that you read the equal sign (=) to be “is the same as”
- number sentences should emerge only as a way to record concrete or pictorial models of equality relationships (Students are not expected to be able to work with symbolic relationships without reference to quantities, or to write and interpret greater than and less than symbols.)

Additional Information

See Appendix A: Additional Information.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Make a duplicate set of dot cards showing numbers to 10. Have students match the cards that are the same. Ask them to explain how they know the cards are the same.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students to fill in the blank, (either a whole number or a combination of numbers showing an operation) to complete the number sentence. Encourage students to explore these number sentences using materials, such as linking cubes of different colours and pan balances.

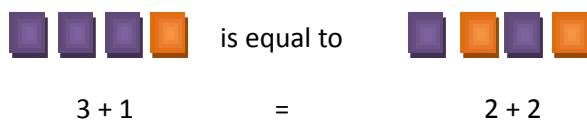
$$\underline{\hspace{2cm}} = 7 + 6$$

$$4 + 2 = \underline{\hspace{2cm}}$$

$$5 + 3 = \underline{\hspace{2cm}}$$

$$\underline{\hspace{2cm}} = 8 + 7$$

- Ask students to use materials and/or pictures to show an equality and then write the matching number sentence. For example,



$$3 + 1 = 2 + 2$$

- Use pan balance pictures to show equalities. Ask students to write a number sentence to represent the equality. This can also be done for inequalities.
- Provide students with pictures of pan balances with number sentences underneath. Ask students to draw objects on the pan balance to match the number sentences.

FOLLOW-UP ON ASSESSMENT**Guiding Questions**

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets K–2 (Bauman 2011)

- No Checkpoint for this outcome.

Planning for Instruction

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of my students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Provide students with a variety of experiences so they will understand that the equal sign represents a relation, not an operation. Use of the words “the same as” for the equal sign will help them see the relation. Avoid misuse of the equal sign, such as $?? = 3$ or $Alisa = 5$.
- Have students create equations and describe the equations verbally.
- Provide students with many opportunities to represent number sentences concretely before recording them symbolically.
- Ensure that students learn to read number sentences from left to right and right to left.

SUGGESTED LEARNING TASKS

- Ask the student to use Cuisenaire rods (or another suitable manipulative material such as linking cubes) to show the pattern for all of the number sentences for 8.
- Create dots cards representing number sentences, like flash cards. Ask students to record the number sentence symbolically. Ask students to represent the number sentence with linking cubes.

- Play a true and false game with students. Show number sentences and ask students to use linking cubes to represent each number sentence. Then, ask them to hold up either a true or a false card in response to each number sentence. Select one student to use linking cubes and a pan balance to prove that the statement is true or false. Number sentences could include
 - $1 + 2 = 1 + 3$
 - $9 = 7 + 2$
 - $7 + 1 = 4 + 4$
 - $4 + 3 = 7$
 - $6 + 4 = 5 + 5$
 - $8 = 8$
- Ask students to work in pairs to show as many different ways as possible to make a number from 1 to 20 and to record them as equalities.

SUGGESTED MODELS AND MANIPULATIVES

- Cuisenaire rods
- dot cards
- linking cubes
- pan balance

MATHEMATICAL LANGUAGE

Teacher	Student (oral language)
<ul style="list-style-type: none"> balance scale compare left, right make sets number expression, number sentence quantity same as, equality, equal sign 	<ul style="list-style-type: none"> balance scale compare left, right make sets quantity same as, equal sign

Resources/Notes

Print Resources

- Making Math Meaningful to Canadian Students, K–8* (Small 2009), pp. 585–586
- Making Math Meaningful to Canadian Students, K–8, Second Edition* (Small 2013), pp. 624–625
- Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 299–302

Notes

Measurement

GCO: Students will be expected to use direct and indirect measure to solve problems.

Specific Curriculum Outcomes

Process Standards Key

[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

- M01** Students will be expected to demonstrate an understanding of measurement as a process of comparing by
- identifying attributes that can be compared
 - ordering objects
 - making statements of comparison
 - filling, covering, or matching [C, CN, PS, R, V]

SCO M01 Students will be expected to demonstrate an understanding of measurement as a process of comparing by

- identifying attributes that can be compared
- ordering objects
- making statements of comparison
- filling, covering, or matching

[C, CN, PS, R, V]

[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- M01.01** Identify common attributes, such as length, mass, volume, capacity, and area that could be used to compare a given set of two objects.
- M01.02** Compare and order two given objects and identify the attributes used to compare.
- M01.03** Predict which object in a set is longest/shortest, determine by matching and explain the reasoning.
- M01.04** Predict which object in a set is heaviest/lightest, determine by comparing and explain the reasoning.
- M01.05** Predict which object in a set is largest/smallest, determine by comparing and explain the reasoning.
- M01.06** Predict which object in a set holds the most/least, determine by filling and explain the reasoning.
- M01.07** Predict which figure in a set has the greatest/least area, determine by covering and explain the reasoning.

Scope and Sequence

Mathematics Primary	Mathematics 1	Mathematics 2
<p>M01 Students will be expected to use direct comparison to compare two objects based on a single attribute, such as length, mass, volume, and capacity.</p>	<p>M01 Students will be expected to demonstrate an understanding of measurement as a process of comparing by</p> <ul style="list-style-type: none"> ▪ identifying attributes that can be compared ▪ ordering objects ▪ making statements of comparison ▪ filling, covering, or matching 	<p>M03 Students will be expected to compare and order objects by length, height, distance around, and mass using non-standard units and make statements of comparison.</p>

Background

Measurement involves identifying and comparing similar attributes of objects, and through measurement activities, students learn that the same object can have many measurable attributes. It is important that students explore measurement in their everyday contexts using direct comparison. For example, they might be asked, Which bean plant grew the tallest? Which ball has the greatest mass? Which box will hold the most treats? Which is the largest block in the set? Who has the largest piece of paper?

When exploring the different measurement attributes, have the students collect items from around the classroom. Students take turns predicting and checking their predictions. More than one student should do the same comparison. Observe whether there is agreement and how disagreements are resolved.

The first step in all these measurement activities should be for students to sort the objects for an attribute from smallest to largest, or vice versa, using their senses. That is, they would use sight to sort lengths, volumes, capacities, and areas; and use touch to sort masses. Then they would check their predictions by direct comparisons. Initially, they should be given only 3 objects to sort, moving on to more objects when they are ready. For the ones that will be sorted by sight, you should place the objects a good distance apart, bringing them together once the students have made their predictions. For mass, students should pick up each item with the same hand, to make their predictions.

Additional Information

See Appendix A: Additional Information.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Set up comparing stations each with two different objects. Ask questions appropriate to the measurement focus for each station. For example,
 - Can you tell me which of these two objects is longer? shorter?
 - Can you tell me which of these two objects is heavier? lighter?
 - Can you tell me which of these two objects is bigger? smaller?
 - Can you tell me which of these two objects holds more? holds less?
- Observe how students decide upon their answers.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Show two different objects that can be compared for mass. Ask students to predict which is heavier. Then, ask them to explain how they could compare them to determine which is heavier. Repeat for other measurement attributes.

- Have students prepare a set of ribbons for first, second, and third places in a race, so that the faster runner gets a longer ribbon.
- Provide students with two objects such as an eraser and a book. Ask,
 - Can you tell which of these two objects is longer?
 - Can you tell which of these two objects is heavier?
 - Can you tell me which of these two objects is bigger?
 - Can you tell which of these two objects takes up the most space?
 - Can you tell which of these two objects holds more?

After each question, have students explain their thinking. (Students should recognize that capacity is an attribute that cannot be used to measure these objects.) Use the questions as identified above, repeating the activity with other sets containing two objects.

- Show the students a coffee mug and a drinking glass. Ask them how they would find out which holds more.
- Give students sets of tangrams and have them compare the areas of the triangles in the sets.
- Provide students with “trains” of various lengths made from linking cubes. Ask them to order the trains from shortest to longest.
- Ask, What does “holds more” mean? Have the student explain his or her thinking.
- Give students a geometric shape, such as an attribute block or a pattern block, and have them draw another shape with a larger area. Have students explain how they know their drawn shape is larger.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets K–2 (Bauman 2011)

- Mathematics 1 Checkpoint 9, pp. 91–92
- Mathematics 1 Checkpoint 10, pp. 99–100

Planning for Instruction

Before introducing new material, consider ways to assess and build on students’ knowledge and skills.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of my students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Have students participate in “dramas” in which someone measures incorrectly and the other students figure out what is wrong. For example, one student could line up pencils of different lengths to measure an item, or could use uniform units, but counts, “1, 2, 4, 5, ...”
- Use two objects of different sizes and ask students how they could compare the objects. For example, two glasses could be compared by height, mass, as well as capacity. Working with a variety of objects will allow many opportunities for students to make comparisons relating to measurement, using many attributes.
- Ask students to order objects from shortest to longest, shortest to tallest, lightest to heaviest, holds least to holds most, or covers least space to covers most space. Include situations in which students are dealing with an independent variable, such as objects that are not straight and objects that are also wide or thick.

SUGGESTED LEARNING TASKS

- Provide students with a pan balance and two items. Ask them to predict and then determine which item is lighter. Observe students’ strategies.
- Provide the students with containers of varying sizes and shapes. Ask students to select two containers, and predict and then determine, by filling with sand or water, which holds more. Students should choose two more containers and repeat the activity. Then, ask students to order the four containers based on how much they hold.
- Ask two students to perform standing long jumps. Encourage them to find a way to determine who jumped farther. Emphasize with the students the importance of a common starting point.
- Have the students make 3 play dough balls. Ask them to predict and then to determine, using a pan scale, which of the balls is the heaviest.
- Display a set of 5 objects of similar size and a 6th “target” object. Ask the students to sort them into groups with masses less than and greater than the “target” object.
- Provide opportunities for students to compare areas by superimposing objects, by comparing different sizes of the same shape, and by comparing different shapes. After comparing, ask, How do you know that the area of this shape is larger?
- Give each student a true and a false card. Make comparative statements and ask students to hold up either the true or false card in response. For example, The fish tank holds more than a juice box. My desk is longer than the white board. The white board eraser is shorter than this paper clip. I am heavier than my jacket. Ask student volunteers to explain their thinking.

SUGGESTED MODELS AND MANIPULATIVES

- pan balance
- play dough
- various everyday objects

MATHEMATICAL LANGUAGE

Teacher	Student (oral language)
<ul style="list-style-type: none"> ▪ area: covers more/covers less, greatest/least area, and largest/smallest area ▪ capacity: holds more/holds less, holds the same ▪ full, empty ▪ length, width, distance, height ▪ length: longest/shortest, shortest/tallest, widest/narrowest, thickest/thinnest, longer/shorter ▪ mass: heaviest/lightest, heavier/lighter ▪ order, compare, fill, cover, match ▪ volume: biggest/smallest, bigger/smaller 	<ul style="list-style-type: none"> ▪ area: covers more/covers less, greatest/least area, and largest/smallest area ▪ capacity: holds more/holds less, holds the same ▪ full, empty ▪ length, width, distance, height ▪ length: longest/shortest, shortest/tallest, widest/narrowest, thickest/thinnest, longer/shorter ▪ mass: heaviest/lightest, heavier/lighter ▪ order, compare, fill, cover, match ▪ volume: takes up more/less space, bigger/smaller

Resources/Notes

Print Resources

- *Making Math Meaningful to Canadian Students, K–8* (Small 2009), pp. 363–368, 370–375, 388–393, 415–419, 421–424, 430–433
- *Making Math Meaningful to Canadian Students, K–8, Second Edition* (Small 2013), pp. 411–415, 418–419, 421–422, 429, 434–437, 451, 461–464, 467–470, 476–479, 481–483
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 223–242

Notes

Geometry

GCO: Students will be expected to describe the characteristics of 3-D objects and 2-D shapes and analyze the relationships among them.

Specific Curriculum Outcomes

Process Standards Key

[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

- G01** Students will be expected to sort 3-D objects and 2-D shapes using one attribute and explain the sorting rule. [C, CN, R, V]
- G02** Students will be expected to replicate composite 2-D shapes and 3-D objects. [CN, PS, V]
- G03** Students will be expected to identify 2-D shapes in 3-D objects. [C, CN, V]

SCO G01 Students will be expected to sort 3-D objects and 2-D shapes using one attribute and explain the sorting rule.

[C, CN, R, V]

[C] Communication

[PS] Problem Solving

[CN] Connections

[ME] Mental Mathematics and Estimation

[T] Technology

[V] Visualization

[R] Reasoning

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- G01.01** Sort a given set of familiar 3-D objects or 2-D shapes using a given sorting rule.
- G01.02** Sort a given set of familiar 3-D objects using a single attribute determined by the student and explain the sorting rule.
- G01.03** Sort a given set of 2-D shapes using a single attribute determined by the student and explain the sorting rule.
- G01.04** Determine the difference between two given pre-sorted sets of familiar 3-D objects or 2-D shapes and explain a possible sorting rule used to sort them.

Scope and Sequence

Mathematics Primary	Mathematics 1	Mathematics 2
<p>G01 Students will be expected to sort 3-D objects using a single attribute.</p> <p>G02 Students will be expected to build and describe 3-D objects.</p>	<p>G01 Students will be expected to sort 3-D objects and 2-D shapes using one attribute and explain the sorting rule.</p>	<p>G02 Students will be expected to recognize, name, describe, compare, and build 3-D objects, including cubes and other prisms, spheres, cones, cylinders, and pyramids.</p> <p>G03 Students will be expected to recognize, name, describe, compare, and build 2-D shapes, including triangles, squares, rectangles, and circles.</p>

Background

Sorting objects into groups is a natural activity that needs to be fostered and extended to sorting three-dimensional (3-D) objects and two-dimensional (2-D) shapes because the study of shapes is essential as students begin to describe, analyze, and understand the world in which they live. Activities selected in geometry should provide students with the opportunity to explore a variety of geometric shapes and objects. They need to see and feel, build and take apart, sort and identify their rule(s), and share their observations with their classmates. Sorting activities are based on students' visual discrimination abilities that should be further developed by discussing how shapes are alike and how they are different, by hearing alternative sorting rules, and by being encouraged to explore alternative ways of sorting.

It is through such activities that students will become familiar with the geometric language of 2-D shapes and 3-D objects. All activities provide opportunities for students to become familiar with those names, and you should use correct names when talking about shapes. It is reasonable that students become familiar with the common names (2-D: triangle, square, rectangle, and circle; 3-D: cylinder, sphere, cone, cube, prisms, and pyramids).

Activities should include a balance of students generating their own sorts and their sorting by assigned sorting rules. Throughout, students should learn to articulate their sorting rules, be challenged to create alternative sorting rules, and develop geometric language.

Additional Information

See Appendix A: Additional Information.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Show students a pre-sorted set of 3-D objects. Ask them to explain the sorting rule.
- Provide students with a set of 3-D objects. Ask them to sort them and to explain their sorting rule.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Provide students with a collection of 2-D shapes or 3-D objects. Tell them a sorting rule. Ask them to sort the collection based on your sorting rule.
- Ask students to work in pairs. Have one student sort a collection of 2-D shapes or 3-D objects. His or her partner must describe the sorting rule.
- Provide students with a collection of 2-D shapes or 3-D objects. Ask them to sort the collection of 2-D shapes or 3-D objects and explain their sorting rule. Then, ask them to resort the collection using a different sorting rule and to explain the new rule.
- Show students a set of 2-D shapes or 3-D objects that you have sorted. Ask students to identify your sorting rule.
- Show students a set of 2-D shapes or 3-D objects that you have sorted. Ask students to sort the shapes or objects in a different way and to explain their new sorting rule.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets K–2 (Bauman 2011)

- Mathematics 1 Checkpoint 12, Task 2, pp. 116–117
- Mathematics 1 Checkpoint 14, pp. 133–14 (Line Master 14.1)

Planning for Instruction

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of my students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Ensure students have many opportunities to represent 2-D shapes and 3-D objects concretely.
- Provide an assortment of 2-D shapes cut from cardboard. Ask students to work in small groups to sort the shapes. Encourage discussion and have the groups share their sorting rules with their classmates.
- Encourage students to use oral language to describe how the 2-D shapes (or 3-D objects) are alike and how they are different.
- Allow students to identify 2-D shapes and 3-D objects in their daily lives. This should include common everyday objects.

SUGGESTED LEARNING TASKS

- Provide several different 2-D shapes or 3-D objects. Ask the student to sort them and to explain the sorting criteria. Ask him or her to sort them again, using different criteria.
- Show students a set of shapes or objects that have been sorted but that contains an object or shape that does not belong. Ask students to remove the object or shape that does not belong and explain why.

- Provide students with a collection of 2-D shapes of various sizes. Ask students to sort the collection into two groups and name the sorting rule. Then, ask students to sort the collection in another way and to explain the new sorting rule.

SUGGESTED MODELS AND MANIPULATIVES

- attribute blocks
- geometric solids
- pattern blocks
- Polydrons
- tangrams

MATHEMATICAL LANGUAGE

Teacher	Student (oral language)
<ul style="list-style-type: none"> ▪ 2-D shapes: triangle, square, rectangle, circle, trapezoid, hexagon, rhombus ▪ 3-D objects: cylinder, sphere, cone, cube, prism, pyramid ▪ attribute ▪ how are they alike/different ▪ roll, stack, slide ▪ sides, edges, corners/vertices, faces ▪ sorting groups of objects ▪ sorting rule 	<ul style="list-style-type: none"> ▪ 2-D shapes: triangle, square, rectangle, circle ▪ 3-D objects: cylinder, sphere, cone, cube, prism, pyramid ▪ how are they alike/different ▪ roll, stack, slide ▪ sides, corners, faces ▪ sorting groups of objects ▪ sorting rule

Resources/Notes

Print Resources

- *Making Math Meaningful to Canadian Students, K–8* (Small 2009), pp. 284–286, 289–290
- *Making Math Meaningful to Canadian Students, K–8, Second Edition* (Small 2013), pp. 340–342, 344–346
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 186–195

Notes

GCO G02 Students will be expected to replicate composite 2-D shapes and 3-D objects.

[CN, PS, V]

[C] Communication [PS] Problem Solving [CN] Connections [ME] Mental Mathematics and Estimation
[T] Technology [V] Visualization [R] Reasoning

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- G02.01** Select 2-D shapes from a given set of 2-D shapes to reproduce a given composite 2-D shape.
G02.02 Select 3-D objects from a given set of 3-D objects to reproduce a given composite 3-D object.
G02.03 Predict and select the 2-D shapes used to produce a composite 2-D shape, and verify by deconstructing the composite shape.
G02.04 Predict and select the 3-D objects used to produce a composite 3-D object, and verify by deconstructing the composite object.

Scope and Sequence

Mathematics Primary	Mathematics 1	Mathematics 2
<p>G02 Students will be expected to build and describe 3-D objects.</p>	<p>G02 Students will be expected to replicate composite 2-D shapes and 3-D objects.</p>	<p>G02 Students will be expected to recognize, name, describe, compare, and build 3-D objects, including cubes and other prisms, spheres, cones, cylinders, and pyramids.</p> <p>G03 Students will be expected to recognize, name, describe, compare, and build 2-D shapes, including triangles, squares, rectangles, and circles.</p> <p>G04 Students will be expected to identify 2-D shapes as parts of 3-D objects in the environment.</p>

Background

This outcome is promoting thinking in geometry that is like part-part-whole thinking in number. For example, determining the two tangram pieces that will solve a two-piece tangram puzzle is like determining two numbers that are parts of a given number. Experimentation and play with 2-D shapes and 3-D objects provide students with opportunities to explore the attributes of shapes and how they can be put together and taken apart to make other shapes. Blocks, tangram pieces, pattern blocks, interlocking cubes, and sets of 3-D solids are useful tools with which students can explore the relationships between and among various objects and shapes.

Creating composite shapes that are out of view and giving instructions to students on how to replicate them will provide opportunities for them to process and use the language of geometry and of position.

Additional Information

See Appendix A: Additional Information.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

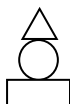
Tasks such as the following could be used to determine students' prior knowledge.

- Show students a 3-D object, such as a can or a small box. Ask them to describe the object to you. Listen to the language they use.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Give each student, or pair of students, a collection of pattern blocks. Say, “I am going to build a design with pattern blocks. I want you to use your pattern blocks to build a design just like mine.” Observe their constructions.
- Provide students with a tangram puzzle outline and ask them to predict, and then select, the tangram pieces required to replicate the shape. Students may then replicate the shape to verify their predictions. Pattern blocks may also be used for this activity.
- Give each student, or pair of students, a collection of pattern blocks. Say to them, “I want you to build the following shape with your pattern blocks.” Give oral directions and have students create your described shape. For example, Place a red trapezoid on your desk. On top of the trapezoid place a green triangle. On the left, place a blue rhombus and on the right, place another blue rhombus.” Observe their constructions and have students share their thinking about the placement of each block.
- Show students a square that has been folded along the diagonal. Ask, What shape will this be when I unfold it?
- Provide students with a variety of 3-D solids. Show a composite 3-D object, such as a tower, and ask students to predict, and then select, the solids they need to replicate the object. Students should build the object using the solids they selected. They may then decompose the given object to verify their predictions.
- Ask students to examine a variety of containers (such as a yogurt container or cereal box). Ask, What shapes would be used to make this container? How do you know?
- Show students a picture, such as the one below, for three to five seconds and ask them to draw it.



- Have students use a computer drawing program to make pictures using 2-D shapes. Then, have them challenge their classmates to replicate them.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets K–2 (Bauman 2011)

- Mathematics Primary Checkpoint 13, pp. 122–123 (Line Master 13.1)
- Mathematics 1 Checkpoint 13, pp. 124–125 (Line Master 13.2)

Planning for Instruction

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of my students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Ensure students have many opportunities to represent 2-D shapes and 3-D objects.
- Provide an assortment of 2-D shapes and/or 3-D objects. Ask students to work in small groups to describe the shapes and/or objects.
- Have students explore the properties of 3-D objects prior to identifying 3-D objects in the environment. Explorations may include tracing the faces of the solids, or pressing the faces in play dough to investigate the attributes of the 2-D shapes and 3-D objects.
- Allow students to identify 2-D shapes and 3-D objects in their daily lives. This should include common everyday objects.

SUGGESTED LEARNING TASKS

- Ask students to cut a square, rectangle, or triangle into three parts. Have them exchange their pieces with a partner. Each partner should rearrange the pieces to make the original shape.

(Activities such as these, in which students are required to assemble a figure from its parts, develops figure-ground perception skills.)

- Have students work in pairs with a geoboard to create a large square with a smaller square inside it.
- Display pictures of various 3-D objects, such as a rocket or sculpture. Ask students to identify which 3-D objects were used to build the object on display. Students can then build their own composite 3-D objects from individual 3-D objects, such as small cardboard boxes or modelling clay. Once the models are built, display the creations in class and ask students to identify the 3-D objects used to build the composite object.
- Ask students to create a design using circles, squares, rectangles and/or triangles. As students are working, ask them to describe their shapes.
- Work with prepared tangram puzzle outlines. Have students predict which shapes they will need to complete the picture and then work to complete the puzzle.
- Ask students to work with a partner. The first student makes a pattern block design with a specified number of pattern blocks. The second student selects the appropriate shapes and replicates the partner’s design.
- Ask students to work with a partner. The first student makes an object with linking cubes. The second student selects the appropriate number of linking cubes and replicates the partner’s design.
- Build a composite 3-D object. Have students select the appropriate 3-D objects and reproduce your object.
- Ask students to work in partners. Set up a screen between the students. Have one student create a composite 3-D object using linking cubes and then describe it to his or her partner. The partner uses the description to reproduce the composite object. They then lift the barrier to check.
- Invite students to take a geometry walk around the school looking for 2-D shapes and 3-D objects.

SUGGESTED MODELS AND MANIPULATIVES

- | | |
|--|---|
| <ul style="list-style-type: none"> ▪ attribute blocks ▪ geoboards ▪ geometric solids ▪ linking cubes | <ul style="list-style-type: none"> ▪ pattern blocks ▪ Polydrons ▪ tangrams |
|--|---|

MATHEMATICAL LANGUAGE

Teacher	Student (oral language)
<ul style="list-style-type: none"> ▪ 2-D shapes: triangle, square, rectangle, circle, trapezoid, hexagon, rhombus ▪ 3-D objects: cylinder, sphere, cone, cube, prism, pyramid 	<ul style="list-style-type: none"> ▪ 2-D shapes: triangle, square, rectangle, circle, ▪ 3-D objects: cylinder, sphere, cone, cube, prism, pyramid

Resources/Notes

Print Resources

- *Making Math Meaningful to Canadian Students, K–8* (Small 2009), pp. 303–304, 309–314
- *Making Math Meaningful to Canadian Students, K–8, Second Edition* (Small 2013), pp. 358–359, 365–368
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 196–200

Notes

SCO G03 Students will be expected to identify 2-D shapes in 3-D objects. [C, CN, V]			
[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

G03.01 Identify the shape of the faces of a 3-D object

G03.02 Identify 3-D objects in the environment that have faces that are a given 2-D shape.

Scope and Sequence

Mathematics Primary	Mathematics 1	Mathematics 2
—	G03 Students will be expected to identify 2-D shapes in 3-D objects.	G04 Students will be expected to identify 2-D shapes as part of 3-D objects in the environment.

Background

Initially, students recognize 3-D objects by their overall appearance, not by their properties. However, with directed activities, students examine 3-D objects to see how they are constructed and to become aware of their attributes. One thing students should notice is that many 3-D objects have faces that are 2-D shapes. For example, when they examine a typical cardboard box, they see that it has faces that are squares or rectangles. Typical classroom sets of 3-D solids include a cube, some prisms and pyramids, a cone, a cylinder, and a sphere. These solids have faces that are triangles, squares, rectangles, or circles—2-D shapes that are familiar to students.

Additional Information

See Appendix A: Additional Information.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Place a collection of 3-D objects in front of students. Ask them to sort the objects and to describe their sorting rule.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students to examine a variety of containers (such as a yogurt container or cereal box). Ask, What shapes would be used to make this container? How do you know?
- Show students a triangle. Ask students to find three things in the classroom that make them think of that shape.
- Provide students with a set of paper 2-D shapes and have them circulate in the classroom or another environment, finding parts of 3-D objects. Students may record their findings in their math journals.
- Provide students with a collection of 3-D objects. Ask them to find all of the objects that have circles for a face.
- Provide students with a collection of 3-D objects. Ask them to sort the objects by the shape of their faces. Ask them to explain their thinking as they sort the objects.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets K–2 (Bauman 2011)

- No Checkpoint for this outcome.

Planning for Instruction

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?

- What teaching strategies and resources should be used?
- How will the diverse learning needs of my students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Ensure students have many opportunities to represent 2-D shapes and 3-D objects.
- Have students explore the properties of 3-D objects prior to identifying 3-D objects in the environment. Explorations may include tracing the faces of the solids, or pressing the faces in play dough to investigate the attributes of the 2-D shapes and 3-D objects.
- Allow students to identify 2-D shapes and 3-D objects in their daily lives. This should include common everyday objects.

SUGGESTED LEARNING TASKS

- Have students examine a collection of objects found in their environment (e.g., cans, cereal boxes, ice cream cones, tissue boxes.) Ask them to identify the 2-D shapes found in each face of each 3-D object. Ask, What shape is the face? Do all of the faces have the same shape?
- Invite children to hunt around the school to find various shapes (e.g., circles, squares, triangles, rectangles) in 3-D objects. If a digital camera is available, take pictures of the objects that are found. Have students share their findings and speculate on why certain shapes are more common than others.
- Ask students to find images of 3-D objects that have a face of a particular 2-D shape; for example a face that is a square. Students can create books entitled “What’s My Face?” using the digital images collected. Other students can read the books and identify the common faces.

SUGGESTED MODELS AND MANIPULATIVES

- attribute blocks
- geometric solids
- pattern blocks
- tangrams

MATHEMATICAL LANGUAGE

Teacher	Student (oral language)
<ul style="list-style-type: none"> ▪ 2-D shapes: triangle, circle, square, rectangle, ▪ 3-D objects: cube, prism, pyramid, cone, cylinder, sphere ▪ faces 	<ul style="list-style-type: none"> ▪ 2-D shapes: triangle, circle, square, rectangle ▪ 3-D objects: cube, prism, pyramid, cone, cylinder, sphere ▪ faces

Resources/Notes

Print Resources

- *Making Math Meaningful to Canadian Students, K–8* (Small 2009), pp. 287–290
- *Making Math Meaningful to Canadian Students, K–8, Second Edition* (Small 2013), pp. 343–346
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 216–217

Notes

Appendices

Appendix A: Additional Information

Number (N)

SCO N01 Students will be expected to say the number sequence by			
<ul style="list-style-type: none"> ▪ 1s, forward and backward between any two given numbers, 0 to 100 ▪ 2s to 20, forward starting at 0 ▪ 5s to 100, forward starting at 0, using a hundred chart or a number line ▪ 10s to 100, forward starting at 0, using a hundred chart or a number line 			
[C, CN, V, ME]			
[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- N01.01** Recite forward by 1s the number sequence between two given numbers, 0 to 100.
- N01.02** Recite backward by 1s the number sequence between two given numbers, 0 to 100.
- N01.03** Record a given numeral, 0 to 100, presented orally.
- N01.04** Read a given presented numeral, 0 to 100.
- N01.05** Skip count by 2s to 20 starting at 0.
- N01.06** Skip count by 5s to 100 starting at 0, using a hundred chart or a number line.
- N01.07** Skip count forward by 10s to 100 starting at 0, using a hundred chart or a number line.
- N01.08** Identify and correct errors and omissions in a given number sequence.

Performance Indicator Background

N01.01 and N01.02 Most students use the auditory patterns that exist in our number system to learn these counting sequences from 20 to 99, including the sequence of decades (20, 30, 40, ..., 90) and the 1 to 9 sequence within each decade (20, 21, 22, 23, ..., 29, 30, 31, 32, 33, ..., 39). Students should be very comfortable counting forward up to 100 before counting backward. Initial backward counting experiences should build on their primary experience counting back from 10, by counting back from a decade, such as 50, 49, 48, ..., 41, 40.

Students should experience a wide variety of situations throughout the school year that require counting so they become familiar with counting patterns. Students can reinforce saying the number sequence forward and backward through various opportunities throughout the day, such as lining up for recess.

N01.03 and N01.04 In order to use a hundred chart, or number line, in support of developing number sequences to 100, students will need to read and record numerals. Students should be able to say “twenty-seven” when shown the numeral “27.” When a number is presented orally, students should be able to identify and record the numeral. For example, say the number “twenty-seven,” ask students to identify where the number “twenty-seven” is on the hundred chart, and to write the numeral “27.”

While students are reading, identifying, and recording numerals to 100, they are doing so through patterning, not through place value knowledge. They will view the two digits in the numerals as a way to “spell” the number. For example, fifty-seven is spelled with a 5 and a 7 to produce 57.

Because it is important that students develop an efficient means of recording numerals, numeral writing should be taught. Allow the students to experiment freely on lined and unlined paper, whiteboards, chart paper, and other mediums. Observe students as they write their numerals, both when copying from a model and when forming them from memory. Students should be encouraged to start at the top when writing numerals. Be very careful not to assume that because a student has learned to write the numerals they are learning anything about the quantities they represent.

N01.05, N01.06, and N01.07 Hundred charts and number lines are essential tools to support skip counting by 2s to 20 and by 5s and 10s to 100. For example, when skip counting by 5s, the student may place a counter on every fifth number, starting at 5, reading the number as each counter is placed on the numeral, and subsequently saying the sequence of numbers hidden by the counters. It is important that students realize that they read the hundred chart left to right and top to bottom. Students will have opportunities to apply these skip counting sequences to count sets of objects to 20 in SCO N03 and SCO N07.

N01.08 After a variety of experiences with saying the number sequences, students should be able to identify errors in counting sequences. For example, students should be able to say which number is omitted or incorrect in the sequence 31, 32, 34, 33, 35, ... or 5, 10, 15, 20, 30, 35, ... and they should be able to correct sequences that have been presented incorrectly.

SCO N02 Students will be expected to recognize, at a glance, and name the quantity represented by familiar arrangements of 1 to 10 objects or dots.

[C, CN, ME, V]

[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

Performance Indicators

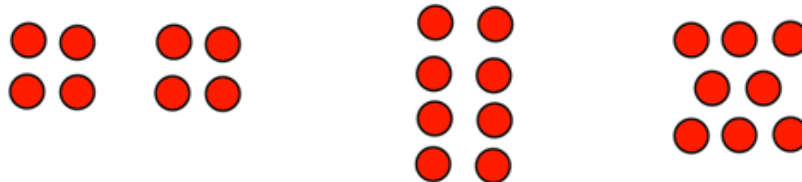
Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- N02.01** Look briefly at a given familiar arrangement of objects or dots and identify the number represented without counting.
- N02.02** Identify the number represented by a given arrangement of counters or dots on a ten-frame.

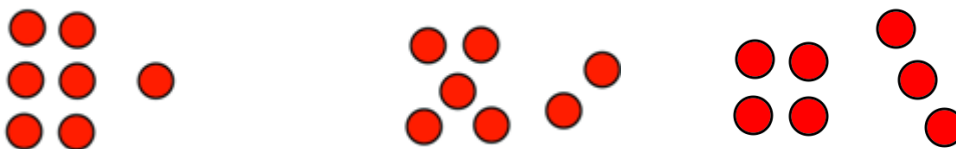
Performance Indicator Background

N02.01 Students must develop the ability to state the quantity when briefly presented with familiar arrangements of dots. They should have just enough time to subitize and not do one-to-one counting. Start with arrangements up to 5 dots and gradually increase the number of dots until students can subitize arrangements of up to 10 dots.

To avoid the misconception that an arrangement can only represent a specific quantity if it is arranged in a certain way, it is very important to vary the arrangement of the objects, dots, or pictures. When asking students to identify the number of fingers, use different combinations of fingers so that children do not believe that there is only one way to represent the number. For example, the number 6 can be represented with 5 fingers on one hand and 1 on the other, 2 fingers on one hand and 4 on the other, and 3 fingers on each hand. Similarly, each of the following three arrangements shows the quantity of 8.



Provide opportunities for students to discover and discuss which configurations are easier to recognize. For example, ask students to show 7 in several ways, and then decide which configuration(s) is (are) easiest to identify and explain why they prefer this arrangement. Possible configurations of 7 may include

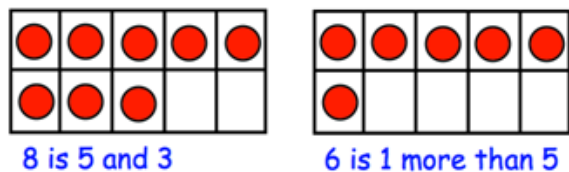


N02.02 Students should also have experiences with modelling numbers on a ten-frame to develop visual representations of numbers to 10. Using five-frames, and then ten-frames, helps to develop part-part-whole thinking. A ten-frame is a rectangle with 2 rows of 5 boxes each large enough to hold a counter.

When using a ten-frame, students should know that

- only one counter is permitted in each box of the ten-frame
- counters are typically placed from left to right in the top row until it is full and then from left to right in the bottom row

Students may think of numbers between 5 and 10 as “5 and some more.” For example, **8 is 5 and 3 more**



When subitizing, it is important to use the familiar (typical) arrangement; however, when students are creating their own representations of a number other arrangements are acceptable.

SCO N03 Students will be expected to demonstrate an understanding of counting to 20 by

- indicating that the last number said identifies “how many”
- showing that any set has only one count
- using the counting-on strategy

[C, CN, ME, R, V]

[C] Communication

[PS] Problem Solving

[CN] Connections

[ME] Mental Mathematics and Estimation

[T] Technology

[V] Visualization

[R] Reasoning

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- N03.01** Answer the question, How many are in the set? using the last number counted in a given set.
- N03.02** Identify and correct counting errors in a given counting sequence.
- N03.03** Show that the count of the number of objects in a given set does not change regardless of the order in which the objects are counted.
- N03.04** Record the number of objects in a set using the numeral symbol.
- N03.05** Determine the total number of objects in a given set, starting from a known quantity and counting on.

Performance Indicator Background

N03.01 Watch students as they count to see if they are touching objects (one-to-one correspondence) and saying the appropriate number when they touch them. Listen to how the students are counting and verify that they start at 1 and say the number words in the correct sequence (stable order). Verify that students realize the last number said is the quantity of objects they have counted (cardinality).

N03.02 Students should be able to identify errors in a counting process. Errors may include counting objects more than once, missing an object, or making a mistake in the number sequence.

N03.03 The goal in Mathematics 1 is for students to see that the order in which they count objects does not change the quantity. Have them count a set of objects from the right. Then ask them how many they think they will get if they start counting from the left or from the middle. Their response should indicate that the count will be the same and there is no need to recount.

N03.04 Students should be able to record numerals presented orally (SCO N01) and those representing the quantity of objects in a set. Opportunities should begin by reviewing counting and recording numbers to 10 before moving to sets of up to 20 objects.

N03.05 Counting on and counting back are fundamental prerequisites for addition and subtraction and their importance should not be underestimated. Counting on requires an ability to say the number sequence, starting at any number (SCO N01). It also requires an understanding that numbers are inclusive; for example, counting on from 6 implies knowing that 1, 2, 3, 4, and 5 are included in the number 6 and there is no reason to recount those numbers (1, 2, 3, 4, 5).

SCO N04 Students will be expected to represent and partition numbers to 20. [C, CN, V]			
[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

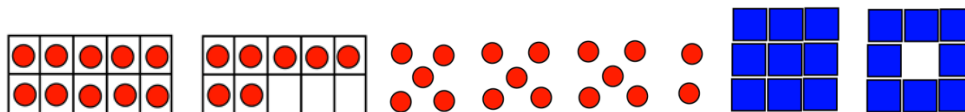
Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- N04.01** Represent a given number up to 20 using a variety of manipulatives, including ten-frames and created materials.
- N04.02** Model a given number up to 20 using a variety of pictorial representations.
- N04.03** Find examples of a given number in the environment.
- N04.04** Place given numerals on a number line with benchmarks 0, 5, 10, 15, and 20.
- N04.05** Partition any given quantity up to 20 into two parts and identify the number of objects in each part.
- N04.06** Model a given number using two different objects.

Performance Indicator Background

N04.01 and N04.02 Students should be able to show and sketch models of quantities to 20 in a variety of ways. For example, a student might show 17 in ten-frames, or with three groups of 5 counters and 2 more counters, or with a set of 9 squares and a set of 8 squares as seen below.



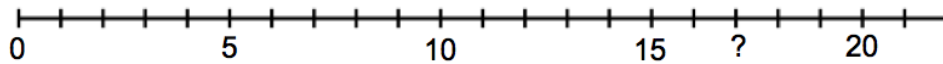
Students may also use created materials to represent a number. For example, to represent 17 they may use a bundle of 10 sticks and 7 single sticks or 3 sticks with 5 beans glued on each and 2 loose beans.

Mathematics 1 is the first time number lines are used to represent numbers. Number lines provide a visual representation of the sequence of numbers and their relative positions to one another. Initial exposure to number lines should include kinesthetic experiences, such as concrete or walk-on number lines that display all numerals, followed by number lines on paper that also display all numerals, and finally to more abstract number lines that display only some numerals. Number lines in Mathematics 1 should always contain tick marks for each number, whether or not the numeral is displayed.

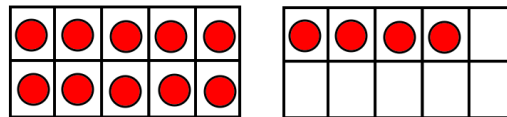
Note: Students will need to be introduced to the concept of zero and its application to a number line. On a number line that starts with zero, a number is represented by the position that is that many spaces from the starting point; for example, 4 is four spaces from the starting point (zero).

N04.03 Students should be able to identify where the quantities of objects, up to 20, may be found outside of math class, such as eggs in a carton and number of crayons in a box. They should also be able to identify where numerals can be found that do not represent a quantity, such as numbers on sports jerseys.

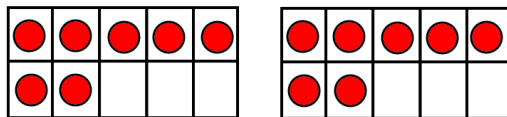
N04.04 Students need a lot of experience using benchmarks to place numbers from 1 to 20 on a number line. For example, students should be able to say what number on the number line is above the “?”.



N04.05 Students should be given opportunities to partition numbers to 20 in a variety of ways. For example, they can use ten-frames to show different ways to model 14.

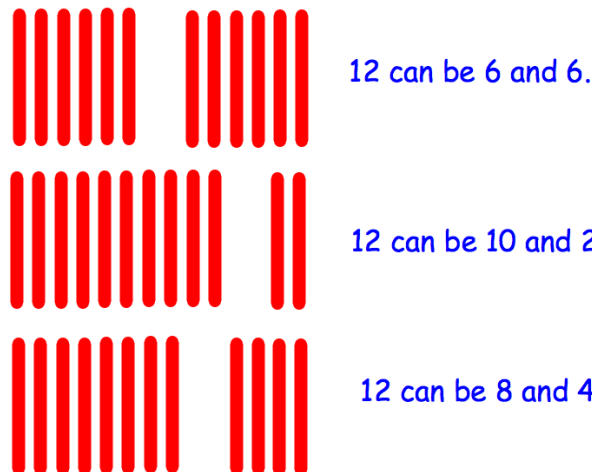


14 can be a set of 10 and a set of 4.



14 can also be a set of 7 and another set of 7.

Students may use objects, such as stir sticks, toothpicks, or counters, to model partitioning in a real context such as the number of boys and the number of girls in a group of 12 students. Students may show three possible ways to partition the group of 12 students using sticks as shown below.



They should be able to relate the arrangements they have created to the context; for example, the 12 students could be 6 boys and 6 girls, 10 boys and 2 girls, or 8 boys and 4 girls. They should be able to explain that no matter how they partition the quantity, there are always 12 students in the group.

N04.06 Students need to understand that the size of the objects does not influence the number that represents the quantity in a set; that is, the objects themselves do not have to be uniform in size and shape, and two sets may both represent the same number even if the objects in one set are larger than those in the other. This is the abstraction principle. For example, they should see that 4 apples, 4 bicycles, or a set of 4 unrelated objects are all sets of 4. To develop the “4-ness” of a set, students should represent a set of 4 with a large variety of different objects, both concretely and pictorially.

SCO N05 Students will be expected to compare sets containing up to 20 objects to solve problems using referents and one-to-one correspondence. [C, CN, ME, PS, R, V]			
[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- N05.01** Build a set that has more, fewer, or as many objects as a given set, up to 20 objects.
- N05.02** Build several sets of different objects that have the same given number of objects in the set.
- N05.03** Compare two given sets using one-to-one correspondence and describe them using comparative words, such as more, fewer, or as many.
- N05.04** Compare a set to a given referent using comparative language.
- N05.05** Solve, using pictures and words, given story problems that involve the comparison of two quantities.

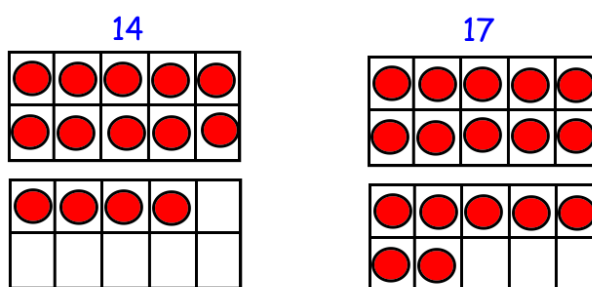
Performance Indicator Background

N05.01 and N05.02 Through planned activities students should have opportunities to build sets of quantities that have more, fewer, or as many objects as a given set.

To reinforce the idea that the size of the objects does not affect the quantity, students may be shown a set of toothpicks and be asked to build a set of cubes that has fewer in the set. Similarly, students should be given opportunities to build sets that contain the same quantity using objects of various sizes.

N05.03 Students should be able to examine two sets that have been presented and be able to compare them by matching items in one set with those in the other to see if each item has a “partner” or if one set has leftover items. Through such one-to-one correspondence, students should be able to explain which set has more and which has fewer, or explain why the two sets are the same.

N05.04 Students should be able to compare a set to a referent, using language such as “less than,” “more than,” or “the same as.” Ten-frames can be very helpful in comparing quantities in general, but also can help students to use referents, such as benchmark numbers, to make comparisons. For example, a student may show that 14 is less than 17 because 14 fills less than 3 rows of the ten-frames (it is less than 15) and 17 fills more than 3 rows (it is more than 15).



Working with referents (benchmarks) is connected to outcome N08 where the relationships of one more than, two more than, one less than, and two less than are explored. Making connections of a quantity to benchmarks of 5 and 10 (and their multiples) using these relationships is critical. For example, students using 15 as a benchmark should know that 17 is 2 more than 15 or 14 is 1 less than 15. Modelling with

ten-frames is a useful way to develop these benchmarks and can be used in conjunction with number lines.

It is interesting to note that although the concept of fewer is logically equivalent to the concept of more, the word **fewer** proves to be more difficult for students than the word **more**. The words **fewer** and **less** are often used incorrectly. The word **fewer** should be used when referring to countable quantities (e.g., there are fewer students in this class). The word **less** is used when referring to measures and non-countable items (e.g., there is less water in this bottle). However, when comparing numbers, the phrases, **less than** and **greater than**, should be used.

N05.05 Students should be orally presented with story problem situations in which they need to compare quantities, and use pictures and words to solve these problems. See the story problem chart and general discussion of story problems in outcome N09. Students may also examine quantities of objects in stories they have been reading (or have been read by the teacher), using comparative language to discuss their observations.

SCO N06 Students will be expected to estimate quantities to 20 by using referents. [C, ME, PS, R, V]			
[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- N06.01** Estimate a given quantity by comparing it to a given referent (known quantity).
N06.02 Select an estimate for a given quantity by choosing between at least two possible choices and explain the choice.

Performance Indicator Background

N06.01 To develop estimation skills, students should be provided with collections of objects and asked to estimate the quantities in these collections. Referents such as 5, 10, 15, and 20 are useful benchmarks to facilitate the development of quantity estimation skills. For example, students could be shown labelled sets of 5 counters and 10 counters, and an unknown set of counters they are to estimate. The labelled sets provide them with referents to which they can visually compare the unknown set in order to arrive at an estimate. After they state their estimates, they should then find the quantity by counting and reflect on whether their estimates were more, fewer, or the same as the actual quantity and which estimates were closest. Through many repetitions of this process, students will refine their estimation skills. Eventually, activities should just involve estimating without checking by counting.

Through other planned activities, students should be presented with opportunities to estimate. For example, display between 10 and 20 cubes. After 5 seconds, cover the display. Ask the students to write down their estimates. Reveal the cubes and move 5 cubes off to the side. Tell students, “Here are 5 cubes. Do you want to change your estimate? If you do, will you change it to be more or less than your first estimate?” Ask students to explain their decisions.

N06.02 Students should experience selecting what they think is the best estimate among given estimates. For example, students could be shown a set of buttons and asked to decide if 5, 10, or 15 is the best estimate of the number of buttons they were shown. For a smaller collection, students may be asked whether it is closer to 5 or 10. Keep in mind that six-year-old students should be able to subitize random sets of 1 to 6 objects; therefore, sets of more than 6 objects should be used for estimation. For larger collections, they may be asked whether the collection is closer to 10 or 20. Include situations in which sets have the same number of items but differ in the amount of physical space they occupy.

SCO N07 Students will be expected to demonstrate an understanding of conservation of number for up to 20 objects.

[C, R, V]

[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- N07.01** Explain why for a given number of counters, no matter how they are arranged, the total number of counters does not change.
- N07.02** Group a set of given counters in more than one way.
- N07.03** Explain why for a given number of counters, no matter how they are grouped, the total number of counters does not change.

Performance Indicator Background

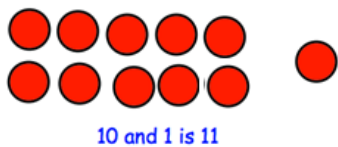
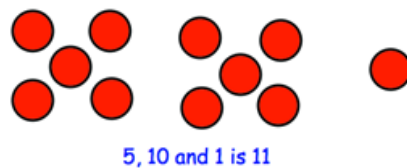
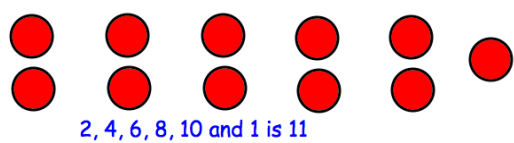
N07.01 Students explore the conservation principle through counting activities. For example, have students count a set of objects that are close together, then push the objects out so they cover a larger area, and ask students to predict how many there are now. (They may be unsure or predict a different number than the one they just counted. These responses indicate that conservation is not yet part of their understanding.) Have them recount the objects and focus their attention on the fact they got the same count both times. It may take several similar experiences before students are convinced they will get the same count regardless of how the objects are arranged. When students have internalized this understanding, they will likely think it is a silly question to ask them again how many there are when you just rearrange the objects after the initial count. They may explain, “It will be the same because you didn’t add any more or take any away!”

N07.02 and N07.03 Once students have demonstrated a competency in one-to-one counting, they should be encouraged to develop flexible thinking about numbers. Students need to explore ways in which previously learned number concepts are connected to the “breaking apart” of numbers (part-whole thinking). This development will lead the students toward increased understanding of the relative size of numbers and will enable students to construct meaning regarding the composing and decomposing of numbers. Therefore, indicators N07.02 and N07.03 should be addressed in conjunction with SCO N04.

Students will learn to trust that a quantity will not change if that quantity is thought of as being made up of parts. Students will also know that the quantity remains the same if some from one of those parts is moved to the other part. Students will need many opportunities to model this concretely.

Students should be encouraged to discuss what they see and talk about the ways they have partitioned the quantity. They should be able to explain how the quantity stays the same regardless of the groupings they have created. Skip-counting strategies can be used to determine the quantity of any collection. For example, encourage students to arrange counters in pairs and skip count by 2s, or organize objects into rows of 5 and skip count by 5s. Planned activities might include giving students a collection of buttons, counters, or cubes and asking them to see if they can organize them into groups and singles. A student might take 11 counters and organize them in some of the following ways. These

groupings show how a student could count a set of 11 by 2s, 5s, or 10s with singles. This reinforces conservation concepts and partitioning ideas.



SCO N08 Students will be expected to identify the number, up to 20, that is one more, two more, one less, and two less than a given number.

[C, CN, ME, R, V]

[C] Communication

[PS] Problem Solving

[CN] Connections

[ME] Mental Mathematics and Estimation

[T] Technology

[V] Visualization

[R] Reasoning

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

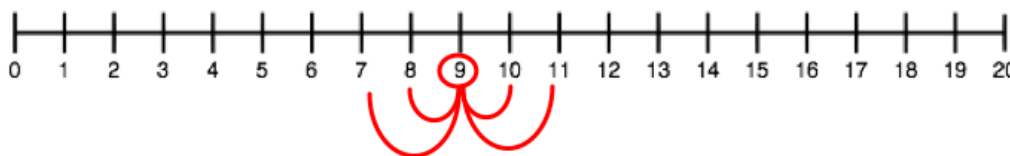
- N08.01** Name the number that is one more, two more, one less, or two less than a given number, up to 20.
- N08.02** Represent a number on ten-frames that is one more, two more, one less, or two less than a given number.

Performance Indicator Background

N08.01 Student's initial exploration of numbers that are one more than, one less than, two more than, and two less than should be done concretely using sets of objects. Have students use counters to create a set equal in number to a given set. Ask them to change their set to equal a number that is one more, one less, two more, or two less than their current set and to orally name the number using the appropriate language, such as 15 is two more than 13. When naming the number that is one more, two more, one less, or two less encourage students to count on or count back rather than beginning the count from one.

Students should also use a number line to explore which numbers come before and after a given number. Students should know the numbers that are one more, one less, two more, and two less than a given number. It might be helpful for students to take steps on a number line that has been placed on the floor, walking one or two more (forward) and walking one or two less (backward). The number line can help students think about a whole number's neighbours—the one-away and two-away neighbours.

9's one-away neighbours are 8 and 10, 9's two-away neighbours are 7 and 11.



N08.02 Have students show a given quantity on a ten-frame or a double ten-frame for numbers from ten to twenty. Ask them to add or remove counters to build and name the number that is one more, two more, one less, or two less. As students' understanding of this concept develops, they can be given tasks where they are shown a quantity in a ten-frame or double ten-frame and asked to visualize and say what number is one more, two more, one less, or two less.

SCO N09 Students will be expected to demonstrate an understanding of the addition of two single-digit numbers and the corresponding subtraction, concretely, pictorially, and symbolically in join, separate, equalize/compare, and part-part-whole situations.

[C, CN, ME, PS, R, V]

[C] Communication

[PS] Problem Solving

[CN] Connections

[ME] Mental Mathematics and Estimation

[T] Technology

[V] Visualization

[R] Reasoning

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N09.01 Act out story problems that are presented orally or through shared reading.

N09.02 Model story problems with manipulatives or pictures, find and share solutions using counting strategies, and record number sentences that represent how they thought about the problems.

N09.03 Create story problems that connect to student experiences.

N09.04 Create story problems for given number sentences.

Performance Indicator Background

N09.01 Students will be more captivated and engaged if initially they act out story situations where they can be the central characters. Present an addition or subtraction story for each structure. Have students act out the story situation to find the solution. For example, five children are sitting on the story mat. Two children left the mat to go back to their seats. How many children stayed on the mat? At this point students are not yet associating these situations with addition and subtraction; rather, they are using counting strategies to act out and find solutions.

N09.02 After acting out story situations, students should model the situations using concrete materials and pictures. They might use story boards or mats to represent the story situations. Students should spend a considerable amount of time modelling and talking about the problems they have solved and should be encouraged to share using language that reflects what they have done, such as 2 and 3 more is equal to 5.

Students will need support when they first start recording these situations and solutions with number sentences that represent how they thought about the problems. For example, consider the story problem: Pat has 8 marbles but she would like to have 12. How many more does she need to get? Some students may count out 8 marbles and add on until they reach 12, counting the 4 that they added on. These students would likely write $8 + 4 = 12$ to represent how they thought about the problem. Other students may count out 12 marbles, remove the 8 they knew they had, and count the 4. These students would likely write $12 - 8 = 4$ to represent how they thought about the problem. Either sentence would be acceptable, but students should be able to explain the number sentence they have written in relation to the situation they are modelling.

N09.03 and N09.04 Students first create story situations using prompts, such as storyboards. Many different problems can be created using the same storyboards. Students should share their story problems with others and record the corresponding number sentence for each of their problems. Students need experiences where they create story problems that will correspond to given number expressions, such as $3 + 7$, and number sentences, such as $3 + 7 = 10$. They should be asked to develop

the script and then act out the story. In this grade, students are required to draw and orally present their stories, rather than writing a script.

By applying their own experiences to the number expressions or sentences, they will create many different scripts. Students more naturally create join and separate (result unknown) types of problems, so they will need to be encouraged to create other types. One possible way to do this is to present three or more problems of one type, such as join (change unknown), discuss how the problems are alike and how they are different, and challenge students to create one like them.

SCO N10 Students will be expected to use and describe strategies to determine sums and differences using manipulatives and visual aids. Strategies include

- counting on or counting back
- one more or one less
- making ten
- doubles
- near doubles

[C, CN, ME, PS, R, V]

[C] Communication

[PS] Problem Solving

[CN] Connections

[ME] Mental Mathematics and Estimation

[T] Technology

[V] Visualization

[R] Reasoning

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N10.01 Use and describe a personal strategy to determine a sum.

N10.02 Use and describe a personal strategy to determine a difference.

N10.03 Use and describe how two different strategies can be used to determine a sum or difference.

Performance Indicator Background

N10.01 and N10.02 Through continued exposure to, discussion of, and practice applying a variety of strategies, students will adopt the strategies that make the most sense to them or with which they are most comfortable. These will be their personal strategies. The most important goal is that students are spontaneously using personal strategies, not count-all strategies, throughout their day.

N10.03 To ensure that students develop more than one personal strategy, they should be expected to describe two ways to find a sum or difference. For example, if asked to find the sum $8 + 7$, they may describe how it could be found by counting on, or by near doubles, or perhaps make 10.

Patterns and Relations (PR)

SCO PR01 Students will be expected to demonstrate an understanding of repeating patterns (two to four elements) by describing, reproducing, extending, and creating patterns using manipulatives, diagrams, sounds, and actions.

[C, PS, R, V]

[C] Communication

[PS] Problem Solving

[CN] Connections

[ME] Mental Mathematics and Estimation

[T] Technology

[V] Visualization

[R] Reasoning

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

PR01.01 Describe a given repeating pattern containing two to four elements in its core.

PR01.02 Identify errors in a given repeating pattern.

PR01.03 Identify the missing element(s) in a given repeating pattern.

PR01.04 Create and describe a repeating pattern using a variety of manipulatives, musical instruments and actions.

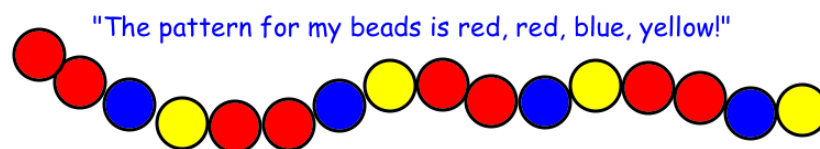
PR01.05 Reproduce and extend a given repeating pattern using manipulatives, diagrams, sounds and actions.

PR01.06 Identify and describe a repeating pattern in the environment (e.g., classroom, outdoors) using everyday language.

PR01.07 Identify repeating events (e.g., days of the week, birthdays, seasons).

Performance Indicator Background

PR01.01 Student descriptions of a repeating pattern should include the attribute and the core. Students should be given the opportunity to describe patterns orally as it helps them interpret the patterns they experience visually and solidify their understanding of the concept. It also allows other students to learn from each other. Repeating the core of a pattern three times is more likely to lead students to the anticipated result; however, there may be alternative extensions unless the pattern rule is described.

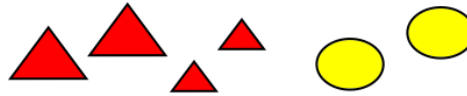


PR01.02 and PR01.03 Once students are confident in identifying patterns, describing them, and extending them, provide repeating patterns containing two to four elements in which there are errors or missing elements. Ask students to identify the errors or omissions, and correct the patterns. Most students may find the obvious errors; however, some students may find less typical errors and these should also be acknowledged.





Place the correct shapes in the empty spaces to correct the pattern.



PR01.04 Students should be encouraged to create and describe their own patterns. In all patterning activities, students should be given the opportunity to describe the patterns verbally. This not only helps them to interpret the specific patterns they experienced, but also to solidify their understanding of the concept of patterning in general. As well, it provides another opportunity for students to learn from each other.

Give children opportunities to make patterns on an informal and independent basis. The choice of manipulatives can affect the difficulty of the task. Connecting cubes and colour tiles are the easiest manipulatives from which children can make patterns, as they have only one visible attribute. They can also create a song that has a pattern word, a sentence, or a beat.

PR01.05 When presenting a pattern for students to reproduce or extend, repeat the core three times (e.g., red, red, blue, red, red, blue, red, red, blue, ...). As students become more efficient reproducing and extending patterns, repeat the core three times and begin the fourth repetition (e.g., red, red, blue, red, red, blue, red, red, blue, red, ...). Observe whether the student is able to continue the pattern from the last element given or repeats the entire core.

PR01.06 and PR01.07 By discussing patterns that occur naturally in students' everyday lives, patterning becomes more meaningful to them. Ask questions that alert students to patterns that occur naturally, such as in classroom routines, games played, songs sung, dance routines, books read, days of the week, and seasons of the year. Have students be on the alert for examples of patterns at home, on the playground, and at school. Students may be able to identify geometric and number patterns, such as tiles on the floor or ceiling, and numbers on doors down a corridor.

SCO PR02 Students will be expected to translate repeating patterns from one representation to another.

[C, R, V]

[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

PR02.01 Represent a given repeating pattern using another mode, e.g., actions to sound; colour to shape; ABC, ABC, ABC to blue, yellow, green, blue, yellow, green, blue, yellow, green,...

PR02.02 Describe a given repeating pattern using a letter code, e.g., ABC.

Performance Indicator Background

PR02.01 When students are given a repeating pattern in one mode, they should be able to represent that pattern in other modes. For example, if presented the action pattern: up, down, up, down, up, down, ..., some students may use cubes to make a colour pattern: red, blue, red, blue, red, blue, ...; other students may use cut-out shapes to make a shape pattern: square, triangle, square, triangle, square, triangle, ...; while still others may use musical instruments to make a sound pattern: drum beat, cymbal crash, drum beat, cymbal crash, drum beat, cymbal crash, ...

PR02.02 Translating two or more modes of the same pattern structure to a common format, such as A, A, B, A, A, B helps children see beyond the materials making up the pattern. In fact, repeating patterns are sometimes described using letter codes that help students to name and compare patterns. For example, all four patterns in the background of SCO PR02 above are AAB patterns. Students should be provided with many experiences describing repeating patterns containing two to four elements such as AB, AAB, ABB, ABC, AABB, ABA, and other combinations. Patterns, such as ABA, can be more challenging for students to identify as they will see the A again and assume the pattern has started over.

SCO PR03 Students will be expected to describe equality as a balance and inequality as an imbalance, concretely and pictorially (0 to 20).

[C, CN, R, V]

[C] Communication

[PS] Problem Solving

[CN] Connections

[ME] Mental Mathematics and Estimation

[T] Technology

[V] Visualization

[R] Reasoning

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- PR03.01** Construct two equal sets using the same objects (same shape and mass) and demonstrate their equality of number using a balance scale.
- PR03.02** Construct two unequal sets using the same objects (same shape and mass) and demonstrate their inequality of number using a balance scale.
- PR03.03** Determine if two given concrete sets are equal or unequal and explain the process used.

Performance Indicator Background

PR03.01 Using concrete materials, students can examine how a balance operates like the seesaw on the playground. Construct two equal sets using the same objects (same shape and mass) and demonstrate their equality of number using a balance scale. Place a set of 6 red cubes on the left pan of a balance scale and 4 yellow cubes on the right pan. Ask students to predict how many more cubes they would need to add, and on which pan, for the scale to balance. Have students place blue cubes, one at a time, onto the scale until it balances. Then have students count the number of cubes in each set, reinforcing the idea that both sides have the same number of cubes, saying, “Six red cubes is the same as four yellow cubes and two blue cubes” or “Six red cubes is equal to four yellow cubes and two blue cubes.”

PR03.02 When one set has more or fewer objects than the other, the sets are unequal. A balance scale can also be used to demonstrate these inequality relationships; for example, place 6 red cubes on the left pan of a balance scale and 4 yellow cubes on the right pan without actually showing students the quantities. Ask students what they observe. Ask them on which pan do they think there are more cubes and why they think so. Ask how this is like children on a seesaw. Have them count the quantities on both pans. Model how they could compare the quantities on the pans, saying, “the set of 6 red cubes is more than the set of 4 yellow cubes” and “the set of 6 red cubes is not the same as the set of 4 yellow cubes” or “the set of 6 red cubes is not equal to the set of 4 yellow cubes.”

When comparing sets, many students may recognize that the set of 6 cubes is more than the set of 4 cubes, but not automatically realize, that the set of 4 cubes is fewer than the set of 6 cubes. Both sides of the relationship need to be considered. Therefore, ask students what they could say about the set of 4 yellow cubes compared to the set of 6 red cubes, bringing out the relationship “is fewer than” and “is not the same as” or “is not equal to.”

PR03.03 Students should have experiences determining equality and inequality relationships through balance scales. For example, provide students with two sets of objects (all the same mass). Have them predict the relationship between the two sets, asking, Are these two sets equal, or unequal? Ask them to use a pan balance to check their predictions, explaining the process they use. Have them express the relationship between the two sets using appropriate language.

Students should also have experiences determining equality and inequality relationships through one-to-one correspondence, another form of balancing comparisons. For example, to show the relationship between the number of boys and the number of girls in a group, the boys and the girls could form two lines across from each other, and make boy-girl partners. If there are leftover boys, discuss the relationship between the number of boys and the number of girls.

One-to-one correspondence is also how students could show or verify equality and inequality relationships between pictures of two sets of objects. They could draw arrows from each object in one set to a corresponding object in the second set, and determine the relationship based on any leftover objects in either set. While the relationships interpreted from this arrow drawing may be obvious to adults, many students in grade one will require many experiences and discussion.

SCO PR04 Students will be expected to record equalities using the equal symbol.			
[C, CN, PS, V]			
[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- PR04.01** Represent a given pictorial or concrete equality in symbolic form.
- PR04.02** Represent a given equality using manipulatives or pictures.
- PR04.03** Provide examples of equalities where the given sum or difference is on either the left or right side of the equal symbol (=).
- PR04.04** Record different representations of the same quantity (0 to 20) as equalities.

Performance Indicator Background

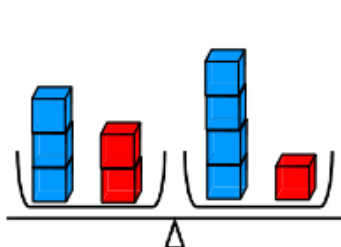
PR04.01 Initially, students should learn to write number sentences to express the equality between the quantities on the two pans of a balance scale. For example, if 2 red cubes and 5 blue cubes are on the left pan of the scale, and 7 yellow cubes are on the right pan, the students see the scale is balanced, say, 2 red cubes and 5 blue cubes is the same as 7 yellow cubes, and learn that $2 + 5 = 7$ is how this is represented in symbols.

PR04.02 After students are comfortable interpreting equalities on balance scales as number sentences, they should be able to create balance scale representations for given number sentences, and draw balance scale pictures to show these equalities. For example, given $8 = 7 + 1$, they could set up a balance scale with, perhaps, 8 red cubes on the left pan, and 7 blue cubes and 1 green cube on the right pan. They could also draw a picture of this balance scale set up.

PR04.03 One possible activity to introduce students to writing equalities of sums on left and right sides of the equality sign involves a balance scale and cube towers. Have students use cubes of two colours, such as red and blue, to build different towers of 5 cubes. They could create these towers: 5 red cubes, 5 blue cubes, 4 red cubes and 1 blue cube, 3 red cubes and 2 blue cubes. Placing these towers, two at a time, on the pans of the balance scale and writing the related number sentences would provide opportunities to write a variety of number sentences, such as $5 = 4 + 1$, $3 + 2 = 5$, $4 + 1 = 3 + 2$, and $5 = 5$.

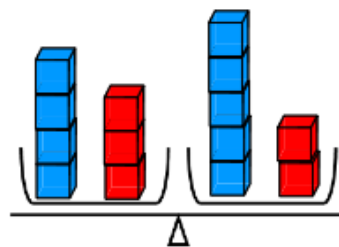
"Three blue blocks and two red blocks is equal to four blue blocks and one red block."

$$3 + 2 = 4 + 1$$



"Four blue blocks and three red blocks is equal to five blue blocks and two red blocks."

$$4 + 3 = 5 + 2$$



Students should also have experiences showing equality between two differences and between a sum and a difference. For example, have students choose a number from 4 to 7 and write their choice at the top of a white board. Suppose they choose 6. Below this number 6, draw a 2-column table, one column labelled “Two Quantities Added,” and the other column labelled “Two Quantities Subtracted.” Provide students with counters, and ask them to suggest something to put in the first column that would have an answer of 6; for example, $5 + 1$. Then ask for something to be put in the second column that would have an answer of 6; for example, $8 - 2$. Continue until there are three or four entries in each column. Orally discuss the possible equalities that could be written using the number 6 and the entries in the table. Then ask the students to write at least five equalities.

PR04.04 This performance indicator is connected to the partitioning outcome N04, where students are concretely and pictorially representing a quantity in two parts. Here they start to symbolize this relationship. For example, ask students to place 11 two-colour counters in a row showing all one colour. Direct them to turn over one counter and write the equality. They would write $11 = 10 + 1$ or $10 + 1 = 11$. Then direct them to turn over another counter and to write this equality. This can be continued until they have written a number of equalities.

Measurement (M)

SCO M01 Students will be expected to demonstrate an understanding of measurement as a process of comparing by			
<ul style="list-style-type: none"> ▪ identifying attributes that can be compared ▪ ordering objects ▪ making statements of comparison ▪ filling, covering, or matching 			
[C, CN, PS, R, V]			
[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- M01.01** Identify common attributes, such as length, mass, volume, capacity, and area that could be used to compare a given set of two objects.
- M01.02** Compare and order two given objects and identify the attributes used to compare.
- M01.03** Predict which object in a set is longest/shortest, determine by matching and explain the reasoning.
- M01.04** Predict which object in a set is heaviest/lightest, determine by comparing and explain the reasoning.
- M01.05** Predict which object in a set is largest/smallest, determine by comparing and explain the reasoning.
- M01.06** Predict which object in a set holds the most/least, determine by filling and explain the reasoning.
- M01.07** Predict which figure in a set has the greatest/least area, determine by covering and explain the reasoning.

Performance Indicator Background

M01.01 and M01.02 Students should learn that any object has multiple attributes that could be used to describe it. Similarly, any two objects can be compared in a variety of ways, depending upon the attributes they have in common. Once the comparisons of two or more attributes of objects have been developed, students should be presented with two objects and asked to describe how these objects could be compared. Then they should compare them in the ways they described, using predictions followed by direct comparisons to check those predictions.

M01.03 Organizing objects by length is usually the easiest attribute for students to do, so this would be the best attribute with which to start. Students should recognize that length describes the measurement of an object in one dimension. These linear measurements include measurements of the height, width, and length of an object, as well as the more abstract concepts of depth and distance that will be discussed in later grades. In Mathematics 1, their predictions of the lengths, widths, and heights of objects are checked by direct comparison. Direct comparison involves lining up items side by side, beginning at a common base. Students should be led to see why a common starting point is important.

Students should order objects from longest to shortest, shortest to tallest, thickest to thinnest, and narrowest to widest, learning the associated vocabulary through the direct comparison activities. Initially, the activities should involve objects for which the attribute being compared is the only variable. For example, if they are organizing a set of erasers from narrowest to widest, the erasers should initially be the same length. Later, students should be able to organize a set of erasers from narrowest to widest when the erasers are also of different lengths, or from longest to shortest when the erasers have different widths, and not be distracted by the other variable.

M01.04 Students should recognize that mass tells about the heaviness of an object. They should explore methods to compare and order masses. The most conceptual way for children to compare the masses of objects is to lift them in their hands. It is recommended that students use the same hand and lift one object after the other because there can be different sensations in their left and right arms. Their predictions can be checked by direct comparison using balance scales. Unlike other attribute development, you should have the objects be very different from one another, so students are not tempted to organize them by their visual characteristics. That is, you should have some objects that are smaller in size but with more mass than larger objects. However, initially the differences in mass should be quite pronounced so students can be successful. While developing measurement skills for mass, students should use terms such as **heaviest** and **lightest**.

M01.05 The volume of an object is the amount of space it occupies. Children naturally compare objects by what they refer to as their size, saying things such as, My toy wagon is bigger than yours. Your book is smaller than mine. That is the largest toy box in the classroom. Of course, they do not realize it is volume they are referring to, but this can make an easy connection for them to the word **volume**. They should predict which objects around the classroom look bigger/smaller or biggest/smallest and then bring them close together to compare them directly to check their predictions. Initially, the differences in size should be obvious. While comparing objects based on volume, students should use language such as “takes up more space” or “takes up less space.”

M01.06 Students should recognize that capacity tells how much something will hold. Identifying differences in capacities of containers is difficult for children; therefore, the differences among the capacities should be obvious so they can be successful. Initially, you should try to keep two of the dimensions of the containers constant. For example, 250-mL, 500-mL, and 1-L milk cartons have the same footprint, so their capacities will be directly related to the heights. These containers can be cut to different heights with their tops off so students can look into them to predict how much each object holds. Then the activities can involve containers of different shapes and varying dimensions.

Students should investigate strategies to compare the capacities of three or more containers. For this attribute, direct comparison involves filling one container and then pouring the contents into another to find which holds more. This method of comparing capacities will be more involved than comparing lengths. For example, suppose students have organized three containers from smallest to largest. They fill the first container with rice, pour it into what they believe to be the next largest, and if there is room for more rice in the second container, it will have to be filled with rice before it is poured into the third container. Obviously, it is important that the containers be organized by sight initially to provide a starting container, and that this will have to be revisited should the first, or subsequent, pourings prove to be incorrect. While developing measurement skills for capacity, students will use terms such as **holds more**, **holds less**, **holds the same**, **full**, and **empty**.

M01.07 In comparing areas, students are examining the amount of surface taken up by an object. For example, one place mat might cover more of the table top than another one, or one piece of paper covers more of the desk than another piece. Students should predict which objects cover more and use direct comparison to verify their prediction. Direct comparison involves placing one surface on top of

another to see if they match or if one sticks out. While developing the concept of area, students should use terms such as **covers more/covers less**, **greatest/least area**, and **largest/smallest area**. As with capacities, for students' initial experiences, you should minimize the differences among the dimensions of surfaces, and make the differences among the areas fairly large.

Geometry (G)

SCO G01 Students will be expected to sort 3-D objects and 2-D shapes using one attribute and explain the sorting rule.			
[C, CN, R, V]			
[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

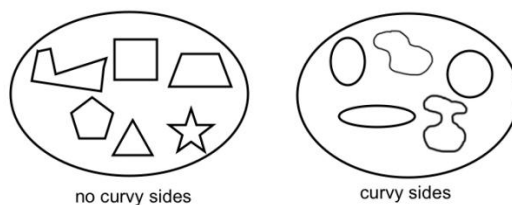
Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- G01.01** Sort a given set of familiar 3-D objects or 2-D shapes using a given sorting rule.
- G01.02** Sort a given set of familiar 3-D objects using a single attribute determined by the student and explain the sorting rule.
- G01.03** Sort a given set of 2-D shapes using a single attribute determined by the student and explain the sorting rule.
- G01.04** Determine the difference between two given pre-sorted sets of familiar 3-D objects or 2-D shapes and explain a possible sorting rule used to sort them.

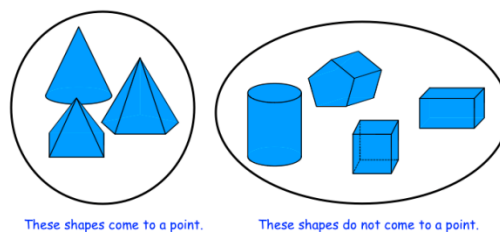
Performance Indicator Background

G01.01 Students should sort familiar objects, such as the objects in their pencil case. They should be able to sort these into two groups based on a given sorting rule, such as things that roll and things that do not roll. They should also have experiences sorting pictures of 2-D shapes. Shapes should not just be a set of normal geometric shapes, but should include a variety of shapes. Students can be asked to sort these based on rules, such as has a curvy side and does not have a curvy side.



G01.02 Students' initial sorts of 3-D objects would likely be by type of shape, by size, and by ability to roll/slide. Through discussions of how two 3-D shapes are alike and how they are different, through other focused questioning, and through sharing of sorting rules, students' sorting should expand to include some other attributes of 3-D shapes:

- come up to a point
- number of edges
- number of corners/vertices
- number of faces



In open-ended sorting experiences, students may also sort based on other attributes of 3-D objects, such as

- the number of edges
- the number of vertices
- the number of faces
- Will it roll? Stack? Slide?

G01.03 Initially, students recognize shapes by their overall appearance rather than by any of their attributes. For example, they would likely say they know something is a square because it looks like a square rather than through an awareness that it has four equal sides and all its angles are right angles. However, through sorting activities students begin to notice attributes of shapes. If students are asked to sort a given set of 2-D shapes, their initial sorts are likely to be by type of shape, by size, or by colour. However, their sorts should expand to involve the attributes of the number of sides and number of corners/vertices.

G01.04 Students need opportunities to recognize and discuss the sorting rule for two pre-sorted sets of familiar 2-D shapes. Sort 2-D shapes while students observe. Have them predict where each object would be placed, explaining the possible sorting rules used. Eventually they should be able to look at two sets and discuss what the difference is between the two sets of objects. Then they should use this difference to determine the rule. Pre-sorted sets should only contain one difference.

Note: The concept of 2-D shapes is very abstract because 2-D shapes do not have thickness. For practical purposes to study 2-D shapes, however, we commonly use pattern blocks, attribute blocks, and cut-out shapes that are actually 3-D objects (prisms and cylinders) that have thickness. These 3-D objects have very prominent 2-D faces that are the shapes to which we refer. For example, the triangle in the set of pattern blocks is actually a triangular prism with two prominent triangular faces. Even when you draw a figure, such as a square, on a piece of paper using a pencil, the square is actually the shape within the pencil segments, not including those pencil segments. It is not intended that you try to explain this to students in grade one unless you have an exceptionally advanced student who you think can understand and benefit from this knowledge. This concept will be clarified for students in later grades. You could, however, very consciously touch the prominent face of a pattern block when referring to its shape name as a subtle way to focus students on the actual shape being discussed.

GCO G02 Students will be expected to replicate composite 2-D shapes and 3-D objects.

[CN, PS, V]

[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

Performance Indicators

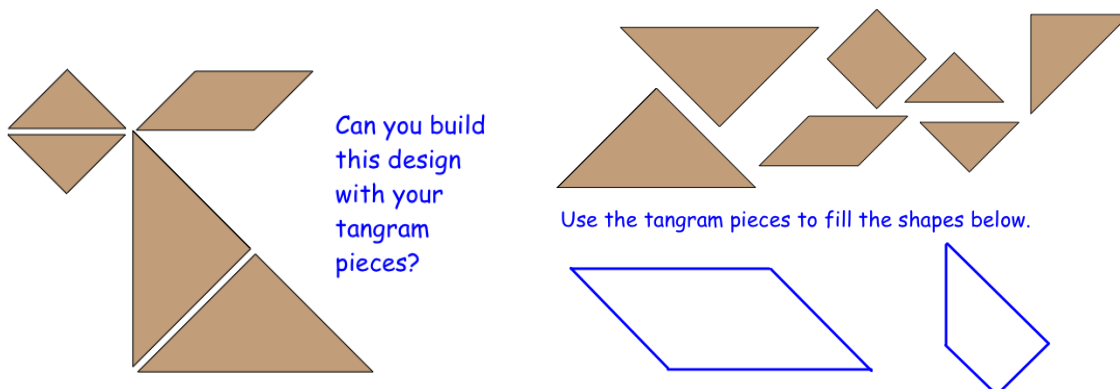
Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- G02.01** Select 2-D shapes from a given set of 2-D shapes to reproduce a given composite 2-D shape.
- G02.02** Select 3-D objects from a given set of 3-D objects to reproduce a given composite 3-D object.
- G02.03** Predict and select the 2-D shapes used to produce a composite 2-D shape, and verify by deconstructing the composite shape.
- G02.04** Predict and select the 3-D objects used to produce a composite 3-D object, and verify by deconstructing the composite object.

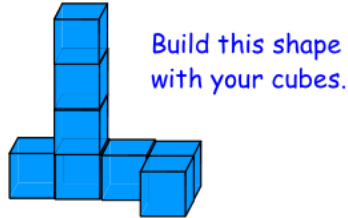
Performance Indicator Background

G02.01 and G02.03 Students should develop the ability to replicate composite 2-D shapes. This can begin by making a block design that matches a presented design involving 4 to 6 blocks. This will require students to select the blocks and arrange them in the same relative positions. Simply presenting a design created with a few blocks for a brief period of time, and then asking students to build that design, will extend their spatial abilities to include visual memory.

Students could create a block design using 2 blocks, trace around the design to create a 2-block puzzle, remove the blocks, and visualize where the 2 blocks would be placed if they returned them. After repeating this a few times, students could share their puzzles with other students to solve. This activity can be extended to puzzles involving more pieces depending upon individual student ability. Using tangram pieces in similar activities will provide students with experience with other shapes of varying sizes. It is through such replication activities that students become familiar with the attributes of various 2-D shapes. Students will use their knowledge of the properties of 2-D shapes to predict and select which shapes are necessary to produce a composite shape. To verify their predictions and selections, they will then deconstruct the original shape and compare the two sets.



G02.02 and G02.04 Students should develop the ability to replicate composite 3-D objects. For example, if students are presented with an object that has been constructed with 8 interlocking cubes, they are able to build the same shape; if they are presented with a display of various combinations of building blocks and 3-D objects, students are able to replicate this display. Such activities will use and develop students' visual discrimination and perception of spatial relationships abilities.



Through replication activities, students become more familiar with the attributes of 3-D objects. They will use these attributes, as well as relationships, to select a set of objects they predict are necessary to produce a composite object. To check their predictions, students should deconstruct the original object and compare the two sets. Any differences between their predictions and the actual set of objects should be discussed to help improve students' prediction abilities.

SCO G03 Students will be expected to identify 2-D shapes in 3-D objects.			
[C, CN, V]			
[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

G03.01 Identify the shape of the faces of a 3-D object

G03.02 Identify 3-D objects in the environment that have faces that are a given 2-D shape.

Performance Indicator Background

G03.01 Students should have opportunities to find 3-D objects in their environment and identify the shape of their faces because these real-world associations are very important in the development of geometric concepts. With this foundation, they could also work with sets of 3-D solids and building blocks to explore the shapes of faces by tracing the different faces of the solids on paper or by pressing the faces in plasticine or sand.

G03.02 Once students have had experiences examining the faces of 3-D objects; they should be better able to concentrate on the component faces of a 3-D object. Prior to identifying 3-D objects in the environment with a face of a particular 2-D shape, students should have some preliminary experiences. For example, place four or five 3-D objects in front of students, ask them to identify an object that has a square face, and select a student to place a finger on the square face; repeat by naming other 2-D shapes. Label a mat with “All these objects have one face that is a triangle,” place a complete set of 3-D solids in view, and ask students to select and place on the mat all the solids that meet this condition.

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Mathematics 2



CURRICULUM

Mathematics 2

**Implementation Draft
May 2013**

Website References

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Mathematics 2, Implementation Draft

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Introduction

Background and Rationale

Mathematics curriculum is shaped by a vision that fosters the development of mathematically literate students who can extend and apply their learning and who are effective participants in society. It is essential that the mathematics curriculum reflect current research in mathematics instruction. To achieve this goal, Western and Northern Canadian Protocol's (WNCP) *The Common Curriculum Framework for K–9 Mathematics* (2006) has been adopted as the basis for the new mathematics curriculum in Nova Scotia.

The Common Curriculum Framework was developed by the seven ministries of education (Alberta, British Columbia, Manitoba, Northwest Territories, Nunavut, Saskatchewan, and Yukon Territory) in collaboration with teachers, administrators, parents, business representatives, post-secondary educators, and others. The framework identifies beliefs about mathematics, general and specific student outcomes, and performance indicators agreed upon by the seven jurisdictions. The outcomes and performance indicators have been adapted for Nova Scotia. This document is based on both national and international research by the WNCP and the National Council of Teachers of Mathematics (NCTM).

There is an emphasis in the Nova Scotia curriculum on particular key concepts at each grade that will result in greater depth of understanding and, ultimately, stronger student achievement. There is also a greater emphasis on number sense and operations concepts in the early grades to ensure students develop a solid foundation in numeracy.

Purpose

This document provides sets of outcomes and performance indicators to be used as a mandated common base for defining mathematics curriculum expectations. This common base should result in consistent student outcomes in mathematics within the province of Nova Scotia. It should also enable easier transfer for students moving within the province or from any jurisdiction that has adopted the WNCP framework. This document is intended to clearly communicate to all education partners across the province the high expectations for students' mathematical learning.

Program Design and Components

Assessment

Ongoing assessment for learning is essential to effective teaching and learning. Research has shown that assessment for learning (formative assessment) practices produce significant and often substantial learning gains, close achievement gaps, and build students' ability to learn new skills (Black & Wiliam 1998; OECD 2006). Student involvement in assessment promotes learning. Timely and effective teacher feedback and student self-assessment allow students to reflect on and articulate their understanding of mathematical concepts and ideas.

Assessment in the classroom includes

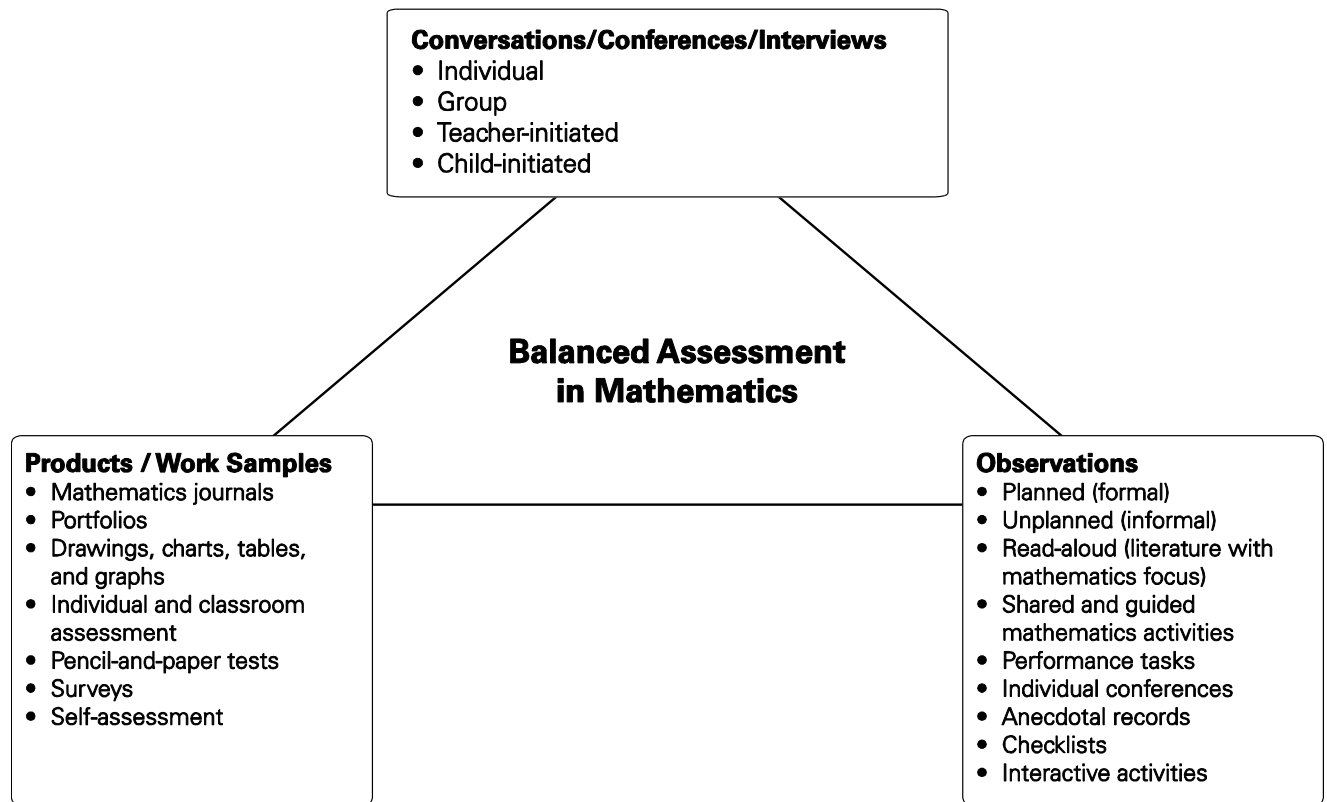
- providing clear goals, targets, and learning outcomes
- using exemplars, rubrics, and models to help clarify outcomes and identify important features of the work
- monitoring progress towards outcomes and providing feedback as necessary
- encouraging self-assessment
- fostering a classroom environment where conversations about learning take place, where students can check their thinking and performance and develop a deeper understanding of their learning

(Davies 2000)

Assessment for learning practices act as the scaffolding for learning, which only then can be measured through assessment of learning (summative assessment). Assessment of learning tracks student progress, informs instructional programming, and aids in decision making. Both forms of assessment are necessary to guide teaching, stimulate learning, and produce achievement gains.

Assessment of student learning should

- align with curriculum outcomes
- clearly define criteria for success
- make explicit the expectations for students' performance
- use a wide variety of assessment strategies and tools
- yield useful information to inform instruction



Time to Learn for Mathematics

The Time to Learn Strategy *Guidelines for Instructional Time: Grades Primary–6* includes time for mathematics instruction in the “Required Each Day” section. In order to support a constructivist approach to teaching through problem solving, it is highly recommended that the 45 minutes required daily in grades primary–2 and the 60 minutes required daily for grades 3–6 mathematics instruction be provided in an uninterrupted block of time.

Time to Learn guidelines can be found at

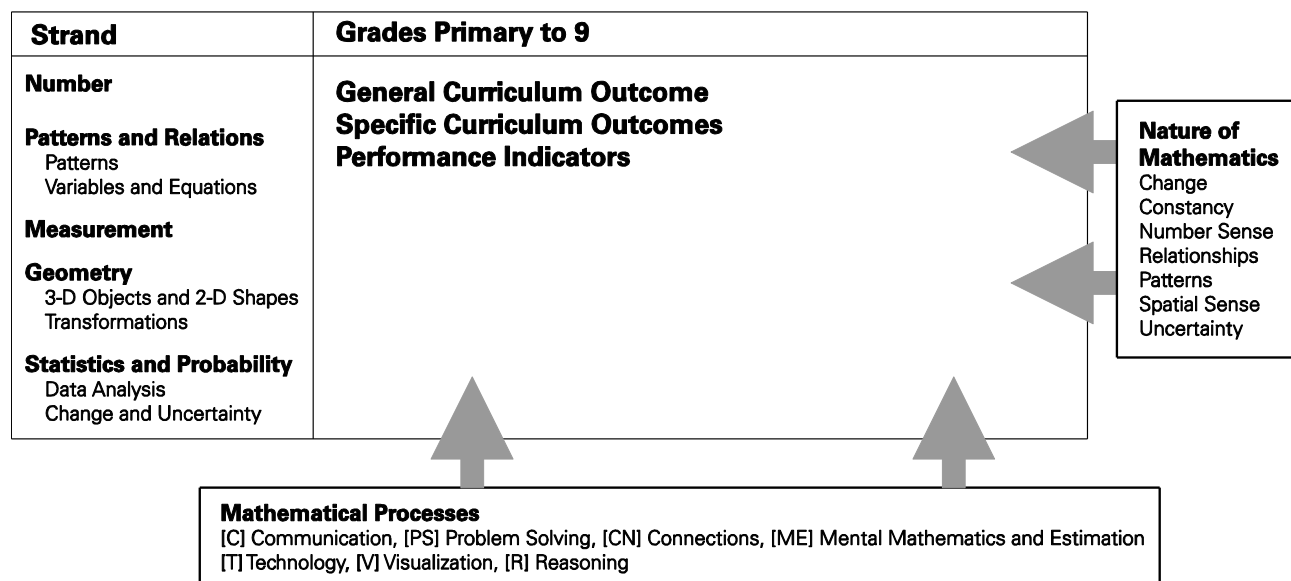
www.ednet.ns.ca/files/ps-policies/semestering.pdf

www.ednet.ns.ca/files/ps-policies/instructional_time_guidelines_p-6.pdf

Outcomes

Conceptual Framework for Mathematics Primary–9

The chart below provides an overview of how mathematical processes and the nature of mathematics influence learning outcomes.



(Adapted with permission from Western and Northern Canadian Protocol, *The Common Curriculum Framework for K–9 Mathematics*, p. 5. All rights reserved.)

Structure of the Mathematics Curriculum

Strands

The learning outcomes in the Nova Scotia Framework are organized into five strands across grades primary to 9.

- Number (N)
- Patterns and Relations (PR)
- Measurement (M)
- Geometry (G)
- Statistics and Probability (SP)

General Curriculum Outcomes (GCO)

Some strands are further subdivided into sub-strands. There is one general outcome (GCO) per sub-strand. GCOs are overarching statements about what students are expected to learn in each strand/sub-strand. The general curriculum outcome for each strand/sub-strand is the same throughout the grades.

NUMBER (N)

GCO: Students will be expected to demonstrate number sense.

PATTERNS AND RELATIONS (PR)

Patterns

GCO: Students will be expected to use patterns to describe the world and solve problems.

Variables and Equations

GCO: Students will be expected to represent algebraic expressions in multiple ways.

MEASUREMENT (M)

GCO: Students will be expected to use direct and indirect measure to solve problems.

GEOMETRY (G)

3-D Objects and 2-D Shapes

GCO: Students will be expected to describe the characteristics of 3-D objects and 2-D shapes and analyze the relationships among them.

Transformations

GCO: Students will be expected to describe and analyze position and motion of objects and shapes.

STATISTICS AND PROBABILITY (SP)

Data Analysis

GCO: Students will be expected to collect, display, and analyze data to solve problems.

Chance and Uncertainty

GCO: Students will be expected to use experimental or theoretical probabilities to represent and solve problems involving uncertainty.

Specific Curriculum Outcomes (SCOs) and Performance Indicators

Specific curriculum outcomes (SCOs) are statements that identify the specific conceptual understanding, related skills, and knowledge students are expected to attain by the end of a given grade.

Performance indicators are statements that identify specific expectations of the depth, breadth, and expectations for the outcome. Teachers use these statements to determine whether students have achieved the corresponding specific curriculum outcome.

Process Standards Key

[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

NUMBER (N)

- N01** Students will be expected to say the number sequence by
- 1s, forward and backward, starting from any point to 200
 - 2s, forward and backward, starting from any point to 100
 - 5s and 10s, forward and backward, using starting points that are multiples of 5 and 10 respectively to 100
 - 10s, starting from any point, to 100 [C, CN, ME, R]

Performance Indicators

- N01.01 Extend counting sequence (by 1s), forward and backward.
 N01.02 Extend a given skip counting sequence (by 2s, 5s, or 10s) forward and backward.
 N01.03 Skip count by 10s, given any number as a starting point.
 N01.04 Identify and correct errors and omissions in a given skip counting sequence.
 N01.05 Count a given sum of money with pennies, nickels, or dimes (to 100¢).
 N01.06 Count quantity using groups of 2s, 5s, or 10s and counting on.

- N02** Students will be expected to demonstrate if a number (up to 100) is even or odd. [C, CN, PS, R]

Performance Indicators

- N02.01 Use concrete materials or pictorial representations to determine if a given number is even or odd.
 N02.02 Identify even and odd numbers in a given sequence, such as on a hundred chart.
 N02.03 Sort a given set of numbers as even numbers and odd numbers.

- N03** Students will be expected to describe order or relative position using ordinal numbers (up to tenth). [C, CN, R]

Performance Indicators

- N03.01 Indicate a position of a specific object in a sequence by using ordinal numbers up to tenth.
 N03.02 Compare the ordinal position of a specific object in two different given sequences.

N04 Students will be expected to represent and partition numbers to 100. [C, CN, V]

Performance Indicators

- N04.01 Represent a given number using concrete materials, such as ten-frames and base-ten materials.
- N04.02 Represent a given number using coins (pennies, nickels, dimes, and quarters).
- N04.03 Represent a given number using tallies.
- N04.04 Represent a given number pictorially.
- N04.05 Find examples of a given number in the environment.
- N04.06 Represent a given number using expressions (e.g., $24 + 6$, $15 + 15$, $40 - 10$)
- N04.07 Read a number (0–100) given in symbolic or word form.
- N04.08 Record in words a given number (0–20).
- N04.09 Record, symbolically, any number (0–100).

N05 Students will be expected to compare and order numbers up to 100. [C, CN, R, V]

Performance Indicators

- N05.01 Compare and order a given set of numbers in ascending or descending order and verify the result using a hundred chart, number line, ten-frames, or by making references to place value.
- N05.02 Identify errors in a given ordered sequence.
- N05.03 Identify missing numbers in a given hundred chart.
- N05.04 Identify errors in a given hundred chart.

N06 Students will be expected to estimate quantities to 100 by using referents. [C, ME, PS, R]

Performance Indicators

- N06.01 Estimate a given quantity by comparing it to a referent (known quantity).
- N06.02 Estimate the number of groups of ten in a given quantity using 10 as a referent.
- N06.03 Select between two possible estimates for a given quantity and explain the choice.

N07 Students will be expected to illustrate, concretely and pictorially, the meaning of place value for numerals to 100. [C, CN, R, V]

Performance Indicators

- N07.01 Explain and show with counters the meaning of each digit for a given 2-digit numeral with both digits the same.
- N07.02 Count the number of objects in a given set using groups of 10s and 1s, and record the result as a 2-digit numeral under the headings of 10s and 1s.
- N07.03 Describe a given 2-digit numeral in at least two ways.
- N07.04 Illustrate using ten-frames and diagrams that a given numeral consists of a certain number of groups of ten and a certain number of ones.
- N07.05 Illustrate using proportional base-ten materials that a given numeral consists of a certain number of tens and a certain number of ones.
- N07.06 Explain why the value of a digit depends on its placement within a numeral.
- N07.07 Represent one unit if shown a pre-grouped model representing ten.

N08 Students will be expected to demonstrate and explain the effect of adding zero to or subtracting zero from any number. [C, R]

Performance Indicators

- N08.01 Add zero to a given number and explain why the sum is the same as the addend.
N08.02 Subtract zero from a given number and explain why the difference is the same as the given number.

N09 Students will be expected to demonstrate an understanding of addition (limited to 1- and 2-digit numerals) with answers to 100 and the corresponding subtraction by

- using personal strategies for adding and subtracting with and without the support of manipulatives
- creating and solving problems that involve addition and subtraction
- explaining and demonstrating that the order in which numbers are added does not affect the sum
- explaining and demonstrating that the order in which numbers are subtracted matters when finding a difference [C, CN, ME, PS, R, V]

Performance Indicators

- N09.01 Solve a given story problem of any type by modelling it with materials or a diagram, and write a number sentence that represents the thinking in the solution.
N09.02 Solve a given story problem of any type by writing a number expression and combining the numbers to complete the number sentences.
N09.03 Match a number sentence to a given story problem.
N09.04 Create an addition or a subtraction number sentence and a story problem for a given solution.
N09.05 Model addition and subtraction using concrete materials or visual representations and record the process symbolically.
N09.06 Add a given set of numbers in two different ways and explain why the sum is the same.
N09.07 Recognize and create equivalent addition and subtraction number sentences.

N10 Students will be expected to apply mental mathematics strategies to quickly recall basic addition facts to 18 and determine related subtraction facts. [C, CN, ME, R, V]

Performance Indicators

- N10.01 Explain the mental mathematics strategy that could be used to determine basic addition facts.
- Doubles Facts
 - Plus One Facts
 - One-Apart (Near Doubles) Facts
 - Plus Two Facts
 - Plus Zero Facts
 - Make-10 Facts
 - Two-Apart Facts
 - Plus Three Facts
- N10.02 Use and describe a personal strategy for determining a sum to 18.
N10.03 Quickly recall basic addition facts to 18 in a variety of contexts.
N10.04 Explain the think-addition strategy used to determine a basic subtraction fact.
N10.05 Use and describe a personal strategy for determining the subtraction facts.

PATTERNS AND RELATIONS (PR)

PR01 Students will be expected to demonstrate an understanding of repeating patterns (three to five elements) by describing, extending, comparing, and creating, patterns using manipulatives, diagrams, sounds, and actions. [C, CN, PS, R, V]

Performance Indicators

- PR01.01 Identify the core of a given repeating pattern.
- PR01.02 Describe and extend a given double attribute pattern.
- PR01.03 Create a repeating non-numerical pattern and explain the rule.
- PR01.04 Predict an element of a given repeating pattern using a variety of strategies and extend the pattern up to the tenth element to verify the prediction.
- PR01.05 Translate a repeating pattern from one mode to another.
- PR01.06 Compare two given repeating patterns, and describe how they are alike/different.

PR02 Students will be expected to demonstrate an understanding of increasing patterns by describing, extending, and creating numerical patterns (numbers to 100) and non-numerical patterns using manipulatives, diagrams, sounds, and actions. [C, CN, PS, R, V]

Performance Indicators

- PR02.01 Identify and describe increasing patterns in a variety of given contexts.
- PR02.02 Represent a given increasing pattern concretely and pictorially.
- PR02.03 Identify errors in a given increasing pattern.
- PR02.04 Explain the rule used to create a given increasing pattern.
- PR02.05 Create an increasing pattern and explain the pattern rule.
- PR02.06 Represent a given increasing pattern using another mode.
- PR02.07 Solve a given problem using increasing patterns.
- PR02.08 Identify and describe increasing patterns in the environment.
- PR02.09 Determine missing terms in a given concrete, pictorial, or symbolic increasing pattern and explain the reasoning.

PR03 Students will be expected to demonstrate and explain the meaning of equality and inequality by using manipulatives and diagrams (0 to 100). [C, CN, R, V]

Performance Indicators

- PR03.01 Determine whether two given quantities of the same object (same shape and mass) are equal by using a balance scale.
- PR03.02 Construct and draw two unequal sets using the same object (same shape and mass) and explain the reasoning.
- PR03.03 Demonstrate how to change two given sets, equal in number, to create inequality.
- PR03.04 Choose from three or more given sets the one that does not have a quantity equal to the others and explain why.

PR04 Students will be expected to record equalities and inequalities symbolically, using the equal symbol or not equal symbol. [C, CN, R, V]

Performance Indicators

- PR04.01 Determine whether two sides of a given number sentence are equal (=) or not equal (\neq). Write the appropriate symbol and justify the answer.
- PR04.02 Model equalities using a variety of concrete representations and record the equality.
- PR04.03 Model inequalities using a variety of concrete representations and record the inequality.

MEASUREMENT (M)

M01 Students will be expected to demonstrate an understanding of the calendar and the relationships among days, weeks, months, and years. [C, CN, PS, R]

Performance Indicators

- M01.01 Read a calendar.
- M01.02 Name and order the days of the week and months of the year.
- M01.03 Communicate the number of days in a week and the number of months in a year.
- M01.04 Solve a given problem involving time which is limited to the number of days in a week and the number of months in a year.

M02 Students will be expected to relate the size of a unit of measure to the number of units (limited to non-standard units) used to measure length and mass. [C, CN, ME, R, V]

Performance Indicators

- M02.01 Explain why one of two given non-standard units may be a better choice for measuring the length of an object.
- M02.02 Explain why one of two given non-standard units may be a better choice for measuring the mass of an object.
- M02.03 Select a non-standard unit for measuring the length or mass of an object and explain why it was chosen.
- M02.04 Estimate the number of non-standard units needed for a given measurement task.
- M02.05 Explain why the number of units of a measurement will vary depending upon the unit of measure used.

M03 Students will be expected to compare and order objects by length, height, distance around, and mass using non-standard units and make statements of comparison. [C, CN, ME, R, V]

Performance Indicators

- M03.01 Estimate, measure, and record the length, height, distance around, or mass of a given object using non-standard units.
- M03.02 Compare and order the measure of two or more objects in ascending or descending order and explain the method of ordering.

M04 Students will be expected to measure length to the nearest non-standard unit by using multiple copies of a unit and using a single copy of a unit (iteration process). [C, ME, R, V]

Performance Indicators

- M04.01 Explain why overlapping or leaving gaps does not result in accurate measures.
- M04.02 Count the number of non-standard units required to measure the length of a given object using a single copy or multiple copies of a unit.
- M04.03 Estimate and measure a given object using multiple copies of a non-standard unit and using a single copy of the same unit many times, and explain the results.
- M04.04 Estimate and measure, using non-standard units, a given length that is not a straight line.

M05 Students will be expected to demonstrate that changing the position of an object does not alter the measurements of its attributes. [C, R, V]

Performance Indicator

- M05.01 Measure a given object, change the position, remeasure, and explain the results.

GEOMETRY (G)

G01 Students will be expected to sort 2-D shapes and 3-D objects using two attributes and explain the sorting rule. [C, CN, R, V]

Performance Indicators

- G01.01 Determine the differences between two given presorted sets and explain the sorting rule.
- G01.02 Identify and name two common attributes of items within a given sorted group.
- G01.03 Sort a given set of 2-D shapes (regular and irregular) according to two attributes and explain the sorting rule.
- G01.04 Sort a given set of 3-D objects according to two attributes and explain the sorting rule.

G02 Students will be expected to recognize, name, describe, compare, and build 3-D objects, including cubes and other prisms, spheres, cones, cylinders, and pyramids. [C, CN, R, V]

Performance Indicators

- G02.01 Sort a given set of 3-D objects and explain the sorting rule.
- G02.02 Identify common attributes of cubes and other prisms, spheres, cones, cylinders, and pyramids from given sets of the same 3-D objects.
- G02.03 Identify and describe given 3-D objects with different dimensions.
- G02.04 Identify and describe given 3-D objects with different positions.
- G02.05 Create and describe a representation of a given 3-D object using materials such as modelling clay.
- G02.06 Identify and name examples of cubes and other prisms, spheres, cones, cylinders, and pyramids found in the environment.

G03 Students will be expected to recognize, name, describe, compare and build 2-D shapes, including triangles, squares, rectangles, and circles. [C, CN, R, V]

Performance Indicators

- G03.01 Sort a given set of 2-D shapes and explain the sorting rule.
- G03.02 Identify common attributes of triangles, squares, rectangles, and circles from given sets of the same type of 2-D shapes.
- G03.03 Identify given 2-D shapes with different dimensions.
- G03.04 Identify given 2-D shapes with different positions.
- G03.05 Identify and name examples of triangles, squares, rectangles, and circles found in the environment.
- G03.06 Create a model to represent a given 2-D shape.
- G03.07 Create a pictorial representation of a given 2-D shape.

G04 Students will be expected to identify 2-D shapes as part of 3-D objects in the environment. [C, CN, R, V]

Performance Indicators

- G04.01 Compare and match a given 2-D shape, such as a triangle, square, rectangle, or circle, to the faces of 3-D objects in the environment.
- G04.02 Name the 2-D faces of a given 3-D object.

STATISTICS AND PROBABILITY (SP)

SP01 Students will be expected to gather and record data about self and others to answer questions. [C, CN, PS, V]

Performance Indicators

- SP01.01 Formulate a question that can be answered by gathering information about self and others.
- SP01.02 Organize data as it is collected using concrete objects, tallies, checkmarks, charts, or lists.
- SP01.03 Answer questions using collected data.

SP02 Students will be expected to construct and interpret concrete graphs and pictographs to solve problems. [C, CN, PS, R, V]

Performance Indicators

- SP02.01 Determine the common attributes of concrete graphs by comparing a given set of concrete graphs.
- SP02.02 Determine the common attributes of pictographs by comparing a given set of pictographs.
- SP02.03 Answer questions pertaining to a given concrete graph or pictograph.
- SP02.04 Create a concrete graph to display a given set of data and draw conclusions.
- SP02.05 Create a pictograph to represent a given set of data using one-to-one correspondence.
- SP02.06 Solve a given problem by constructing and interpreting a concrete graph or pictograph.

Mathematical Processes

There are critical components that students must encounter in a mathematics program in order to achieve the goals of mathematics education and encourage lifelong learning in mathematics.

Students are expected to

- communicate in order to learn and express their understanding of mathematics (Communication [C])
- develop and apply new mathematical knowledge through problem solving (Problem Solving [PS])
- connect mathematical ideas to other concepts in mathematics, to everyday experiences, and to other disciplines (Connections [CN])
- demonstrate fluency with mental mathematics and estimation (Mental Mathematics and Estimation [ME])
- select and use technologies as tools for learning and solving problems (Technology [T])
- develop visualization skills to assist in processing information, making connections, and solving problems (Visualization [V])
- develop mathematical reasoning (Reasoning [R])

The Nova Scotia curriculum incorporates these seven interrelated mathematical processes that are intended to permeate teaching and learning. The key to these process standards is presented in a box, as shown below, with each specific curriculum outcome within the strands.

Process Standards Key

[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

Communication [C]

Students need opportunities to read about, represent, view, write about, listen to, and discuss mathematical ideas. These opportunities allow students to create links between their own language and ideas, and the formal language and symbols of mathematics. Communication is important in clarifying, reinforcing, and modifying ideas, knowledge, attitudes, and beliefs about mathematics. Students should be encouraged to use a variety of forms of communication while learning mathematics.

Students also need to communicate their learning using mathematical terminology. Communication can help students make connections between and among the different representational modes—contextual, concrete, pictorial, linguistic/verbal, and symbolic—of mathematical ideas. Students must communicate *daily* about their mathematics learning. This enables them to reflect, to validate, and to clarify their thinking and provides teachers with insight into students' interpretations of mathematical meanings and ideas.

Problem Solving [PS]

Learning through problem solving should be the focus of mathematics at all grade levels. When students encounter new situations and respond to questions of the type, How would you ...? or How could you ...? the problem-solving approach is being modelled. Students develop their own problem-solving strategies by being open to listening, discussing, and trying different strategies.

In order for an activity to be problem-solving based, it must ask students to determine a way to get from what is known to what is sought. If students have already been given ways to solve the problem, it is not a problem, but practice. A true problem requires students to use prior learning in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement, perseverance, and collaboration.

Problem solving is also a powerful teaching tool that fosters multiple, creative, and innovative solutions. Creating an environment where students openly look for and engage in finding a variety of strategies for solving problems empowers students to explore alternatives and develops confident, cognitive, mathematical risk takers.

When students are exposed to a wide variety of problems in all areas of mathematics, they explore various methods for solving and verifying problems. In addition, they are challenged to find multiple solutions for problems and to create their own problem.

Connections [CN]

Contextualization and making connections to the experiences of learners are powerful processes in developing mathematical understanding. When mathematical ideas are connected to one another or to real-world phenomena, students can begin to view mathematics as useful, relevant, and integrated. Learning mathematics within contexts and making connections relevant to learners can validate past experiences and increase student willingness to participate and be actively engaged. The brain is constantly looking for and making connections.

“Because the learner is constantly searching for connections on many levels, educators need to orchestrate the experiences from which learners extract understanding. ... Brain research establishes and confirms that multiple complex and concrete experiences are essential for meaningful learning and teaching.” (Caine and Caine 1991, 5).

Mathematics should be viewed as an integrated whole rather than as the study of separate strands or units. Connections must also be made between and among the different representational modes—contextual, concrete, pictorial, linguistic/verbal, and symbolic. The process of making connections, in turn, facilitates learning. Concepts and skills should also be connected to everyday situations and other curricular areas. For example, when developing literacy skills students learn to make text-to-world, text-to-text, and text-to-self connections. Students can also make connections to make mathematics come alive through math-to-world, math-to-math, and math-to-self connections.

Mental Mathematics and Estimation [ME]

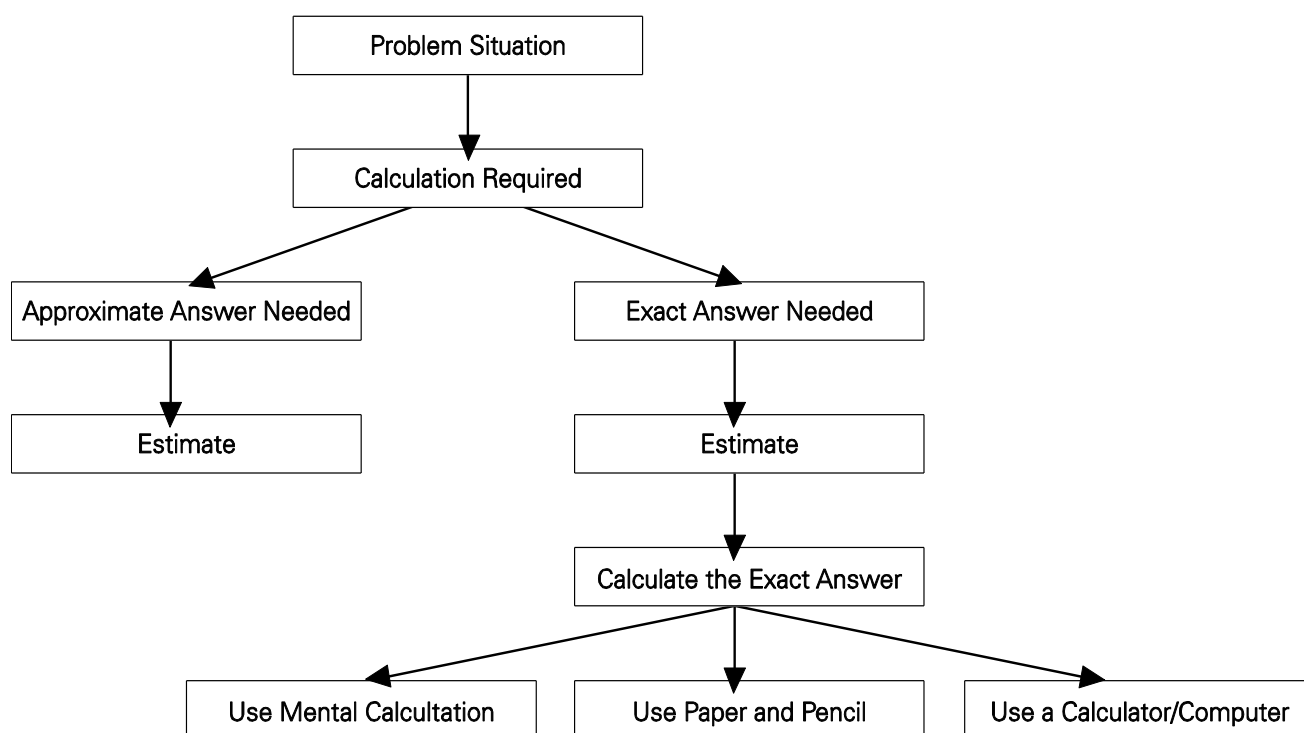
Mental mathematics is a combination of cognitive strategies that enhance flexible thinking and number sense. It is calculating mentally without the use of external aids. Mental mathematics enables students to determine answers without paper and pencil. It improves computational fluency by developing efficiency, accuracy, and flexibility. “Even more important than performing computational procedures or using calculators is the greater facility that students need—more than ever before—with estimation and mental math.” (National Council of Teachers of Mathematics, May 2005).

Students proficient with mental mathematics “become liberated from calculator dependence, build confidence in doing mathematics, become more flexible thinkers, and are more able to use multiple approaches to problem solving.” (Rubenstein 2001) Mental mathematics “provides a cornerstone for all

estimation processes, offering a variety of alternative algorithms and nonstandard techniques for finding answers.” (Hope 1988, v)

Estimation is a strategy for determining approximate values or quantities, usually by referring to benchmarks or using referents, or for determining the reasonableness of calculated values. Students need to know how, when, and what strategy to use when estimating. Estimation is used to make mathematical judgments and develop useful, efficient strategies for dealing with situations in daily life.

Students need to develop both mental mathematics and estimation skills through context and not in isolation so they are able to apply them to solve problems. Whenever a problem requires a calculation, students should follow the decision-making process as illustrated below.



The skill of estimation requires a sound knowledge of mental mathematics. Both are necessary to many everyday experiences, and students should be provided with frequent opportunities to practise these skills.

Technology [T]

Technology can be effectively used to contribute to and support the learning of a wide range of mathematical outcomes and enables students to explore and create patterns, examine relationships, test conjectures, and solve problems.

Technology can be used to

- explore and demonstrate mathematical relationships and patterns
- organize and display data
- extrapolate and interpolate
- assist with calculation procedures as part of solving problems
- decrease the time spent on computations when other mathematical learning is the focus
- reinforce the learning of basic facts and test properties

- develop personal procedures for mathematical operations
- create geometric displays
- simulate situations
- develop number sense

The use of calculators is recommended to enhance problem solving, to encourage discovery of number patterns, and to reinforce conceptual development and numerical relationships. They do not, however, replace the development of number concepts and skills. Carefully chosen computer software can provide interesting problem-solving situations and applications.

Technology contributes to a learning environment in which the growing curiosity of students can lead to rich mathematical discoveries at all grade levels. While technology can be used in grades primary to 3 to enrich learning, it is expected that students will achieve all outcomes without the use of technology.

Visualization [V]

Visualization “involves thinking in pictures and images, and the ability to perceive, transform and recreate different aspects of the visual-spatial world.” (Armstrong 1999). The use of visualization in the study of mathematics provides students with opportunities to understand mathematical concepts and make connections among them. Visual images and visual reasoning are important components of number, spatial, and measurement sense. Number visualization occurs when students create mental representations of numbers. These mental images are needed to develop concepts and understand procedures. Images and explanations help students clarify their understanding of mathematical ideas in all strands.

Being able to create, interpret, and describe a visual representation is part of spatial sense and spatial reasoning. Spatial visualization and reasoning enable students to describe the relationships among and between 3-D objects and 2-D shapes.

Measurement visualization goes beyond the acquisition of specific measurement skills. Measurement sense includes the ability to determine when to measure, when to estimate and to know several estimation strategies. (Shaw and Cliatt 1989)

Visualization is fostered through the use of concrete materials, technology, and a variety of visual representations.

Reasoning [R]

Mathematical reasoning helps students think logically and make sense of mathematics. Students need to develop confidence in their abilities to reason and justify their mathematical thinking. High-order questions challenge students to think and develop a sense of wonder about mathematics. Mathematical experiences in and out of the classroom provide opportunities for inductive and deductive reasoning. Inductive reasoning occurs when students explore and record results, analyze observations, make generalizations from patterns, and test these generalizations. Deductive reasoning occurs when students reach new conclusions based upon what is already known or assumed to be true.

Mathematics reasoning involves informal thinking, conjecturing, and validating—these help students understand that mathematics makes sense. Students are encouraged to justify, in a variety of ways, their solutions, thinking processes, and hypotheses. In fact, good reasoning is as important as finding correct answers.

Nature of Mathematics

Mathematics is one way of trying to understand, interpret, and describe our world. There are a number of components that define the nature of mathematics, and these are woven throughout this document. These components include change, constancy, number sense, relationships, patterns, spatial sense, and uncertainty.

Change

It is important for students to understand that mathematics is dynamic and not static. As a result, recognizing change is a key component in understanding and developing mathematics. Within mathematics, students encounter conditions of change and are required to search for explanations of that change. To make predictions, students need to describe and quantify their observations, look for patterns, and describe those quantities that remain fixed and those that change. For example, the sequence 4, 6, 8, 10, 12, ... can be described as

- skip counting by 2s, starting from 4
- an arithmetic sequence, with first term 4 and a common difference of 2
- a linear function with a discrete domain

(Steen 1990, 184).

Constancy

Different aspects of constancy are described by the terms **stability**, **conservation**, **equilibrium**, **steady state**, and **symmetry** (AAAS–Benchmarks 1993, 270). Many important properties in mathematics and science relate to properties that do not change when outside conditions change. Some problems in mathematics require students to focus on properties that remain constant. The recognition of constancy enables students to solve problems. Examples of constancy include the following:

- The area of a rectangular region is the same regardless of the methods used to determine the solution.
- The sum of the interior angles of any triangle is 180° .
- The theoretical probability of flipping a coin and getting heads is 0.5.

Number Sense

Number sense, which can be thought of as intuition about numbers, is the most important foundation of numeracy (British Columbia Ministry of Education 2000, 146). A true sense of number goes well beyond the skills of simply counting, memorizing facts, and the situational rote use of algorithms. Number sense develops when students connect numbers to real-life experiences and use benchmarks and referents. This results in students who are computationally fluent, flexible with numbers, and have intuition about numbers. The evolving number sense typically comes as a by-product of learning rather than through direct instruction. However, number sense can be developed by providing rich mathematical tasks that allow students to make connections.

Relationships

Mathematics is used to describe and explain relationships. As part of the study of mathematics, students look for relationships among numbers, sets, shapes, objects, and concepts. The search for possible relationships involves the collection and analysis of data, and describing relationships visually, symbolically, orally, or in written form.

Patterns

Mathematics is about recognizing, describing, and working with numerical and non-numerical patterns. Patterns exist in all strands, and it is important that connections are made among strands. Working with patterns enables students to make connections within and beyond mathematics. These skills contribute to students' interaction with an understanding of their environment. Patterns may be represented in concrete, visual, or symbolic form. Students should develop fluency in moving from one representation to another. Students must learn to recognize, extend, create, and use mathematical patterns. Patterns allow students to make predictions and justify their reasoning when solving problems. Learning to work with patterns in the early grades helps develop students' algebraic thinking, which is foundational for working with more abstract mathematics in higher grades.

Spatial Sense

Spatial sense involves visualization, mental imagery, and spatial reasoning. These skills are central to the understanding of mathematics. Spatial sense enables students to reason and interpret among and between 3-D and 2-D representations and identify relationships to mathematical strands. Spatial sense is developed through a variety of experiences and interactions within the environment. The development of spatial sense enables students to solve problems involving 3-D objects and 2-D shapes. Spatial sense offers a way to interpret and reflect on the physical environment and its 3-D or 2-D representations. Some problems involve attaching numerals and appropriate units (measurement) to dimensions of objects. Spatial sense allows students to make predictions about the results of changing these dimensions. For example,

- knowing the dimensions of an object enables students to communicate about the object and create representations
- the volume of a rectangular solid can be calculated from given dimensions
- doubling the length of the side of a square increases the area by a factor of four

Uncertainty

In mathematics, interpretations of data and the predictions made from data may lack certainty. Events and experiments generate statistical data that can be used to make predictions. It is important to recognize that these predictions (interpolations and extrapolations) are based upon patterns that have a degree of uncertainty. The quality of the interpretation is directly related to the quality of the data. An awareness of uncertainty allows students to assess the reliability of data and data interpretation. Chance addresses the predictability of the occurrence of an outcome. As students develop their understanding of probability, the language of mathematics becomes more specific and describes the degree of uncertainty more accurately.

Curriculum Document Format

This guide presents the mathematics curriculum so that a teacher may readily view the scope of the outcomes that students are expected to achieve during that year. Teachers are encouraged, however, to examine what comes before and what follows after, to better understand how students' learning at a particular grade level is part of a bigger picture of concept and skill development.

The order of presentation in no way assumes or prescribes a preferred order of presentation in the classroom, but simply lays out the specific curriculum outcomes in relation to the overarching general curriculum outcomes (GCOs).

The footer of the document shows the name of the course, and the strand name is presented in the header. When a specific curriculum outcome (SCO) is introduced, it is followed by the mathematical processes and performance indicators for that outcome. A scope and sequence is then provided, which relates the SCO to previous and next grade SCOs. Also for each SCO, there is background information, assessment strategies, suggested instructional strategies, suggested models and manipulatives, mathematical language, and a section for resources and notes. For each section, the guiding questions should be used to help with unit and lesson preparation.

SCO**Mathematical Processes**

[C] Communication [PS] Problem Solving [CN] Connections
 [ME] Mental Mathematics and Estimation
 [T] Technology [V] Visualization [R] Reasoning

Performance Indicators

Describes observable indicators of whether students have achieved the specific outcome.

Scope and Sequence

Previous grade or course SCOs	Current grade SCO	Following grade or course SCOs

Background

Describes the “big ideas” to be learned and how they relate to work in previous grade and work in subsequent courses.

Additional Information

A reference to Appendix A, which contains further elaborations for the performance indicators.

Assessment, Teaching, and Learning**Assessment Strategies****Guiding Questions**

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Sample tasks that can be used to determine students’ prior knowledge.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Some suggestions for specific activities and questions that can be used for both instruction and assessment

FOLLOW-UP ON ASSESSMENT**Guiding Questions**

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Correlations to related resources.

Planning for Instruction**Guiding Questions**

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcome and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Suggested strategies for planning daily lessons.

SUGGESTED LEARNING TASKS

Suggestions for general approaches and strategies suggested for teaching this outcome.

Guiding Questions

- How can the scope and sequence be used to determine what prior knowledge needs to be activated prior to beginning new instruction?

SUGGESTED MODELS AND MANIPULATIVES**MATHEMATICAL LANGUAGE**

Teacher and student mathematical language associated with the respective outcome.

Resources/Notes

Contexts for Learning and Teaching

Beliefs about Students and Mathematics Learning

“Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge.” (National Council of Teachers of Mathematics 2000, 20).

The Nova Scotia mathematics curriculum is based upon several key assumptions or beliefs about mathematics learning that have grown out of research and practice. These beliefs include the following:

- Mathematics learning is an active and constructive process.
- Learning is most effective when standards of expectation are made clear with ongoing assessment and feedback.
- Learners are individuals who bring a wide range of prior knowledge and experiences and who learn via various styles and at different rates.
- Learning is most likely to occur when placed in meaningful contexts and in an environment that supports exploration, risk taking, and critical thinking and that nurtures positive attitudes and sustained effort.

Students are curious, active learners with individual interests, abilities, and needs. They come to classrooms with varying knowledge, life experiences, and backgrounds. A key component in successfully developing numeracy is making connections to these backgrounds and experiences.

Students develop a variety of mathematical ideas before they enter school. Children make sense of their environment through observations and interactions at home and in the community. Mathematics learning is embedded in everyday activities, such as playing, reading, storytelling, and helping around the home. Such activities can contribute to the development of number and spatial sense in children. Curiosity about mathematics is fostered when children are engaged in activities such as comparing quantities, searching for patterns, sorting objects, ordering objects, creating designs, building with blocks, and talking about these activities. Positive early experiences in mathematics are as critical to child development as are early literacy experiences.

Students learn by attaching meaning to what they do and need to construct their own meaning of mathematics. This meaning is best constructed when learners encounter mathematical experiences that proceed from the simple to the complex and from the concrete to the abstract. The use of models and a variety of pedagogical approaches can address the diversity of learning styles and developmental stages of students and enhance the formation of sound, transferable, mathematical concepts. At all levels, students benefit from working with and translating through a variety of materials, tools, and contexts when constructing meaning about new mathematical ideas. Meaningful discussions can provide essential links among concrete, pictorial, contextual, and symbolic representations of mathematics.

The learning environment should value and respect all students’ experiences and ways of thinking, so that learners are comfortable taking intellectual risks, asking questions, and posing conjectures. Students need to explore problem-solving situations in order to develop personal strategies and become mathematically literate. Learners must realize that it is acceptable to solve problems in different ways and that solutions may vary.

Goals for Mathematics Education

The main goals of mathematics education are to prepare students to

- use mathematics confidently to solve problems
- communicate and reason mathematically
- appreciate and value mathematics
- make connections between mathematics and its applications
- become mathematically literate adults, using mathematics to contribute to society

Students who have met these goals will

- gain understanding and appreciation of the contributions of mathematics as a science, a philosophy, and an art
- exhibit a positive attitude toward mathematics
- engage and persevere in mathematical tasks and projects
- contribute to mathematical discussions
- take risks in performing mathematical tasks
- exhibit curiosity about mathematics and situations involving mathematics

Opportunities for Success

A positive attitude has a profound effect on learning. Environments that create a sense of belonging, encourage risk taking, and provide opportunities for student success help develop and maintain positive attitudes and self-confidence. Students with positive attitudes toward learning mathematics are likely to be motivated and prepared to learn, participate willingly in classroom activities, persist in challenging situations, and engage in reflective practices.

To experience success, students must be taught to set achievable goals or assess their progress as they work toward these goals. Striving toward success and becoming autonomous and responsible learners are ongoing, reflective processes that involve revisiting the setting and assessing of personal goals.

Engaging All Learners

“No matter how engagement is defined or which dimension is considered, research confirms this truism of education: *The more engaged you are, the more you will learn.*” (Hume 2011, 6)

Student engagement is at the core of learning. This is critical for teachers to take into account when planning and implementing instruction. Effective instruction engages, embraces, and supports all learners through a range of learning experiences. This curriculum is designed to provide learning opportunities that reflect culturally proficient instructional and assessment practices and are equitable, accessible, and inclusive of the multiple facets of diversity represented in today’s classrooms.

Engagement in learning occurs when students are provided with opportunities to become more invested in their learning. When teachers know their students as individual learners and as individual people, their students are more likely to be motivated to learn, participate in classroom activities, persist in challenging situations, and engage in reflective practices. Students often become more engaged when teachers demonstrate a genuine belief in each student’s potential to learn.

SUPPORTIVE LEARNING ENVIRONMENTS

A supportive and positive learning environment has a profound effect on students' learning. In classrooms where students feel a sense of belonging, are encouraged to actively participate, are challenged without being frustrated, and feel safe and supported to take risks with their learning, students are more likely to experience success. It is realized that not all students will progress at the same pace or be equally positioned in terms of their prior knowledge of and skill with particular concepts and outcomes. Teachers provide all students with equitable access to learning by integrating a variety of instructional approaches and assessment activities that consider all learners and align with the following key principles:

- Instruction must be flexible and offer multiple means of representation.
- Students must have opportunities to express their knowledge and understanding in multiple ways.
- Teachers must provide options for students to engage in learning through multiple ways.

Teachers who know their students well become aware of individual learning differences and infuse this understanding into planned instructional and assessment decisions. They organize learning experiences to accommodate the many ways in which students learn, create meaning, and demonstrate their knowledge and understanding. Teachers use a variety of effective teaching approaches that may include

- providing all students with equitable access to appropriate learning strategies, resources, and technology
- offering a range of ways students can access their prior knowledge to connect with new concepts
- scaffolding instruction and assignments so that individual or groups of students are supported as needed throughout the process of learning
- verbalizing their thinking to model comprehension strategies and new learning
- balancing individual, small-group, and whole-class approaches to learning activities
- involving students in the co-creation of criteria for assessment and evaluation
- providing students with choice in how they demonstrate their understanding according to learning styles and preferences, building on individual strengths, and including a range of difficulty and challenge
- providing frequent and meaningful feedback to students throughout their learning experiences

LEARNING STYLES AND PREFERENCES

The ways in which students make sense of, receive, and process information, demonstrate learning, and interact with peers and their environment both indicate and shape learning preferences, which may vary widely from student to student. Learning preferences are influenced also by the learning context and purpose and by the type and form of information presented or requested. Most students tend to favour one learning style and may have greater success if instruction is designed to provide for multiple learning styles, thus creating more opportunities for all students to access learning. The three most commonly referenced learning styles are

- auditory (such as listening to teacher-presented lessons or discussing with peers)
- kinesthetic (such as using manipulatives or recording print or graphic/visual text)
- visual (such as interpreting information with text and graphics or viewing videos)

While students can be expected to work using all modalities, it is recognized that one or some of these modalities may be more natural to individual students than the others.

A GENDER-INCLUSIVE CURRICULUM

It is important that the curriculum respects the experiences and values of all students and that learning resources and instructional practices are not gender-biased. Teachers promote gender equity and inclusion in their classrooms when they

- articulate equally high expectations for all students
- provide equal opportunity for input and response from all students
- model gender-fair language and respectful listening in their interactions with students

VALUING DIVERSITY: TEACHING WITH CULTURAL PROFICIENCY

Teachers understand that students represent diverse life and cultural experiences, with individual students bringing different prior knowledge to their learning. Therefore, teachers build upon their knowledge of their students as individuals and respond by using a variety of culturally-proficient instruction and assessment strategies. “Instruction that is embedded in socially meaningful contexts, and tasks that are meaningful and relevant to the lives of students, will engage students in high-level problem-solving and reasoning and enhance students’ engagement (Frankenstein 1995; Gutstein 2003; Ladson-Billings 1997; Tate 1995).” (Herzig 2005)

STUDENTS WITH LANGUAGE, COMMUNICATION, AND LEARNING CHALLENGES

Today’s classrooms include students who have diverse backgrounds, abilities, levels of development, and learning challenges. By observing and interacting with students as they work on assigned activities, teachers can identify areas where students may need additional support to achieve their learning goals. Teachers can then respond with a range of effective instructional strategies. Students who have English as an Additional Language (EAL) may require curriculum outcomes at different levels, or temporary individualized outcomes, particularly in language-based subject areas, while they become more proficient in their English language skills. For students who are experiencing difficulties, it is important that teachers distinguish between students for whom curriculum content is challenging and students for whom language-based issues are at the root of apparent academic difficulties.

STUDENTS WHO DEMONSTRATE GIFTED AND TALENTED BEHAVIOURS

Some students are academically gifted and talented with specific skill sets or in specific subject areas. Most students who are gifted and talented thrive when challenged by problem-centred, inquiry-based learning and open-ended activities. Teachers may challenge students who are gifted and talented by adjusting the breadth, the depth, and/or the pace of instruction. Learning experiences may be enriched by providing greater choice among activities and offering a range of resources that require increased cognitive demand and higher-level thinking at different levels of complexity and abstraction. For additional information, refer to *Gifted Education and Talent Development* (Nova Scotia Department of Education 2010).

Connections across the Curriculum

The teacher should take advantage of the various opportunities available to integrate mathematics and other subjects. This integration not only serves to show students how mathematics is used in daily life, but it helps strengthen the students’ understanding of mathematical concepts and provides them with opportunities to practise mathematical skills. There are many possibilities for integrating mathematics in health education, literacy, music, physical education, science, social studies, and visual arts.

Number

GCO: Students will be expected to demonstrate number sense.

Specific Curriculum Outcomes

Process Standards

[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

- N01** Students will be expected to say the number sequence by
- 1s, forward and backward, starting from any point to 200
 - 2s, forward and backward, starting from any point to 100
 - 5s and 10s, forward and backward, using starting points that are multiples of 5 and 10 respectively to 100
 - 10s, starting from any point, to 100 [C, CN, ME, R]
- N02** Students will be expected to demonstrate if a number (up to 100) is even or odd. [C, CN, PS, R]
- N03** Students will be expected to describe order or relative position using ordinal numbers (up to tenth). [C, CN, R]
- N04** Students will be expected to represent and partition numbers to 100. [C, CN, V]
- N05** Students will be expected to compare and order numbers up to 100. [C, CN, R, V]
- N06** Students will be expected to estimate quantities to 100 by using referents. [C, ME, PS, R]
- N07** Students will be expected to illustrate, concretely and pictorially, the meaning of place value for numerals to 100. [C, CN, R, V]
- N08** Students will be expected to demonstrate and explain the effect of adding zero to or subtracting zero from any number. [C, R]
- N09** Students will be expected to demonstrate an understanding of addition (limited to 1- and 2-digit numerals) with answers to 100 and the corresponding subtraction by
- using personal strategies for adding and subtracting with and without the support of manipulatives
 - creating and solving problems that involve addition and subtraction
 - explaining and demonstrating that the order in which numbers are added does not affect the sum
 - explaining and demonstrating that the order in which numbers are subtracted matters when finding a difference [C, CN, ME, PS, R, V]
- N10** Students will be expected to apply mental mathematics strategies to quickly recall basic addition facts to 18 and determine related subtraction facts. [C, CN, ME, R, V]

SCO N01 Students will be expected to say the number sequence by

- 1s, forward and backward, starting from any point to 200
- 2s, forward and backward, starting from any point to 100
- 5s and 10s, forward and backward, using starting points that are multiples of 5 and 10 respectively to 100
- 10s, starting from any point, to 100

[C, CN, ME, R]

[C] Communication [PS] Problem Solving [CN] Connections [ME] Mental Mathematics and Estimation
[T] Technology [V] Visualization [R] Reasoning

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- N01.01** Extend counting sequence (by 1s), forward and backward.
N01.02 Extend a given skip counting sequence (by 2s, 5s, or 10s) forward and backward.
N01.03 Skip count by 10s, given any number as a starting point.
N01.04 Identify and correct errors and omissions in a given skip counting sequence.
N01.05 Count a given sum of money with pennies, nickels, or dimes (to 100¢).
N01.06 Count quantity using groups of 2s, 5s, or 10s and counting on.

Scope and Sequence

Mathematics 1	Mathematics 2	Mathematics 3
<p>N01 Students will be expected to say the number sequence by</p> <ul style="list-style-type: none"> ▪ 1s, forward and backward between any two given numbers, 0 to 100 ▪ 2s to 20, forward starting at 0 ▪ 5s to 100, forward starting at 0, using a hundred chart or a number line ▪ 10s to 100, forward starting at 0, using a hundred chart or a number line 	<p>N01 Students will be expected to say the number sequence by</p> <ul style="list-style-type: none"> ▪ 1s, forward and backward, starting from any point to 200 ▪ 2s, forward and backward, starting from any point to 100 ▪ 5s and 10s, forward and backward, using starting points that are multiples of 5 and 10 respectively to 100 ▪ 10s, starting from any point, to 100. 	<p>N01 Students will be expected to say the number sequence forward and backward by</p> <ul style="list-style-type: none"> ▪ 1s through transitions to 1000 ▪ 2s, 5s, 10s, or 100s, using any starting point to 1000 ▪ 3s, using starting points that are multiples of 3 up to 100 ▪ 4s, using starting points that are multiples of 4 up to 100 ▪ 25s, using starting points that are multiples of 25 up to 200

Background

In Mathematics 2, students are continuing to develop an understanding of number and counting. In previous grades, students have had experience counting by 1s, forward and backward, between any two given numbers (0 to 100); 2s to 20, forward starting at 0; and 5s and 10s to 100, forward starting at 0. They also have demonstrated an understanding of counting by using the counting-on strategy and skip counting to count sets. Students will extend their experience with skip-counting patterns to 100 and will extend their knowledge of the number sequence by 1s to 200.

Additional Information

See Appendix A: Additional Information.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

Ask students to count by

- 1s, forward and backward, between any two given numbers (0 to 100)
- 2s, to 20, forward starting at 0
- 5s and 10s to 100, forward starting at 0, using a hundred chart or number line

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students to start at
 - 92 and count forward by 1s (stop at 121)
 - 42 and count forward by 2s (stop at 60)
 - 13 and count by 2s (stop at 35)
 - 78 and count backward by 2s (stop at 58)
 - 30 and count by 10s (stop at 100)
 - 8 and count by 10s (stop at 58)
 - 100 and count backward by 10s (stop at 40)
 - 15 and count by 5s (stop at 60)
 - 85 and count backward by 5s (stop at 55)
- Ask students to count a collection of counters. Ask them to find the total number of counters and to explain their method of counting (e.g., by 2s, 5s, 10s).
- Ask students to count by 2s (5s or 10s) as you clap. Ask students to record the final number when you finish clapping.
- Show students a counting sequence with an error or missing number. Ask students to identify and correct the error.
- Provide students with a collection of pennies, nickels, or dimes and ask them to skip count to find the total.
- Provide students with a collection of different coins (pennies, nickels, and dimes) and ask them to count the coins to find the total.
- Tell students that you have 35 cents in your “piggy bank.” Ask them to skip count to find the total as you add nickels or dimes to your piggy bank.

- Have students listen as you count, 5, 10, 15, 16, 17. Ask, What coins am I counting? Repeat for different coins.
- Tell students that you have some coins in your hand that total 44 cents. Ask them to record possible combinations of coins.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets K–2 (Bauman 2011)

- Grade 2 Checkpoint 1, Task 4 (counting forward by 2s only), pp. 25–26
- Grade 2 Checkpoint 3 (counting forward by 1s only), pp. 42–43, (Line Master 3.1)

Numeracy Nets 3 (Bauman 2009)

- Grade 3 Checkpoint 2, pp. 23–24

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Use daily routines to reinforce number sense to 100.
- Use literature that involves skip counting.
- Use the repeat (constant) function (press 0, +, 2, =, =, =, ...) on a calculator to skip count to a target number. For example, if you start at 0 and want to end at 40, by which number(s) could you skip count (2, 5, 10)? What if you started at a different point? What if you wanted to end at a different point?
- Use a variety of problems including open-ended (more than one solution) questions. For example, tell students that you have some coins that total 61 cents. Ask students to identify possible combinations of coins that would result in that total.

- Provide students with a variety of authentic tasks that require them to recognize when it is more efficient to skip count.
- Use number patterns, such as 65, $_$, 55, $_$, 45, $_$, ... or 1, 3, $_$, 7, $_$, $_$, 13, ... to encourage skip counting.

SUGGESTED LEARNING TASKS

- **Orchestra counting:** Put students into groups. The first group begins counting forward by 1s (or 2s, 5s, or 10s) from a given number. When the teacher points to another group, they continue the count.
- Use a horizontal number line, hundred chart, or a large floor chart with numbers 1 to 100. Ask the class to skip count by 2s (or 5s or 10s), and ask a student volunteer to step on each number as it is said. Discuss the movements made by the student volunteer.
- Provide students with a number of counters (60, for example). Ask students to separate the counters from the pile as they count them by groups. Have them discuss which method is the most efficient way of counting the counters.
- Ask students to reach into a bag and take a handful of counters. Then, ask them to spin a spinner labelled “by 2s, by 5s, by 10s.” Students group their counters by the number shown on the spinner and then count them to find the total.
- Have students create visuals to represent counting by 2s, 5s, and 10s. This could be done using a digital camera, drawings (by hand or computer), or prints. For example, skip counting by 5s could be represented by digital pictures of each student’s hand with the fingers spread apart, or counting by 2s could be represented by digital pictures of each student’s eyes, etc.
- Use ten-frames to model skip counting by 10s. Begin with a ten-frame representing a single-digit number, such as 3. Have students count by 10s, and add a full ten-frame to the display as they say the next number.
- Ask students to correct the following “counting by 2s” sequence: ..., 82, 84, 86, 87, 88, 90, 92, ...
- Ask students to work in pairs to create a number sequence with a missing number and then exchange their sequence with another pair of students. Each pair identifies the missing number in the sequence they received.
- Invite students to use calculators to count using the repeat (constant) function. For example, as a student places counters or coins in a bag and skip counts aloud, others may repeatedly add one on calculators to keep track of the count electronically. The calculator repeat function may also be used when skip counting by 2s, 5s, or 10s. Calculators may also be used with starting points other than 0 to model counting on.
- Give students 3 dimes, 2 nickels, and 6 pennies. Ask them to count the coins.
- Provide coins for the students. Ask, Can you use 7 of these coins to make 43 cents? Have students share their strategies for solving the problem.

SUGGESTED MODELS AND MANIPULATIVES

- calculators
- coins
- counters
- hundred chart
- number line
- ten-frames

MATHEMATICAL LANGUAGE

Teacher	Student
<ul style="list-style-type: none"> ▪ coins: penny, nickel, dime ▪ forward, backward ▪ hundred chart, number line, calculator ▪ numbers: zero to two hundred ▪ position words: next, before, after, between ▪ sequence ▪ skip count 	<ul style="list-style-type: none"> ▪ coins: penny, nickel, dime ▪ forward, backward ▪ hundred chart, number line, calculator ▪ numbers: zero to two hundred ▪ position words: next, before, after, between ▪ skip count

Resources/Notes

Print

- *Making Math Meaningful to Canadian Students, K–8* (Small 2009), pp. 86–87, 138–140
- *Making Math Meaningful to Canadian Students, K–8, Second Edition* (Small 2013), pp. 139, 142
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 39–41, 56–58, 123–127, 134–138, 150–152

Videos

- *An Introduction to Using Number Lines* (13:04 min.) (ORIGO Education 2010)
- *Analyzing Patterns (Skip Counting) on a Hundred Board* (27:16 min.) (ORIGO Education 2010)
- *Teaching Number: 0 to 9* (14:47 min.) (ORIGO Education 2010)
- *Teaching Number: Counting* (10:49 min.) (ORIGO Education 2010)

Notes

SCO N02 Students will be expected to demonstrate if a number (up to 100) is even or odd. [C, CN, PS, R]			
[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- N02.01** Use concrete materials or pictorial representations to determine if a given number is even or odd.
- N02.02** Identify even and odd numbers in a given sequence, such as on a hundred chart.
- N02.03** Sort a given set of numbers as even numbers and odd numbers.

Scope and Sequence

Mathematics 1	Mathematics 2	Mathematics 3
—	N02 Students will be expected to demonstrate if a number (up to 100) is even or odd.	—

Background

Mathematicians use the term **even** to describe a quantity that can be shared equally into groups of two with no leftovers. The terms **odd** and **even** need to be carefully developed in the context of classifying numbers because they are also used in real-life situations with different meanings. Outside of mathematics, odd may mean strange, extraordinary, or unusual. Students may also have heard the term **even** used in relation to a scale being balanced. They may have also heard the term **evenly** being used in sharing situations (e.g., if 12 candies are shared evenly with 3 people, then they all have the same number of candies). It is important that the vocabulary be explored with students so that they have a clear understanding of the mathematical definitions.

Additional Information

See Appendix A: Additional Information.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

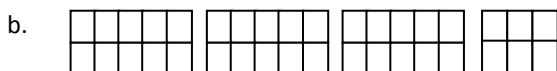
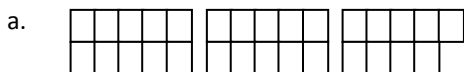
Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my Assessment Strategies with my teaching strategies?

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students to tell you if the following representations are even or odd and explain why.



- Ask students to sort the following numbers into even and odd by using linking cubes or ten-frames (or other materials): 11, 23, 30, 39, 40, 48.
- Ask students to use a hundred chart to explain if the following numbers are even or odd: 3, 18, 37, 55, 71.
- Ask students to fill in the missing numbers on a hundred chart. Ask, Are they even or odd? How do you know?
- Provide students with a hundred chart. Ask them to describe the odd and even patterns they see on the chart.
- Ask students to select any two numbers between 10 and 100. Ask them to tell whether each of the numbers they have selected is even or odd and to explain their thinking.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets K–2 (Bauman 2011)

- No Checkpoint for this outcome.

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

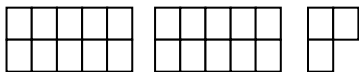
- Use concrete models to represent numbers to demonstrate their evenness or oddness.



7 (groups of 2 with one left over)

7 (2 equal groups with one left over)

- Use hundred charts and number lines to show even and odd number patterns.
- Use children's literature, such as *Even Stephen, Odd Todd* by K. Cristaldi (1996), to introduce the terms **odd** and **even** as they relate to number.
- Include discussions as part of the daily routine that relate to even and odd numbers (e.g., Do we have an even number of people here today?).
- Clarify misconceptions regarding numbers with both an even and odd digit (e.g., 23) by representing the number with ten-frames or square tiles grid pictures arranged as shown below.



SUGGESTED LEARNING TASKS

- Ask students to use sets of coloured tiles to determine whether a given number of tiles is even or odd.
- Show a set of counters on the overhead. Ask students to use ten-frames to determine whether the set of counters shown is odd or even by placing counters on their ten-frames. Ask students to explain their thinking.
- Work as a class to represent numbers with counters to determine their evenness or oddness. Then, ask students to colour the corresponding numeral on the hundred chart to begin creating a pattern (red for even, blue for odd). Ask students to continue discovering the even/odd pattern on the hundred chart.
- Ask students to sort the following numbers into even and odd: 9, 24, 30, 51, 67, 78, 86, and to explain their thinking.
- Ask students to explore the evenness or oddness of multiples of 10. Ask, Is 50 an even or an odd number? Explain how you know.
- Ask students to solve problems, such as Dave's class has 23 students. He is planning snacks for snack time and wants to know if he will need an even or odd number of snacks if he gives each student 1 snack? 2 snacks? 3 snacks?

SUGGESTED MODELS AND MANIPULATIVES

- colour tiles
- dot cards
- hundred chart
- linking cubes
- ten-frames

MATHEMATICAL LANGUAGE

Teacher	Student
<ul style="list-style-type: none"> ▪ even, odd numbers ▪ groups of two, two equal groups ▪ hundred chart, number line ▪ numerals ▪ ones digit, tens digit ▪ quantity ▪ shared, left over 	<ul style="list-style-type: none"> ▪ even, odd numbers ▪ groups of two, two equal groups ▪ hundred chart, number line ▪ numerals ▪ quantity ▪ shared, left over

Resources/Notes**Print**

- *Making Math Meaningful to Canadian Students, K–8* (Small 2009), p. 149
- *Making Math Meaningful to Canadian Students, K–8, Second Edition* (Small 2013), p. 204
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 291–293

Notes

SCO N03 Students will be expected to describe order or relative position using ordinal numbers (up to tenth).

[C, CN, R]

[C] Communication [PS] Problem Solving [CN] Connections [ME] Mental Mathematics and Estimation
[T] Technology [V] Visualization [R] Reasoning

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N03.01 Indicate a position of a specific object in a sequence by using ordinal numbers up to tenth.

N03.02 Compare the ordinal position of a specific object in two different given sequences.

Scope and Sequence

Mathematics 1	Mathematics 2	Mathematics 3
—	N03 Students will be expected to describe order or relative position using ordinal numbers (up to tenth).	PR01 Students will be expected to demonstrate an understanding of increasing patterns by describing, extending, comparing, and creating numerical (numbers to 1000) patterns and non-numerical patterns using manipulatives, diagrams, sounds, and actions.

Background

Ordinal numbers are used to describe the relative position of an object or event that have been ordered or ranked based on a criterion, such as size, importance, or chronology. Students are likely familiar with some ordinal numbers through everyday experiences, such as the second Tuesday, the third activity, and the first in line.

Students should discover important ideas about ordinal numbers including

- position matters — the ordinal number used to describe an object depends on the position of the object in relation to what has been deemed as being first
- For every cardinal number, there is an ordinal number. For example, counting a set of 9 objects, the last object touched will be the ninth object touched.
- There are two ways to record an ordinal number, in words and in symbolic form, such as third and 3rd.
- The position of first is not always fixed. It depends on the point of view. For example, the circle below can be described as the first from the left and the fifth from the right.



Additional Information

See Appendix A: Additional Information.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

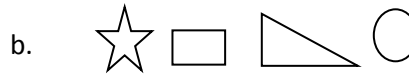
Tasks such as the following could be used to determine students' prior knowledge.

- Provide students with a triangle, a square, and a circle from the attribute blocks. Ask students to arrange them so that a triangle is first in line, a circle is second, and a square is last.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students to name the position of the star in each row of shapes starting from the right. Then, ask them to name the position of the star in each row of shapes starting from the left.



- Ask students to make a “train” of 10 linking cubes in which the 3rd and 7th cars from the front of the train are different colours from the rest of the train. Ask, What position are these cars if you are standing at the other end of the train?
- Provide students with a calendar. Ask them to tell you the date of the third Thursday of the month.
- Provide students with a selection of attribute blocks and directions for ordering them in a sequence. For example, The first shape in the row is a triangle. The third shape from the right is a square. The fifth shape is a rectangle. The second shape is a circle. The fourth shape is a circle.
- Ask students to create a sequence of shapes using attribute blocks or pattern blocks and to record their sequence using pictures. Then, ask them to write a description of their sequence so that another student could recreate that sequence. Have students exchange descriptions with one another and attempt to use the written description he or she received to recreate the sequence.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets K–2 (Bauman 2011)

- No Checkpoint for this outcome.

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Use calendars to provide a useful context for ordinal numbers.
- Have students observe position in line-ups. Who is third? If there are 6 people in front of you, where are you in the line? **Note:** Students who are seated can see these relationships more clearly than students in the line.
- Use literature, such as *Ten Little Rubber Ducks* by Eric Carle (2005) to explore ordinal numbers. Discuss the use of ordinal numbers in the literature.

SUGGESTED LEARNING TASKS

- Ask a child to place a set of objects in a line by following directions given orally or written on cards.
- Ask for ten student volunteers. Give each volunteer a card with an ordinal number (1st to 10th) on it. Ask students to place themselves in order from 1st to 10th.
- Ask students to create a sequence of shapes using attribute blocks or pattern blocks and to record the sequence using pictures. Then, ask them to write a description of their sequence so that another student could recreate that sequence. Have students exchange descriptions with one another and attempt to use the written description he or she received to recreate the sequence.
- Provide students with a set of pictures of sequences of objects and a set of ordinal descriptions of those same sequences. Ask students to match the pictures with the correct descriptions.
- Ask students to use counters to create a pattern in which the number of counters in the 4th position is fewer than the number of counters in the 3rd position.
- After reading literature that involves ordinal numbers, invite students to write and illustrate their own ordinal stories.
- Ask the student to use pattern blocks to make a row of five different shapes in which the first shape is a triangle and the third is a square. Ask, What would you have to do to make the square the fourth shape?

SUGGESTED MODELS AND MANIPULATIVES

- attribute blocks
- calendar
- pattern blocks

MATHEMATICAL LANGUAGE

Teacher	Student
<ul style="list-style-type: none"> ▪ objects and events ▪ ordinal numbers: first to tenth ▪ position ▪ sequence 	<ul style="list-style-type: none"> ▪ objects and events ▪ first to tenth ▪ position

Resources/Notes**Print**

- *Making Math Meaningful to Canadian Students, K–8* (Small 2009), pp. 89, 95
- *Making Math Meaningful to Canadian Students, K–8, Second Edition* (Small 2013), pp. 145, 152

Videos

- *Teaching Number: Relative Position* (20:41 min.) (ORIGO Education 2010)

Notes

SCO N04 Students will be expected to represent and partition numbers to 100. [C, CN, V]			
[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- N04.01** Represent a given number using concrete materials, such as ten-frames and base-ten materials.
- N04.02** Represent a given number using coins (pennies, nickels, dimes, and quarters).
- N04.03** Represent a given number using tallies.
- N04.04** Represent a given number pictorially.
- N04.05** Find examples of a given number in the environment.
- N04.06** Represent a given number using expressions (e.g., $24 + 6$, $15 + 15$, $40 - 10$)
- N04.07** Read a number (0–100) given in symbolic or word form.
- N04.08** Record in words a given number (0–20).
- N04.09** Record, symbolically, any number (0–100).

Scope and Sequence

Mathematics 1	Mathematics 2	Mathematics 3
N04 Students will be expected to represent and partition numbers to 20.	N04 Students will be expected to represent and partition numbers to 100.	N02 Students will be expected to represent and partition numbers to 1000.

Background

Numbers, like all mathematical concepts, can be represented in five ways—contextually, concretely, pictorially, symbolically, and verbally. Students need numerous experiences representing numbers to 100 in all of these ways, and translating between and among them.

In addition, to develop flexibility in working with number when performing operations, it is very important for students to understand that numbers can be broken down into parts, in many different ways. For example, 75 can be partitioned into $70 + 5$, $50 + 25$, or $60 + 12 + 3$. It is important that students understand that these are three ways of writing 75 and not just three expressions that have an answer of 75.

While there are nine indicators, classroom tasks should provide opportunities to observe many or all of the indicators simultaneously. For example, a specific number such as 75 can be represented with pictures, coins, base-ten materials, tallies, ten-frames, words, parts, and contexts. According to John Van de Walle (2006), to conceptualize a number as being made up of two or more parts is the most important understanding that can be developed about number relationships.

In previous grades, students have had experiences with representing and describing numbers to 20 concretely, pictorially, and symbolically; reading number words to 20; determining compatible number pairs for 5, 10, and 20; and placing numerals on a number line with benchmarks 0, 5, 10, 15, and 20. Students have had no formal instruction in place value. This will be introduced later in this grade with outcome N07. In Mathematics 1, students demonstrated with concrete materials and with pictures, how

a number, up to 20, could be partitioned into two or more smaller groups. For example, they may have taken a number like 16 and grouped it into three groups of five and one single using five-frames, or one group of 10 using a ten-frame with six singles, which sets the stage for learning place value.

Additional Information

See Appendix A: Additional Information.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Ask children to show the number 15 in as many different ways as they can using manipulatives, words, pictures, and symbols.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students to represent 52 (or any 2-digit number) with
 - ten-frames
 - tallies
 - coins
 - base-ten blocks
 - a picture
 - an expression
- Ask students to choose a 2-digit number. Ask them to represent their chosen 2-digit numbers with base-ten blocks in as many ways as they can.
- Ask students, Which of the expressions in the box represent 36?

$30 + 6$	$28 + 8$
$3 + 6$	$66 - 30$
$40 - 4$	$20 + 26$
$35 + 2$	

- Ask students to create two more expressions that would also equal 36. (This activity could initially be assessed using models, but students may be able to complete it symbolically after working on SCO N09.)
- Give students two representations of the same number, point to one and ask, Is this number more, less, or are they the same? Have students explain their thinking.



- Ask students to represent the number of students in their class as many different ways as they can.
- Ask students to use words to describe a number (e.g., thirty-six is four less than forty) using comparative language.
- Ask students to read a given number presented in symbolic form (e.g., 47) or in word form (e.g., forty-seven).
- Ask students to identify where they might see the numeral 26 in their neighbourhood.
- Say a number or show a number of base-ten blocks or ten-frames and ask students to record that number in symbolic form.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets 3 (Bauman 2009)

- Grade 3 Checkpoint 3, pp. 26–27

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Use a variety of representations to explore numbers. Different representations of the same number can be compared for equivalency.
- Have available a class or individual chart / word wall that matches pictorial, and/or symbolic representations with the words to support students as they learn number words.

SUGGESTED LEARNING TASKS

- Give each student a different number (e.g., 25, 36, 42, 48, ...) of counters. Ask students to arrange their counters in groups to make it easier for a classmate to count them; for example, 25 might be grouped as 5 groups of 5 or as 2 groups of 10 and 1 group of 5. Have students rotate around the room determining the number of counters each classmate has displayed.
- Ask students to represent a given 2-digit number, for example 53, using ten-frames. Ask them to make as many statements about the number as possible. For example, 53 is 3 more than 50; 53 is 7 less than 60; 53 is the same as 50 and 3 more; 53 is 2 less than 55; 53 is 13 more than 40. This may also be done using a hundred chart.
- Begin with the first day of school in September and do “Number of the Day” tasks. Have students express a given number in as many ways as they can. For example, Day 26, probably sometime in October, may be expressed as $5 + 5 + 5 + 5 + 5 + 1$; $20 + 6$; $10 + 10 + 6$; $10 + 16$; 26 ones; 2 tens and 6 ones; 1 quarter, 1 penny, etc.
- Have students choose a price card, for example 37 cents, and represent that amount with coins (create a context by setting up a class “store”). Ask, How many ways can you pay for one item with coins?
- Ask students to work in a group of four. Students take turns drawing a numeral card. Each student in the group takes a turn representing the chosen number with base-ten blocks and with symbols. Each block display within the group must be different. For example, if 46 was selected, it might be represented as 4 rods and 6 small cubes, 3 rods and 16 small cubes, 25 small cubes and 21 small cubes, 3 rods and 3 groups of 5 small cubes and 1 small cube.
- Ask students to select a 2-digit number and describe it in different ways. Have a student read his or her description to the class and ask other students to identify the number.

SUGGESTED MODELS AND MANIPULATIVES

- base-ten blocks
- coins
- counters
- hundred charts
- ten-frames

MATHEMATICAL LANGUAGE

Teacher	Student
<ul style="list-style-type: none"> ▪ coins: penny, nickel, dime, quarter ▪ expression ▪ hundred chart, number line ▪ number words: zero to twenty (zero, one, two, ..., twenty) ▪ numeral ▪ parts ▪ partition numbers/quantities ▪ represent ▪ tally ▪ ten-frames, base-ten blocks 	<ul style="list-style-type: none"> ▪ coins: penny, nickel, dime, quarter ▪ expression ▪ hundred chart, number line ▪ number words: zero to twenty (zero, one, two, ..., twenty) ▪ numeral ▪ parts ▪ quantity ▪ tally ▪ ten-frames

Resources/Notes**Print**

- *Making Math Meaningful to Canadian Students, K–8* (Small 2009), pp. 138–140, 142–143, 146
- *Making Math Meaningful to Canadian Students, K–8, Second Edition* (Small 2013), pp. 194–196, 198–199, 202
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 47–51, 54–55, 56–58, 122–125, 146–148

Notes

SCO N05 Students will be expected to compare and order numbers up to 100.

[C, CN, R, V]

[C] Communication

[PS] Problem Solving

[CN] Connections

[ME] Mental Mathematics and Estimation

[T] Technology

[V] Visualization

[R] Reasoning

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- N05.01** Compare and order a given set of numbers in ascending or descending order and verify the result using a hundred chart, number line, ten-frames, or by making references to place value.
- N05.02** Identify errors in a given ordered sequence.
- N05.03** Identify missing numbers in a given hundred chart.
- N05.04** Identify errors in a given hundred chart.

Scope and Sequence

Mathematics 1	Mathematics 2	Mathematics 3
N05 Students will be expected to compare sets containing up to 20 objects to solve problems using referents and one-to-one correspondence.	N05 Students will be expected to compare and order numbers up to 100.	N03 Students will be expected to compare and order numbers to 1000.

Background

Students should encounter a variety of numbers in context. These contexts help them develop an understanding of number size. Students will be able to order a set of numbers in ascending and descending order. They will be able to justify their solutions using benchmarks, hundred charts, number lines, ten-frames, and/or place value. Visual models encourage reasoning, as students consider how to compare and order numbers. As with all concepts, begin with concrete models. Initially, use groupable materials such as beans or wooden stir sticks, and then move to pre-grouped materials such as base-ten blocks or ten-frames. Students should be given many experiences with concrete materials before moving to more pictorial and symbolic representations.

Additional Information

See Appendix A: Additional Information.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Provide two different sets of counters, each with 20 or fewer. Ask students which set has more and which has fewer (e.g., one set has 19 and the other has 12) and to explain how they know.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Show a number of base-ten unit cubes (e.g., 34). Beside the cubes, show 5 rods and 6 unit cubes. Ask, Which has more? Is one easier to count than the other? Explain.
- Give students two representations of the same number, point to one and ask, Is this number more, less, or are they the same? Have students explain their thinking.



- Ask students to explain using concrete and/or pictorial representations, why 42 is greater than 29.
- Ask students to use base-ten materials to show why 24 is less than 42.
- Ask students to record a number in the blank so that the three numbers are in order. For example, 25, __, 31 or 75, __, 68.
- Ask students to choose three numbers, each less than 100. Ask them to put their numbers in order from smallest to largest. Ask them to explain their thinking.
- Ask students, Is a number with a 7 in it always greater than a number with a 6 in it? Ask students to explain their thinking.
- Show a number line with some numbers placed incorrectly. Ask students to identify the errors, explain their thinking, and place the numbers correctly.
- Ask the students to rearrange the numbers below to correct the errors in the following ascending sequence: 7, 13, 20, 32, 28, 56, 69, 71, 44
- Ask students to explain how they compare two numbers.
- Provide students with a hundred chart and cubes of different colours. Ask them to place cubes on the hundred chart as directed. For example, ask students to place
 - a red cube on a number less than 36
 - a green cube on a number greater than 84
 - a blue cube on a number between 53 and 60
- Provide students with a hundred chart with missing numbers. Ask them to explain what number would be placed in each of the empty spaces and to explain their thinking.
- Provide students with a hundred chart containing errors. Ask them to correct the errors and to explain their thinking.
- Ask students to record a number that is greater than 34 but less than 43.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets K–2 (Bauman 2011)

- Grade 2 Checkpoint 4, Task 2, pp. 51–52 (Line Master 4.2)

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Compare benchmark quantities of the same item (e.g., 50 counters and 10 counters) to provide a visual of the relative size of numbers.
- Ask students to compare numbers during classroom routines (e.g., Are there more boys than girls in class today?).
- Expect students to compare and order numbers in a variety of ways including concrete, pictorial, and symbolic representations and to explain their thinking.
- Encourage students to use number lines and open number lines to compare and order numbers.

SUGGESTED LEARNING TASKS

- Ask students to order a given set of numbers, such as 34, 43, 17, and 21, in ascending or descending order and verify the result using a hundred chart, number line, or ten-frames. Verification may also make reference to place value when working on SCO N07.
- Prepare a set of 100 stir sticks numbered from 1 to 100. Select a group of five students. Ask each student in the group to draw out a stir stick and then to work co-operatively to order themselves in ascending (or descending) order.

- Provide students with 9 rods and 9 small cubes (base-ten blocks). Ask students to create two different amounts, each using exactly 5 of the base-ten blocks. Ask, Which amount is greater than the other? How is it possible to use the same number of blocks, but still have one amount greater than the other?
- Provide a teacher-made flyer in which prices are less than one dollar. Ask students to circle the item that costs the most, the least, more than 50 cents, etc.
- Provide each student with a deck of 2-digit numeral cards. Students work in pairs. Each student turns over the top card in his or her deck. The student with the greatest number collects both cards and sets them aside. Play continues until all cards have been compared. The student who has collected the greatest number of cards wins.
- Ask students to identify missing numbers or errors in a given hundred chart or ordered sequence. Ask them to fill in the missing numbers or correct the errors and to explain their thinking.
- Play “Guess My Number.” The teacher selects a target number and identifies the range for students (e.g., “I am thinking of a number between 1 and 60.”). Students take turns asking questions in order to identify the number. The teacher can only answer “yes,” “no,” “too small,” or “too large.” It is important to model effective questions rather than have students guess random numbers.
Example: Is the number greater than ___? Is it less than ___? Is it an even or odd number? A number line may be used to record the information determined after each question. In the example, the game would start with two markers placed at 1 and the 60. If the first student asks if the number is 20 and the teacher answers, “too small,” the first marker can then be moved from the 1 to the 20. The range is then narrowed to between 20 and 60.
- Use labels on which numerals from 0 to 100 are written. Place a label on each student’s back. Each student then moves around the room and asks classmates questions similar to those above in order to identify the numeral written on his or her label. Each question may only be answered with “yes,” “no,” or “I don’t know.” Once the student has identified the numeral written on his or her label, he or she works with other students to place themselves in order creating a number line.

SUGGESTED MODELS AND MANIPULATIVES

- base-ten blocks
- hundred chart
- number line
- ten-frames

MATHEMATICAL LANGUAGE

Teacher	Student
<ul style="list-style-type: none"> ascending, descending order compare, order numbers hundred chart, number line, ten-frame, identify missing numbers, errors place value: tens 	<ul style="list-style-type: none"> compare, order numbers hundred chart, number line, ten-frame tens

Resources/Notes

Print

- Making Math Meaningful to Canadian Students, K–8* (Small 2009), pp. 142–143, 145
- Making Math Meaningful to Canadian Students, K–8, Second Edition* (Small 2013), pp. 198–199, 201
- Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 38, 42–47, 54–55, 56–58, 142–143

Notes

SCO N06 Students will be expected to estimate quantities to 100 by using referents. [C, ME, PS, R]			
[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- N06.01** Estimate a given quantity by comparing it to a referent (known quantity).
N06.02 Estimate the number of groups of ten in a given quantity using 10 as a referent.
N06.03 Select between two possible estimates for a given quantity and explain the choice.

Scope and Sequence

Mathematics 1	Mathematics 2	Mathematics 3
N06 Students will be expected to estimate quantities to 20 by using referents.	N06 Students will be expected to estimate quantities to 100 using referents.	N04 Students will be expected to estimate quantities less than 1000 using referents.

Background

The ability to estimate, a key reasoning skill in mathematics, should develop with regular practice over the course of the year. To develop estimation skills, students should be provided with collections of objects and be asked to estimate the size of the group, using a referent. Estimation helps students develop flexible, intuitive ideas about numbers, further developing number sense. Estimating is difficult for students; therefore, time must be spent developing an understanding of the language associated with estimating such as “more than,” “less than,” “closer to,” and “about.” Model this language and encourage students to use it when talking about estimating experiences.

Additional Information

See Appendix A: Additional Information.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Place between 10 and 20 counters on the table. Ask students to record an estimate for the number of counters shown. Then, move 5 counters (referents) to the side of the table. Tell students, Here are 5 counters. Do you want to adjust your estimate? If you want to adjust it, will it be more or less than your first estimate? Ask students to explain their thinking.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Place a pile of paper clips on a desk. Ask students to estimate the number of paper clips. Observe and/or interview students to determine if they are using a referent. A guiding question might include, How did you choose that number?
- Give students a train of four linking cubes. Ask them to estimate the number of cubes in a longer train that is on display in the room. Have students explain their thinking.
- Show students a group of items and ask them to choose between two given estimates. Have them explain their reasoning for their choice.
- Show students a jar containing about 100 beads. Tell them that one student estimated there were 90 beads in the jar. Another student estimated there were 25 beads in the jar. Ask, Which estimate is closer to the actual number of beads in the jar? Explain your thinking.
- Ask students to estimate the number of steps it will take to walk a specified distance. (e.g., How many steps will it take to walk from the pencil sharpener to the classroom door?).

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets K–2 (Bauman 2011)

- No Checkpoint for this outcome.

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?

- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Use the same unit (e.g., footsteps) to estimate and check the attributes of a variety of things. If students are able to explore these types of tasks in succession with the same unit, they refine their estimating skills.
- Use children’s literature with an estimation focus, such as *Counting on Frank* by Rod Clement (1994) or *The Penny Pot* by Stuart Murphy (1998).
- Provide opportunities in the classroom for estimation. These may include
 - using estimation jars filled with different objects
 - determining the number of mathematics materials in a container
 - determining the number of pages in a book

SUGGESTED LEARNING TASKS

- Ask students to estimate how many footsteps
 - from the door to the window
 - from the door to the top of the stairs
 - from the door to your desk
- Show 10 paper clips on the overhead for the students to have a visual reference. Then display a larger group of paper clips. Ask students to estimate how many paper clips there are. Ask, Why do you think that?
- Give a small group of students a series of baggies with a number of bread ties. In each baggie place a card asking, Is it closer to ___ or ___? (e.g., is it closer to 20 or 50?). Bags are considered one at a time by the group, with students explaining their choice. The group may then count the quantity to determine the closest estimate.
- Challenge the students to estimate how many times they can print their name in one minute. Individual considerations include length of name or speed of printing.
- Have students draw a card that indicates how many cubes they need to get from a container (less than 20, between 30 and 50, about 20). They must choose which size scoop to use to get that many items. Students count to check. **Variation:** Have only one scoop and several buckets of different-sized objects. Students have to decide which object to scoop to get their target range. Decide whether students can use the scoop only once or a smaller scoop more than once. Emphasis should be placed on estimating to scoop the target number.
- Provide three different sets of small objects such as centicubes, pennies, or beans. Ask student to take a handful of one of the objects, estimate the number of objects, and then count to check. Students should then self-assess and indicate whether their estimate was too small, just right, or too large. Students should record their results in a chart with labels Object, Estimate, Count, My Estimate Was Too Small, Just Right, Too Large. Students should then repeat the process with each of the remaining objects.

SUGGESTED MODELS AND MANIPULATIVES

- variety of containers and objects (baggies, beads, buckets, cubes, marbles, paper clips)

MATHEMATICAL LANGUAGE

Teacher	Student
<ul style="list-style-type: none">▪ estimate▪ group of ten▪ more than, less than, closer to, about▪ referent	<ul style="list-style-type: none">▪ estimate▪ group of ten▪ more than, less than, closer to, about

Resources/Notes**Print**

- *Making Math Meaningful to Canadian Students, K–8* (Small 2009), p. 144
- *Making Math Meaningful to Canadian Students, K–8, Second Edition* (Small 2013), p. 144
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 132

Notes

SCO N07 Students will be expected to illustrate, concretely and pictorially, the meaning of place value for numerals to 100.

[C, CN, R, V]

[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- N07.01** Explain and show with counters the meaning of each digit for a given 2-digit numeral with both digits the same.
- N07.02** Count the number of objects in a given set using groups of 10s and 1s, and record the result as a 2-digit numeral under the headings of 10s and 1s.
- N07.03** Describe a given 2-digit numeral in at least two ways.
- N07.04** Illustrate using ten-frames and diagrams that a given numeral consists of a certain number of groups of ten and a certain number of ones.
- N07.05** Illustrate using proportional base-ten materials that a given numeral consists of a certain number of tens and a certain number of ones.
- N07.06** Explain why the value of a digit depends on its placement within a numeral.
- N07.07** Represent one unit if shown a pre-grouped model representing ten.

Scope and Sequence

Mathematics 1	Mathematics 2	Mathematics 3
—	N07 Students will be expected to illustrate, concretely and pictorially, the meaning of place value for numerals to 100.	N05 Students will be expected to illustrate, concretely and pictorially, the meaning of place value for numerals to 1000.

Background

The foundation for the development of the concept of place value centres around grouping tasks, and students' understandings of unitizing (the concept that ten ones equal one ten). In order to understand and use place value, students need to be able to think in groups or to unitize. Ten is both a single entity with a value of 10 and a collection of 10 single units each with a value of 1. Students should have many opportunities to explore counting and organizing larger sets of materials into groups of ten to build connections between 10 ones and 1 ten.

Groupable, proportional models should be used first; that is, materials that can be put together, or taken apart, to make, or unmake, tens that will be ten times the size of the ones. Suggested materials include wooden stir sticks that can be placed together in groups of 10 using rubber bands, linking cubes which can be connected to make strips of 10, or beans that can be bagged or placed in cups in groups of 10. It is important that this stage not be rushed. Many problems that students encounter with place value concepts are believed to stem from inadequate attention to these early place value tasks in which the students create the tens. When students make that important connection between all that they know about counting by ones and the concept of grouping by tens, they should notice how much easier it is to count.

Base-ten blocks are an efficient and valuable model, as they are proportional in size. This means that a rod is ten times the size of a small cube (the rod is scored to show the joining of 10 small cubes), and a flat is ten times the size of a rod, and one hundred times the size of a small cube (the flat is scored to show the 100 small cubes). This helps with developing number sense as a number like 80 is ten times as great as the number 8. While these multiplicative relationships among the blocks exist, it is unlikely that most students in Mathematics 2 will think in this way; they are more likely to count the number of small cubes they see joined together to make a rod and a flat, and they will need to place 10 rods on top of a flat to be convinced of that relationship.

When working with numbers, students should always have base-ten blocks, ten-frames, or other materials available to represent numbers, and should be expected to use those materials to support their thinking. This consistent use of manipulatives solidifies their understanding of number. It is essential that students be provided plenty of opportunity to manipulate and work with base-ten blocks and ten-frames. For example, these materials could be used daily in morning routines, such as calendars or number of the day, to reinforce base-ten concepts.

If they begin working with pre-grouped models prematurely, students can easily attach words such as tens to both materials and groups without realizing what the materials or symbols represent. Such students may learn by rote to represent a number, such as 46, by showing 4 rods and 6 small cubes without realizing the rods represent the 40 in the number. If they were given 46 counters and asked to show what the 4 and 6 each represent, they may very well point to 4 counters and 6 counters. These students have what is often referred to as a face-value interpretation of a number rather than a place-value interpretation.

The big idea must be to understand the counting of groups and the units within the groups of ten as the foundation of place value. Ensure that although physical models play a key role, they in themselves are not the concept. Students must construct the concept and relate it to the model. Using a variety of materials allows students to construct a deep understanding of place-value concepts.

Additional Information

See Appendix A: Additional Information.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Ask students to represent a given number up to 20 using ten-frames. Ask them to use ten-frames to represent another number that is greater than the first one they represented.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students to choose a 2-digit number and to record it symbolically. Ask them to represent their 2-digit number using ten-frames. Ask them to explain the value of each digit in relation to the ten-frames.
- Ask students to represent a given 2-digit number using base-ten materials. Ask them to explain the value of each digit in relation to the base-ten materials.
- Show students a number represented with small cubes and the same number represented with rods and small cubes. For example, 23 represented as 23 small cubes and as 2 rods and 3 small cubes. Ask, Are these two numbers the same? Have students explain their thinking.
- Show students a rod from the base-ten materials. Tell them the rod is worth 10. Ask them to show you the base-ten material that would have a value of 1.
- Ask students to pick up a handful of counters and represent the total using ten-frames. Ask, How else could you show this number?
- Show the students a 2-digit numeral with both digits the same (e.g., 44). Have students model the value of each digit. Ask students to explain why these digits do not represent the same value.
- Provide students with a large number of objects (less than 100). Ask students to count the objects and to record the result as a 2-digit numeral. Observe whether students grouped the objects by tens or used other efficient strategies.
- Give students a place-value chart and a group of objects. Have them determine how many objects they have and record their results in the chart.
- Ask students to model numbers using pre-grouped materials, to record numerals from existing models, and to show how to count an amount that is modelled.

FOLLOW-UP ON ASSESSMENT**Guiding Questions**

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets K–2 (Bauman 2011)

- No Checkpoint for this outcome.

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

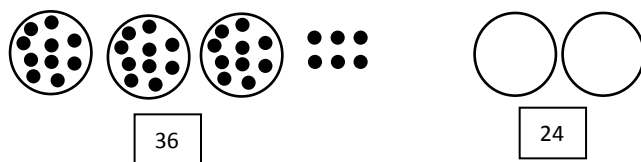
Consider the following strategies when planning daily lessons.

- Precede tasks with pre-grouped models with tasks using groupable models so students will know that a rod and ten small cubes are the same amount. The best base-ten models are proportional and can be classified as “groupable” or “pre-grouped.”
- Give students many opportunities to count and sort large numbers of objects within a relevant context (e.g., take inventory of classroom objects). Note whether students count the number of objects using groups of 10s and 1s. Ask them to record the result as a 2-digit numeral.
- Students should be expected to express numbers symbolically, verbally, and with pictures and models. They should translate among these representations.
- Ensure that although physical models play a key role, they are not the concept. Students must construct the concept and relate it to the model. Using a variety of materials allows students to construct a deep understanding of place-value concepts.

SUGGESTED LEARNING TASKS

- Place different amounts of wooden stir sticks into paper bags. Ask students to count the sticks by grouping them into bundles of 10s with elastic bands. Ask them to record the total as a 2-digit numeral under the headings of tens and ones. Ask, How many sticks are there? Is it easier to count the sticks by ones or by tens when they are grouped? How do you know there are that many sticks? (Note how the students respond. Do they count by groups of ten?)
- Give students a 2-digit numeral card. Ask them to use ten-frames to represent it. Ask them to show the ten-frames that represent the first digit in the numeral. Then, ask them to show the ten-frames that represent the second digit in the numeral.
- Give students a pre-grouped model and ask, If this is ten, what would one look like (what would 3 or 17 or 85 look like)?

- Set out a number of arrangements of beans grouped by tens (in cups, on sticks, on plates, ten-frames, etc.) and individual units. Also have some empty “stations” as shown below. Provide the students with a set of prepared 2-digit numeral cards. Direct them to place the matching card in front of the appropriate prepared display and to build the arrangements for the other cards they have. Use a large collection of numbers (e.g., 13, 16, 18, 24, 26, 28, 33, 36, 38, and 40).



- Show students a 2-digit numeral. Ask them to read the numeral and model it with base-ten materials.
- Provide students with a set of 33 counters. Ask them to record the 2-digit numeral that represents the set. Ask them to show the counters represented by the first digit in the numeral (thirty counters) and by the second digit (three counters).
- Have students work in groups. Give each group a piece of chart paper on which is written a 2-digit numeral. Ask students to record all the ways they can represent or describe the numeral. For example, students might record 24 as two 10s and four 1s, 20 and 4, two groups of 10 and 4 left over, one 10 and fourteen 1s, or twenty-four 1s.

SUGGESTED MODELS AND MANIPULATIVES

- base-ten blocks
- beans
- counters
- Cuisenaire rods
- ten-frames
- wooden stir stick

MATHEMATICAL LANGUAGE

Teacher	Student
<ul style="list-style-type: none"> base-ten blocks, ten-frame numeral, digit ones, tens place value to 100 quantity small cube, rod, flat two-digit numeral 	<ul style="list-style-type: none"> base-ten blocks, ten-frame numeral, digit ones, tens quantity small cube, rod

Resources/Notes

Print

- Making Math Meaningful to Canadian Students, K–8* (Small 2009), pp. 138–144
- Making Math Meaningful to Canadian Students, K–8, Second Edition* (Small 2013), pp. 194–200
- Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 127–141, 145–149

Videos

- *Teaching Place Value: 20 to 99* (29:08 min.) (ORIGO Education 2010)
- *Teaching Place Value: Teen Numbers* (8:30 min.) (ORIGO Education 2010)
- *Using a Hands-on Approach to Represent Tens and Ones* (10:05 min.) (ORIGO Education)

Notes

SCO N08 Students will be expected to demonstrate and explain the effect of adding zero to or subtracting zero from any number.

[C, R]

[C] Communication

[PS] Problem Solving

[CN] Connections

[ME] Mental Mathematics and Estimation

[T] Technology

[V] Visualization

[R] Reasoning

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N08.01 Add zero to a given number and explain why the sum is the same as the addend.

N08.02 Subtract zero from a given number and explain why the difference is the same as the given number.

Scope and Sequence

Mathematics 1	Mathematics 2	Mathematics 3
N08 Students will be expected to identify the number, up to 20, that is one more, two more, one less, and two less than a given number.	N08 Students will be expected to demonstrate and explain the effect of adding zero to or subtracting zero from any number.	—

Background

Zero is conceptually different from any other number. Zero cannot be connected to an actual object or represented by a concrete item (until later grades when they explore negative numbers). Zero indicates an absence of quantity or the quantity before the count begins. Therefore, addition and subtraction with zero do not change the original value.

The no-change nature of adding and subtracting zero can be difficult for some students; therefore, it is important that students engage in discussion and tasks involving the number zero to further their understanding of this concept. Students should be encouraged to develop story problems involving zero to help them understand zero in addition and subtraction. Students can also role-play tasks where they receive or give away zero. They should model the addition and subtraction of zero frequently to develop a generalization that addition does not always increase the quantity and subtraction does not always decrease the quantity.

Additional Information

See Appendix A: Additional Information.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

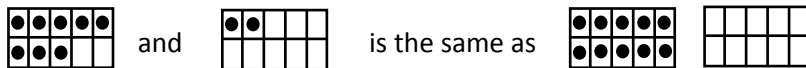
Tasks such as the following could be used to determine students' prior knowledge.

- Show students 5 counters on top of an inverted opaque container. Tell students, I have 5 counters in all. How many counters are hidden under the container?

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Use ten-frames to construct addition and subtraction situations for the students that require them to use a blank ten-frame to complete the operation. How many counters will I need to add to make the two sides equal?



- Give students a prepared number line with a start point (e.g., 24). Ask students to show the hops on the number line as you dictate to them. For example, add 2, subtract 0. Where are you now? (e.g., 26) Explain your thinking.
- Ask students to create a word problem with 0 and 36 in it.
- Tell students, Billy said that he started with 16 and added a number to it. His sum was 16. Ask, What number did Billy add to 16 to get a sum of 16?

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets K–2 (Bauman 2011)

- No Checkpoint for this outcome.

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Develop story problems involving zero to help students understand zero in addition and subtraction.
- Model the addition and subtraction of zero frequently to develop student's understanding that addition does not always increase the quantity and subtraction does not always decrease the quantity.
- Role-play tasks where students receive / give away zero.

SUGGESTED LEARNING TASKS

- Use a horizontal number line, hundred chart or a large floor chart with numbers 1 to 100. Ask a student volunteer to step on 57. Tell him or her to take zero steps forward. Tell him or her to take zero steps backward. Discuss the movements made by the student volunteer.
- Read children's literature such as, *A Place for Zero: A Math Adventure* by Angeline Sparagna LoPresti and Phyllis Hornug (2003) and discuss with students.
- Show students a number for a few seconds using dot patterns or your fingers. Tell students that a number has been added. Flash the sum (same as original value) and ask students to name that addend (0). This may also be done with ten-frames.
- Tell students, After I subtracted zero I have this many (illustrate value with dot cards, fingers, etc.). How many did I start with?
- Use a pan balance with equal amounts on each side and have the students explore what needs to be added or subtracted to keep the balance.
- Have students predict the answer when zero is added to a number. Use the constant function on a calculator and add zero several times to a number to show the constancy of the number. Repeat with different numbers. Repeat using the subtract function. Discuss the pattern that results.

SUGGESTED MODELS AND MANIPULATIVES

- calculator
- dot cards
- number lines
- pan balance
- ten-frames

MATHEMATICAL LANGUAGE

Teacher	Student
<ul style="list-style-type: none">▪ adding, subtracting zero▪ difference, take away▪ no-change nature▪ zero	<ul style="list-style-type: none">▪ adding, subtracting zero▪ difference, take away ▪ zero

Resources/Notes**Print**

- *Making Math Meaningful to Canadian Students, K–8* (Small 2009), p. 110
- *Making Math Meaningful to Canadian Students, K–8, Second Edition* (Small 2013), p. 166
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 75–77

Notes

SCO N09 Students will be expected to demonstrate an understanding of addition (limited to 1- and 2-digit numerals) with answers to 100 and the corresponding subtraction by

- using personal strategies for adding and subtracting with and without the support of manipulatives
- creating and solving problems that involve addition and subtraction
- explaining and demonstrating that the order in which numbers are added does not affect the sum
- explaining and demonstrating that the order in which numbers are subtracted matters when finding a difference

[C, CN, ME, PS, R, V]

[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- N09.01** Solve a given story problem of any type by modelling it with materials or a diagram, and write a number sentence that represents the thinking in the solution.
- N09.02** Solve a given story problem of any type by writing a number expression and combining the numbers to complete the number sentences.
- N09.03** Match a number sentence to a given story problem.
- N09.04** Create an addition or a subtraction number sentence and a story problem for a given solution.
- N09.05** Model addition and subtraction using concrete materials or visual representations and record the process symbolically.
- N09.06** Add a given set of numbers in two different ways and explain why the sum is the same.
- N09.07** Recognize and create equivalent addition and subtraction number sentences.

Scope and Sequence

Mathematics 1	Mathematics 2	Mathematics 3
<p>N09 Students will be expected to demonstrate an understanding of the addition of two single-digit numbers and the corresponding subtraction, concretely, pictorially, and symbolically in join, separate, equalize/compare, and part-part-whole situations.</p>	<p>N09 Students will be expected to demonstrate an understanding of addition (limited to 1- and 2-digit numerals) with answers to 100 and the corresponding subtraction by</p> <ul style="list-style-type: none"> ▪ using personal strategies for adding and subtracting with and without the support of manipulatives ▪ creating and solving problems that involve addition and subtraction ▪ explaining and demonstrating that the order in which numbers are added does not affect the sum ▪ explaining and demonstrating that the order in which numbers are subtracted matters when finding a difference 	<p>N06 Students will be expected to describe and apply mental mathematics strategies for adding two 2-digit numerals.</p> <p>N07 Students will be expected to describe and apply mental mathematics strategies for subtracting two 2-digit numerals.</p> <p>N08 Students will be expected to apply estimation strategies to predict sums and differences of 1-, 2-, and 3-digit numerals in a problem-solving context.</p> <p>N09 Students will be expected to demonstrate an understanding of addition and subtraction of numbers (limited to 1-, 2- and 3-digit numerals) with answers to 1000 by</p> <ul style="list-style-type: none"> ▪ using personal strategies for adding and subtracting with and without the support of manipulatives ▪ creating and solving problems in context that involve addition and subtraction of numbers, concretely, pictorially, and symbolically

Background

Students develop an understanding of addition and subtraction of numbers by modelling, acting out, building, drawing, and using appropriate mathematics language in creating number sentences and solving number stories. Through these experiences, students will also begin to develop personal strategies for adding and subtracting. New vocabulary words can be added to the mathematics word wall to reinforce the use of terminology, such as **together**, **part**, **sum**, **difference**, **add**, **subtract**, and **take away**. Addition and subtraction should be taught simultaneously to enable students to see the relationship between the two operations.

Students should have experience with the many different types of addition and subtraction problems. The meanings of, and the relationship between, addition and subtraction are developed using situations (see chart below) that are first modelled concretely, then pictorially, and lastly, symbolically.

Join			Part-Part-Whole	Compare
Result Unknown	Change Unknown	Start Unknown	Whole Unknown	Difference Unknown
Pat has 8 marbles. Her brother gives her 4. How many does she have now? $8 + 4 = ?$	Pat has 8 marbles but she would like to have 12. How many more does she need to get? $8 + ? = 12$ or $12 - 8 = ?$	Pat has some marbles. Her brother gave her 4 and now she has 12. How many did she have to start? $? + 4 = 12$ or $12 - 4 = ?$	Pat has 8 blue marbles and 4 green marbles. How many does she have in all? $8 + 4 = ?$	Pat has 8 blue marbles and 4 green marbles. How many more blue marbles does she have? $8 - 4 = ?$ or $4 + ? = 8$
Separate			Part-Part-Whole	Compare
Result Unknown	Change Unknown	Start Unknown	Part Unknown	Smaller or Larger Unknown
Pat has 12 marbles. She gives her brother 4 of them. How many does she have left? $12 - 4 = ?$	Pat has 12 marbles. She gives her brother some. Now she has 8. How many marbles did she give to her brother? $12 - ? = 8$ or $12 - 8 = ?$	Pat has some marbles. She gives her brother 4 of them. Now she has 8. How many marbles did she have to start? $? - 4 = 8$ or $8 + 4 = ?$	Pat has 12 marbles. Eight are blue and the rest are green. How many are green? $8 + ? = 12$ or $12 - 8 = ?$	Pat has 8 blue marbles and some green marbles. She has 4 more blue marbles than green ones. How many green marbles does she have? $8 - 4 = ?$ or $? + 4 = 8$

The strategies and symbols that students use should reflect how they thought about the problem. For example, consider the story problem: Pat has 12 marbles. She gives her brother some. Now she has 8. How many marbles did she give to her brother? This problem represents a separate situation and could be solved by some students by starting with 12 counters, counting back as counters are removed until they reach 8, and counting the counters that were removed to get 4. These students are likely to write $12 - 4 = 8$ to represent what they did. However, some students could start with 12 counters, remove the 8 that they knew were left, and count the remaining counters to get 4. These students are likely to write $12 - 8 = 4$ to represent what they did. Whichever number sentence is used, it is essential that students understand how each part of the number sentence relates to the situation. This story problem illustrates that one situation can be represented in different ways symbolically.

Additional Information

See Appendix A: Additional Information.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Present students with a pictorial representation of a story problem involving two single-digit numbers. Ask them to tell a story that matches the picture. Ask them to solve the problem and to record a number sentence that matches the story problem and solution.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Give students an addition or subtraction sentence. Ask them to model the number sentence using concrete materials or pictures.
- Use base-ten blocks to model an addition or subtraction story problem. Ask students to record a number sentence that would match the base-ten model.
- Ask students to write the addition sentence that would help them solve the following:
 $? = 16 - 8$
 $18 - 9 = ?$
 $50 - ? = 20$
- Ask students to add the following equations two different ways and explain why the sum is the same regardless of the order that they used.
 $65 + 28 = ?$
 $7 + 4 + 3 + 6 = ?$
- Ask students to solve problems, such as the following, and to record their work with pictures and numbers.
 - My dad made 43 chocolate chip cookies and some peanut butter cookies. There were 92 cookies on the cupboard. How many were peanut butter? Solve and explain your thinking.
 - My mom used 28 nails to make a birdhouse. There are 55 nails left in the box. How many nails were in the box before she started? Solve and explain your thinking.
- Tell students that the answer to a problem is 31 balloons. Ask students to make up a story problem and give the number sentence that matches this answer.
- Give students an addition and/or subtraction number sentence and ask them to show different strategies to solve it. Encourage them to show as many different ways as they can.
- Ask students to add a given set of numbers in two different ways and to explain why the sum is the same. For example, if asked to add $2 + 5 + 3 + 8$, a student could add $2 + 3 + 5 + 8$ or $8 + 2 + 5 + 3$.
- Ask students to explain using models or pictures whether they would get the same answer for $9 - 3$ and for $3 - 9$.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets 3 (Bauman 2009)

- Grade 3 Checkpoint 3, pp. 25–27
- Grade 3 Checkpoint 5, pp. 33–34 (Line Masters 5.1 and 5.2)
- Grade 3 Checkpoint 6, pp. 36–37 (Line Master 6.1)

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

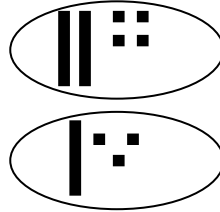
- Have students explore the fact that a story problem could be represented with an addition or a subtraction number sentence. For example: $2 + ? = 7$ describes the same situation as $7 - 2 = ?$ Either of these is acceptable.
- Continue to use models and other representations. Encourage students to match various representations of the same problem.
- Write number sentences horizontally to encourage more divergent thinking and development of personal strategies. Students using personal strategies find solving problems with numbers requiring “regrouping” as easy as those problems that do not require “regrouping.”
- Encourage students to create and solve the four different types of addition and subtraction problems: join, separate, part-part-whole, and compare (Van de Walle and Lovin, 2006, 67–69).

SUGGESTED LEARNING TASKS

- Tell students that Janet read 18 books and Fred read 42. Ask them to record a number sentence and then explain how to find the difference using an open number line (or other representation).
- Tell students that someone told you that you do not have to learn to subtract if you know how to add. Ask, Do you agree? Why or why not?
- Provide students with a variety of models and ask how they would add $42 + 29$. Have students explore different ways to find the sum. Have students explore additional examples that require regrouping.
- Have students explore different ways to find the difference between 22 and 6.
- Have students make a booklet and on the front cover write, “The Answer is 25.” Have students create their own addition and subtraction word problems that would result in an answer of 25. They

should record one problem on each page of their booklet. Students could illustrate each page to match the story problem.

- Ask students what the difference is between 6 and 12. Gradually extend the activity to use 2-digit numbers. Number lines are a good model to support students' thinking.
- Let students choose a favourite story and create addition and subtraction problems related to the story. These could be shared through dramatizations, pictures, or writing.
- Show two numbers modelled with base-ten blocks. After showing the students one pre-grouped model (top model) and the other pre-grouped model (bottom model), ask the students to record addition and/or subtraction sentences these represent and to explain their thinking (e.g., $37 - 23 = 14$).



SUGGESTED MODELS AND MANIPULATIVES

- base-ten blocks
- hundred charts
- number lines
- open number lines
- ten-frames

MATHEMATICAL LANGUAGE

Teacher	Student
<ul style="list-style-type: none"> ▪ add, together, plus, sum, total ▪ compatible numbers ▪ how many more, how many less ▪ number sentence ▪ ones, tens ▪ order and grouping addends ▪ part, whole ▪ story problem, situation ▪ strategy ▪ strip diagram ▪ subtract, minus, difference, take away 	<ul style="list-style-type: none"> ▪ add, together, plus, sum, total ▪ friendly numbers ▪ how many more, how many less ▪ number sentence ▪ ones, tens ▪ order and grouping ▪ part, whole ▪ story problem ▪ strip diagram ▪ subtract, minus, difference, take away

Resources/Notes

Print

- *Making Math Meaningful to Canadian Students, K–8* (Small 2009), pp. 104–110, 162–173
- *Making Math Meaningful to Canadian Students, K–8, Second Edition* (Small 2013), pp. 159–166, 215–217
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 65–75, 157–172

Videos

- *Comparing Mental Strategies: Addition* (14:42 min.) (ORIGO Education 2010)
- *Questions for Developing Mental Computation Strategies* (13:42 min.) (ORIGO Education 2010)
- *Using a Hands-On Approach to Develop Mental Strategies for Addition* (11:04 min.) (ORIGO Education 2010)
- *Using a Hands-On Approach to Develop Mental Strategies for Subtraction* (6:45 min.) (ORIGO Education 2010)
- *Using Language Stages to Develop Addition Concepts* (15:38 min.) (ORIGO Education)
- *Using Language Stages to Develop Subtraction Concepts* (18:32 min.) (ORIGO Education 2010)
- *Using Mental Strategies to Add* (26:15 min.) (ORIGO Education 2010)
- *Using Static Problems to Relate Addition and Subtraction and Introduce Equality* (13:25 min.) (ORIGO Education 2010)
- *Using Static Problems to Relate Addition and Subtractions and Introduce Functions* (18:59 min.) (ORIGO Education 2010)

Notes

SCO N10 Students will be expected to apply mental mathematics strategies to quickly recall basic addition facts to 18 and determine related subtraction facts.

[C, CN, ME, R, V]

[C] Communication [PS] Problem Solving [CN] Connections [ME] Mental Mathematics and Estimation
[T] Technology [V] Visualization [R] Reasoning

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N10.01 Explain the mental mathematics strategy that could be used to determine basic addition facts.

- Doubles Facts
- Plus-One Facts
- One-Apart (Near Doubles) Facts
- Plus-Two Facts
- Plus-Zero Facts
- Make-10 Facts
- Two-Apart Facts
- Plus-Three Facts

N10.02 Use and describe a personal strategy for determining a sum to 18.

N10.03 Quickly recall basic addition facts to 18 in a variety of contexts.

N10.04 Explain the think-addition strategy used to determine a basic subtraction fact.

N10.05 Use and describe a personal strategy for determining the subtraction facts.

Scope and Sequence

Mathematics 1	Mathematics 2	Mathematics 3
<p>N10 Students will be expected to use and describe strategies to determine sums and differences using manipulatives and visual aids.</p> <p>Strategies include</p> <ul style="list-style-type: none"> ▪ counting on or counting back ▪ one more or one less ▪ making ten ▪ doubles ▪ near doubles 	<p>N10 Students will be expected to apply mental mathematics strategies to quickly recall basic addition facts to 18 and determine related subtraction facts.</p>	<p>N10 Students will be expected to apply mental mathematics strategies and number properties to develop quick recall of basic addition facts to 18 and related basic subtraction facts.</p>

Background

The development of mental mathematics needs to be a major goal of any mathematics program for two major reasons. First, in their day-to-day tasks, most people's computational, measurement, and spatial needs can be met by having well-developed mental mathematics strategies. Secondly, because technology has replaced paper-and-pencil as the major tool for complex tasks, people need to have well-developed mental mathematics strategies to be alert to the reasonableness of the results generated by this technology.

Fact learning refers to the acquisition of the 100 number facts related to the single digits 0 to 9 for each of the four operations. When students know these facts, they can quickly retrieve them from memory (usually in 3 seconds or less). Ideally, through practice, over time students will achieve automaticity; that is, they will have instant recall without using strategies.

Initially, students develop and use strategies to get quick recall of the facts. These strategies and the facts themselves are the foundations for the development of other mental mathematics strategies. When the facts are automatic, students are no longer employing strategies to retrieve them from memory. In turn, the facts and mental mathematics strategies are the foundations for computational estimation strategies. Actually, attempts at computational estimation are often thwarted by the lack of knowledge of the related facts and mental mathematics strategies.

In general, a strategy should be introduced in isolation from other strategies, a variety of different reinforcement tasks should be provided until students have become proficient, the strategy should be assessed in a variety of ways, and then it should be combined with other previously learned strategies.

Additional Information

See Appendix A: Additional Information.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Ask students to explain what strategies they might use to solve each of the following:
 $7 + 1$ $5 - 1$ $7 + 2$ $8 - 2$ $8 + 9$ $8 - 4$ $6 + 4$ $6 - 4$ $6 + 7$ $5 - 1$ $9 + 6$

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students to describe in as many ways as possible why $15 - 8 = 7$.
- Ask, What other facts could $4 + 4 = 8$ help you know?
- Ask students to list three other facts (addition or subtraction) that would be easier to remember if they know that $6 + 5 = 11$.

- Have students list all the subtraction questions they can for which both numbers are less than ten and their difference is 3 (or other similar problems).
- Ask the student to tell why $\square + 5$ has to be 2 greater than $\square + 3$. (This assumes that the \square represents the same number in both expressions.)
- Ask students to explain how they could find the sum of $5 + 5$ or any other addition fact.
- Give students a set of addition and subtraction fact cards and a set of strategy cards. Ask students to sort the fact cards under the strategy headings. Ask students to explain how they would use the strategy to arrive at the answer.
- Record observations of student's explanations of their strategies that they use daily to solve computational problems. This can also be done through individual interviews, which can provide insights into a student's thinking and help identify groups of students that can all benefit from the same kind of instruction and practice.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets K–2 (Bauman 2011)

- Grade 2 Checkpoint 6, pp. 68–69 (Line Masters 6.1 and 6.2)

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Ask students to use as many representations as possible to determine sums and differences, including dramatization, drawing pictures, verbally explaining their ideas, using concrete materials, and writing number sentences.
- Use ten-frames to develop the concept of bridging through 10 (adding and subtracting).

- Facilitate the learning of addition and subtraction facts by having students solve problems with familiar contexts. Encourage students to create their own problems.
- Encourage ongoing discussion and exploration of the most efficient ways to add and subtract numbers. The discussion should focus more on how students obtained their answer rather than the actual sum or difference. The strategies will vary depending on the problem and individual students.
- Use “missing part” (e.g., $6 + \square = 8$) and “join type” problems to develop the connection between addition and subtraction (e.g., Sally had some marbles. Sonya gave her 5 more. Now she has 11 marbles.).
- Provide opportunities for strategy practice, using games and meaningful context as much as possible, rather than only requiring the memorization of facts in isolation. Board games in which students are required to find the sum of two number cubes to determine how far to move are good examples.
- Provide students with time to learn basic facts, so they understand the operation and can invent their strategies rather than memorizing.

SUGGESTED LEARNING TASKS

- Use ten-frames to develop number relationships. For example, to solve $9 + 4$, students can use the ten-frame to see that 9 is 1 less than 10, and they can take 1 from the 4 to fill the ten-frame. They can then see that the sum is 10 and 3 more or 13.
- Use dot cards and dominoes to practise addition facts. For example, show a dot card representing “4” and ask the students to say the ten fact that goes with it ($4 + 6 = 10$).
- Use multiple representations of numbers (ten-frames, dot cards, etc.) to reinforce number relationships.
- Have students work in pairs to sort addition facts into groups of facts that are related. Give them time to share with others and to explain their sorting based upon the strategy used to solve them.

SUGGESTED MODELS AND MANIPULATIVES

- addition chart
- counters
- dominoes
- dot cards
- linking cubes
- number cubes
- ten-frames
- visual cue cards to show doubles

MATHEMATICAL LANGUAGE

Teacher	Student
<ul style="list-style-type: none"> ▪ 1-apart facts—near doubles ▪ 2-apart facts—smaller, larger ▪ addition facts, subtraction facts ▪ double facts—double ▪ make-10 facts ▪ make-10 facts with a 7 ▪ mental mathematics strategy ▪ plus one facts—plus one, next number, number after ▪ plus three facts ▪ plus two facts—skip count by 2, next even number, next odd number ▪ plus zero facts ▪ think addition 	<ul style="list-style-type: none"> ▪ 1-apart, near doubles ▪ 2-apart, smaller, larger ▪ facts ▪ double ▪ make-10 ▪ plus one, next number, number after ▪ skip count by 2, next even number, next odd number ▪ plus zero ▪ think addition

Resources/Notes

Print

- *Making Math Meaningful to Canadian Students, K–8* (Small 2009), pp. 110–115
- *Making Math Meaningful to Canadian Students, K–8, Second Edition* (Small 2013), pp. 166–170
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 94–110

Videos

- *An Introduction to Teaching Addition Number Facts* (15:51 min.) (ORIGO Education 2010)
- *Comparing Mental Strategies: Addition* (14:42 min.) (ORIGO Education 2010)
- *Questions for Developing Mental Computation Strategies* (13:42 min.) (ORIGO Education 2010)
- *Teaching the Bridge-to-10 Strategy for Addition Number Facts* (17:11 min.) (ORIGO Education 2010)
- *Teaching the Count-on Strategy for Addition Number Facts* (17:49 min.) (ORIGO Education 2010)
- *Teaching the Think-Addition Subtraction Fact Strategy* (13:41 min.) (ORIGO Education 2010)
- *Teaching the Use-Doubles Strategy for Addition Number Facts* (14:20 min.) (ORIGO Education 2010)
- *Using a Hands-On Approach to Develop Mental Strategies for Addition* (11:04 min.) (ORIGO Education 2010)
- *Using a Hands-On Approach to Develop Mental Strategies for Subtraction* (6:45 min.) (ORIGO Education 2010)
- *Using Language Stages to Develop Addition Concepts* (15:38 min.) (ORIGO Education)
- *Using Language Stages to Develop Subtraction Concepts* (18:32 min.) (ORIGO Education 2010)
- *Using Mental Strategies to Add* (26:15 min.) (ORIGO Education 2010)

Notes

Patterns and Relations

GCO: Students will be expected to use patterns to describe the world and solve problems.

GCO: Students will be expected to represent algebraic expressions in multiple ways.

Specific Curriculum Outcomes

Process Standards

[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

- PR01** Students will be expected to demonstrate an understanding of repeating patterns (three to five elements) by describing, extending, comparing, and creating patterns using manipulatives, diagrams, sounds, and actions. [C, CN, PS, R, V]
- PR02** Students will be expected to demonstrate an understanding of increasing patterns by describing, extending, and creating numerical patterns (numbers to 100) and non-numerical patterns using manipulatives, diagrams, sounds, and actions. [C, CN, PS, R, V]
- PR03** Students will be expected to demonstrate and explain the meaning of equality and inequality by using manipulatives and diagrams (0 to 100). [C, CN, R, V]
- PR04** Students will be expected to record equalities and inequalities symbolically, using the equal symbol or not equal symbol. [C, CN, R, V]

SCO PR01 Students will be expected to demonstrate an understanding of repeating patterns (three to five elements) by describing, extending, comparing, and creating patterns using manipulatives, diagrams, sounds, and actions.

[C, CN, PS, R, V]

[C] Communication

[PS] Problem Solving

[CN] Connections

[ME] Mental Mathematics and Estimation

[T] Technology

[V] Visualization

[R] Reasoning

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

PR01.01 Identify the core of a given repeating pattern.

PR01.02 Describe and extend a given double attribute pattern.

PR01.03 Create a repeating non-numerical pattern and explain the rule.

PR01.04 Predict an element of a given repeating pattern using a variety of strategies and extend the pattern up to the tenth element to verify the prediction.

PR01.05 Translate a repeating pattern from one mode to another.

PR01.06 Compare two given repeating patterns, and describe how they are alike/different.

Scope and Sequence

Mathematics 1	Mathematics 2	Mathematics 3
<p>PR01 Students will be expected to demonstrate an understanding of repeating patterns (two to four elements) by describing, reproducing, extending, and creating patterns using manipulatives, diagrams, sounds, and actions.</p> <p>PR02 Students will be expected to translate repeating patterns from one representation to another.</p>	<p>PR01 Students will be expected to demonstrate an understanding of repeating patterns (three to five elements) by describing, extending, comparing, and creating patterns using manipulatives, diagrams, sounds, and actions.</p>	<p>PR01 Students will be expected to demonstrate an understanding of increasing patterns by describing, extending, comparing, and creating numerical (numbers to 1000) patterns and non-numerical patterns using manipulatives, diagrams, sounds, and actions.</p> <p>PR02 Students will be expected to demonstrate an understanding of decreasing patterns by describing, extending, comparing, and creating numerical patterns (numbers to 1000) and non-numerical patterns using manipulatives, diagrams, sounds, and actions.</p>

Background

In Mathematics 1, students examined repeating patterns with a core of two to four elements. This patterning concept is essential to help students understand repeating patterns as they continue to study patterning with a core of up to five elements and work with double attributes in Mathematics 2. In Mathematics 1, students also had experiences translating patterns to other modes, such as letters. For example, the pattern red, blue, green, red, blue, green, red, blue, green could be also written as ABCABCABC. This led them to the convention of describing repeating patterns using letters to describe the elements in a core, such as an AB pattern, an AAB pattern, or an ABC pattern. Students in Mathematics 2 will continue using these letters to describe repeating patterns and extend them to cores of five elements.

The foundation of algebraic thinking is investigating patterns and their representations. Contextual, open-ended, and situation-specific problem solving should be an integral part of everyday mathematics instruction, tasks, and assessment. Patterns could be integrated in physical education, music, visual arts, science and other subject areas to provide a context. Students need to recognize and extend many different forms of the same pattern, including those constructed or in their environment.

Students should be able to identify the core of the repeating pattern, and predict elements in repeating patterns. These patterns may be found in a variety of contexts, such as a hundred chart, calendars, number lines, and tiling patterns. Students' predictions should be verified by extending the pattern concretely, pictorially, and symbolically. Students should describe, extend, compare, and create

- sound patterns, such as clap, snap, tap, clap, snap, tap, clap, snap, tap, ...
- action patterns, such as sit, sit, stand, hop, sit, sit, stand, hop, sit, sit, stand, hop, ...
- concrete and pictorial shape patterns, such as

□ △ △ □ □ △ △ □ □ △ △ □ ...

- concrete and pictorial size patterns, such as large, small, small, large, small, small, large, small, small, ...

Additional Information

See Appendix A: Additional Information.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

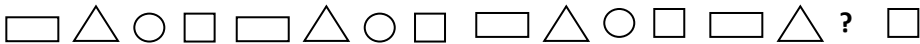
Tasks such as the following could be used to determine students' prior knowledge.

- Show students a repeating pattern that you have begun. Ask them to continue the pattern in two different ways and explain the different pattern rules they used to continue the pattern.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students to use three colours of tiles to create a repeating pattern.
- Ask students to create a repeating pattern that has a core of three elements. Ask them to describe the pattern.
- Present students with a repeating pattern. Ask them to identify the core of the pattern.
- Present students with a repeating pattern. Ask them to describe the rule used to create that pattern.
- Ask students to create a repeating pattern where a specific element is to be identified (e.g., Create a pattern in which the 4th element is green.).
- Present the following pattern: –| | –| | –| | Ask students to translate this pattern into a different mode (sounds, shapes, etc.).
- Ask students to determine the missing element in a given repeating pattern. For example,


- Present students with a repeating pattern. Ask them to predict the position of a specific element. Ask them to extend the pattern to verify their answer.
- Show students three patterns, two of which have the same repeating pattern represented using different elements. Ask students to look at the patterns, and to explain which of the patterns are alike. Then, ask students to choose the pattern that is different from the others, and to represent it using actions.
- Present a double attribute pattern to students. Ask them to describe the pattern and identify the core. Ask them to extend the pattern (e.g., use a double attribute pattern involving colour and size).



FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets K–2 (Bauman 2011)

- Grade 2 Checkpoint 7, Task 1, pp. 76–77

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?

- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Include concrete materials in patterning tasks.
- Integrate patterns in physical education, music, visual arts, science, and other subject areas to provide a context.
- Encourage students to see that patterns may continue in many different ways unless a pattern rule is defined.
- Provide opportunities for students to make predictions about patterns using a variety of materials.
- Use concrete materials for students to build double attribute patterns that can be described in more than one way.
- Expect students to communicate their thinking about patterns, verbally and in writing.

SUGGESTED LEARNING TASKS

- Provide the start of a pattern. Ask students to continue the pattern in more than one way and to describe the pattern rule in each case.
- Provide students with a repeating pattern to extend. Before students begin to extend the pattern, ask them to predict exactly what element will be in a specific position (e.g., What would be in the sixth position?). Have students provide a reason for their prediction before extending their pattern to check their prediction. If their prediction is incorrect, have them examine their reasoning and try to figure out why the prediction was not accurate.
- Ask students to describe and extend a given double attribute pattern, such as



- Ask the students to create two patterns that are similar, but not exactly the same. Ask them to comment on the similarities and differences.
- Tell students that the first two elements in a pattern are a red square and a blue triangle. Ask for several different ways in which the pattern might be continued.
- Tell students that another child continued the pattern blue square, red triangle, blue circle, red square, ... , by saying “blue triangle, red circle.” Ask, Do you think that the other child is incorrect or is there a rule that might explain continuing the pattern in this way?

SUGGESTED MODELS AND MANIPULATIVES

- | | |
|--------------------|------------------|
| ▪ addition chart | ▪ colour tiles |
| ▪ attribute blocks | ▪ hundred chart |
| ▪ base-ten blocks | ▪ number line |
| ▪ calendar | ▪ pattern blocks |

MATHEMATICAL LANGUAGE

Teacher	Student
<ul style="list-style-type: none"> ▪ core, element ▪ describe, extend, compare, create, predict ▪ mode ▪ pattern rule ▪ repeating pattern 	<ul style="list-style-type: none"> ▪ core, element ▪ describe, extend, compare, create, predict ▪ pattern rule ▪ repeating pattern

Resources/Notes

Print

- *Making Math Meaningful to Canadian Students, K–8* (Small 2009), pp. 568–570, 571–576
- *Making Math Meaningful to Canadian Students, K–8, Second Edition* (Small 2013), pp. 605–612
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 276–280, 313–315

Notes

SCO PR02 Students will be expected to demonstrate an understanding of increasing patterns by describing, extending, and creating numerical patterns (numbers to 100) and non-numerical patterns using manipulatives, diagrams, sounds, and actions. [C, CN, PS, R, V]			
[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- PR02.01** Identify and describe increasing patterns in a variety of given contexts.
- PR02.02** Represent a given increasing pattern concretely and pictorially.
- PR02.03** Identify errors in a given increasing pattern.
- PR02.04** Explain the rule used to create a given increasing pattern.
- PR02.05** Create an increasing pattern and explain the pattern rule.
- PR02.06** Represent a given increasing pattern using another mode.
- PR02.07** Solve a given problem using increasing patterns.
- PR02.08** Identify and describe increasing patterns in the environment.
- PR02.09** Determine missing terms in a given concrete, pictorial, or symbolic increasing pattern and explain the reasoning.

Scope and Sequence

Mathematics 1	Mathematics 2	Mathematics 3
—	PR02 Students will be expected to demonstrate an understanding of increasing patterns by describing, extending, and creating numerical patterns (numbers to 100) and non-numerical patterns using manipulatives, diagrams, sounds, and actions.	<p>PR01 Students will be expected to demonstrate an understanding of increasing patterns by describing, extending, comparing, and creating numerical patterns (numbers to 1000) and non-numerical patterns using manipulatives, diagrams, sounds, and actions.</p> <p>PR02 Students will be expected to demonstrate an understanding of decreasing patterns by describing, extending, comparing, and creating numerical patterns (numbers to 1000) and non-numerical patterns using manipulatives, diagrams, sounds, and actions.</p>

Background

Increasing patterns is a new concept for students in Mathematics 2. An increasing pattern is a growing pattern where each term is increasing by a specific quantity. The following pattern shows each term increasing by one zero: 0, 00, 000, 0000, ...

Through examples and discussions, students should understand how an increasing pattern is different from a repeating pattern. It should be noted that any growing pattern involving numbers should not exceed the number 100.

The foundation of algebraic thinking is investigating patterns and their representations. Contextual, open-ended, and situation-specific problem solving should be an integral part of everyday mathematics instruction, tasks, and assessment. Students need to recognize and extend many different forms of the same pattern, including those constructed or in their environment. They must be able to predict a term in increasing patterns using many strategies within a variety of contexts, such as a hundred chart, calendars, number lines, and tiling patterns. These predictions should be verified by extending the pattern concretely, pictorially, and symbolically.

Additional Information

See Appendix A: Additional Information.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

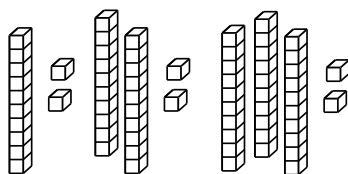
- Present students with two patterns. One should be a repeating pattern and one should be an increasing pattern. Ask them to point to the repeating pattern and explain the pattern rule. Ask them to tell why the other pattern is not a repeating pattern.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students to use tiles to create an increasing pattern. Ask them to explain their pattern rule.
- Ask students to identify an increasing pattern they see in the world and to describe that pattern.
- Ask students to create an increasing pattern in which a specific element is identified (e.g., the 5th element is 50).
- Present students with an increasing pattern and ask them to represent it in a different mode.

- Present students with the following base-ten blocks.



- Ask students to identify what would be next in the pattern and explain how they know.
- Ask students to use a number line to show an increasing pattern. Ask them to describe the pattern.
- Present an increasing pattern in which there is an error. Ask students to identify and correct the error, and explain their thinking.
- Present the following increasing pattern: Ask students to identify what would come next in the pattern and to explain the pattern rule.
- Ask students to create an increasing pattern that begins 1, 2, 3, ... Ask them to explain how they developed their patterns.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets 3 (Bauman 2009)

- Grade 3 Checkpoint 10, Task 2, pp. 50–51

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

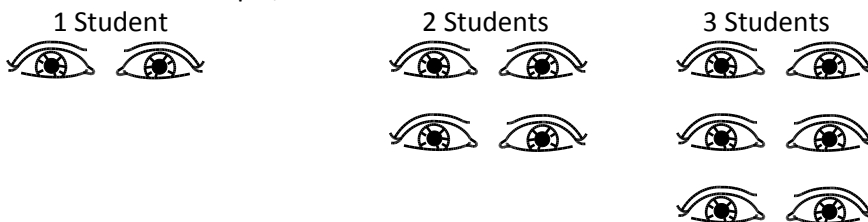
CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Include concrete materials in patterning tasks.
- Integrate patterns in physical education, music, visual arts, science, and other subject areas to provide a context.
- Encourage students to see that patterns may continue in many different ways unless a pattern rule is defined. For example, a pattern that begins 1, 2, 3, ... might continue in the following ways: 1, 2, 3, 1, 2, 3, 1, 2, 3, ... (repeating pattern) or 1, 2, 3, 4, 5, 6, 7, ... (increasing pattern).
- Provide opportunities for students to make predictions about patterns using a variety of materials.
- Expect students to communicate their thinking about patterns, verbally and in writing.

SUGGESTED LEARNING TASKS

- Provide students with a pattern rule for an increasing pattern (e.g., each number is two more than the one before). Ask students to build the pattern with concrete materials.
- Ask students to work in small groups to identify and describe increasing patterns that they see in the hundred chart or the calendar. Have each group present their discoveries to the class.
- Ask students to represent skip counting by 2s by showing jumps on a number line. Ask students to describe the increasing pattern that results.
- Provide the start of an increasing pattern using base-ten blocks. Ask students to continue the pattern and explain the pattern rule.
- Ask students to create two different increasing patterns with concrete materials that are similar, but not exactly the same. Ask them to comment on the similarities and differences.
- Tell students that the first two numbers in an increasing pattern are 5 and 10. Ask them to describe how the pattern could continue.
- Show students an increasing pattern such as 1, 4, 7, 10, ... Ask them to recreate the pattern with pictures of counters using a computer drawing program and to continue the pattern.
- Show students the following increasing pattern: 2, 12, 22, 32, 42, 52, ... Ask students to describe the pattern and to model it with base-10 blocks.
- Tell students that they are going to explore increasing patterns using the people in the classroom. Ask, “How many eyes are there in our classroom? How can we use increasing patterns to find the answer?” Model the process using digital pictures of students’ eyes and then translate them to numbers. For example,



Number of Students	1	2	3	4
Number of Eyes	2	4	6	8

SUGGESTED MODELS AND MANIPULATIVES

- base-ten blocks
- calendar
- colour tiles
- counters
- hundred chart
- number lines

MATHEMATICAL LANGUAGE

Teacher	Student
<ul style="list-style-type: none"> ▪ describe, reproduce, extend, create, predict ▪ growing pattern ▪ increases, grows ▪ increasing pattern ▪ mode ▪ pattern rule ▪ start number, amount of increase ▪ term 	<ul style="list-style-type: none"> ▪ describe, reproduce, extend, create, predict ▪ growing pattern ▪ increases, grows ▪ increasing pattern ▪ pattern rule ▪ start number, amount of increase ▪ term

Resources/Notes**Print**

- *Making Math Meaningful to Canadian Students, K–8* (Small 2009), pp. 570–577
- *Making Math Meaningful to Canadian Students, K–8, Second Edition* (Small 2013), pp. 608–609, 613–614
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 281–282, 284–288

Notes

SCO PR03 Students will be expected to demonstrate and explain the meaning of equality and inequality by using manipulatives and diagrams (0 to 100).

[C, CN, R, V]

[C] Communication

[PS] Problem Solving

[CN] Connections

[ME] Mental Mathematics and Estimation

[T] Technology

[V] Visualization

[R] Reasoning

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

PR03.01 Determine whether two given quantities of the same object (same shape and mass) are equal by using a balance scale.

PR03.02 Construct and draw two unequal sets using the same object (same shape and mass) and explain the reasoning.

PR03.03 Demonstrate how to change two given sets, equal in number, to create inequality.

PR03.04 Choose from three or more given sets the one that does not have a quantity equal to the others and explain why.

Scope and Sequence

Mathematics 1	Mathematics 2	Mathematics 3
PR03 Students will be expected to describe equality as a balance and inequality as an imbalance, concretely and pictorially (0 to 20).	PR03 Students will be expected to demonstrate and explain the meaning of equality and inequality by using manipulatives and diagrams (0 to 100).	PR03 Students will be expected to solve one-step addition and subtraction equations involving symbols representing an unknown number.

Background

Balance tasks form a basis for understanding equality. Working with balance scale problems involving concrete materials and pictures helps students build foundations for further study in operations, algebra, and equation solving. Using concrete materials will help students see the equality or inequality relationship between the quantities on each pan of the balance scale.

Throughout balance tasks, students should answer questions, such as

- What is the relationship between the two quantities if the scale is balanced? (This provides opportunities to use the phrases “is the same as” and “is equal to”.)
- What is the relationship between the quantities if the left balance pan is lower than the right pan? (This provides opportunities to use the phrases “is more than” and “is not equal to,” and to discuss the “is-less-than” relationship between the right and left quantities on the pans.)
- What is the relationship between the quantities if the left balance pan is higher than the right pan? (This provides opportunities to use the phrases “is less than” and “is not equal to,” and to discuss the is-more-than relationship between the right and left quantities on the pans.)

This outcome should be addressed in conjunction with the development of the symbolic representations in outcome PR04.

Additional Information

See Appendix A: Additional Information.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Ask students to create two sets of cubes (each less than 20) that are equal. Ask them to use a pan balance to prove that they are equal. Then, ask them to change the sets so that they are not equal.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Show students three different balance scales with opaque bags of cubes placed on them. The first balance should represent an equality. The second balance should represent an inequality with the left pan lower than the right. The third balance should represent an inequality with the right pan lower than the left. Tell students that the bags contain cubes of the same mass and shape.
 - **Task 1:** Ask students to identify which balance represents equal sets and to explain their thinking. Ask them to tell you how many cubes might be in each bag.
 - **Task 2:** Ask students to identify which balance shows unequal sets. Ask them to explain their thinking. Ask them to tell you how many cubes might be in each of the bags.
 - **Task 3:** If during Task 2 the student identified only one of the two remaining balances as showing unequal sets, ask them to tell you about the remaining balance.
- Use a set of dominoes. Select three dominoes—two that are equal in value and one that is unequal. Ask students to identify the domino that is not equal to the others and to explain how they know.
- Ask students to represent two numbers (each less than 100) on a balance scale using counters. Ask them to tell if the two quantities are equal and to explain how they know.
- Have students build an equal or an unequal relationship using Cuisenaire rods and explain their thinking.
- Show students a pan balance with 25 cubes on one side and 52 cubes on the other side. Ask students to tell if the two sets are equal or not equal. Ask them to explain their thinking. Ask students to explain what to do to make the two sets equal.

- Ask students to use concrete materials and a pan balance to prove that $9 + 5 = 7 + 7$ and that $4 \neq 5 - 3$.
- Ask students to use concrete materials to prove that $35 \neq 20 + 20$.
- Ask students to use base-ten blocks to represent two numbers that are not equal. Ask them to explain their thinking.
- Ask students to use base-ten blocks to represent two numbers that are equal. Ask them to explain their thinking.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets K–2 (Bauman 2011)

- No Checkpoint for this outcome.

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?





CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Use of the words “the same as” for the equal sign and “not the same as” for the unequal sign will help students see that the symbols represent a relationship. Students should become familiar with the terminology that “inequality” means “is greater than” or “is less than” and “equality” means “the same as.”
- Give students opportunities to develop their own equalities and inequalities on a pan balance.
- Give students many opportunities to construct equal and unequal sets and to translate their findings to diagrams and then to number sentences.

SUGGESTED LEARNING TASKS

- Present students with a pan balance on which you have placed two opaque bags containing quantities of cubes. Ask students to tell which bag contains the greatest number of cubes and to explain how they know. Students should answer questions, such as
 - What is the relationship between the two quantities if the scale is balanced? (This provides opportunities to use the phrases “is the same as” and “is equal to.”)
 - What is the relationship between the quantities if the left balance pan is lower than the right pan? (This provides opportunities to use the phrases “is more than” and “is not equal to,” and to discuss the “is-less-than relationship” between the right and left quantities on the pans.)
 - What is the relationship between the quantities if the left balance pan is higher than the right pan? (This provides opportunities to use the phrases “is less than” and “is not equal to,” and to discuss the “is-more-than” relationship between the right and left quantities on the pans.)
- Use True or False tasks. Present various relationships, such as $8 + 4$ is the same as $12 + 5$, or $13 = 13$, or 16 is the same as $25 - 9$. Have students use pan balances to determine whether the statement is true or false and to justify their answers.
- Students work in pairs. One partner uses ten-frames to represent two 2-digit numbers. The other partner must determine whether the ten-frame display represents equal or unequal sets.
- Students work in pairs. Each pair is given a set of cards with numerals to 100 and a set of word cards “is the same as” and “is not the same as.” One student draws a numeral card and creates a set of cubes to represent that number. The same student draws a word card and places it to the right of the set of cubes. The second student must create a second set of cubes to make the statement true. Both students record the statement in pictorial and/or symbolic form.
- Ask students to find dominoes that satisfy relationships such as “is the same as” or “is not the same as.” For example,


 is the same as
 

 is not the same as
 
- Have students compare three or more given sets to determine which are equal/not equal and explain their reasoning.
- Ask students to write a problem that involves the number 12 as shown below in the number sentence (the blanks are numbers): $\underline{\quad} + 12 = \underline{\quad}$ and $\underline{\quad} - 12 = \underline{\quad}$.

SUGGESTED MODELS AND MANIPULATIVES

- Cuisenaire rods
- five-frames and ten-frames
- dominoes
- pan balance
- dot cards

MATHEMATICAL LANGUAGE

Teacher	Student
<ul style="list-style-type: none"> ▪ balance ▪ balance scale ▪ equal sets, unequal sets ▪ equality, inequality ▪ is more than, is less than ▪ is not the same as, is not equal to ▪ is the same as, is equal to 	<ul style="list-style-type: none"> ▪ balance ▪ balance scale ▪ equal sets, unequal sets ▪ is more than, is less than ▪ is not the same as, is not equal to ▪ is the same as, is equal to

Resources/Notes

Print

- *Making Math Meaningful to Canadian Students, K–8* (Small 2009), pp. 585–586
- *Making Math Meaningful to Canadian Students, K–8, Second Edition* (Small 2013), p. 625
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 70, 125, 133–134, 299–301

Notes

SCO PR04 Students will be expected to record equalities and inequalities, symbolically, using the equal symbol or not equal symbol. [C, CN, R, V]			
[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- PR04.01** Determine whether two sides of a given number sentence are equal (=) or not equal (\neq). Write the appropriate symbol and justify the answer.
- PR04.02** Model equalities using a variety of concrete representations and record the equality.
- PR04.03** Model inequalities using a variety of concrete representations and record the inequality.

Scope and Sequence

Mathematics 1	Mathematics 2	Mathematics 3
PR04 Students will be expected to record equalities using the equal symbol.	PR04 Students will be expected to record equalities and inequalities, symbolically, using the equal symbol or not equal symbol.	PR03 Students will be expected to solve one-step addition and subtraction equations involving symbols representing an unknown number.

Background

Students need to understand that the equal sign (=) is a symbol that represents a relationship between quantities and does not mean “the answer is.” It tells us the quantity on the left-hand side is the same as the quantity on the right-hand side; that is, the two sides are balanced. Often, when students only have experience using the equal sign in addition and subtraction number sentences, they mistakenly believe the sign (=) means is the answer to and, therefore, do not accept as possible, number sentences such as $8 = 5 + 3$, $6 = 6$, and $4 + 5 = 2 + 7$. However, through balance tasks, students see that such number sentences express equality relationships between quantities on the two sides of a balance scale, or in a one-to-one correspondence. As such, these number sentences are not only possible, but are necessary to express the relationships.

This outcome should be addressed in conjunction with the development of the meaning of equality and inequality addressed in outcome PR03.

The focus of this outcome is to have students interpret the = and \neq signs. This is the first time students will have experiences with the \neq sign and it means two quantities are not equal. They must be able to write these symbols to describe the relationship between quantities with or without an operation, such as $72 = 72$, $7 \neq 5$, $21 = 18 + 3$, and $10 - 6 \neq 3 + 3$. It may be easier balancing one addition or subtraction sentence first, such as $10 + 4 = 14$ and $12 - 7 \neq 7$, and then moving to balancing two addition or subtraction sentences, such as $1 + 4 \neq 7 + 3$ and $5 - 2 = 6 - 3$. Lastly, students should be ready to deal with addition and subtraction sentences in combination, such as $3 + 9 = 13 - 1$.

When the focus is on the number relationships between sides of number sentences, students become more flexible in their thinking and may find more efficient ways to solve problems. Exploring the relationship between the expressions on either side of an equal sign enables students to develop an understanding of relationships that may exist between these expressions. It is important that students discover these relationships on their own. For example, they may recognize that $16 + 18 = 18 + 16$ without adding both sides because they realize both sides involve the addition of the same two numbers. They may recognize that $4 + 8 = 3 + 9$ without adding both sides because they realize 1 has been subtracted from 4 and added to 8 to create 3 and 9. Such thinking is called relational thinking, and it should be encouraged and explored; however, it will likely not be typical of most students in Mathematics 2. One way to encourage relational thinking is by providing examples that encourage students to use their number sense and to examine the numbers in a computation to determine if it can be simplified. For example, in the sentence $8 + 7 - 7 = \underline{\quad}$, students may recognize that adding 7 and then subtracting 7 will leave 8 unaffected; therefore, the computation is not necessary to know the solution. Other equations, such as $8 + 4 = \underline{\quad} + 5$, can be solved only if students have a broad understanding of the equal sign. Students can solve this sentence using relational thinking by noticing that 5 is 1 more than 4, so the unknown number has to be one less than 8.

Informal work with inequalities and equalities may lead students to discover that there could be more than one possible answer to satisfy a relationship, such as $5 + ? + ? = 9$, $6 + 7 \neq ? + 5$.

Note: A common error made by students in solving $8 + 4 = \underline{\quad} + 5$ is to place 12 in the blank. These students have not yet internalized that the equal sign means is the same as; rather, they think it is a call for an answer to $8 + 4$. Students need to regularly explore four types of equality relationships: whole is equal to whole ($6 = 6$); whole is equal to part-part ($6 = 4 + 2$); part-part is equal to whole ($4 + 2 = 6$), and part-part is equal to part-part ($4 + 2 = 5 + 1$).

Additional Information

See Appendix A: Additional Information.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Ask students to fill in the blank (with a whole number or a combination of numbers showing an operation) to complete the number sentence. $\underline{\quad} = 7 + 6$ or $4 + 2 = \underline{\quad}$ or $5 - 3 = \underline{\quad}$ or $\underline{\quad} = 5 - 1$

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students to build number sentences to demonstrate an understanding of equality using ten-frames. Place a ten-frame on either side of the equal sign and have students place different coloured counters to show more than one combination (e.g., $5 + 5 = 8 + 2$ or $10 = 2 + 2 + 6$ or $6 + 4 = 7 + 3$).
- Complete the number sentence so that it is true:
 $5 + 1 = \underline{\quad} + 2$; or $4 + \underline{\quad} = 2 + 2 + 2$; or $\underline{\quad} + 0 = 30 - 1$; or $16 + 5 \neq \underline{\quad} - 7$
- Provide students with a number sentence representing equality and ask students to model the number sentence using a pan balance and explain their thinking.
- Have students build an equal or an unequal relationship using Cuisenaire rods and explain their thinking to a partner.
- Provide students with the following expressions:
 $3 + 7$ $6 + 4$ $9 + 1$ $8 + 2$ $4 + 7$ $5 + 5$
Ask students to identify, using concrete materials, which expression does not have a quantity equal to the others and explain their thinking.
- Ask a student to write number sentences using the equal sign (=) and/or the not equal sign (\neq) and then explain their reasoning.
- Ask a student to use concrete materials and pictures to determine whether the following statements are true:
 $3 + 2 = 9 - 4$ $0 \neq 1 + 0$ $0 = 8 - 8$ $7 - 4 \neq 6 - 3$

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets K–2 (Bauman 2011)

- No Checkpoint for this outcome.

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Introduce students to a variety of equation types. For example: $6 + 7 = 13$ or $68 = 61 + 7$ or $8 - 5 = 7 - 4$.
- Use of the words “the same as” for the equal sign and “not the same as” for the unequal sign will help students see that the symbols represent a relationship. Students should become familiar with the terminology that **inequality** means “is greater than” or “is less than” and **equality** means “the same as.”
- Give students opportunities to develop their own equations and inequalities.
- Give students many opportunities to construct equal and unequal sets and to translate their findings to diagrams and then to number sentences.

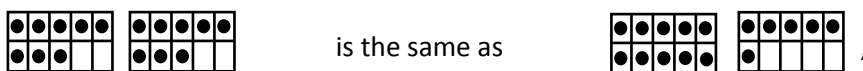
SUGGESTED LEARNING TASKS

- Challenge students to find different ways to express a particular number encouraging the use of different operations or relationships. For example: $10 \neq 10 + 2$, $10 = 2 + 8$, 10 is less than 24 . Include examples with more than two numbers on the same side. For example:

$$10 = 4 + 1 + 5 \quad 17 - 2 - 5 = 10$$

Ask students to use concrete materials to prove that they are correct.

- Provide students with pictures of ten-frames showing equalities and inequalities. Ask students to write number sentences that match the pictures. For example, if shown



students would record $8 + 8 = 10 + 6$.

- Provide each pair of students with a sheet of chart paper and a 2-digit number. Ask each pair to work co-operatively to decide whether they will create representations that are equal to or not equal to their number. Pairs of students then work to create a pictorial display for their number in accordance with the decision they made. After all students have completed their charts, post them. Ask students to read each of the charts and to decide whether the display shows “is equal to” or “is not equal to.”
- Use True or False tasks. Present various relationships, such as $8 + 4 = 12 + 5$ or $13 = 13$ or $16 = 25 - 9$. Have students tell whether it is true or false and justify their answers using concrete materials and/or pictures.
- Ask students to write true sentences in several different forms. For example, $_ + _ = _ + _$; or $_ - _ = _ - _$, or $_ + _ = _ - _$, or $_ + _ \neq _ - _$, or $_ = _ - _$, or $_ = _ + _$.
- Have students compare three or more given sets to determine which are equal/not equal and explain their reasoning.
- Ask students to write a problem that involves the number 12 as shown below in the number sentence (the blanks are numbers): $_ + 12 = _$, $_ + 12 \neq _$, $_ - 12 = _$, $_ - 12 \neq _$

SUGGESTED MODELS AND MANIPULATIVES

- Cuisenaire rods
- dominoes
- dot cards
- five-frames and ten-frames
- pan balance

MATHEMATICAL LANGUAGE

Teacher	Student
<ul style="list-style-type: none"> ▪ addition sentence, subtraction sentence ▪ balance ▪ equal sign, not equal sign (= and \neq) ▪ equal, not equal ▪ equality, inequality ▪ is not the same as, is not equal to ▪ is the same as, is equal to ▪ number sentence 	<ul style="list-style-type: none"> ▪ balance ▪ equal sign, not equal sign (= and \neq) ▪ equal, not equal ▪ is not the same as, is not equal to ▪ is the same as, is equal to ▪ number sentence

Resources/Notes

Print

- *Making Math Meaningful to Canadian Students, K–8* (Small 2009), pp. 585–586
- *Making Math Meaningful to Canadian Students, K–8, Second Edition* (Small 2013), pp. 624, 626–627
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 70, 125, 133–134, 299–301

Notes

Measurement

GCO: Students will be expected to use direct and indirect measure to solve problems.

Specific Curriculum Outcomes: Measurement (M)

Process Standards

[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

- M01** Students will be expected to demonstrate an understanding of the calendar and the relationships among days, weeks, months, and years. [C, CN, PS, R]
- M02** Students will be expected to relate the size of a unit of measure to the number of units (limited to non-standard units) used to measure length and mass. [C, CN, ME, R, V]
- M03** Students will be expected to compare and order objects by length, height, distance around, and mass using non-standard units and make statements of comparison. [C, CN, ME, R, V]
- M04** Students will be expected to measure length to the nearest non-standard unit by using multiple copies of a unit and using a single copy of a unit (iteration process). [C, ME, R, V]
- M05** Students will be expected to demonstrate that changing the position of an object does not alter the measurements of its attributes. [C, R, V]

SCO M01 Students will be expected to demonstrate an understanding of the calendar and the relationships among days, weeks, months, and years.

[C, CN, PS, R]

[C] Communication

[PS] Problem Solving

[CN] Connections

[ME] Mental Mathematics and Estimation

[T] Technology

[V] Visualization

[R] Reasoning

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

M01.01 Read a calendar.

M01.02 Name and order the days of the week and months of the year.

M01.03 Communicate the number of days in a week and the number of months in a year.

M01.04 Solve a given problem involving time which is limited to the number of days in a week and the number of months in a year.

Scope and Sequence

Mathematics 1	Mathematics 2	Mathematics 3
—	<p>M01 Students will be expected to demonstrate an understanding of the calendar and the relationships among days, weeks, months, and years.</p>	<p>M01 Students will be expected to relate the passage of time to common activities using non-standard and standard units (minutes, hours, days, weeks, months, years).</p> <p>M02 Students will be expected to relate the number of seconds to a minute, the number of minutes to an hour, the number of hours to a day, and the number of days to a month in a problem-solving context.</p>

Background

Although calendar work has not been an outcome in previous grades, some students will have some previous knowledge with a calendar either in school or at home. Calendar tasks should be incorporated into daily routines, helping students make connections between their experiences and real-world scenarios. It is important that students in Mathematics 2 be given opportunities to explore and learn different calendar units (days, weeks, and months) and relationships among these units. Each month brings a new calendar page to explore. It is expected that students will communicate their understanding of calendar skills through questioning, games, problem solving, journal writing, and group discussions.

In the real world, the calendar is used to plan, keep track of appointments, and measure time. This is how it should be used in the classroom. In order to focus on the structure of the month and numerical patterns, engage students, at the beginning of the month, in a teacher-directed activity where students build and create their own monthly calendar. They will need to write the month, write the days of the week in order, number the days, and fill in any special dates for that month, such as class trips and holidays. Having a one page, year-long calendar nearby, will help students see and understand where the current month fits into a year's progression.

Additional Information

- See Appendix A: Additional Information.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Ask students, What day of the week is it today?

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Point to a date on the calendar. Ask students to read the date. They should tell you the month, the day of the week, and the date (e.g., Tuesday, April 6).
- Sequence a set of events occurring in a familiar story involving days or months.
- Ask students to name the days in order beginning with Sunday.
- Ask students to name the months of the year beginning with January.
- Tell students, Today is Wednesday. Ask, What day was it yesterday? What day will it be tomorrow?
- Tell students, Billy says that 7 days is longer than a week. Ask students to explain whether Billy is correct or not and to explain their thinking.
- Have students solve problems involving time, such as
 - a) If Grandma eats an apple every second day for three weeks, how many apples will she eat?
 - b) It is April 16th. In two weeks, Bethany is going to the dentist. On what date will Bethany go to the dentist?
 - c) You go swimming in the lake in July. What might you be doing at the lake seven months later?
 - d) It is the 11th day of the month. If our class is going on a field trip in 16 days, what is the date of our field trip?
- Ask students, How might you use a calendar to help subtract 14 from a number?

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets K–2 (Bauman 2011)

- No Checkpoint for this outcome.

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Use a calendar daily to explore days and months.
- Give students problematic situations to work on, such as pointing out the date on the calendar and asking how many days/weeks until a specific holiday or event.
- Provide students with experiences involving comparisons between events of differing durations to help them understand time. For example, How many days in a school week as compared to a calendar week? How many months until your friend’s birthday compared to your birthday?
- Use school activity calendars and lunch menus to explore days and months.
- Read children’s literature such as *Chicken Soup with Rice: A Book of Months* by Maurice Sendak (1991) to your students. Provide students with a random set of the sequence of months from the poem and have them order the months. This activity could be duplicated with the days of the week using *Today Is Monday* by Eric Carle (2002) or a poem such as “Monday’s Child” (Mother Goose nursery rhyme).
- Assign two students each day to the calendar routine. Routines could include, Write today’s date and be prepared to give the dates for yesterday and tomorrow; highlight any special events listed for the day or locate today’s date on the year-long calendar.

SUGGESTED LEARNING TASKS

- List the days of the week in order along a seven-section number line. Attach the ends of the number line to complete a circle. This unit (one week) demonstrates the cyclical property of how 7 days equals 1 week (unitizing). This activity can be extended to include several weeks. A similar activity can be developed to show the months of the year.
- Show students a calendar for the year. Ask him or her to point out the day's date and to find out what date it will be in 6 weeks.
- Show students a calendar for the year and ask him or her to identify ways in which months are the same and ways in which they differ.
- Have students solve problems where they must determine what day it would be 3 days from now. This activity could be varied by using months instead of days and changing the number.
- Have students prepare a schedule on a blank calendar starting on a particular day/date, for a specific job (e.g., recycling pick up), that is repeated regularly (e.g., biweekly).
- Ask different types of calendar questions such as, Is the date of the third Wednesday an even or an odd number? On which day of the week is the first 2-digit number? On which day of the week is the last 1-digit number? On which day of the week is the 9th? How many full weeks are in this month? Which day of the week is 9 days after the 6th? Find the second week. What is Friday's date that week?

SUGGESTED MODELS AND MANIPULATIVES

- calendar
- number lines

MATHEMATICAL LANGUAGE

Teacher	Student
<ul style="list-style-type: none"> ▪ calendar ▪ date ▪ day, week, month, year ▪ days of the week: Sunday, Monday, ... ▪ months of the year: January, February, ... ▪ next, before ▪ today's, yesterday's, tomorrow's date 	<ul style="list-style-type: none"> ▪ calendar ▪ date ▪ day, week, month, year ▪ days of the week: Sunday, Monday, ... ▪ months of the year: January, February, ... ▪ next, before ▪ today's, yesterday's, tomorrow's date

Resources/Notes

Print

- *Making Math Meaningful to Canadian Students, K–8* (Small 2009), pp. 573–575
- *Making Math Meaningful to Canadian Students, K–8, Second Edition* (Small 2013), pp. 612–613

Notes

SCO M02 Students will be expected to relate the size of a unit of measure to the number of units (limited to non-standard units) used to measure length and mass.

[C, CN, ME, R, V]

[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- M02.01** Explain why one of two given non-standard units may be a better choice for measuring the length of an object.
- M02.02** Explain why one of two given non-standard units may be a better choice for measuring the mass of an object.
- M02.03** Select a non-standard unit for measuring the length or mass of an object and explain why it was chosen.
- M02.04** Estimate the number of non-standard units needed for a given measurement task.
- M02.05** Explain why the number of units of a measurement will vary depending upon the unit of measure used.

Scope and Sequence

Mathematics 1	Mathematics 2	Mathematics 3
<p>M01 Students will be expected to demonstrate an understanding of measurement as a process of comparing by</p> <ul style="list-style-type: none"> ▪ identifying attributes that can be compared ▪ ordering objects ▪ making statements of comparison ▪ filling, covering, or matching 	<p>M02 Students will be expected to relate the size of a unit of measure to the number of units (limited to non-standard units) used to measure length and mass.</p> <p>M03 Students will be expected to compare and order objects by length, height, distance around, and mass using non-standard units and make statements of comparison.</p>	<p>M03 Students will be expected to demonstrate an understanding of measuring length (cm, m) by</p> <ul style="list-style-type: none"> ▪ selecting and justifying referents for the units centimetre and metre (cm, m) ▪ modelling and describing the relationship between the units centimetre and metre (cm, m) ▪ estimating length using referents ▪ measuring and recording length, width, and height <p>M04 Students will be expected to demonstrate an understanding of measuring mass (g, kg) by</p> <ul style="list-style-type: none"> ▪ selecting and justifying referents for the units gram and kilogram (g, kg) ▪ modelling and describing the relationship between the units gram and kilogram (g, kg) ▪ estimating mass using referents ▪ measuring and recording mass

Background

In previous grades the measurement outcomes focused on direct comparisons of attributes of objects. Students realized that an object has several attributes that can be compared to other objects or figures. For example, a 3-D object has mass; length, width, and height; volume; and capacity. Any one of these attributes can be directly compared to the same attribute of another object. This can be done by lifting both objects to compare masses; placing objects side-by-side to compare lengths, widths, or heights; filling one object with sand or water and pouring into the other object to compare capacities; and placing objects side-by-side to compare volumes.

In this Mathematics 2 outcome, students will begin to measure the attributes of mass and length using non-standard units, getting numerical measurements that vary according to the size of the unit used. This relationship between the size of the unit and the number of units is an inverse relationship (the larger the non-standard unit, the smaller the number of units or the smaller the non-standard unit, the larger the number of units). While this inverse relationship is obvious to adults, it is often a difficult concept for young students and will need to be carefully developed.

Once the attribute to be measured is understood, a unit of measure can be chosen to measure that attribute. Students should understand that they must use multiple uniform copies of their chosen unit of measure to measure the attribute. For example, if they are going to measure the lengths of their desks using paper clips, all those paper clips have to be the same length. Students should concentrate on the measurement process. For example, to measure the length of their desks using paper clips, they must place the paper clips in a straight line against the length of the desks with no gaps or overlaps. Research has shown that using non-standard units first allows students to concentrate on the attribute of the object being measured and the measurement process, both of which seem to get lost when the focus is on a standard unit of measure. Estimation is a key component to the measurement process and should be developed while students are using non-standard units. Prior to actually measuring an attribute of an object with non-standard units, have students estimate how many of those units they think they will need to use.

Note: When addressing this outcome, note that the terms **mass** and **weight** are similar, but they are not the same. Weight measures how heavy an object is and depends upon gravity, so it will vary with height above sea level. For example, weight is the number showing on bathroom scales when you weigh yourself. On the other hand, mass measures the amount of matter in an object and will be the same at all heights above sea level. For example, mass is the value you get when an object is measured on a balance scale. Students should be exposed to the correct term **mass**; therefore, mass measurements should be made using non-standard units on balance scales.

Additional Information

See Appendix A: Additional Information.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Provide students with two objects such as an eraser and a book. Ask, Can you tell which of these two objects is longer? Can you tell which of these two objects is heavier? After each question, have students explain their thinking.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Give students an object to hold in one hand. Ask each student to estimate how many of another non-standard unit would have the same mass and explain their thinking.
- Provide students with containers filled with various sizes of paper clips. Ask students to estimate the length of a table using large paper clips. Then, ask them to estimate the length of the table using small paper clips. Note whether their estimates reflect the impact of the different sized units of measure. This can also be done using cubes of various masses. Students can be asked to estimate the mass of an object.
- Have students choose one of two non-standard units to measure a particular length/mass and to defend their choice (e.g., linking cubes, a toothpick, and a straw as choices of non-standard units to measure the length of a table).
- Ask questions to demonstrate the reasonableness of estimates (e.g., Is this pencil 6 paper clips or 60 paper clips long?).
- Ask students to use several different non-standard units to measure the same object. Ask, Which would give you the closest measure? How do you know? (e.g., if students are given paper clips, pencils, and linking cubes, they may identify the cubes as being the most accurate if they fit almost exactly on the item they are measuring.)
- Tell students that Mark and Eli measured the length of the teacher's desk. Mark said that the teacher's desk was 45 units long. Eli said that the teacher's desk was 15 units long. Both students measured correctly. Ask students to explain why they might have gotten different answers?

FOLLOW-UP ON ASSESSMENT**Guiding Questions**

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets K–2 (Bauman 2011)

- Grade Primary Checkpoint 11, pp. 105–106
- Grade 1 Checkpoint 11, pp. 107–108
- Grade 2 Checkpoint 10, Task 2, pp. 101–102 (Line Master 10.1)
- Grade 2 Checkpoint 11, Task 1, pp. 109–110

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Provide a variety of non-standard measurement tools, such as stir sticks, paper clips of various sizes, toothpicks, straws, unifix or linking cubes, colour tiles, pennies, and blocks.
- Provide students the opportunity to make their own simple measuring instruments using objects such as paper clips. The units are lined up and marked off on a length of paper. This will link the understanding that it is the space and not the mark that indicates a measure of length on a ruler.
- Give students many opportunities to use a variety of non-standard units to measure an object and then to discuss which is most appropriate and why.
- Students require practise making and accepting estimates. Model the use of referents at all times, (e.g., placing one paper clip beside a pencil before estimating).
- Encourage students to recognize that the mass of objects is not related to its size (e.g., compare a golf ball with an inflated balloon).

SUGGESTED LEARNING TASKS

- Have students work in small groups. Provide each group with a different non-standard unit, such as small paper clips, stir sticks, straws, counters, or toothpicks. Have each group use the given unit to measure the length of their table or desk and record their results. Debrief the activity with the whole class. After each group presents their results, ask questions such as, Did any group have problems using their measuring unit? Did you find your measuring unit easy to use? Why or why not? Each group measured the same object. Why did groups get a different number of units? Does the size of the measuring unit make a difference? This can be repeated for mass using different non-standard units.
- Provide students with a variety of non-standard units and an object to measure. Ask them to determine the best (most efficient and accurate) non-standard measurement unit to use and to justify their choice of unit.
- Ask students to search for items that have estimates of a given number of paper clips long or a mass of two bags of pennies.
- Provide students with a length of string and have them estimate how many paper clips long it is.
- Provide students with a large object such as an inflated balloon and a small object such as a rock. Ask them to estimate how many pennies it would take to match the mass of each of the objects.
- Have students develop a book on length and/or mass that they can add to over time. Students can draw a picture of the object they measured and record below it the number of non-standard units used to measure it. Alternately, they could create a book for a specific measurement, for example, “Objects That Are Four Paper Clips Long” or “Objects That Have a Mass of 10 Linking Cubes.”
- Read the book, *How Big Is a Foot?* by Rolf Myller (1991), and relate the story to non-standard measurement.

SUGGESTED MODELS AND MANIPULATIVES

- | | |
|------------------------------------|----------------|
| ▪ blocks | ▪ paper clips |
| ▪ everyday objects such, as string | ▪ paper strips |
| ▪ hand spans | ▪ pennies |
| ▪ linking cubes | |

MATHEMATICAL LANGUAGE

Teacher	Student
<ul style="list-style-type: none"> ▪ about, little less than, little more than ▪ balance scale ▪ compare ▪ estimate, measure ▪ length, width, height, mass ▪ non-standard unit ▪ object 	<ul style="list-style-type: none"> ▪ about, little less than, little more than ▪ balance scale ▪ compare ▪ estimate, measure ▪ length, width, height, mass ▪ unit ▪ object

Resources/Notes

Print

- *Making Math Meaningful to Canadian Students, K–8* (Small 2009), pp. 370–375, 430–433
- *Making Math Meaningful to Canadian Students, K–8, Second Edition* (Small 2013), pp. 414–415, 418–422, 478–479
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 58–59, 223–231, 241–242

Notes

SCO M03 Students will be expected to compare and order objects by length, height, distance around, and mass using non-standard units and make statements of comparison.

[C, CN, ME, R, V]

[C] Communication

[PS] Problem Solving

[CN] Connections

[ME] Mental Mathematics and Estimation

[T] Technology

[V] Visualization

[R] Reasoning

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

M03.01 Estimate, measure, and record the length, height, distance around, or mass of a given object using non-standard units.

M03.02 Compare and order the measure of two or more objects in ascending or descending order and explain the method of ordering.

Scope and Sequence

Mathematics 1	Mathematics 2	Mathematics 3
<p>M01 Students will be expected to demonstrate an understanding of measurement as a process of comparing by</p> <ul style="list-style-type: none"> ▪ identifying attributes that can be compared ▪ ordering objects ▪ making statements of comparison ▪ filling, covering, or matching 	<p>M02 Students will be expected to relate the size of a unit of measure to the number of units (limited to non-standard units) used to measure length and mass.</p> <p>M03 Students will be expected to compare and order objects by length, height, distance around, and mass using non-standard units and make statements of comparison.</p>	<p>M03 Students will be expected to demonstrate an understanding of measuring length (cm, m) by</p> <ul style="list-style-type: none"> ▪ selecting and justifying referents for the units centimetre and metre (cm, m) ▪ modelling and describing the relationship between the units centimetre and metre (cm, m) ▪ estimating length using referents ▪ measuring and recording length, width, and height <p>M04 Students will be expected to demonstrate an understanding of measuring mass (g, kg) by</p> <ul style="list-style-type: none"> ▪ selecting and justifying referents for the units gram and kilogram (g, kg) ▪ modelling and describing the relationship between the units gram and kilogram (g, kg) ▪ estimating mass using referents ▪ measuring and recording mass <p>M05 Students will be expected to demonstrate an understanding of perimeter of regular, irregular, and composite shapes by</p> <ul style="list-style-type: none"> ▪ estimating perimeter using referents for centimetre or metre (cm, m) ▪ measuring and recording perimeter (cm, m) ▪ creating different shapes for a given perimeter (cm, m) to demonstrate that many shapes are possible for a perimeter

Background

Children should recognize that length tells about the extent of an object along one dimension. Initially, they would compare lengths informally by simply viewing the two lengths. Later, they should investigate strategies to compare the lengths of two or more objects directly and indirectly. Direct comparison involves comparing lengths by lining up items side by side, beginning at a common base. Students should be led to see why a common starting point is important. Indirect comparison involves comparing lengths using another object. This is particularly useful when it is not possible to physically line up the objects. For example, to compare length of hand to wrist size, students may cut pieces of string the length of their hands, and then wind the strings around their wrists for comparison. Students are encouraged to compare and order the length and height using appropriate mathematical vocabulary, such as **length**, **height**, **longer than**, **shorter than**, and **number of units**.

Distance around refers to perimeter and circumference. **Distance around** is a term that students in Mathematics 2 may be more comfortable using instead of the formal terminology, **perimeter**. Estimating distance around, especially when that distance cannot be easily treated as linear parts (such as a circle), is very difficult for students. They need many experiences such as the one that follows. Initially, students will learn to measure the perimeter of a shape, such as their desk, by fitting a string around it and cutting the string to that length. Once students are comfortable measuring the distance around a linear object (one that has straight edges), students are more capable of understanding the notion of measuring around curves. When measuring round or curvy objects is first introduced, students should use material such as string, ribbon, or wool.

Students should recognize that mass tells about the heaviness of an object. They should explore methods to compare and order masses, including situations involving both direct and indirect comparisons. Direct comparisons involve, for instance, placing two objects on a balance simultaneously and comparing their masses. Indirect comparisons involve comparing the masses of two objects by using another object as a referent.

After objects are ordered by an attribute using either direct or indirect comparisons, students should estimate and measure the attribute using a non-standard unit. It is the numerical values assigned to the attribute that are the measurements of those attributes. If students estimate and measure the smallest or largest of the objects, they can use that result to refine their estimates of the other objects before they are measured.

While students are likely to use a variety of language phrases when referring to comparisons of attributes of objects, they should hear correct language they can begin to model. For example, one object may be said to be longer than another object because it has a greater measure. Students may say “bigger measure”; however, measures are numerical, and numbers are compared using greater and less.

Additional Information

See Appendix A: Additional Information.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Provide students with a set of ribbons that are of various lengths. Ask them to show you the ribbons that are the shortest and the longest. Point to a particular ribbon and ask them to tell show you a ribbon that is longer than yours. Point to a particular ribbon and ask them to show you a ribbon that is shorter than yours.
- Provide students with a set of objects that have different masses. Ask them to show you the object that is the heaviest and the object that is the lightest.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Give students an object to hold in one hand. Ask each student to show how many of another non-standard unit would have the same mass and to explain their thinking.
- Provide a set of objects for students. Ask them to measure the mass of each object using non-standard units and then to order them from heaviest to lightest.
- Ask students to estimate and measure the distance around a water bottle.
- Ask students how they would compare the height of a garbage can to the distance around the top of the can and explain.
- Ask students to prepare a set of ribbons for first, second, and third places in a race, so that the faster runner gets a longer ribbon.
- Ask students, If three kiwis have the same mass as one apple, what do you know about the mass of the apple?
- Provide a group of objects to be measured; for example, a book, a piece of paper, a length of string, and a pencil. Ask students to use non-standard units to measure each object and then to identify the object that is longest and the one that is shortest.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets K–2 (Bauman 2011)

- Grade 2 Checkpoint 9, pp. 93–94
- Grade 2 Checkpoint 10, Task 1, pp. 101–102
- Grade 2 Checkpoint 11, Task 2, pp. 109–110

Numeracy Nets 3 (Bauman 2009)

- Grade 3 Checkpoint 12, Task 1, pp. 58–59
- Grade 3 Checkpoint 13, pp. 62–63

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Provide students with experiences using a broad measurement vocabulary. For example, they should hear and use words such as **heavier**, **lighter**, **has the same mass**, **longer**, **taller**, and **wider**.
- Ask students to order objects from longest to shortest. Include situations in which students are dealing with other attributes, such as objects that are not straight or that are also wide or thick.
- Ask students to estimate and measure objects in non-standard units for the purpose of answering relevant and practical questions (e.g., Do people with longer legs usually jump farther?)
- Focus should be on answering, in every measurement situation, questions that focus on the attributes, such as, Which book is the tallest? How do you know?
- Use the same non-standard unit to measure a variety of items in order to compare them (e.g., How many wooden blocks would balance a sneaker? a book? a grapefruit? Which object is heaviest? lightest?)

SUGGESTED LEARNING TASKS

- Ask students to search for items that are a given number of paper clips long or that have a mass of two bags of marbles.
- Provide students with a length of string and have them estimate how many paper clips long it is. Ask them to measure the string using the paper clips. Ask them to explain whether their estimates were reasonable or not. This can also be repeated for mass.

- Have students compare the masses of two objects using a pan balance.
- Create an activity centre where students estimate, measure, compare, and sort various objects based on length, distance around, or mass. Comparison should include longer, shorter or about the same length as a specified object. The comparison for mass would be heavier, lighter, or about the same mass.
- Collect cylinders of different sizes. Ask students to order them according to their height or according to the distance around.
- Provide students with opportunity to measure, in non-standard units, the distance around a variety of classroom objects such as water bottles, cylinders, pumpkins, apples, the recycling bin, 2-D shapes, and 3-D objects.
- Have students work with a partner to measure a variety of classroom objects and record their results.
- Provide small groups of students with three or four objects, each with a different length and mass. If possible, try to have one of the shorter objects have a mass greater than the others. Ask students to measure the length of the objects and then order them from longest to shortest. Students should record their results. Then ask students to find the mass of the objects and order them from heaviest to lightest and record their results. Have students compare their results. Ask questions such as, Is the longest object the heaviest? Is the tallest object the heaviest? Is the shortest object the lightest? Do you think you can predict the mass of an object by looking at its height? Why or why not? If someone told you the mass of a hidden object, do you think you could predict the height? Why or why not?
- Give students pieces of string to use to measure the distance around their heads. Ask each student to find an object in the classroom that is about the same length as the piece of string.

SUGGESTED MODELS AND MANIPULATIVES

- | | |
|--|--|
| <ul style="list-style-type: none"> ▪ blocks ▪ everyday objects such as string ▪ hand spans ▪ linking cubes | <ul style="list-style-type: none"> ▪ paper clips ▪ paper strips ▪ pennies |
|--|--|

MATHEMATICAL LANGUAGE

Teacher	Student
<ul style="list-style-type: none"> ▪ balance, referent ▪ compare, order ▪ estimate, measure ▪ length, height, distance around, mass, heaviness ▪ length: longer than, shorter than ▪ mass: heavier than, lighter than ▪ non-standard unit ▪ object 	<ul style="list-style-type: none"> ▪ balance, referent ▪ compare, order ▪ estimate, measure ▪ length, height, distance around, mass ▪ length: longer than, shorter than ▪ mass: heavier than, lighter than ▪ object ▪ unit

Resources/Notes

Print

- *Making Math Meaningful to Canadian Students, K–8* (Small 2009), pp. 370–375, 430–433
- *Making Math Meaningful to Canadian Students, K–8, Second Edition* (Small 2013), pp. 414–415, 418–422, 478–479
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 58–59, 223–231, 241–242

Notes

SCO M04 Students will be expected to measure length to the nearest non-standard unit by using multiple copies of a unit and using a single copy of a unit (iteration process).

[C, ME, R, V]

[C] Communication

[PS] Problem Solving

[CN] Connections

[ME] Mental Mathematics and Estimation

[T] Technology

[V] Visualization

[R] Reasoning

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- M04.01** Explain why overlapping or leaving gaps does not result in accurate measures.
- M04.02** Count the number of non-standard units required to measure the length of a given object using a single copy or multiple copies of a unit.
- M04.03** Estimate and measure a given object using multiple copies of a non-standard unit and using a single copy of the same unit many times, and explain the results.
- M04.04** Estimate and measure, using non-standard units, a given length that is not a straight line.

Scope and Sequence

Mathematics 1	Mathematics 2	Mathematics 3
<p>M01 Students will be expected to demonstrate an understanding of measurement as a process of comparing by</p> <ul style="list-style-type: none"> ▪ identifying attributes that can be compared ▪ ordering objects ▪ making statements of comparison ▪ filling, covering, or matching 	<p>M04 Students will be expected to measure length to the nearest non-standard unit by using multiple copies of a unit and using a single copy of a unit (iteration process).</p> <p>M05 Students will be expected to demonstrate that changing the position of an object does not alter the measurements of its attributes.</p>	<p>M03 Students will be expected to demonstrate an understanding of measuring length (cm, m) by</p> <ul style="list-style-type: none"> ▪ selecting and justifying referents for the units centimetre and metre (cm, m) ▪ modelling and describing the relationship between the units centimetre and metre (cm, m) ▪ estimating length using referents ▪ measuring and recording length, width, and height <p>M05 Students will be expected to demonstrate an understanding of perimeter of regular, irregular, and composite shapes by</p> <ul style="list-style-type: none"> ▪ estimating perimeter using referents for centimetre or metre (cm, m) ▪ measuring and recording perimeter (cm, m) ▪ creating different shapes for a given perimeter (cm, m) to demonstrate that many shapes are possible for a perimeter

Background

In order to measure the length of an object, a series of uniform units must be used, or a single unit must be used repeatedly (iteration). The measure is dependent on the type of unit used, the placement of the units, the uniformity of the unit, and the unit's positioning. Students should have many experiences

measuring lengths of objects with multiple copies of non-standard units that they can correctly place and count before they attempt to measure lengths with a single unit as a measuring device.

It is essential that students understand how to measure length of an object using non-standard units. For example, to measure the length of a straw using toothpicks, students should start at one end of the straw placing congruent toothpicks tip to tip along the edge of the straw, making sure there are no gaps or overlaps between any of the toothpicks. The number of toothpicks used to match the edge of the straw will be the measure of the length of the straw. Students often struggle to understand that it is the number of intervals that are counted; therefore, they could place the straw and toothpicks on paper, use a pencil to mark the endpoints of the toothpicks, remove the toothpicks, and count the intervals between the pencil marks. Opportunities to explore and compare counting intervals will result in a greater understanding of measurement before standard units are introduced.

Students should measure length and height using non-standard units. Initially, use objects to measure that show a large difference from the object that is being measured, such as using a pencil to measure the length of a desk, then moving to objects similar in length or height, such as using a pencil to measure the length of a straw. Students should use everyday classroom objects to measure lengths, such as paper clips, crayons, straws, toothpicks, markers, or pencils. Students should be exposed to measuring larger objects in the classroom, such as doors, windows, or white boards, to help students further visualize length and height attributes.

Before any measurement activity, students should be encouraged to provide an estimate. Through continual estimating, measuring, and comparing estimates with actual measurements, followed by discussions of estimation strategies, students will develop their estimation abilities.

Additional Information

See Appendix A: Additional Information.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Provide students with two objects such as an eraser and a book. Ask, Can you tell which of these two objects is longer? heavier? bigger? takes up the most space? holds more? After each question, have students explain their thinking. (Children should recognize that capacity is an attribute that cannot be used to measure the objects.)

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students to compare two or more crooked paths, made with skipping ropes, lengths of rope, string, etc., and to estimate and then determine which path is the longest and which is the shortest.
- Ask students to solve problems, such as
 - a) Navaeh used a tile to measure the table in the classroom. Tyrel used the same tile to measure the same side of the table but they have different results. How is this possible? You may use a diagram to show your thinking.
 - b) Susan measured the length of her hand with linking cubes on three different days. On the first day, she said that her hand was 9 cubes long. On the second day, she said that her hand was 5 cubes long. On the third day, she said that her hand was 3 cubes long. Ask students to explain possible measurement errors Susan was making.
- Show students a straw and tell them that it is one unit. Ask students to estimate the length of various objects (e.g., a book, a table, the width of a door, etc.). Have students check their estimates.
- Ask students to estimate the number of cubes it would take to measure the length of a big book. Provide students with a collection of linking cubes and ask them to measure the length of the big book (multiple copies of a non-standard unit). Then, ask them to measure the length of the big book again using only one cube (single copy of the same unit many times). Ask them to explain the results.
- As students observe you, use linking cubes to measure the width of the window ledge but make an error in your measurement. For example, leave gaps between the cubes if using multiple units or overlap if using a single cube. Tell students the width of the window ledge. Ask them to tell if your measurement is accurate or not and explain their thinking.
- Provide students with containers of objects that can be used to measure length. Ask students to select one of the objects and measure a line segment. Note how students measure. Do they select only one type of object to do the measurement? Do they select objects of the same size (multiple uniform copies of this unit) or do they mix the sizes of the objects? Do they overlap the objects or do they leave gaps between them?
- Ask students to draw a line segment that they think is 6 cubes long. Then, have them measure the line segment with the cubes.

FOLLOW-UP ON ASSESSMENT**Guiding Questions**

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets K–2 (Bauman 2011)

- Grade 2 Checkpoint 10, Task 1, pp. 101–102
- Grade 2 Checkpoint 11, pp. 109–110

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Ask students to predict the results prior to making their measurements. Students should understand that the way a measurement unit is used is as important as the attribute being measured.
- Provide students with opportunities to measure an object using multiple copies and using single copies of a non-standard unit. They should compare results and discuss why (or why not) their measure is the same or different.
- Use children's literature, such as *David's Father* by Robert Munsch (1983), to have students discuss non-standard measurement units and estimating lengths and distances.

SUGGESTED LEARNING TASKS

- Present students with the following problem. I want to measure the length of the table with this whiteboard brush, but I only have one brush. How do you think we can measure the table using only one brush? Brainstorm ideas and have students try their suggestions.
- Estimate and measure a given object using multiple copies of a non-standard unit and using a single copy of the same unit many times, and explain the results. For example, ask students to estimate and measure the length of their arms by first measuring with straws laid end to end and then by measuring using only one straw. Ask them to give reasons for discrepancies between the two measures.
- Have students work in pairs. One student will measure the length, width, height, or distance around an object using non-standard unit. He or she may choose to measure correctly or he or she may choose to make an error. His or her partner observes the measuring process and explains whether the measurement is accurate and correct or not, and explains the error made.
- Students work in pairs to determine an effective measure of the height of their partner. Students should begin by selecting a unit of measure, and then they should estimate their partner's height in that non-standard unit. Explain that it might be easier to consider measuring a classmate when he or she is lying down. Discuss the similarity or difference in results from various groups of students.

- Provide students with containers filled with various sizes of paper clips. Ask students to measure the length of a book using paper clips. Note how students measure. Do they select paper clips of the same size (multiple uniform copies of this unit) or do they mix the sizes of the paper clips? Do they overlap the paper clips or do they leave gaps between them? Do they measure from one edge of the book to the other? This task can also be done using cubes of various masses. Students can be asked to measure the mass of an object.
- Draw curved or zigzagged line segments on large pieces of paper or make them with masking tape on the floor. Ask students how they might measure the length of the line segments. Try their suggestions to determine effective ways of measuring the line segments.

SUGGESTED MODELS AND MANIPULATIVES

- colour tiles
- Cuisenaire rods
- linking cubes
- paper clips
- pattern blocks
- straws

MATHEMATICAL LANGUAGE

Teacher	Student
<ul style="list-style-type: none"> ▪ estimate, measure, compare ▪ length ▪ multiple copies, single copies ▪ non-standard unit 	<ul style="list-style-type: none"> ▪ estimate, measure, compare ▪ length ▪ unit

Resources/Notes

Print

- *Making Math Meaningful to Canadian Students, K–8* (Small 2009), pp. 370–375
- *Making Math Meaningful to Canadian Students, K–8, Second Edition* (Small 2013), pp. 418–422
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 58–59, 223–231

Notes

SCO M05 Students will be expected to demonstrate that changing the position of an object does not alter the measurements of its attributes.

[C, R, V]

[C] Communication [PS] Problem Solving [CN] Connections [ME] Mental Mathematics and Estimation
[T] Technology [V] Visualization [R] Reasoning

Performance Indicator

Use the following indicator to determine whether students have achieved the corresponding specific curriculum outcome.

M05.01 Measure a given object, change the position, remeasure, and explain the results.

Scope and Sequence

Mathematics 1	Mathematics 2	Mathematics 3
<p>M01 Students will be expected to demonstrate an understanding of measurement as a process of comparing by</p> <ul style="list-style-type: none"> ▪ identifying attributes that can be compared ▪ ordering objects ▪ making statements of comparison ▪ filling, covering, or matching 	<p>M04 Students will be expected to measure length to the nearest non-standard unit by using multiple copies of a unit and using a single copy of a unit (iteration process).</p> <p>M05 Students will be expected to demonstrate that changing the position of an object does not alter the measurements of its attributes.</p>	<p>M03 Students will be expected to demonstrate an understanding of measuring length (cm, m) by</p> <ul style="list-style-type: none"> ▪ selecting and justifying referents for the units centimetre and metre (cm, m) ▪ modelling and describing the relationship between the units centimetre and metre (cm, m) ▪ estimating length using referents ▪ measuring and recording length, width, and height

Background

It is very common even for adults to misjudge the size of objects when those objects are in different positions. For example, a carpet hung on a wall can appear to be larger than one of the same size lying on the floor. It is important to provide many opportunities for students to play with, examine, and measure different objects in different positions and directions. By providing students with these experiences, students will have the opportunity to conclude that the position and direction of an object does not change its measurements. Students need to recognize that whether an object is standing up, lying flat, or tilted, its dimensions and mass will remain the same. Furthermore, students need to begin to realize that if a solid object is restructured so that its dimensions are altered, its mass will not be changed.

Additional Information

See Appendix A: Additional Information.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

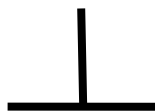
Tasks such as the following could be used to determine students' prior knowledge.

- Set up a measurement centre. Provide a collection of objects each with a different length and mass. Have students choose two objects and compare their length or their mass.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students to measure the length of a piece of paper with a tile. Record the result. Ask students to make a second measurement along the same side of the paper after repositioning the paper. Record the result. Discuss your findings.
- Show students a big book and tell them, "I measured the length of this big book and it was 25 cubes long." Reposition the same big book and ask, "If I measure the length of the big book now, how many cubes long will it be? Explain your thinking."
- Show students an apple and tell them, "I measured the mass of this apple and it has the same mass as 37 cubes." Turn the apple over and ask, "If I measure the mass of the apple now, what will it be? Explain your thinking."
- Ask, "If you change the position of an object (flip it, turn it, or slide it) do its measurements change? Explain your thinking."
- Show students a picture such as the one below.



Ask students to predict and then measure to determine which line segment is longer.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets K–2 (Bauman 2011)

- Grade 2 Checkpoint 11, Task 2, pp. 109–110

Numeracy Nets 3 (Bauman 2009)

- Grade 3 Checkpoint 12, Task 1, pp. 58–59

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Ask students to predict the results prior to making their measurements.
- Provide students with opportunities to measure an object and then remeasure the same object after it has been reoriented. They should compare results and discuss whether their measure is the same or different. Indicate that sometimes reorientation assists in the ease but not the value of the measurement.

SUGGESTED LEARNING TASKS

- Ask students to measure the length of an object in non-standard units, such as linking cubes. Put the object on display, with the linking cubes along the side that was measured, and record the measure. Ask students to place the object in a new position and state the length of that object.
- Have students measure their height in non-standard units while standing up and while lying down. Ask them why the measure of their height didn't change. Ask them to explain which measurement was easier to complete.
- Ask students to use linking cubes to measure the distance around a triangle. Record the measure. Then, ask them to predict the measure of the distance around that triangle if they turn the triangle or flip it to a new position. Allow them to measure the distance around the triangle and to explain the new measure.

SUGGESTED MODELS AND MANIPULATIVES

- colour tiles
- Cuisenaire rods
- linking cubes
- paper clips
- pattern blocks
- straws

MATHEMATICAL LANGUAGE

Teacher	Student
<ul style="list-style-type: none"> ▪ estimate, measure ▪ position, direction 	<ul style="list-style-type: none"> ▪ estimate, measure

Resources/Notes**Print**

- *Making Math Meaningful to Canadian Students, K–8* (Small 2009), pp. 370–375
- *Making Math Meaningful to Canadian Students, K–8, Second Edition* (Small 2013), pp. 412–415
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 58–59, 223–231

Notes

Geometry

GCO: Students will be expected to describe the characteristics of 3-D objects and 2-D shapes and analyze the relationships among them.

Specific Curriculum Outcomes

Process Standards

[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

- G01** Students will be expected to sort 2-D shapes and 3-D objects using two attributes and explain the sorting rule. [C, CN, R, V]
- G02** Students will be expected to recognize, name, describe, compare, and build 3-D objects, including cubes and other prisms, spheres, cones, cylinders, and pyramids. [C, CN, R, V]
- G03** Students will be expected to recognize, name, describe, compare, and build 2-D shapes, including triangles, squares, rectangles, and circles. [C, CN, R, V]
- G04** Students will be expected to identify 2-D shapes as part of 3-D objects in the environment. [C, CN, R, V]

SCO G01 Students will be expected to sort 2-D shapes and 3-D objects using two attributes and explain the sorting rule.

[C, CN, R, V]

[C] Communication

[PS] Problem Solving

[CN] Connections

[ME] Mental Mathematics and Estimation

[T] Technology

[V] Visualization

[R] Reasoning

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- G01.01** Determine the differences between two given presorted sets and explain the sorting rule.
- G01.02** Identify and name two common attributes of items within a given sorted group.
- G01.03** Sort a given set of 2-D shapes (regular and irregular) according to two attributes and explain the sorting rule.
- G01.04** Sort a given set of 3-D objects according to two attributes and explain the sorting rule.

Scope and Sequence

Mathematics 1	Mathematics 2	Mathematics 3
<p>G01 Students will be expected to sort 3-D objects and 2-D shapes using one attribute and explain the sorting rule.</p>	<p>G01 Students will be expected to sort 2-D shapes and 3-D objects using two attributes and explain the sorting rule.</p>	<p>G01 Students will be expected to describe 3-D objects according to the shape of the faces and the number of edges and vertices.</p> <p>G02 Students will be expected to name, describe, compare, create, and sort regular and irregular polygons, including triangles, quadrilaterals, pentagons, hexagons, and octagons, according to the number of sides.</p>

Background

“Children need experiences with a rich variety of both two- and three-dimensional shapes. It is useful for students to be able to identify common shapes, notice likenesses and differences among shapes, become aware of the properties that different shapes have, and eventually use these properties to further define and understand their geometric world.” (Van De Walle and Lovin 2006, 193) “The [Van Hiele] levels [of geometric thought] describe how we think and what types of geometric ideas we think about, rather than how much knowledge we have.” (Van de Walle and Lovin 2006, 188)

Sorting tasks help develop visual discrimination. It is important to encourage students to look for alternative ways of sorting; this necessitates the further investigation of objects. According to the Van Hiele model, most students in Mathematics 2 are at the visualization level. They rely mostly on the appearance of the shape or object when doing sorting tasks. There are many different attributes or characteristics of shapes and objects. Students are most familiar with attributes that describe the whole shape, such as colour, size, shape, or texture.

Other attributes refer to parts of the shape or object, such as number of sides or faces, shapes of faces, number of vertices (corners), or lengths of sides or edges. Students in Mathematics 2 may need help focussing their attention on these attributes if their sorting is to include such attributes. Students will often use non-geometric language to describe the attributes, such as crooked or wavy. Students may also sort according to the position of the shape, such as a square turned may be seen as a diamond. Again, students may need help to be convinced that a shape or object in a different position in space is not a different one.

In earlier grades, students will have had many opportunities to explore shapes through sorting by one attribute, patterning, and building tasks. In Mathematics 2, tasks that will further develop these skills should be provided. In particular, this outcome extends students' sorting of 2-D shapes and 3-D objects to sorts involving two attributes. Students will begin at different levels of development so it will be necessary to provide tasks with a range of complexity.

In Mathematics 2, the 2-D geometric shapes being explored include triangles, squares, rectangles, and circles, while 3-D geometric objects being explored include, spheres, cones, cylinders, cubes and other prisms, and pyramids with rectangular and triangular bases. However, other shapes and objects may also be used in sorting tasks without an emphasis on naming them. When discussing 3-D objects with students, it is important to refer to them as simply objects or 3-D solids. This will assist students in differentiating between 2-D shapes and 3-D objects.

Additional Information

See Appendix A: Additional Information.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Provide students with a collection of 2-D shapes or 3-D objects. Tell them a sorting rule based on one attribute. Ask them to sort the collection based on your sorting rule.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students to describe a given 2-D shape or 3-D object. Note the attributes described by the students.
- Provide several different 3-D objects. Ask the student to sort them using two attributes and to explain the sorting criteria. Ask him or her to sort them again, using different criteria.
- Show students a set of 3-D objects or 2-D shapes that have been sorted. Ask students to explain the differences between the two sets and to explain the sorting rule.
- Provide several different 2-D shapes. Ask the student to sort them using two attributes and to explain the sorting criteria. Ask him or her to sort them again, using different criteria.
- Show students a set of 2-D shapes or 3-D objects that have been sorted, but which contains a shape or an object that does not belong. Ask students to remove the shape or object that does not belong and to explain their thinking.
- Show students a set of 3-D objects that you have sorted based on two common attributes. Have additional 3-D objects available that have yet to be sorted. Ask students to select an object and to add it to the set that you sorted. Ask them to explain their thinking.
- Note the language students use to describe their sorting rules. As students develop a higher level of thinking, they will begin to use more geometric language and focus more on the properties of the shapes.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets K–2 (Bauman 2011)

- No Checkpoint for this outcome.

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Include a variety of sorting and classifying tasks. Use collections of real objects, such as cans, boxes, or balls, for students to sort.
- Focus on how shapes and objects are alike and how they are different. Provide many opportunities for students to use oral and written language to describe the attributes of 2-D shapes and 3-D objects.
- Make a collection of 2-D shapes cut from poster board. These should include not only shapes with which students are familiar, but also some less common shapes that would fit in potential sorting categories (e.g., a semi-circle or a jellybean shape could be classified under “things with curved edges”). Because it is important for students to share their thinking with others, it is recommended that they work in small groups, talking about their ideas and strategies.
- Challenge students to test their ideas about shapes and objects. For example, Can you find a triangle that has one square corner?

SUGGESTED LEARNING TASKS

- Hold up a geometric solid, like a cone, and ask the students to find other objects that could be sorted into the same group as the cone. When a student adds an object to the group, have them explain what attribute it shares with the original object.
- Provide a collection of about eight shapes or objects. Have students take turns sorting the shapes while the other group members try to guess the sorting rule being used. Each student in the group should have a turn to sort the shapes. As a class, discuss all the ways that the shapes were sorted and try to determine if there are any ways that were missed.
- Ask students to select two 2-D shapes or 3-D objects and tell how they are the same or how they are different.
- Create a set of 3-D objects or 2-D shapes (about five) that have a secret sorting rule based on two attributes. Ask the students to add to your set (a drawing or a real item) and explain the rule.
- Play game of “One Way Different.” Select an object or shape as a starter piece. Students take turns placing objects on either side of the original shape. These objects must differ by one attribute from the one it is placed beside. Students need to explain why their piece works. For example, if the original object was a pyramid, the next object placed beside it could be a cone and the student could justify that the cone has curved faces.
- Sort a small number of students into two groups without telling the class how they are being sorted. One at a time, the remaining students go to the group to which they think they belong. Tell each student whether he or she is in the correct group. When all students are correctly sorted in groups, ask, What’s my sorting rule? Repeat using other attributes.
- Sort a set of buttons using two attributes; for example, buttons that are round and have two holes; and buttons that are not round and do not have two holes. Have students guess the sorting rule.

SUGGESTED MODELS AND MANIPULATIVES

- | | |
|-------------------------|------------------|
| ▪ attribute blocks | ▪ pattern blocks |
| ▪ dot and/or grid paper | ▪ pentominoes |
| ▪ geoboards | ▪ polydrons |
| ▪ geometric solids | ▪ tangrams |

MATHEMATICAL LANGUAGE

Teacher	Student
<ul style="list-style-type: none"> ▪ 2-D shapes: triangle, square, rectangle, circle ▪ 3-D objects: cube, sphere, cone, cylinder, pyramid, prism ▪ attribute ▪ regular, irregular shapes ▪ sides, edges, corners/vertices, faces ▪ sorting rule ▪ sorting sets of shapes ▪ straight, curved, large, small, points, square, roll, stack 	<ul style="list-style-type: none"> ▪ 2-D shapes: triangle, square, rectangle, circle ▪ 3-D objects: cube, sphere, cone, cylinder, pyramid, prism ▪ sides, edges, corners/vertices, faces ▪ sorting rule ▪ sorting sets of shapes ▪ straight, curved, large, small, points, square, roll, stack

Resources/Notes**Print**

- *Making Math Meaningful to Canadian Students, K–8* (Small 2009), pp. 284–291, 516–520
- *Making Math Meaningful to Canadian Students, K–8, Second Edition* (Small 2013), pp. 340–347, 560–564
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 192–195, 206–208

Notes

SCO G02 Students will be expected to recognize, name, describe, compare, and build 3-D objects, including cubes and other prisms, spheres, cones, cylinders, and pyramids.

[C, CN, R, V]

[C] Communication

[PS] Problem Solving

[CN] Connections

[ME] Mental Mathematics and Estimation

[T] Technology

[V] Visualization

[R] Reasoning

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- G02.01** Sort a given set of 3-D objects and explain the sorting rule.
- G02.02** Identify common attributes of cubes and other prisms, spheres, cones, cylinders, and pyramids from given sets of the same 3-D objects.
- G02.03** Identify and describe given 3-D objects with different dimensions.
- G02.04** Identify and describe given 3-D objects with different positions.
- G02.05** Create and describe a representation of a given 3-D object using materials such as modelling clay.
- G02.06** Identify and name examples of cubes and other prisms, spheres, cones, cylinders, and pyramids found in the environment.

Scope and Sequence

Mathematics 1	Mathematics 2	Mathematics 3
<p>G01 Students will be expected to sort 3-D objects and 2-D shapes using one attribute and explain the sorting rule.</p> <p>G02 Students will be expected to replicate composite 2-D shapes and 3-D objects.</p> <p>G03 Students will be expected to identify 2-D shapes in 3-D objects.</p>	<p>G02 Students will be expected to recognize, name, describe, compare and build 3-D objects, including cubes and other prisms, spheres, cones, cylinders, and pyramids.</p>	<p>G01 Students will be expected to describe 3-D objects according to the shape of the faces and the number of edges and vertices.</p> <p>G02 Students will be expected to name, describe, compare, create, and sort regular and irregular polygons, including triangles, quadrilaterals, pentagons, hexagons, and octagons, according to the number of sides.</p>

Background

Students will continue to develop their depth of understanding of 3-D objects. Students in Mathematics 2 need many varied opportunities to manipulate 3-D objects. Tasks in which they describe, compare, and build 3-D objects and discuss their observations help to develop essential geometric skills. In order to describe, compare, and construct 3-D objects, students must first have multiple opportunities to explore concrete models of cubes and other prisms with rectangular faces, spheres, cones, cylinders, and pyramids. Students should be exploring prisms and pyramids with triangular and rectangular bases. As part of the exploration process, students should be able to touch, feel, build, and observe a wide variety of geometric solids in the classroom as well as familiar 3-D objects in their home and school environment. It is through such tasks that students will learn the names of 3-D objects and begin to recognize their characteristics.

“As students develop mathematically, they are increasingly able to identify and name a shape by examining its properties and using reasoning.” (Small 2009, 287) However, initially students recognize a 3-D object by its overall appearance and through association with objects like it in their environment or in books. It is not unusual for students at this level of geometric development to state, for example, “This object is a cube because it looks like a cube,” or “This object is a cube because it looks like a box.”

Through hands-on exploration, students will discover the various components used to classify 3-D solids. These components would include faces, curved surfaces, edges, and vertices. While it is very important to encourage students to use accurate language when naming objects such as cube, prism, sphere, cone, cylinder, and pyramid, it is important to accept the language that students use to describe these components; however, you should consistently model the appropriate mathematical terminology and display these words in the classroom environment. They should also discover that an object has certain attributes regardless of its position or size. For example, even though a pyramid may look different lying on one of its triangular faces, it is still a pyramid with the same attributes. They should also recognize that it is still a pyramid whether it is tall or short.

Additional Information

See Appendix A: Additional Information.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Provide students with a collection of 3-D objects. Ask them to sort the 3-D objects and to explain their sorting rule.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Provide students with a set of 3-D objects that share common attributes. Ask them to describe how the objects are the same and how they are different. Note the attributes that students describe.
- Ask students to sort a given set of 3-D objects and to explain their sorting rule. Ask them to sort the set a second time using a new sorting rule.

- Ask students to choose a 3-D object and tell how they would describe the object to someone who did not know what it was.
- Provide students with modelling clay. Ask them to create a particular 3-D object, such as a sphere or a pyramid.
- Ask students to look around the classroom and identify at least three different rectangle-based prisms that they see and to explain why the objects they identify are all rectangle-based prisms.
- Ask students to describe how a cube and square-based pyramid are alike and how they differ. Repeat the question using two other 3-D objects such as a cone and cylinder.
- Place a set of 3-D objects in front of students. There should be a mix of pyramids and prisms. Ensure that, within the set, the triangle-based pyramids are placed in different positions (e.g., resting on the base, resting on a face). Ask students to identify all the triangle-based pyramids in the set. Note whether students select only those triangle-based pyramids that are positioned on their bases.
- Tell students that you traced around one of the faces of a 3-D object, and the shape that you drew was a square. Ask students to tell what the object could be and to explain their thinking.
- Show students a picture of a 2-D shape (circle, square, or triangle). Provide a variety of 3-D objects and ask the student to choose one that has a face that matches the picture.
- Label each side of a cube with the words: “cube,” “sphere,” “cone,” “cylinder,” “pyramid” and “free choice.” Have students roll the cube and find, in a book or in the environment, an example of something that is the shape shown on the cube.
- Provide students with a blackline master that has drawings of a variety of 2-D shapes. Ask students to use a familiar 3-D object from their environment (toy or article from home) and match one of its faces to one or more of the shapes from the sheet.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets K–2 (Bauman 2011)

- Grade Primary Checkpoint 12, Task 2, pp. 114–115
- Grade 2 Checkpoint 12, Task 2, pp. 118–119

Numeracy Nets 3 (Bauman 2009)

- Grade 3 Checkpoint 15, Task 1, pp. 70–71 (Line Master 15.1–15.5)
- Grade 3 Checkpoint 16, Task 1, pp. 73–74 (Line Master 16.1)

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Have students bring in 3-D objects that could be described as a cube, sphere, cone, cylinder, or pyramid. Students should explain their reasons for its classification focusing on particular properties. For example, a student may bring in a funnel and classify it as both a cone and a cylinder.
- Have the students explore how they can create a cone, sphere, or cylinder using materials in the classroom. Expect to see use of modelling clay, play dough, pipe cleaners, toothpicks, paper, and circles from the attribute blocks. Have the students describe their methods to a partner, and present to the class, or write about their methods in their journals.
- Provide opportunities for students to build with 3-D objects by following oral directions; for example, Place the cube between the small cylinder and the cone, and place the large cylinder behind the cube. Invite students to make their own designs and then challenge their classmates to build the same structure from their oral directions.
- “Students need ample opportunities to draw, build, make, put together, and take apart shapes. These activities should be built around specific characteristics or properties so that students develop an understanding of geometric properties ...” (Van de Walle and Lovin, 2006, 192)
- Ensure that students are exposed to 3-D objects in a variety of sizes and positions so they can discover that an object has certain attributes regardless of its position or size.

SUGGESTED LEARNING OPPORTUNITIES

- Give the student some toothpicks and clay and ask him or her to build a cube or a pyramid. Ask how many toothpicks were needed.
- Provide pairs of students with a small collection of 3-D objects. Have them build a creature or robot or something else of their choosing. Have them present their creation to the class and describe it using the names of the 3-D objects they used in their creation.
- Tell students that you have a 3-D object in a bag. One of its faces is round (a circle). Ask what the object could be. Follow up with having the students find an object in the class that has the same face.
- Have students trace one of the faces of a 3-D object and then search for another 3-D object in the classroom that that has the same face.
- Challenge students to build your “secret object.” Give them clues such as, My object uses 10 cubes and has a rectangle base and it looks like stair steps, or My object uses 8 cubes and has a square base and it looks like the letter L.
- Ask students to look around the room and see what 3-D shapes they can see and name. List the shape and the object name on chart paper.
- Put students in small groups and have each group create a book of 3-D shapes by cutting pictures from catalogues and magazines.
- Provide students with sets of cubes and other prisms, spheres, cones, cylinders, or pyramids. Each set should contain objects of different sizes. Have students compare the objects and identify common attributes. Record attributes on a chart.

SUGGESTED MODELS AND MANIPULATIVES

- geometric solids
- polydron

MATHEMATICAL LANGUAGE

Teacher	Student
<ul style="list-style-type: none"> ▪ 3-D objects: cube, sphere, cone, cylinder, pyramid, prism ▪ attributes: faces/surfaces, edges, vertices/corners, ▪ different positions: slide, flip, turn ▪ different sizes/dimensions ▪ recognize, name, describe, compare, build 	<ul style="list-style-type: none"> ▪ 3-D objects: cube, sphere, cone, cylinder, pyramid, prism ▪ faces/surfaces, edges, vertices/corners, ▪ slide, flip, turn ▪ different sizes ▪ recognize, name, describe, compare, build

Resources/Notes**Print**

- *Making Math Meaningful to Canadian Students, K–8* (Small 2009), pp. 287–292, 303–304
- *Making Math Meaningful to Canadian Students, K–8, Second Edition* (Small 2013), pp. 343–346, 358–359
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 196–200, 204–206

Notes

SCO G03 Students will be expected to recognize, name, describe, compare, and build 2-D shapes, including triangles, squares, rectangles, and circles.

[C, CN, R, V]

[C] Communication

[PS] Problem Solving

[CN] Connections

[ME] Mental Mathematics and Estimation

[T] Technology

[V] Visualization

[R] Reasoning

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- G03.01** Sort a given set of 2-D shapes and explain the sorting rule.
- G03.02** Identify common attributes of triangles, squares, rectangles, and circles from given sets of the same type of 2-D shapes.
- G03.03** Identify given 2-D shapes with different dimensions.
- G03.04** Identify given 2-D shapes with different positions.
- G03.05** Identify and name examples of triangles, squares, rectangles, and circles found in the environment.
- G03.06** Create a model to represent a given 2-D shape.
- G03.07** Create a pictorial representation of a given 2-D shape.

Scope and Sequence

Mathematics 1	Mathematics 2	Mathematics 3
<p>G01 Students will be expected to sort 3-D objects and 2-D shapes using one attribute and explain the sorting rule.</p> <p>G02 Students will be expected to replicate composite 2-D shapes and 3-D objects.</p> <p>G03 Students will be expected to identify 2-D shapes in 3-D objects.</p>	<p>G03 Students will be expected to recognize, name, describe, compare, and build 2-D shapes, including triangles, squares, rectangles, and circles.</p>	<p>G01 Students will be expected to describe 3-D objects according to the shape of the faces, and the number of edges and vertices.</p> <p>G02 Students will be expected to name, describe, compare, create, and sort regular and irregular polygons, including triangles, quadrilaterals, pentagons, hexagons, and octagons, according to the number of sides.</p>

Background

A student's ability to conceptualize shape develops gradually. Initially, younger students identify and name 2-D shapes by their overall appearance and by association with objects in their environments, not by examining their parts or properties. For example, they may know a 2-D shape they are examining is a square because it looks like a geoboard that they know is a square. In this early stage, however, students may not recognize a square that has been rotated, thinking instead that it is a diamond. As students develop mathematically, they are able to name shapes according to properties and develop spatial abilities to recognize a shape regardless of its position or size. Tasks in which they sort, make, and recognize patterns, build and draw shapes, and talk about what is happening help to develop these essential geometric skills.

It is very important to encourage students to use accurate language when naming 2-D shapes. Students should be comfortable using the names **triangle**, **square**, **rectangle**, and **circle**. Describing shapes allows students to focus on their basic characteristics. Use questioning to focus student thinking; for example, What can you say about its sides? Is there anything special about its corners? What other shapes are like this one? In what ways are they alike?

Additional Information

See Appendix A: Additional Information.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Provide a set of 2-D shapes that have been sorted based on one attribute. Ask students to explain the sorting rule. Then ask them to resort the shapes and to explain their new sorting rule.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Listen to students' observations as they work on tasks to determine whether they understand common attributes of shapes.
- Provide students with a set of 2-D shapes. Ask students to sort the shapes and to explain their thinking.
- Show students a set of triangles in various sizes and positions. Tell them that one student thinks these shapes are all triangles but another student doesn't agree. Ask them to explain who is correct.
- Ask students to describe or draw different triangles (rectangles).
- Have students create a picture using only two shapes (e.g., circles and squares). Have them share their pictures with the class and discuss what they used.
- Make a rectangle (or triangle or square) on a geoboard and have students make two different rectangles on their geoboards.
- Provide students with a set of tangrams. Have students use two or more of the pieces to create a square or a triangle.

- Provide students with a design that includes a variety of 2-D shapes, such as a tangram puzzle. Ask them to identify for you the different shapes they can find. They can use different colours to trace around each shape. Real-world examples that include various 2-D shapes could also be used (e.g., a hockey rink or a curling rink).
- Give students 10 triangular pattern blocks. Ask them to make as many different-sized triangles as they can and to record their answers by making a drawing of each one.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets K–2 (Bauman 2011)

- Grade Primary Checkpoint 12, Task 1, pp. 114–115 (Line Master 12.1)
- Grade 1 Checkpoint 12, Task 1, pp. 116–117
- Grade 2 Checkpoint 12, Task 1, pp. 118–119 (Line Master 12.2–12.5)
- Grade 2 Checkpoint 13, pp. 126–127 (Line Master 13.3)

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Use a variety of models of the 2-D shapes to ensure students are provided with many different examples of triangles, squares, rectangles, or circles.
- Use 5×5 geoboards for students to explore how many different squares, rectangles, or triangles they can make.
- Encourage students to provide explanations beyond simple naming of shapes. For example, if the student says it is a triangle because it looks like a triangle, ask them to describe the characteristics of triangles.

- Include tasks that focus on the entire class of shapes (e.g., all rectangles) and their likenesses and differences.
- Provide opportunities for students to create 2-D shapes by following oral directions (e.g., Place a square below a triangle). Invite students to make their own designs and then challenge their classmates to build the same design from their oral directions.
- Ensure that students are exposed to 2-D shapes in a variety of sizes and positions so they can discover that a shape has certain attributes regardless of its position or size.
- Explore a variety of tangram puzzles.

SUGGESTED LEARNING TASKS

- Provide students with the large triangle, square, and rectangle of an attribute-block set. Ask them to work in pairs, comparing the three shapes, and listing all the ways in which they are the same and how they differ. They should be able to properly identify the shapes and to compare such things as number of sides or length of sides.
- Provide a drawing that incorporates many shapes in different positions and of various sizes. Ask the student to point out the triangles among the shapes.
- Give students a combination of 2-D shapes and 3-D objects. Ask them to investigate which of the 2-D shapes appears most often as a face of a 3-D object.
- Include tasks in which students are required to find shapes in pictures of objects and their environment.
- Have children hunt around the school to find various shapes (e.g., squares, triangles, circles, rectangles). Have them share their findings and discuss why certain shapes are more common than others.
- Put students in small groups and have each group create a book of 2-D shapes by cutting pictures from catalogues and magazines.
- Have students use shapes to spell the name of the shape or make the shape. For example, use a collection of triangles to form the letters that spell “triangle” or create the sides of a larger triangle.
- Have students physically build the shapes using their hands or fingers or in groups using their bodies.

SUGGESTED MODELS AND MANIPULATIVES

- | | |
|-------------------------|------------------|
| ▪ attribute blocks | ▪ pattern blocks |
| ▪ dot and/or grid paper | ▪ pentominoes |
| ▪ geoboards | ▪ polydrons |
| ▪ geometric solids | ▪ tangrams |

MATHEMATICAL LANGUAGE

Teacher	Student
<ul style="list-style-type: none"> ▪ 2-D shapes: triangle, square, rectangle, circle ▪ attributes: sides, corners, square corners ▪ different positions: slide, flip, turn ▪ different sizes/dimensions ▪ recognize, name, describe, compare, build 	<ul style="list-style-type: none"> ▪ 2-D shapes: triangle, square, rectangle, circle ▪ sides, corners, square corners ▪ slide, flip, turn ▪ different sizes ▪ recognize, name, describe, compare, build

Resources/Notes

Print

- *Making Math Meaningful to Canadian Students, K–8* (Small 2009), pp. 287–290, 292–294, 303–304, 309–314, 516–520
- *Making Math Meaningful to Canadian Students, K–8, Second Edition* (Small 2013), pp. 343–346, 348–351, 560–567
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 196–200, 202–204

Notes

SCO G04 Students will be expected to identify 2-D shapes as part of 3-D objects in the environment.

[C, CN, R, V]

[C] Communication

[PS] Problem Solving

[CN] Connections

[ME] Mental Mathematics and Estimation

[T] Technology

[V] Visualization

[R] Reasoning

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

G04.01 Compare and match a given 2-D shape, such as a triangle, square, rectangle, or circle, to the faces of 3-D objects in the environment.

G04.02 Name the 2-D faces of a given 3-D object.

Scope and Sequence

Mathematics 1	Mathematics 2	Mathematics 3
<p>G02 Students will be expected to replicate composite 2-D shapes and 3-D objects.</p> <p>G03 Students will be expected to identify 2-D shapes in 3-D objects.</p>	<p>G04 Students will be expected to identify 2-D shapes as part of 3-D objects in the environment.</p>	<p>G01 Students will be expected to describe 3-D objects according to the shape of the faces and the number of edges and vertices.</p>

Background

Students have had opportunities to explore shapes through sorting, patterning, and building tasks. Considering the attributes of various 2-D shapes and 3-D objects, students identified, compared, and sorted them by one attribute. Students had many varied opportunities to manipulate both 2-D shapes and 3-D objects to help make connections to objects in their environment.

Many of the 3-D shapes students see or explore have faces (pyramids and cubes), but others have curved surfaces (cylinders, spheres, and cones). Students should begin to include in their descriptions of a 3-D shape the surfaces and faces that make it up. For example, a cylinder has two circle faces and a curved surface, and a sphere has one curved surface. Through experiences with 3-D objects, students should be comfortable naming a cylinder, sphere, cone, pyramid, and cube and naming their curved surfaces and their 2-D faces that are squares, triangles, rectangles, or circles.

Pressing the faces of 3-D objects in sand to see the 2-D footprints made by the objects helps students focus on these faces. They should also trace on paper the faces of a 3-D solid to see these footprints. Pasting paper copies of 2-D shapes on 3-D objects also focuses students' attention on the faces. For example, if students are provided with six different coloured 10 cm x 10 cm squares and a large cube from base-ten blocks, they could tape those squares to the faces of the cube and describe the result.

Additional Information

See Appendix A: Additional Information.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Provide students with a set of 3-D objects. Ask them to sort the objects by the shape of their faces. Ask them to explain their thinking as they sort the objects.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Tell students that you traced around one of the faces of a 3-D object and the shape that you drew was a square. Ask students to tell what the object could be and to explain their thinking.
- Show students a picture of a 2-D shape (circle, square, rectangle, or triangle). Provide a variety of 3-D objects and ask the student to choose one object that has a face that matches the picture.
- Provide students with a blackline master that has drawings of a variety of 2-D shapes. Ask students to use a familiar 3-D object from their environment (toy or article from home) and match one of its faces to one or more of the shapes from the sheet.
- Show students a given 3-D object. Ask them to name the faces of the object.
- Show students a rectangle-based prism. Ask them to find other 3-D objects in the set they are given that have at least one face that is the same. Ask them to explain their thinking.
- Provide students with 3-D objects. Ask them to write a description of the faces of the object they've been given.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets K–2 (Bauman 2011)

- No Checkpoint for this outcome.

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Have students bring in 3-D objects that could be described as a cube, sphere, cone, cylinder, or pyramid. Students should explain their reasons for its classification focusing on particular properties, including faces.
- When creating 3-D solids using a variety of materials such as modelling clay, pipe cleaners, toothpicks, paper, and circles from the attribute blocks, have students focus on the shapes of the faces. Have students describe the faces of the 3-D objects to a partner, present their findings to the class, or write about their discoveries in their journals.
- Provide opportunities for students to build with 3-D objects by following oral directions; for example, Make a 3-D object that has two faces that are circles. Invite students to make their own objects and then challenge their classmates to build the same structure from their oral directions.
- “Students with need ample opportunities to draw, build, make, put together, and take apart shapes in both two and three dimensions. These activities should be built around specific characteristics or properties so that students develop an understanding of geometric properties ...” (Van de Walle and Lovin 2006, 192).
- Ensure that students are exposed to 3-D objects in a variety of sizes and positions so they can discover that an object has certain attributes regardless of its position or size.

SUGGESTED LEARNING TASKS

- Give students a combination of 2-D shapes and 3-D objects. Ask them to investigate which of the 2-D shapes appears most often as a face of a 3-D object.
- Include tasks in which students are required to find shapes in pictures of objects and their environment. For example, provide students with a chart showing 2-D shapes. Take students on a walk around the school or neighbourhood and ask students to record in the chart the number of times they see each shape. Discuss why some certain shapes occur more frequently than others.
- Have students identify all of the faces for a triangular prism and a triangular pyramid by pressing the prisms and pyramids in sand, coating them in paint, and making face prints or by tracing them on paper. Ask them to compare the impressions, face prints, or drawings and to describe how they are the same and how they are different.

- Place a variety of 3-D objects in a bag. Invite students to reach in the bag, feel the objects, and pull out an object with a square face. Ask them to name the object they pull out of the bag. Ask, Are there other 3-D objects that could have the same shaped face?
- Provide students with a set of 3-D objects. Ask them to select only those 3-D objects that have rectangular faces.
- Have students create riddles that focus on the faces of 3-D objects. For example, I am a 3-D object. Four of my faces are triangles. My base is a square. Who am I?

SUGGESTED MODELS AND MANIPULATIVES

- attribute blocks
- cones
- cubes and other prisms
- cylinders
- modelling clay
- pipe cleaners
- pyramids
- spheres
- toothpicks

MATHEMATICAL LANGUAGE

Teacher	Student
<ul style="list-style-type: none"> 2-D shapes: square, triangle, rectangle, circle 3-D objects: cube, sphere, cone, cylinder, pyramid faces, footprint, curved surface 	<ul style="list-style-type: none"> 2-D shapes: square, triangle, rectangle, circle 3-D objects: cube, sphere, cone, cylinder, pyramid faces, footprint, curved surface

Resources/Notes

Print

- Making Math Meaningful to Canadian Students, K–8* (Small 2009), pp. 288–289, 292
- Making Math Meaningful to Canadian Students, K–8, Second Edition* (Small 2013), pp. 344–347
- Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 193–195, 216–217

Notes

Statistics and Probability

GCO: Students will be expected to collect, display, and analyze data to solve problems.

GCO: Students will be expected to use experimental or theoretical probabilities to represent and solve problems involving uncertainty.

Specific Curriculum Outcomes: Statistics and Probability (SP)

Process Standards

[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

- SP01** Students will be expected to gather and record data about self and others to answer questions. [C, CN, PS, V]
- SP02** Students will be expected to construct and interpret concrete graphs and pictographs to solve problems. [C, CN, PS, R, V]

SCO SP01 Students will be expected to gather and record data about self and others to answer questions.

[C, CN, PS, V]

[C] Communication

[PS] Problem Solving

[CN] Connections

[ME] Mental Mathematics and Estimation

[T] Technology

[V] Visualization

[R] Reasoning

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

SP01.01 Formulate a question that can be answered by gathering information about self and others.

SP01.02 Organize data as it is collected using concrete objects, tallies, checkmarks, charts, or lists.

SP01.03 Answer questions using collected data.

Scope and Sequence

Mathematics 1	Mathematics 2	Mathematics 3
—	<p>SP01 Students will be expected to gather and record data about self and others to answer questions.</p> <p>SP02 Students will be expected to construct and interpret concrete graphs and pictographs to solve problems.</p>	<p>SP01 Students will be expected to collect first-hand data and organize it using tally marks, line plots, charts, and lists to answer questions.</p> <p>SP02 Students will be expected to construct, label, and interpret bar graphs to solve problems.</p>

Background

Students often find themselves collecting and organizing data. Even though data management has not been formally taught in mathematics prior to grade 2, it is reasonable to expect that students will have had experiences collecting data in other content areas. The focus of data management explorations should be to answer questions. The contexts should be realistic and of interest to the students.

Additional Information

See Appendix A: Additional Information.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- How could we find out the favourite flavour of ice cream for our class? Explain your thinking.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Provide students with a set of 2-D shapes. Ask them to create a list or chart to show the types of shapes in the given set and how many of each type of shape there are.
- Provide students with a collection of about 25 linking cubes in three or four different colours. Ask them to organize the cubes and record the data in a chart using tally marks or another method. Ask them to write two questions that the tally marks would answer.
- Show students tallies or a chart on a topic of interest to students. Have them answer questions about the tallies or chart. Ask them to make up their own question that the tally marks or chart would answer.
- Have students formulate their own question that can be answered by collecting information within the school. Students should be able to write their question, gather their data, and organize it. In SCO SP02, students will be expected to display their information in the form of a concrete or pictograph and write about what they have learned.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets K–2 (Bauman 2011)

- Grade 2 Checkpoint 14, pp. 135–136 (Line Master 14.1)
- Grade 2 Checkpoint 15, pp. 143–144

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions




- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Emphasis should be placed on answering real questions and ways to organize and interpret the data collected.
- Model the formulation of questions such as, I wonder..., How can we find out?, or Whom shall we ask?
- Use everyday events to formulate questions and collect data about the children's environment such as How do you travel to school? Which kind of pizza did you order? Which author should we read this week?
- Model questions on the same topic in several ways and allow students to choose the best question for its purpose; for example, How did you travel to school today? or Did you walk to school today? or How many students in our class used the school bus today?
- Ensure that data management tasks are relevant and of interest to the students in your class. Each class member should be able to participate and contribute data to the investigation.
- Encourage students to conduct small surveys to collect data. Encourage students to organize the data as they collect it using tallies, checkmarks, charts, or lists.
- Make use of opportunities to integrate data collection into other subject areas, such as You and Your World.
- Read a book such as *The Best Vacation Ever* by Stuart J. Murphy (1997) or *Charlie's Checklist* by Rory S. Lerman (1997). Both books have characters that formulate questions and gather data. Discuss the questions chosen and the methods used to gather and record the data.

SUGGESTED LEARNING TASKS

- Ask students to collect data about the type of snacks brought for recess. Display that data as tally marks. Ask students questions that relate to the tally marks; for example, How many students brought an apple for recess? or How many more students brought apples than brought granola bars?
- Ask students to collect data about a question of interest to them. This may include questions such as, What is your favourite day of the week? What is your favourite type of book? What is your favourite colour? Ask students to display the data they collect using charts, tally marks, checkmarks, or lists. Ask students to create questions that relate to the data they have collected. Students can share their work with a partner, and the partner can interpret the data and answer the questions.
- Ask students to create a chart to show the colours of tops/shirts/blouses the students are wearing and how many students are wearing each colour. Complete the activity again but ask students to organize the data using tally marks instead of a chart. Ask students to compare the two data displays and to explain how they are the same and how they are different.
- Ask the student what is wrong with the following conclusion drawn from the tally marks below:
Just as many kids like pizza as burgers.
Pizza:  
Burgers: 

- Show students a set of data represented in two ways, such as tally marks and as a chart. Ask students to describe how the data is the same and how it is different.
- Show students a sheet of 2-D shapes including squares, triangles, rectangles, and circles. Show them three sets of tally marks meant to represent the number of each type of shape on the sheet. Ask students to identify which set of tally marks correctly represents the sheet of 2-D shapes.

SUGGESTED MODELS AND MANIPULATIVES

- colour tiles
- linking cubes
- links

MATHEMATICAL LANGUAGE

Teacher	Student
<ul style="list-style-type: none"> ▪ collect/gather, organize, record ▪ data ▪ tallies, checkmarks, charts, lists, tables 	<ul style="list-style-type: none"> ▪ data ▪ tallies, checkmarks, charts, lists, tables

Resources/Notes

Print

- *Making Math Meaningful to Canadian Students, K–8* (Small 2009), pp. 525–528
- *Making Math Meaningful to Canadian Students, K–8, Second Edition* (Small 2013), pp. 568–570
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 310–312, 317–319

Notes

SCO SP02 Students will be expected to construct and interpret concrete graphs and pictographs to solve problems.

[C, CN, PS, R, V]

[C] Communication

[PS] Problem Solving

[CN] Connections

[ME] Mental Mathematics and Estimation

[T] Technology

[V] Visualization

[R] Reasoning

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- SP02.01** Determine the common attributes of concrete graphs by comparing a given set of concrete graphs.
- SP02.02** Determine the common attributes of pictographs by comparing a given set of pictographs.
- SP02.03** Answer questions pertaining to a given concrete graph or pictograph.
- SP02.04** Create a concrete graph to display a given set of data and draw conclusions.
- SP02.05** Create a pictograph to represent a given set of data using one-to-one correspondence.
- SP02.06** Solve a given problem by constructing and interpreting a concrete graph or pictograph.

Scope and Sequence

Mathematics 1	Mathematics 2	Mathematics 3
—	<p>SP01 Students will be expected to gather and record data about self and others to answer questions.</p> <p>SP02 Students will be expected to construct and interpret concrete graphs and pictographs to solve problems.</p>	<p>SP01 Students will be expected to collect first-hand data and organize it using tally marks, line plots, charts, and lists to answer questions.</p> <p>SP02 Students will be expected to construct, label, and interpret bar graphs to solve problems.</p>

Background

Concrete graphs organize objects into carefully aligned rows or columns in order that one-to-one correspondence can be used to compare quantities of objects in those rows or columns. The objects used can progress from the real objects to toy models of the real objects to representative concrete objects such as counters or cubes. These concrete graphs can lead to pictographs as pictures of objects are used rather than the real objects, and as a way to record concrete graphs on paper. When constructing concrete graphs and pictographs, model and discuss the importance of aligning objects accurately.

It is important to draw students' attention to the different layouts and formats of graphs. Students should create and interpret graphs that run horizontally and those that run vertically. It would be helpful to show the same data in two different formats.

Additional Information

See Appendix A: Additional Information.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Provide a set of prisms and pyramids. Ask students to sort the shapes into two groups. Ask students, Are there more prisms or pyramids in the set? Ask them to explain how they know.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Have students use colour tiles to create a concrete graph showing the number of boys and girls in the classroom. Ask them to use the graph to determine whether there are more girls or boys in class.
- Have students create a pictograph to show how many students in the class know how to swim (or skate).
- Provide students with a set of data about a topic of interest to them. Ask them to create a pictograph and a concrete graph that represents the data.
- Ask students to explain how they would use a set of data to create a pictograph.
- Show students a concrete graph or pictograph on a topic of interest to students. Have them answer questions about the graph and have them make up their own question that the graph would answer.
- Provide students with a set of concrete graphs or a set of pictographs, one of which has an error in it. Ask students to identify the error and to explain how to correct it.
- Present students with a pictograph or a concrete graph. Ask them to write three questions that could be answered by reading the graph. Ask them to write one question that could not be answered by reading the graph.
- Show students two graphs, one horizontal and one vertical, that represent the same data. Ask students to compare the two displays and to explain how they are the same.
- Provide students with a collection of about 25 linking cubes in three or four different colours. Ask them to organize the cubes and record the data in a chart using tally marks or another method. Provide them with grid paper and have them create a graph to display the data.
- Have students formulate their own question that can be answered by collecting information within the school. Students should be able to write their question, gather their data, display their information in the form of a concrete or pictograph, and write about what they have learned.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets K–2 (Bauman 2011)

- No Checkpoint for this outcome.

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Emphasis should be placed on answering real questions and ways to present data and how to interpret the data that is collected. Provide meaningful opportunities for students to collect, represent, and interpret data; for example, ask students to vote on a class book to read, collect data on the number of sunny, cloudy, rainy, or snowy days in a particular month, or vote on games to play during indoor recess.
- Ensure that data management tasks are relevant and of interest to the students in your class. Each class member should be able to participate and contribute data to the investigation.
- Encourage students to conduct small surveys to collect and display data.
- Use a piece of vinyl and tape to create a floor mat grid on which children can stand to form a graph. Have students place name cards on the grid before they step off it so everyone can see the whole graph that was created.
- Make use of opportunities to integrate graphing concepts into other subject areas, such as You and Your World.

SUGGESTED LEARNING TASKS

- Construct two different pictographs. Have students compare the graphs and identify the common attributes.

- Construct two different concrete graphs. Have students compare the graphs and identify the common attributes.
- Ask students to arrange themselves into a concrete people graph to compare the number of students who are wearing shoes with laces to those wearing shoes with Velcro.
- Have students create three different representations of the same set of data. For example, ask them to glue coloured pasta on the first graph. On the second graph, ask them to draw and colour pasta pieces to represent the same information that is on the first graph. Lastly, ask them to represent the same data with tallies. Discuss how the three representations are the same and how they are different.
- Ask the student what is wrong with the following conclusion drawn from the graph below.
Just as many kids like pizza as burgers.
Pizza: □ □ □ □ □ □
Burgers: □ □ □ □
- Ask the students to create a pictograph to show the number of children in the class who play various games (sports) or musical instruments. Have them write two questions that their graph will answer.
- Show students a graph without a title or labels. Ask them to create different sets of data that the graph could represent.
- Show students a graph without a title or labels and have them match the graph to a set of tally marks.
- Attach a photograph of each child to a strip of magnetic tape. Have each student place his or her photograph on the whiteboard to create a graph.

SUGGESTED MODELS AND MANIPULATIVES

- colour tiles
- grid paper
- linking cubes
- links
- various objects to use in a concrete graph

MATHEMATICAL LANGUAGE

Teacher	Student
<ul style="list-style-type: none"> ▪ compare, similar, different ▪ graphs: concrete/people, pictographs/picture ▪ one-to-one correspondence ▪ title, labels, columns, rows 	<ul style="list-style-type: none"> ▪ graphs: people, picture ▪ title

Resources/Notes

Print

- *Making Math Meaningful to Canadian Students, K–8* (Small 2009), pp. 471–478
- *Making Math Meaningful to Canadian Students, K–8, Second Edition* (Small 2013), pp. 517–523
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 310–312, 317–319

Notes

Appendices

Appendix A: Additional Information

Number (N)

SCO N01 Students will be expected to say the number sequence by

- 1s, forward and backward, starting from any point to 200
- 2s, forward and backward, starting from any point to 100
- 5s and 10s, forward and backward, using starting points that are multiples of 5 and 10 respectively to 100
- 10s, starting from any point, to 100

[C, CN, ME, R]

[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- N01.01** Extend counting sequence (by 1s), forward and backward.
- N01.02** Extend a given counting sequence (by 2s, 5s, or 10s) forward and backward.
- N01.03** Skip count by 10s, given any number as a starting point.
- N01.04** Identify and correct errors and omissions in a given skip counting sequence.
- N01.05** Count a given sum of money with pennies, nickels, or dimes (to 100¢).
- N01.06** Count quantity using groups of 2s, 5s, or 10s and counting on.

Performance Indicator Background

N01.01 It is important for students to learn the sequence of number names in advance of attaching meaning to these numbers. Students should be provided with experiences that enable them to rote count beyond 100 to 200. They should examine how the patterns of words said in the number sequence 100 to 200 are similar to the patterns from 0 to 100. A hundred chart can easily be extended to 200 to aid exploration of the counting sequence to 200.

Students should be able to count forward and backward by 1s from any number in the interval 0 to 200, and extend a given forward and backward sequence. This will help to reinforce understandings of what comes next and what comes before in a given counting sequence. When exploring numbers between 0 and 200, considerable time should be spent focusing on the numbers between 100 and 200, so that students see that the pattern of the decades 10 to 90 repeats and the 1 to 9 patterns within each decade repeats. For example, they should see that 110, 120, 130, ..., 190 is the same pattern as 10, 20, 30, ..., 90 and that 130, 131, 132, 133, ..., 139 is the same as 30, 31, 32, 33, ..., 39. In reciting the numbers over 100, a common misconception for students is to think that 200 comes after 109 because of their experience with numbers to 100 where they observed that the next number after the one that ends in 9 is the next big number name, such as 30 after 29 and 50 after 49.

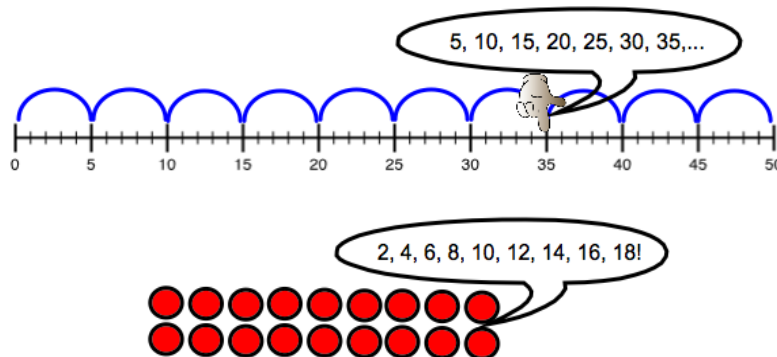
N01.02 Proficiency with skip counting will support later work with operations. In Mathematics 2, it is expected that students will develop the following skip counting skills:

- Counting by 2s, forward and backward, starting at any number within 0 to 100.

- Counting by 5s and 10s, forward and backward, starting from multiples of 5 and 10 within 0 to 100.
- Counting using coins (pennies, nickels, dimes).

Not only should students be able to respond to requests to skip count in any of these three ways, they should also be able to detect patterns in given sequences and extend those sequences. For example, if presented with the sequence 75, 70, 65, ... , students recognize that there is a skip count by 5 backward and continue the sequence 60, 55, 50, 45, 40, ...

It is helpful to students to use models, such as a number line or counters, as they are developing skip counting skills. With number lines, students need to realize that they are counting spaces, while with counters they are counting quantities.

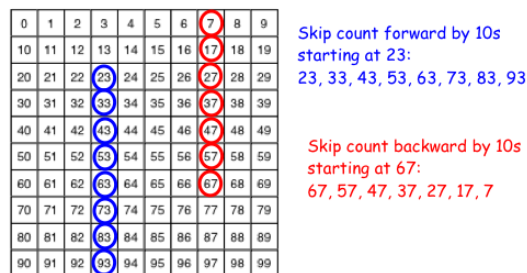


Students should be given frequent opportunities to develop these skills so they become proficient and are able to skip count without models.

When planning opportunities for skip counting, consider using appropriate related children’s literature; having students walk on a number line or on a 100 mat; and using rhythmic skip counting activities such as clapping, marching, drumming with hands on the desk, and striking instruments. Students may also use the constant function on a calculator (press 0, +, 2, =, =, =, ...) to skip count to a target number.

Students should be asked reflective questions to solidify their understanding of skip counting. For example, if you start at 0 and want to end at 40, by which number(s) could you skip count? (2, 5, 10) What if you started at a different point? What if you wanted to end at a different point?

N01.03 Students should be able to start at any number between 0 and 100 and skip count by 10s, forward or backward. This will help to develop strategies for addition and subtraction that will be used in later outcomes. A hundred chart is an especially helpful tool for supporting the development of skip counting by 10 from any number because they are able to see that from any starting point they simply continue to count down or up the column.



N01.04 Students should be presented with incorrect skip counting sequences and be asked to identify and correct errors; for example, Mary was saying the numbers 2, 4, 6, 8, 10, 11, 12, 14, 17, 20. Ask if Mary made any errors and, if so, what were they and what should she have said instead. Similarly, students should be presented with skip counting sequences with missing terms.

N01.05 One use of skip counting is for counting coins. Students need sufficient experience counting pennies, nickels, and dimes separately using skip counting by 5s and 10s.

N01.06 It is important that students understand that skip counting is a way to find out how many objects are in a set, and that they will get the same quantity as counting by 1s. It is important that students have many and varied experiences with materials that they can group and count in a variety of ways. These activities should be purposeful, and presented throughout the year as students build their sense of number.

“As students get older, the numbers they deal with in their everyday lives become more complex. Students need strategies for representing and making sense of these greater numbers. Although it is possible to count, say, 87 items individually, it is not practical. When items are grouped, counting is made easier and probably more accurate.” (Small 2009, 138).

Provide students with a variety of activities that require students to recognize when it is more efficient to skip count quantities. For example, a student may be given a quantity of counters or blocks and asked to find a way to count them efficiently. Teachers should watch to see if students group in 2s, 5s, or 10s to make counting easier. Encouraging students to group to count will help them to appreciate grouping ideas that will be foundational to later place-value concepts.

SCO N02 Students will be expected to demonstrate if a number (up to 100) is even or odd.			
[C, CN, PS, R]			
[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

Performance Indicators

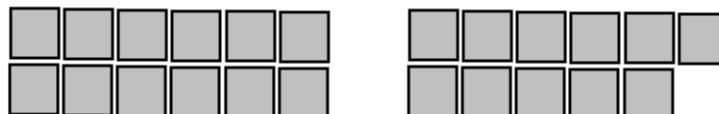
Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- N02.01** Use concrete materials or pictorial representations to determine if a given number is even or odd.
- N02.02** Identify even and odd numbers in a given sequence, such as on a hundred chart.
- N02.03** Sort a given set of numbers as even numbers and odd numbers.

Performance Indicator Background

N02.01 It is important to provide students with a variety of concrete and pictorial representations of even and odd numbers. These concepts can be shown using counters. If all of the counters can be paired up, that number is even; if all the counters but one can be paired up, then the number is odd. In other words, students should discover

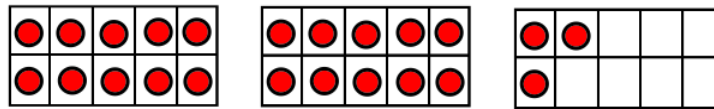
- even numbers can be used to make groups of two with none leftover or two equal whole number groups
- odd numbers can be used to make groups of two with one left over or cannot be shared into two equal whole number groups



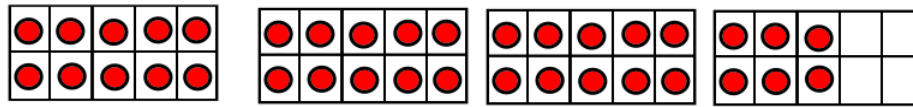
N02.02 Once students have experienced small even and odd numbers using concrete and pictorial models, the concept can be extended to larger numbers. Students should come to discover that even-number quantities have numerals that end in the digits 0, 2, 4, 6, or 8, and odd-number quantities have numerals that end in the digits 1, 3, 5, 7, or 9. Students should arrive at these generalizations through hands-on experiences. Students will learn that even numbers are the numbers they say when they count by 2s from 0. Odd numbers are the numbers they say when they count by 2s from 1.

Often students will mistakenly consider the tens digit when looking at the symbolic representation of numbers, such as 14, 23, and 55, to decide whether it is even or odd. An important concept in understanding even and odd is that the ones digit is the determining feature. By examining the hundred chart and using models to consider whether quantities can be shared in two equal groups, students will come to discover that the tens digit does not affect the evenness of a number; the ones digit will be the deciding factor.

A possible way to clarify the misconception regarding the tens digits in 2-digit numbers is to encourage students to represent the number with ten-frames or square tiles to determine if it can be shared in two equal parts.



23 is odd, it cannot be organized into two equal rows.



36 is even, it can be organized into two equal rows.

Note: Since using counters to convince students that zero is even is too abstract, noting its position on the hundred chart in an even column and having an odd number as a neighbour introduces this idea more convincingly.

N02.03 Students should be able to take a given set of numbers and sort them into even and odd numbers and be able to explain how they determined the sort.

SCO N03 Students will be expected to describe order or relative position using ordinal numbers (up to tenth). [C, CN, R]			
[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- N03.01** Indicate a position of a specific object in a sequence by using ordinal numbers up to tenth.
N03.02 Compare the ordinal position of a specific object in two different given sequences.

Performance Indicator Background

N03.01 Students should be able to identify orally, in words, and using symbols, the relative position of objects or events in sets that have been ordered by different criteria. They should also identify the object or event that corresponds to a stated ordinal number.

While calendars show cardinal numbers, they are actually ordinal numbers because the numbers refer to the days' positions within the month; for example, we write November 10, but say November tenth. Students will experience ordinal numbers up to 31st through exposure to the calendar; however, they need only be assessed on ordinal numbers to tenth.

N03.2 Students should be asked to identify the position of the same object in different sequences so that they realize the ordinal number is not dependent upon the object itself, rather its position in a sequence. Therefore, the ordinal number used will change if the sequence changes.

SCO N04 Students will be expected to represent and partition numbers to 100.			
[C, CN, V]			
[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- N04.01** Represent a given number using concrete materials, such as ten-frames and base-ten materials.
- N04.02** Represent a given number using coins (pennies, nickels, dimes, and quarters).
- N04.03** Represent a given number using tallies.
- N04.04** Represent a given number pictorially.
- N04.05** Find examples of a given number in the environment.
- N04.06** Represent a given number using expressions (e.g., $24 + 6$, $15 + 15$, $40 - 10$)
- N04.07** Read a number (0–100) given in symbolic or word form.
- N04.08** Record in words a given number (0–20).
- N04.09** Record, symbolically, any number (0–100).

Performance Indicator Background

N04.01 Students need to represent and describe numbers using a variety of concrete materials, such as wooden stir sticks, straws, beans, counters, ten-frames, and books. Students may bundle a quantity of stir sticks or toothpicks to show partitions of a given quantity. For example, a student may take 50 stir sticks and show this can be bundled in five groups of 10, $10 + 10 + 10 + 10 + 10$. They may also group the bundles of ten to show that it could be expressed as $30 + 20$ or $10 + 40$. They may also open one bundle and show that 50 could be expressed as $25 + 25$ or $21 + 29$.

After experiences with groupable materials bundled in 10s, students can be introduced to base-ten blocks. These commercially made materials differ from the counters and bundled materials because they involve trading a set of ten 1s (small cubes) for a single representation of ten (rod). Take time to discuss this difference with students and introduce these materials carefully.

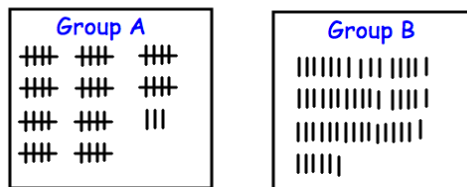
N04.02 Counting coins is an important skill for students to develop and also provides an everyday context for representing and partitioning a number. Students will need to be introduced to the names and values of pennies, nickels, dimes, and quarters in order to be able to use them in their representations.

Students could be asked to show different ways to make 78¢ with coins. They can describe how the coins show different partitions of the number. Students should be given numerous opportunities to count coins in context and discuss various ways to partition amounts using coins. The emphasis should be on using whole numbers with the ¢ symbol and not decimal numbers with the \$ symbol.

Note: Most materials used to represent numbers have a visible one-to-one correspondence with the quantity they represent. Coins do not. Each coin represents a value that is not easily seen or proportional to its value.

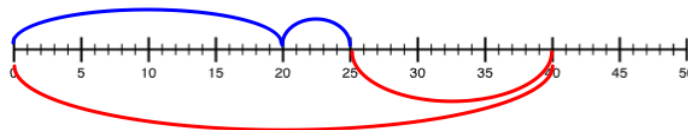
N04.03 Students should be introduced to tally marks as a way to count in groups of 5. This can be especially useful when students are counting larger objects that cannot be easily moved into nice arrangements of equal groups. For example, students may use tally marks while counting cars in a

parking lot, posters in the school hallways, or windows in the school. Students should be shown how to make tally marks and should discuss why counting in this way makes counting easier. For example, students may be asked to talk about why the 53 tally marks in group A are easier to count than those in group B.

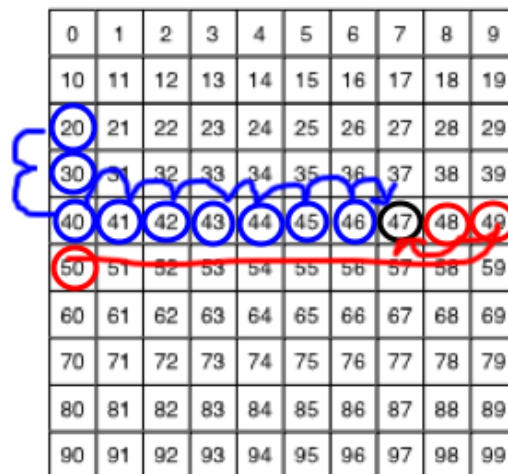


N04.04 Students should be able to use pictorial models to represent and partition number. Initially, students will use pictures to record their concrete models. Additional pictorial representations can be added throughout the year.

Both number lines and hundred charts are important pictorial models for students to use when partitioning numbers. For example, a student may show that 25 can be represented on the number line by a jump of 20 followed by a jump of 5 ($20 + 5$) or a jump of 40 followed by a jump back of 15 ($40 - 15$).



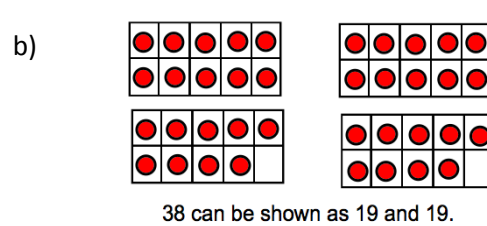
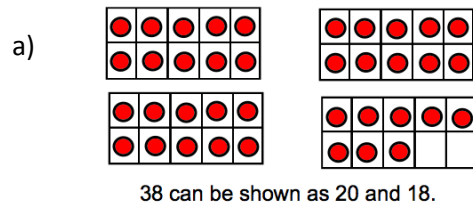
Similarly, a student may show that to get to 47 in a hundred chart a student may start at 20 and jump down 20 and over 7, showing that 47 can be represented by $20 + 27$. Similarly a student may say that if you start at 50 and go back 3 you will also get to 47, which can be represented by the expression, $50 - 3$.



N04.05 An important part of number development is to be able to identify and explain how numbers and quantities are used in everyday contexts. Students should be able to identify real-world examples of where they may see quantities of 0 to 100 objects. For example, they may be able to say that there were about 50 fans at the hockey game on the weekend, or they saw 10 to 15 ducks on a pond.

Students should also discuss numbers they see being used in their environment that are not connected to quantity, such as house numbers, sports jerseys, and clothing sizes.

N04.06 Students should represent a quantity in a variety of ways using partitioning. For example, a student may be asked to show 38 in two separate parts using counters and ten-frames and may show this in one of the two ways below:



Addition and subtraction symbols were introduced to students in Mathematics 1. Students should be encouraged to use words and symbolic notation to describe what they have done concretely or pictorially. There is no expectation that students should be able to partition numbers symbolically without having used concrete or pictorial models first. For example, a student may partition 63 objects, saying and writing that 63 can be shared in a group of 30 and a group of 33 before recording this symbolically as $30 + 33$. Also a student may use a number line and say and write that jumping forward 70 and back 7 will also get us to 63 before writing $70 - 7$.

N04.07 Students should be given lots of exposure to the written form of numerals to 100. This may be in the form of written instructions on how to complete tasks, charts displayed in the classroom with pictorial and symbolic representations, or as entries into a personal mathematics dictionary or on a classroom mathematics word wall.

N04.08 Provide ample opportunities for students to record number words up to 20. For example, there are seventeen boys in the classroom. Number words are often written in contexts of language arts, science, and social studies.

SCO N05 Students will be expected to compare and order numbers up to 100. [C, CN, R, V]			
[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- N05.01** Compare and order a given set of numbers in ascending or descending order and verify the result using a hundred chart, number line, ten-frames, or by making references to place value.
- N05.02** Identify errors in a given ordered sequence.
- N05.03** Identify missing numbers in a given hundred chart.
- N05.04** Identify errors in a given hundred chart.

Performance Indicator Background

N05.01 Students should develop strategies for ordering numbers. Seeing the relative position of numbers on number lines and in hundred charts will help students to solidify their understanding of the order and size of numbers. Students should know that as they count forward, the numbers increase and the quantity they represent is greater. Students may use a hundred chart to show that 87 is more than 63 because 87 is in the 80s and 80 is more than 60.

An open number line (no marked increments) provides opportunity for students to refine their knowledge of number relationships. Students may be asked to place the numbers 49, 18, 25, 37, and 42 on a number line where 0 and 50 are the only numbers shown. Students will need to use benchmarks and what they know about the relative position of numbers to accurately place numbers on that number line.

Another strategy for ordering numbers is to consider place value. Students may refer to the number of tens when ordering numbers. For example, 47 is more than 21 since all numbers in the 40s are greater than all numbers in the 20s. Students should focus on the fact that the digit 4 in 47 has a value of 40 and the digit 2 in 21 has a value of 20. This knowledge will emerge from having represented numbers using ten-frames and groupable materials.

N05.02 Students should be able to identify when a given sequence of numbers is not in the correct order and be able to correct it. For example, they may be told that a student was asked to sort numbers in ascending order and gave the following sequence: 13, 17, 26, 24, 28, 42, 38, 56. They should be able to identify which numbers are not in the correct order and rearrange them. They should be encouraged to talk about how they made their corrections.

N05.03 Students should have enough familiarity with the hundred chart that they are able to identify the values of missing numbers. A student could be given a hundred chart with numbers missing and be asked to fill in the missing values and to explain how they decided what number went in each empty position. They may also be given a hundred chart, cut into pieces, and be asked to put it back together in the correct way.

N05.04 Students should be able to identify errors in a hundred chart and explain why they believe that there are errors. For example, as a routine morning activity, display a hundred chart with individual numbers in the wrong positions, or entire decades in the wrong order. Have students come up one at a time and identify the error and explain to the class why it is wrong. Students should correct the hundred chart as they explain the error.

SCO N06 Students will be expected to estimate quantities to 100 by using referents. [C, ME, PS, R]			
[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

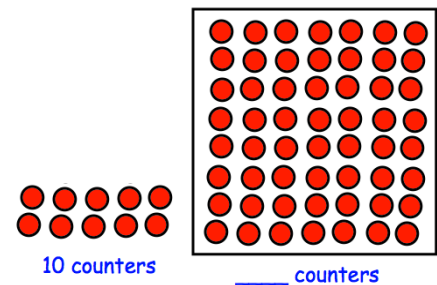
- N06.01** Estimate a given quantity by comparing it to a referent (known quantity).
- N06.02** Estimate the number of groups of ten in a given quantity using 10 as a referent.
- N06.03** Select between two possible estimates for a given quantity and explain the choice.

Performance Indicator Background

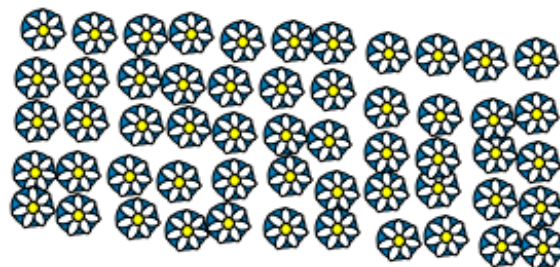
N06.01 A referent (a known quantity) is useful as a benchmark or anchor in the development of estimation skills. Students use referents to determine the amount in a large group of similar objects. For example, if I know what a group of 5 people looks like, I can estimate the number of people in a classroom. Ask students to explain their decision. Dot cards or similar models are helpful in providing visual referents and building estimation skills. Subitizing quantities addressed in Mathematics Primary and Mathematics 1 should be continued as they provide helpful referents.

N06.02 To estimate quantities to 100, 10 is an important referent. For example, if the small picture shows 10 counters, how many counters are in the larger picture?

Students need a strong sense of 10-ness in order to use ten as a referent. Provide opportunities for students to see 10 in a variety of different contexts and arrangements; for example, 10 people, 10 chairs, 10 counters.



N06.03 Students should also be able to judge the reasonableness of an estimate and be able to explain the reasons for their choice. For example, they may be asked to say whether 50 or 80 would be a good estimate for the collection of flowers below. To justify their estimate, a student may circle the first two columns (ten flowers) and use this as a referent.



Students should be provided with numerous opportunities to choose between two possible estimates for a given set of objects and to explain how they have chosen their estimate. The more they do this, the more refined their estimates will become.

SCO N07 Students will be expected to illustrate, concretely and pictorially, the meaning of place value for numerals to 100.			
[C, CN, R, V]			
[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

Performance Indicators

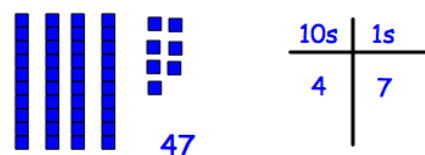
Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- N07.01** Explain and show with counters the meaning of each digit for a given 2-digit numeral with both digits the same.
- N07.02** Count the number of objects in a given set using groups of 10s and 1s, and record the result as a 2-digit numeral under the headings of 10s and 1s.
- N07.03** Describe a given 2-digit numeral in at least two ways.
- N07.04** Illustrate using ten-frames and diagrams that a given numeral consists of a certain number of groups of ten and a certain number of ones.
- N07.05** Illustrate using proportional base-ten materials that a given numeral consists of a certain number of tens and a certain number of ones.
- N07.06** Explain why the value of a digit depends on its placement within a numeral.
- N07.07** Represent one unit if shown a pre-grouped model representing ten.

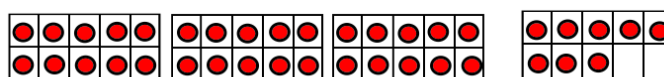
Performance Indicator Background

N07.01 Students should be given many opportunities to build groupable models of 2-digit numbers in which the digits are the same and explain which parts of the model represent the tens and which parts represent the ones. For example, a student may build 22 by showing 2 full ten-frames and 2 extra counters on an additional ten-frame. They should be able to explain that the 2 full ten-frames represents the 20, shown by a 2 in the tens place, and the 2 counters represent the 2 in the ones place. They may also use bundled sticks to show 22 by showing that there are two full bundles of 10 and 2 extra sticks.

N07.02 Students need to be able to count quantities by groups of tens and ones and be able to explain how many tens and how many ones the number represents. For example, if students are shown the base-ten model below they should be able to identify the quantity by counting the rods in 10s (10, 20, 30, 40) and then counting the small cubes (41, 42, 43, 44, 45, 46, 47). They should have opportunities to record this in a chart under 10s and 1s, and as the number 47.



Students need to see different models of 2-digit numbers and become comfortable with using counting strategies learned in SCO N01 to count the quantity and discuss what this tells them about the place value of each digit. For example, they may be presented with ten-frame models or bundled sticks. They should be able to identify the quantity of tens and ones in each model.

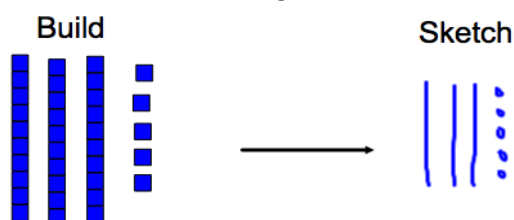


38 is 3 sets of 10 and 8 more.

N07.03 When students understand place value they are able to think of a number, such as 37, not only as 3 tens and 7 ones or $30 + 7$, but also as 2 tens and 17 ones or $20 + 17$, and as 37 ones. Students should be asked to model 2-digit numbers in at least two different ways. This connects place value concepts to work done in SCO N04 with partitioning. This flexibility with numbers will provide good preparation for later work with operations. For example, a student should be able to show and explain why 43 can be shown using 4 tens and 3 ones or as 3 tens and 13 ones. This may be needed later in subtraction situations such as $43 - 37$ or $43 - 15$.

N07.04 Students should be able to sketch models and diagrams to show how a certain numeral can be shown in groups of ten and groups of one. Ten-frames can be very effective ways for students to sketch a model for a 2-digit number, such as the model for 38 shown above. These ten-frame models clearly show groups of ten and ones.

Students should also develop strategies for sketching base-ten models. Rather than spending time drawing ten small cubes attached together to represent a rod, students can simply draw a line. To represent a small cube, they can draw a circle or a large dot.



Of course, such representational pictures often do not visually show the relationship between a rod and a small cube. Therefore, careful consideration should be given as to the best time to introduce such pictures.

N07.05 Students should have plenty of opportunities to manipulate and work with base-ten blocks to represent numbers to 100. Have students model 2-digit numbers in various ways, such as 46 as 46 ones, or 4 tens and 6 ones, or 3 tens and 16 ones. Ask students to find out how many ways they can make a given number using base-ten blocks.

N07.06 Students should be able to say that 45 can be modelled with four groups of 10 and five 1s. They should be able to say that the number in the tens place represents 40 and the number in the ones place represents 5. This will enable students to develop the understanding that the position of a digit matters and they will be able to explain why the value of a digit depends on its placement within a numeral. For example, they should be able to use this reasoning to explain the difference between the 2 in 42 and the 2 in 27.

SCO N08 Students will be expected to demonstrate and explain the effect of adding zero to or subtracting zero from any number.

[C, R]

[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N08.01 Add zero to a given number and explain why the sum is the same as the addend.

N08.02 Subtract zero from a given number and explain why the difference is the same as the given number.

Performance Indicator Background

N08.01 To experience the no-change nature of adding zero, students should be encouraged to think about building a set of a certain amount and then adding zero to it to see that the amount remains unchanged. They can also stand on a certain spot on a number line and take no steps forward; they have not moved position. These experiences show that adding zero to a quantity results in no change, such as $12 + 0 = 12$ and $25 + 0 = 25$.

N08.02 Students need similar experiences with subtracting zero. They should be able to explain that if they build a set of a certain amount and take away zero, the quantity stays the same. If a student is standing on a number line at 6 and does not take any steps backwards, he or she stays on the same spot, and $6 - 0 = 6$ represents this action. Students should be able to record subtraction of zero as a no-change nature such as $37 - 0 = 37$ and $21 - 0 = 21$. In subtraction, students can also consider the difference between a number and zero. This difference meaning of subtraction provides an alternative way for students to think about subtracting zero. For example, if one student is standing at 0 on the number line and another student is standing at 6, the students are 6 units apart, and $6 - 0 = 6$.

SCO N09 Students will be expected to demonstrate an understanding of addition (limited to 1- and 2-digit numerals) with answers to 100 and the corresponding subtraction by			
<ul style="list-style-type: none"> ▪ using personal strategies for adding and subtracting with and without the support of manipulatives ▪ creating and solving problems that involve addition and subtraction ▪ explaining and demonstrating that the order in which numbers are added does not affect the sum ▪ explaining and demonstrating that the order in which numbers are subtracted matters when finding a difference 			
[C, CN, ME, PS, R, V]			
[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- N09.01** Solve a given story problem of any type by modelling it with materials or a diagram, and write a number sentence that represents the thinking in the solution.
- N09.02** Solve a given story problem of any type by writing a number expression and combining the numbers to complete the number sentences.
- N09.03** Match a number sentence to a given story problem.
- N09.04** Create an addition or a subtraction number sentence and a story problem for a given solution.
- N09.05** Model addition and subtraction using concrete materials or visual representations and record the process symbolically.
- N09.06** Add a given set of numbers in two different ways and explain why the sum is the same.
- N09.07** Recognize and create equivalent addition and subtraction number sentences.

Performance Indicator Background

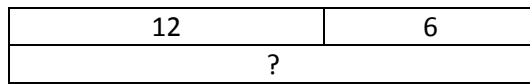
N09.01 Students should be presented with many opportunities to model story problems of all types (see chart), and find solutions while modelling using counting, addition, and/or subtraction. They should be able to write appropriate number sentences that reflect how they thought about the problems. Through sharing of strategies and discussions, students will begin to realize that the same situation, such as a join (change unknown), can be thought about as addition by some students and subtraction by others.

For example, consider the following story problem. Pat has 8 marbles but she would like to have 12. How many more does she need to get? Some students may count out 8 marbles and add on until they reach 12, counting the 4 that they added on. These students would write $8 + 4 = 12$ to represent how they thought about the problem. Other students may count out 12 marbles, remove the 8 they knew they had, and count the 4. These students would write $12 - 8 = 4$ to represent how they thought about the problem. Either sentence would be acceptable, but they should be able to explain the number sentence they have written in relation to the situation they are modelling.

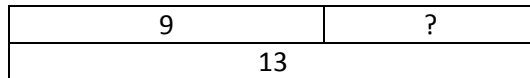
After students have modelled and solved a number of addition and subtraction situations, they may be introduced to pictorial representations such as those described below in performance indicator 9.05. As well, Van de Walle and Lovin offer four possible pictorial representations for addition and subtraction.

Refer to *Teaching Student-Centered Mathematics, Grades K–3, Volume 1*, page 67 (Van de Walle and Lovin 2006) for an example.

In addition to the pictorial representations described by Van de Walle and Lovin, students may be introduced to strip diagrams as another way to represent the situations. For example, Bobby was given 6 green stamps. He already had 12 stamps. How many does he have now? The strip diagram for this problem is



For example, Bobby had 9 stamps. He was given more by his friend. Then he had 13 stamps. How many stamps did his friend give him? The strip diagram for this problem is



Because students have to decide where to place in the diagram the two given numbers in the story problem, they have to carefully read the problem to determine whether each given quantity is a part or a whole. If the quantity is a part, it would be placed in one section of the top rectangle; if the quantity is a whole it would be placed in the bottom rectangle. They should put a question mark in the bottom rectangle or one of the sections in the top rectangle, depending upon what is missing (what they are asked to find).

The principal use of strip diagrams is as a strategy to help students interpret story problems. Students will solve the problems using their personal strategies; however, through extensive use of strip diagrams some students may generalize that subtraction is the operation that will always find a missing part and that addition will always find a missing whole.

Students in Mathematics 2 may be less familiar with the types of comparison problems than the other types of problems. As such, these comparison problems should be the subject of specific lessons and discussions. Subtracting two quantities to find how much more one quantity is than another is conceptually difficult for many students because they associate “moreness” with addition, and subtraction with “take away” rather than “difference.”

N09.02 By the end of Mathematics 2, after many experiences modelling story problems of all types, students should be able to read some story problems, write open number sentences, and work with the numbers to solve the problem. For example, consider the following story problem. Pat has 28 marbles. Her brother gives her 14. How many does she have now? Students should be able to read this problem, write $28 + 14 = \underline{\quad}$ without modelling the story, and combine the 28 and 14 using their personal strategies to get the answer 42.

N09.03 When presented with a story problem of any type, students should be able to match a number sentence to the story problem. For example, read students a story problem that includes a missing addend such as, There were 9 students that drank milk during recess. If 3 students drank white milk, how many had chocolate milk? Then display two or three number sentences on the board or on chart paper.

$$9 + 3 = 12$$

$$12 - 3 = 9$$

$$9 - 3 = 6$$

Have them match the appropriate number sentence with the problem. Encourage students to discuss their strategies to solve the problem.

N09.04 Creating story problems is an important part of the development of students' understanding of addition and subtraction. As students create their own number stories, they may use different materials, such as snap cubes, ten-frames, toys, or other students to model the situations. If they are told that the solution to a story problem is a specified number, such as 12, they should be able to create addition and subtraction number sentences and corresponding story problems involving this number as the solution.

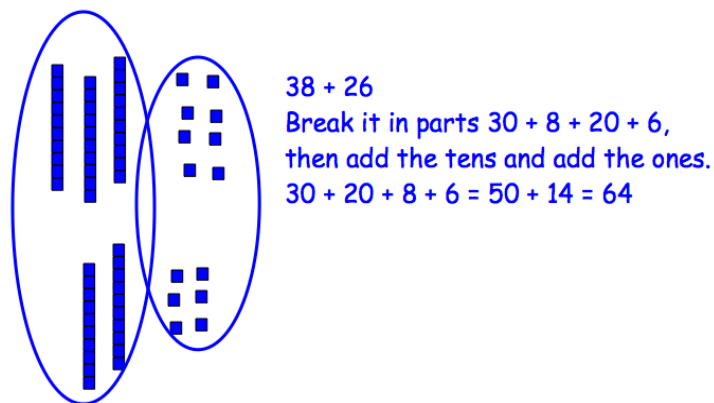
Students will need directed lessons to create story problems of various types. For example, students could be given four different story problems of one type, such as join (start unknown), asked to solve them, and discuss how the four problems are alike. Then they should create other story problems that are like these four.

N09.05 It is essential that students model addition and subtraction using a variety of models that may include counters, base-ten materials, number lines, and hundred charts. The more models students become comfortable with, the more flexibility they have with open number sentences. Students should not be taught a variety of strategies and be expected to memorize them; rather, they should develop their own personally meaningful strategies through exploration, modelling, and sharing. The strategies should emerge from the models students use through focused questions and discussions.

Initially, students will get the answer using models but may not be clear on what strategy they used. Encourage students to use symbols to record what they did with the models, so there is a strong connection between the symbolic and concrete representations. This will encourage students to reflect on what they are doing in order to realize that they are using a strategy that could be applied to the addition or subtraction of any two numbers.

While there are many strategies that could be used for addition and subtraction, two possible strategies for each operation are illustrated below.

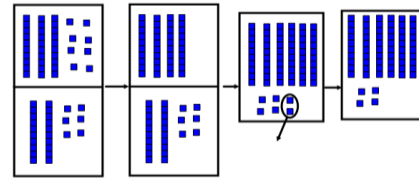
A student may use base-ten blocks to model $38 + 26$ as shown below.



This method requires place value understanding and should be developed with consistent use of concrete models.

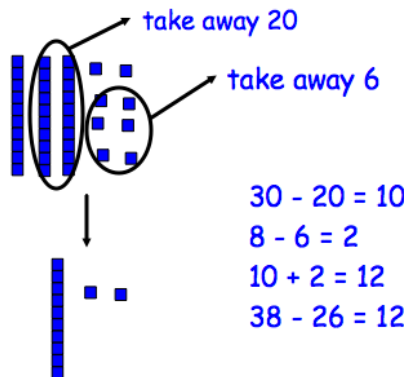
Another student may use a compensating (making nice or friendly numbers) strategy.

Students need to use their knowledge of compatible number pairs for 10 to use this strategy.



I noticed that 38 is almost 40 so I added $40 + 26$, but this was 2 too many so I took 2 away.
 $38 + 26 = 64$ because $40 + 26 = 66$ and $66 - 2 = 64$

Students may use base-ten blocks and place-value strategies to subtract the tens and subtract the ones. This strategy is most efficient when no regrouping is required. When regrouping is required, students will need to recognize that a trade of one ten for ten ones may be needed. In the example to the right, the student is modelling $38 - 26$ using base-ten models, and started by showing 38. They have recorded the calculation to the side.

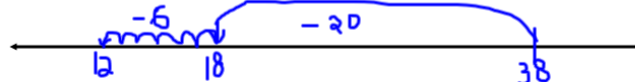


Another approach to solving this problem involves taking away 10s, then 1s. This can be modelled using the number line or hundreds chart.

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

I started with 38 and jumped back 20 to 18, then I went back 6 more to get to 12.

$38 - 26$
 $38 - 20 = 18$
 $18 - 6 = 12$

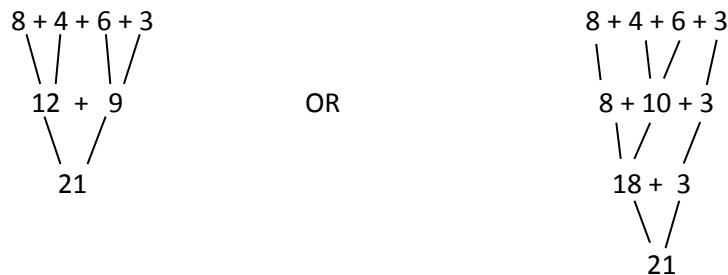


When two 2-digit numbers are added and subtracted, standard algorithms may become part of the discussion, if students share how their parents do it. While these standard algorithms may be shared and discussed, there is no expectation that all students should use these strategies for all addition and subtraction questions. The goal is to have students develop a repertoire of personal strategies and be flexible, efficient, and accurate in their application. In Mathematics 2, strategies that use the quantity represented by the tens digit (as in the examples above) should be given emphasis.

N09.06 Students should understand and use the fact that two numbers can be added in any order (commutative property of addition). For example, once they find $45 + 36$ is 81, they should realize that $36 + 45$ is also 81. Students should extend this understanding that two addends can be combined in any order to situations that have three or four addends. The focus should continue to be that the order in which you add does not matter. For example, $2 + 5 + 8$ could be added left to right, $2 + 5$ is 7 and $7 + 8$ is 15 or it can be thought of as $2 + 8 + 5$, so again going left to right $2 + 8 = 10$ and $10 + 5$ is 15. This may be modelled with Cuisenaire rods or illustrated with a strip diagram to see that the order does not matter.

2	5	8
2	8	5
?		

Students should be able to explain that, in addition, the addends can be added in any order and the result will be the same. The associative property tells us that three or more addends can be grouped in different ways to make adding easier. For example, $8 + 4 + 6 + 3$ can be thought of in both of the following ways:



As with order, students should be able to explain that different groupings will not change the sum.

It is not necessary that students use the vocabulary **commutative property** or **associative property**; they should understand and apply these to make adding more efficient. For example, they may combine these two properties by changing the order to group friendly (compatible) numbers.

N09.07 Through sharing and discussion of story problems, students should realize that often the same problem may be thought of as an addition situation by some students and as a subtraction situation by others. Strip diagrams provide a graphic illustration of the relationship between addition and subtraction. There are four number sentences that can be written from a given strip diagram.

9	4
13	

For example, for this strip diagram, the possible number sentences would be $9 + 4 = 13$, $4 + 9 = 13$, $13 - 4 = 9$, $13 - 9 = 4$. These four number sentences that represent the relationships among the three numbers are said to be equivalent.

When presented with a set of number sentences, students should be able to recognize those that are equivalent and those that are not. Given a number sentence, students should be able to write other number sentences that are equivalent to it.

SCO N10 Students will be expected to apply mental mathematics strategies to quickly recall basic addition facts to 18 and determine related subtraction facts.

[C, CN, ME, R, V]

[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- N10.01** Explain the mental mathematics strategy that could be used to determine basic addition facts.
- Doubles Facts
 - Plus-One Facts
 - One-Apart (Near Doubles) Facts
 - Plus-Two Facts
 - Plus-Zero Facts
 - Make-10 Facts
 - Two-Apart Facts
 - Plus-Three Facts
- N10.02** Use and describe a personal strategy for determining a sum to 18.
- N10.03** Quickly recall basic addition facts to 18 in a variety of contexts.
- N10.04** Explain the think-addition strategy used to determine a basic subtraction fact.
- N10.05** Use and describe a personal strategy for determining the subtraction facts.

Performance Indicator Background

N10.01, **N10.02**, and **N10.03** The following strategies are presented in a sequence for introduction, reinforcement, and subsequent integration with previous strategies. In this provided sequence, all facts involving zeros are left until 51 other facts are known. This approach is partly for motivational reasons because students get to add 19 relatively easy facts to their charts to get a total of 70. This approach is also suggested to help counter a common misconception that causes many students to make errors with these facts—they overgeneralize that addition involves an action that results in a larger quantity; thus, they are reluctant to give a response that shows no change. As well, addition sentences with zeros are not likely encountered in response to story problems or in situations in their everyday lives.

DOUBLES FACTS

There are nine *doubles* facts: $1 + 1$, $2 + 2$, $3 + 3$, $4 + 4$, $5 + 5$, $6 + 6$, $7 + 7$, $8 + 8$, $9 + 9$.

The strategy for the double facts is to use association to common things with which students have experience. The table below gives examples of possible associations. Use these or others with which students can strongly identify.

Double Fact	Possible Association
1 + 1	Number of tires on two unicycles.
2 + 2	Number of tires on two bicycles. Number of eyes on two people.
3 + 3	Number of tires on two tricycles. Number of sides on two triangles. Six pack of pop.
4 + 4	Number of tires on two cars. Number of sides on two squares. Number of legs on a spider.
5 + 5	Number of fingers on two hands.
6 + 6	Dozen eggs in a carton.
7 + 7	Number of days in two weeks.
8 + 8	Number of crayons in two rows in a box. Number of legs on two octopuses.
9 + 9	Number of tires on an 18-wheeler truck.

PLUS-ONE FACTS

There are 16 *plus-one* facts: $2 + 1$, $3 + 1$, $4 + 1$, $5 + 1$, $6 + 1$, $7 + 1$, $8 + 1$, $9 + 1$ and their commutative pairs $1 + 2$, $1 + 3$, $1 + 4$, $1 + 5$, $1 + 6$, $1 + 7$, $1 + 8$, $1 + 9$ (**Note:** $1 + 1$ was already a double but could also be thought of as next number.)

These strategies are learned with a call for the next number. That is, whenever students see a 1 in an addition phrase, they should look at the other number and think, What number comes after this number? For example, for $6 + 1$, think, The number after 6 is 7.

Time should be spent first with the set of $___ + 1$ facts, and then with the set of $1 + ___$ facts, being careful that students are convinced about the commutative nature of addition. Finally, mix both sets.

ONE-APART (NEAR-DOUBLES) FACTS

There are 14 *near-doubles* facts: $2 + 3$, $3 + 4$, $4 + 5$, $5 + 6$, $6 + 7$, $7 + 8$, $8 + 9$ and their commutative pairs $3 + 2$, $4 + 3$, $5 + 4$, $6 + 5$, $7 + 6$, $8 + 7$, $9 + 8$. (**Note:** $1 + 2$ and $2 + 1$ were already included in the plus-one facts but could also be thought of as 1-aparts.)

This strategy is a combination of the doubles facts and the plus-one facts. It involves doubling the smaller number and adding one. (Some students may double the larger and subtract 1.) Students need to be convinced that the larger number can be partitioned without changing the sum. For example, they need to see that for $4 + 5$, the 5 can be partitioned into 4 and 1, and that the 4 can be combined with the other 4 before adding the 1. For example, for $2 + 3$, think, double 2 is 4 and the next number is 5; or 2 and 2 is 4, and plus 1 is 5.

PLUS-TWO FACTS

There are 12 *plus-two* facts: $4 + 2$, $5 + 2$, $6 + 2$, $7 + 2$, $8 + 2$, $9 + 2$ and their commutative pairs $2 + 4$, $2 + 5$, $2 + 6$, $2 + 7$, $2 + 8$, $2 + 9$. (**Note:** $1 + 2$, $2 + 2$, $3 + 2$, $2 + 1$, and $2 + 3$ have already been included in other strategies but could also be thought of as next even or odd numbers.)

This strategy involves associating the addition of 2 with skip counting by 2, or with getting the next even or odd number. You could help students get ready for this strategy by conducting skip-counting chants starting at different numbers. For example, for $5 + 2$, think, 5 is an odd number and the next odd number is 7; or skipping 2 starting at 5 is 7.

PLUS-ZERO FACTS

There are 19 *plus-zero* facts: $0 + 0$, $1 + 0$, $2 + 0$, $3 + 0$, $4 + 0$, $5 + 0$, $6 + 0$, $7 + 0$, $8 + 0$, $9 + 0$ and their commutative pairs $0 + 1$, $0 + 2$, $0 + 3$, $0 + 4$, $0 + 5$, $0 + 6$, $0 + 7$, $0 + 8$, $0 + 9$.

This strategy involves the association of adding zero with making no change to the other addend. This should be introduced by reference to story problems that would be represented by the addition of zero. For example, I had 5 stickers. I met my friend who would not give me any more stickers. How many stickers did I have after I met my friend? Stories, such as this one, are not likely to have been encountered before, and while students probably think of them as silly, they do make the point about the role of zero in addition. For example, for $5 + 0$, think, adding zero will make no change to 5, so the answer is 5.

When reinforced in isolation, these facts are not problematic; however, when integrated with other facts, some students treat them as if they were plus-one facts because they overgeneralize that addition is an action that makes a larger quantity and are reluctant to record no change as a result. Consequently, you will probably not have to spend much time reinforcing these 19 facts in isolation; rather, more time should be spent reinforcing the total of 70 facts.

MAKE-10 FACTS

There are 10 *Make-10* facts involving 9: $9 + 3$, $9 + 4$, $9 + 5$, $9 + 6$, $9 + 7$ and their commutative pairs $3 + 9$, $4 + 9$, $5 + 9$, $6 + 9$, $7 + 9$.

There are 8 *Make-10* facts involving 8: $8 + 3$, $8 + 4$, $8 + 5$, $8 + 6$ and their commutative pairs $3 + 8$, $4 + 8$, $5 + 8$, $6 + 8$.

There are 6 *Make-10* facts involving 7: $7 + 3$, $7 + 4$, $7 + 5$ and their commutative pairs $3 + 7$, $4 + 7$, $5 + 7$.

This strategy involves taking 1 or 2 or 3 from one addend to make the 9 or 8 or 7 addend a 10, and then adding this 10 to what was left from the other addend. A strategy is to visualize a ten-frame and note how many it would take to make 10, and then add on any leftovers from the other number.

Students need to be convinced that adding 9 or 8 or 7 is as easy as adding 10. Start with the facts that involve 9s, and reinforce them. Once the response time is achieved for these facts, move on to isolate the facts involving 8s. Once the response time for these facts is achieved, move on to isolate the facts involving 7s. Finally, isolate the facts involving 7s.

Instead of this Make-10 strategy, some students may naturally add 10, instead of 9, to the other number and then adjust the answer by subtracting 1. It is not necessary that students know the name of this strategy; however, it is the Compensation Strategy.

TWO-APART FACTS

There are 6 *two-apart* facts: $3 + 5$, $4 + 6$, $5 + 7$ and $5 + 3$, $6 + 4$, $7 + 5$.

If 1 from the larger of the two numbers is transferred to the smaller, a double is produced; this double is the number between the two given numbers. If students made Unifix towers for the two numbers, they could move one cube from the larger to the smaller and see the resultant double. Some students have trouble with this strategy because both numbers change so they are doubling a number that they don't actually see in front of them.

Another strategy for these facts involves removing 2 from the larger to make a double of the smaller, so the smaller is doubled and the 2 is added back on. Again, if students used Unifix cubes to model this strategy, they would see the double plus 2.

PLUS-THREE FACTS

There are 6 *plus-three* facts: $5 + 3$, $6 + 3$, $7 + 3$ and $3 + 5$, $3 + 6$, $3 + 7$.

A strategy for these facts is to add the 3 in two steps: first add 2 and then add 1.

N10.04 and N10.05 In Mathematics 1, through visualization of ten-frames, students learned to subtract with minuends of 10 or less. This strategy involves visualizing the ten-frame dot configuration for a number, mentally removing the required number of dots, visualizing the resultant dot configuration, and naming this number. For some facts with minuends of 5 or 10, some students may just visualize the ten-frame configuration of the subtrahend, and know the difference is the number of empty cells in first row or the second row. A full top row that represents 5 and a full ten-frame that represents 10 provide students with anchors for visualization of the numbers and operations.

THINK ADDITION

This strategy involves students finding answers to subtraction facts by using their knowledge of addition facts and their understanding of the inverse relationship of addition and subtraction. Instead of counting back to subtract, students ask themselves what they would have to add to the subtrahend to get the minuend.

Knowing the subtraction facts with a quick recall has been assigned to Mathematics 3 mental mathematics; however, the strategies should be discussed and applied with no time constraints in Mathematics 2.

Patterns and Relations (PR)

SCO PR01 Students will be expected to demonstrate an understanding of repeating patterns (three to five elements) by describing, extending, comparing, and creating patterns using manipulatives, diagrams, sounds, and actions. [C, CN, PS, R, V]			
[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- PR01.01** Identify the core of a given repeating pattern.
- PR01.02** Describe and extend a given double attribute pattern.
- PR01.03** Create a repeating non-numerical pattern and explain the rule.
- PR01.04** Predict an element of a given repeating pattern using a variety of strategies and extend the pattern up to the tenth element to verify the prediction.
- PR01.05** Translate a repeating pattern from one mode to another.
- PR01.06** Compare two given repeating patterns, and describe how they are alike/different.

Performance Indicator Background

PR01.01 As students identify the core of a pattern in activities, be sure to use appropriate patterning vocabulary with students, such as **core** (the repeating part of the pattern) and **elements** (the actual objects used in the pattern). It is important to create patterns that have the core repeating at least three times, and to remind students that repeating patterns can be extended in both directions. Encourage students to reference the position of the elements using ordinal numbers.

The core of this shape pattern is—circle, square, triangle.

There are three elements in this pattern, namely a circle, a square, and a triangle.



The pattern to the right is also a three-element pattern.



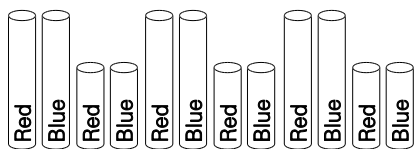
The core of this three-element pattern is heart (1st element), heart (2nd element), star (3rd element). To help students identify the pattern core, it is suggested students highlight, or isolate, the core each time it repeats.

PR01.02 Although students have had experiences with repeating patterns in earlier grades, it is important to provide them with more challenging repeating patterns in Mathematics 2, such as double attribute patterns. A double attribute pattern (or a two attribute pattern) is a pattern that consists of like objects with two different attributes, such as colour and size, or shape and position. For example, this pattern is made with 2-D shapes that have two different attributes (size and shape):



In describing this pattern in relation to size, it is an ABB pattern—small, big, big. In describing it in relation to shape, it is an ABC pattern—square, circle, triangle.

Another example of a double attribute with like objects (straws) with two different attributes (colour and length):



Introduce double attribute patterns with your class by using student attributes as the elements of the pattern. You may wish to create a pattern by using students with blonde, brown, and black hair, as well as, students with glasses and no glasses. Ask students to describe the pattern and invite them to choose classmates to extend the pattern.

PR01.03 Students should create their own repeating patterns without a given pattern rule using manipulatives, sounds, diagrams, and actions. Some possible manipulatives include snap cubes, counters, coloured tiles, rubber stamps, pattern blocks, and collections of small items. Students should explain how the pattern is repeating by identifying the core of the pattern and the particular way the pattern repeats. To help students identify the pattern, ask students questions such as, What comes first in your pattern? and What comes next?

Although students find it easier to demonstrate rather than articulate patterns, they must learn to explain, in both words and symbols, the rule used to create a repeating pattern.

PR01.04 Students should have experiences predicting missing elements within the pattern and also elements at the end of the repeating pattern. It is important that students have practice predicting the next element in a pattern up to the tenth element. Some strategies that would encourage students to predict and verify elements of the pattern would be to

- build the pattern with manipulatives or act it out (if applicable)
- identify the core of the pattern before they predict the element
- say the pattern aloud

PR01.05 When students are given a repeating pattern in one mode, they should be able to represent that pattern in other modes. For example, if presented the action pattern up, down, around, up, down, around, up, down, around, ..., some students may use cubes to make a colour pattern—red, blue, green, red, blue, green, red, blue, green, ...

Translating two or more modes of the same pattern to a common format helps children see beyond the materials making up the pattern. In fact, repeating patterns are sometimes described using letter codes that help students to name and compare patterns. Students should be provided with many experiences describing repeating patterns containing three to five elements such as AAB, ABB, ABC, AABB, ABBBC, and other combinations.

PR01.06 When comparing two repeating patterns, students describe the similarities and differences between number of elements, attributes of the elements, and the core of the pattern.

SCO PR02 Students will be expected to demonstrate an understanding of increasing patterns by describing, extending, and creating numerical patterns (numbers to 100) and non-numerical patterns using manipulatives, diagrams, sounds, and actions.

[C, CN, PS, R, V]

[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- PR02.01** Identify and describe increasing patterns in a variety of given contexts.
- PR02.02** Represent a given increasing pattern concretely and pictorially.
- PR02.03** Identify errors in a given increasing pattern.
- PR02.04** Explain the rule used to create a given increasing pattern.
- PR02.05** Create an increasing pattern and explain the pattern rule.
- PR02.06** Represent a given increasing pattern using another mode.
- PR02.07** Solve a given problem using increasing patterns.
- PR02.08** Identify and describe increasing patterns in the environment.
- PR02.09** Determine missing terms in a given concrete, pictorial, or symbolic increasing pattern and explain the reasoning.

Performance Indicator Background

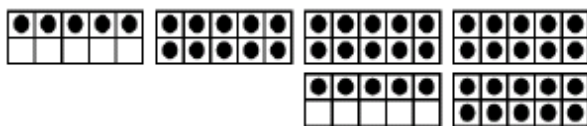
PR02.01 There are a variety of contexts in which increasing patterns can be demonstrated.

- On a number line, you can identify a start number and a jump number. For example, the start number could be 2, and the jump number could be 3.



Students should find the pattern in these numbers (an increase of three).

- On a hundred chart, students could colour start-and-jump sequences, making a visual pattern. Students can be encouraged to investigate how patterns change when only the start number changes, or how they change when only the skip number changes. They should investigate which skip counts make diagonal patterns and/or which make column patterns.
- On ten-frames, build an increasing pattern by placing counters on the ten-frames and have students identify how the pattern is growing. For example, for the pattern 5, 10, 15, 20, ... These ten-frames show that the numbers increase by 5 because another full row of 5 is filled each time.



- On a calendar, students can be shown how the days of the week form an increasing pattern of 1, by looking at the rows, and an increasing pattern of 7 by looking at the columns. The calendar can be used to find other kinds of number patterns, such as repeating digits, skip counting, and patterns in rows, columns, and diagonals.

PR02.02 When presenting increasing patterns to students, always provide at least the first three terms. Some students have difficulty identifying an increasing pattern, and may think of the first term as being the core of a repeating pattern. Although students may use other language to describe patterns, it is important to model mathematical language and thinking.

Students may represent increasing patterns using base-ten blocks. For example, for the number pattern 20, 40, 60, ..., they could make the capital letter T with two rods, then make the T grow bigger by adding one more rod to the top of the T and one to the bottom, and finally make it grow again by adding one more rod to the top and one more to the bottom. Students could also be asked to predict how big their 7th T would be if they continue this pattern.

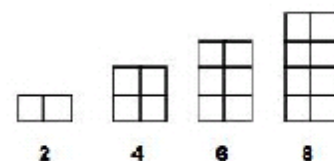
Students may also use coins to represent increasing patterns. For example, they could use pennies and dimes to represent the pattern 11, 22, 33, 44, ..., where the pattern grows by one penny and one dime.

Students could be provided with increasing number patterns and asked to represent them using interlocking cubes.

Students will need many concrete experiences representing increasing patterns before they begin to represent them using pictures, numbers, and words, orally and written. It is suggested that students first create patterns as a whole class and be shown how to model them pictorially.

PR02.03 When students are trying to find any possible errors in an increasing pattern, encourage them first to identify the pattern rule to describe how the pattern is increasing and then to check that each term is increasing by the stated rule.

PR02.04 Students must be able to explain, in both words and symbols, the rule used to create a given non-numerical or numerical increasing pattern, and to represent the pattern in another mode, such as materials, actions, or sounds. Because students find it easier to demonstrate rather than articulate patterns, they must learn that when describing the rule of an increasing pattern, they need to identify the starting term and how it is increasing. Without indicating these, the pattern rule is incomplete. Students should learn that a pattern rule must describe how each and every term of the pattern is increasing. For example, the following block pattern starts with 2 blocks and increases by 2 blocks each time.



PR02.05 Students may find it easier to begin creating increasing patterns using small increments of one or two. Encourage students to first build their pattern and then record. They may create their increasing patterns in many ways, such as using a variety of concrete materials, sounds, actions, pictures, or numbers. Regardless of the mode of representation, when explaining the rule, remind students the pattern rule should state the start term and by how the pattern increases.

PR02.06 Students need to see that they are able to recreate patterns using different materials, and in different ways (modes). For example, a student could make an increasing pattern using numbers and then show this pattern using hand clapping, or vice versa. Students should be encouraged to look for other students' patterns that are the same as theirs but represented in different ways.

PR02.07 Give students many opportunities to solve problems using increasing patterns. This real-world connection using increasing patterns may help them to better understand this concept. For example, tell students that Tommy brought 4 candies to school on Monday, 6 candies on Tuesday, and 8 candies on Wednesday. If he continues in this same way of bringing more candies each school day, how many candies will he bring next Monday?

PR02.08 Provide students with opportunities to recognize naturally occurring increasing patterns in the world around them, and encourage them to look for patterns in their environment at all times. For example, students could be taken on a pattern scavenger hunt around the school and invited to be pattern detectives, observing and recording patterns that they find.

PR02.09 To help students determine missing terms in an increasing pattern, encourage them to first determine the pattern rule. Once they know how much it should grow each time, they can make sure the missing term(s) grows according to that rule.

SCO PR03 Students will be expected to demonstrate and explain the meaning of equality and inequality by using manipulatives and diagrams (0 to 100).

[C, CN, R, V]

[C] Communication

[PS] Problem Solving

[CN] Connections

[ME] Mental Mathematics and Estimation

[T] Technology

[V] Visualization

[R] Reasoning

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

PR03.01 Determine whether two given quantities of the same object (same shape and mass) are equal by using a balance scale.

PR03.02 Construct and draw two unequal sets using the same object (same shape and mass) and explain the reasoning.

PR03.03 Demonstrate how to change two given sets, equal in number, to create inequality.

PR03.04 Choose from three or more given sets the one that does not have a quantity equal to the others and explain why.

Performance Indicator Background

PR03.01 Using concrete materials, students can examine how a balance operates. Construct two equal sets using the same objects (same shape and mass) and demonstrate their equality of number using a balance scale. Prepare a set of identical covered containers, each one holding 6 cubes in different colour combinations, such as 6 red cubes, 4 yellow, and 2 blue cubes; 3 red and 3 blue cubes; 5 red and 1 yellow cubes. Have students place a container on each pan of the balance. Ask, Are the containers equal? How do you know? Then have students open the containers, count the number of different coloured cubes in each, and describe the equality, saying, for example, "six red cubes is the same as four yellow cubes and two blue cubes or six red cubes is equal to four yellow cubes and two blue cubes." Repeat with other combinations of containers.

PR03.02 A balance scale can also be used to demonstrate inequality relationships; for example, place 6 red cubes on the left pan of a balance scale and four yellow cubes on the right pan without actually showing students the quantities. Ask them to explain on which pan they think there are more cubes and why they think so. Have them count the quantities on both pans. Model how they could compare the quantities on the pans, saying, "six red cubes is more than four yellow cubes and six red cubes is not the same as four yellow cubes or six red cubes is not equal to four yellow cubes.

When comparing sets, many students may recognize that 6 cubes is more than 4 cubes, but not automatically realize that 4 cubes is less than 6 cubes. Both sides of the relationship need to be considered. Therefore, ask students what they could say about the 4 yellow cubes compared to the 6 red cubes, bringing out the relationship is less than and is not the same as or is not equal to.

After students have had many concrete experiences interpreting the relationships between quantities on two pans of a balance scale, they could be asked to interpret and draw balance scale pictures. Finally, they can draw two unequal sets of the same object and, using their understanding of inequality of quantities, can explain their reasoning.

PR03.03 Students should be able to demonstrate changing an equality situation to make it an inequality. For example, place 12 cubes on both pans of a balance scale to show equality. Ask students to remove two cubes from the left pan, ask them to describe the relationship between the quantities on the pans and to explain their reasoning.

PR03.04 Student should be able to distinguish which set from multiple given sets is not like the others. For example, place 6 cubes in each of three small paper bags and 8 cubes in a fourth bag. Ask the students, using a balance scale, to find the bag that is not equal to the others, and to explain how they know if it is more or less.

SCO PR04 Students will be expected to record equalities and inequalities, symbolically, using the equal symbol or not equal symbol.

[C, CN, R, V]

[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- PR04.01** Determine whether two sides of a given number sentence are equal (=) or not equal (\neq). Write the appropriate symbol and justify the answer.
- PR04.02** Model equalities using a variety of concrete representations and record the equality.
- PR04.03** Model inequalities using a variety of concrete representations and record the inequality.

Performance Indicator Background

PR04.01 Present the two sides of equal and unequal number sentences with a (?) where the sign should be. Have students indicate if they think the number sentences are equal or unequal, and to explain how they made their decisions. Use opportunities to encourage relational thinking in decision making.

PR04.02 and PR04.03 Manipulatives, such as ten-frames, snap cubes, balance scales, and marbles, can all be used to help students develop an understanding of equal and unequal number sentences. Allowing students to model number sentences and situations with manipulatives, particularly with balance scales, gives them a visual to see whether, or not, the two sides of the number sentence are equal.

Measurement (M)

SCO M01 Students will be expected to demonstrate an understanding of the calendar and the relationships among days, weeks, months, and years.			
[C, CN, PS, R]			
[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- M01.01** Read a calendar.
- M01.02** Name and order the days of the week and months of the year.
- M01.03** Communicate the number of days in a week and the number of months in a year.
- M01.04** Solve a given problem involving time which is limited to the number of days in a week and the number of months in a year.

Performance Indicator Background

M01.01 Through modelling and much practise, students should be able to identify the day of the week, the month, and the year when a day on a calendar is pointed to, or when a special event that is marked on a calendar is referenced. Similarly, when given a date, such as December 12, 2013, students should be able to locate it on a calendar and state that it is a Thursday. As well, if students are told today's date, such as October 24, 2013, they should be able to spontaneously state yesterday's and tomorrow's dates.

M01.02 and M01.03 By the end of Mathematics 2, students should know the days of the week and the months of the year in order. Learning the days of the week and months of the year can be taught simultaneously and not as separate entities. It is important to involve students in daily problem-solving activities with the calendar in order to help students gain a deeper understanding of how the days of the week and months of the year are organized.

Students may have more difficulty with the months for which they have less experience in their own lives. Ask questions about the months regularly, such as, It is March. Which month comes next? Start reciting the months beginning with a month other than January and have students continue. Recite a sequence of five months with an error, such as an incorrect order or an omission, and ask students to detect the error.

The calendar is one of the first places where young students are exposed to a ready-made pattern; that is, the days of the week form a seven-element repeating pattern, and the months of the year is the core of a 12-element repeating pattern.

M01.04 Students should move beyond just identifying the day and date by engaging in problem-solving activities to make deeper connections between the calendar, patterning, and their own lives. For example, If it is now May 25, then is last or next Christmas closer?

SCO M02 Students will be expected to relate the size of a unit of measure to the number of units (limited to non-standard units) used to measure length and mass. [C, CN, ME, R, V]			
[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- M02.01** Explain why one of two given non-standard units may be a better choice for measuring the length of an object.
- M02.02** Explain why one of two given non-standard units may be a better choice for measuring the mass of an object.
- M02.03** Select a non-standard unit for measuring the length or mass of an object and explain why it was chosen.
- M02.04** Estimate the number of non-standard units needed for a given measurement task.
- M02.05** Explain why the number of units of a measurement will vary depending upon the unit of measure used.

Performance Indicator Background

M02.01 and M02.03 Provide a selection of non-standard measuring units for students to explore. Have them work in pairs to choose appropriate non-standard units to measure the lengths of various objects found in their environment. Ask students to present their findings to their classmates, explaining what was measured, which non-standard unit was used, how many units they used, and why it was an appropriate unit.

M02.02 Students should see the relationship between the mass of the object that is being measured and the mass of the non-standard measuring unit. For example, if students want to find the mass of a rock, they must first understand the rock is a heavy object and it would be best to choose a non-standard unit that is also heavy, such as a marble as opposed to a toothpick.

M02.03 Students should be provided with opportunities to discover why certain non-standard units may be more efficient and more accurate to use when measuring the mass of an object. Through guided instruction and exploration, students can further develop thinking about choosing appropriate non-standard units of measure. Provide groups of students with two types of non-standard units of measurement that clearly differ in size, such as toothpicks and blocks. Instruct them to measure, on a balance scale, the mass of an eraser using both units and to record their answers. Discuss what students discovered during this activity. Ask, If you wanted to know the mass of your shoe, would it be best to use toothpicks or blocks? Why? Through activities, such as this one, students will develop their abilities to select the more appropriate non-standard unit when given two choices.

M02.04 There is always value to use estimation in teaching measurement because estimation is an effective way to get students to focus on the attribute being measured. Sometimes an estimate, or approximate measurement, is all you need, whereas other times it is a useful check on the reasonableness of a more precise answer. Students should be encouraged to estimate the number of non-standard units they will use before they actually measure the attribute of an object. After they measure the objects and get actual numbers of units, they should compare these to their estimates.

With time and experiences like this, students will refine their estimation abilities. When talking to students, try to use phrases such as, The desk is about 3 unsharpened pencils long, or The tile is a little less than two unsharpened pencils.

M02.05 Students should understand that the size of the non-standard unit used to measure an object's length will affect the results of the measurement. For example, have each student measure the length of his or her desk using two different non-standard units, such as paper clips and straws. Students should discover that more paper clips than straws were used because straws are a larger unit. Similarly, with mass, have students work in pairs to measure the mass of a small book on a balance scale, one student using pennies and the other student using boxes of paper clips. Have them record their answers on paper, compare the numbers, and discuss what they notice. Try to get students to generalize that the larger or heavier the unit used, the fewer the number of units and the smaller or lighter the units used, the greater the number of units.

SCO M03 Students will be expected to compare and order objects by length, height, distance around, and mass using non-standard units and make statements of comparison.

[C, CN, ME, R, V]

[C] Communication

[PS] Problem Solving

[CN] Connections

[ME] Mental Mathematics and Estimation

[T] Technology

[V] Visualization

[R] Reasoning

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- M03.01** Estimate, measure, and record the length, height, distance around, or mass of a given object using non-standard units.
- M03.02** Compare and order the measure of two or more objects in ascending or descending order and explain the method of ordering.

Performance Indicator Background

M03.01 In measurement, approximations are important. When talking to students about measuring, use phrases such as, The length of the desk is about 12 pencils long; The mass of the eraser is about 3 blocks; and The distance around my head is about 3 hand spans. Students need some understanding of measuring using non-standard units, prior to estimating. Once they start estimating, they should always estimate prior to doing a measurement.

M03.02 Students should be given opportunities to order measurements from least to greatest and greatest to least. They should be able to explain why and how they have ordered the measurements. For example, given a set of objects, students could measure different attributes using non-standard units and order those objects in different ways. They could order the objects by length from least to greatest numbers of non-standard units, such as paper clips; or they could order the same objects by mass from greatest to least numbers of non-standard units, such as yellow pattern blocks.

SCO M04 Students will be expected to measure length to the nearest non-standard unit by using multiple copies of a unit and using a single copy of a unit (iteration process).

[C, ME, R, V]

[C] Communication

[PS] Problem Solving

[CN] Connections

[ME] Mental Mathematics and Estimation

[T] Technology

[V] Visualization

[R] Reasoning

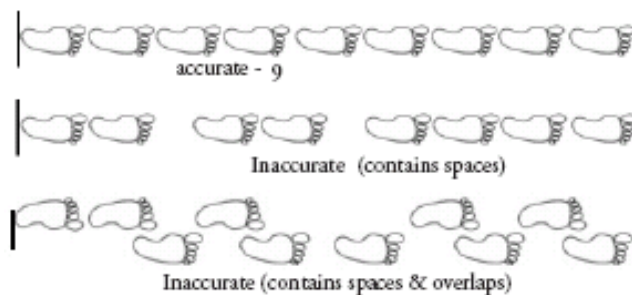
Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- M04.01** Explain why overlapping or leaving gaps does not result in accurate measures.
- M04.02** Count the number of non-standard units required to measure the length of a given object using a single copy or multiple copies of a unit.
- M04.03** Estimate and measure a given object using multiple copies of a non-standard unit and using a single copy of the same unit many times, and explain the results.
- M04.04** Estimate and measure, using non-standard units, a given length that is not a straight line.

Performance Indicator Background

M04.01 Activities should be provided where students are shown measurements, some of which are correct and others of which have obvious gaps and overlapping, and asked to explain which measurements are accurate and which are not. For example, students could be shown these cut outs of feet measuring the length of a closet and be asked to explain how two students who both measured the length using the same cut outs of feet could have gotten completely different answers.



M04.02 It is suggested when using non-standard units to measure length, students should first be provided with multiple copies of a non-standard unit before using a single copy of a unit. For example, to measure the length of a desk, a student should first use several cubes placed along the edge of a desk before using a single cube repeatedly. Using a single copy of a unit to measure is the same as using a measuring tool and there are many possible sources of error, such as not advancing in a straight path, or unwittingly leaving gaps or overlapping.

M04.03 Before students make any measurement of length, have them examine the object that they are going to measure and the non-standard unit they will use, and get them to commit in writing their estimates of the number of units they will use. After they actually measure the length, get them to compare it to their estimates, and have them discuss strategies that could be used to get closer estimates.

M04.04 Give students common objects found in the classroom that can be easily bent into curvy lines, such as pipe cleaners, modelling clay, and wool. Have students first estimate and measure the objects straight and then curvy. They could also measure all around an object, such as their desk or a picture frame. Through these experiences, students will realize that measuring objects that are not straight requires them to make estimation decisions as they measure since the rigid non-standard units can't always bend to match the object, and they have to decide what to count when they turn and go in a different direction.

SCO M05 Students will be expected to demonstrate that changing the position of an object does not alter the measurements of its attributes.

[C, R, V]

[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

Performance Indicator

Use the following indicator to determine whether students have achieved the corresponding specific curriculum outcome.

M05.01 Measure a given object, change the position, remeasure, and explain the results.

Performance Indicator Background

M05.01 Students need to recognize that whether an object is standing up, lying flat, or tilted, its dimensions, and mass will remain the same. For example, display two congruent cereal boxes, one standing in normal position and one lying down, ask students which box is larger, and explain how they decided. Then have students check their decisions by handling the two boxes.

Furthermore, students need to begin to realize that if a solid object is restructured so that its dimensions are altered, its mass will not be changed.

Geometry (G)

SCO G01 Students will be expected to sort 2-D shapes and 3-D objects using two attributes and explain the sorting rule. [C, CN, R, V]			
[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- G01.01** Determine the differences between two given presorted sets and explain the sorting rule.
- G01.02** Identify and name two common attributes of items within a given sorted group.
- G01.03** Sort a given set of 2-D shapes (regular and irregular) according to two attributes and explain the sorting rule.
- G01.04** Sort a given set of 3-D objects according to two attributes and explain the sorting rule.

Performance Indicator Background

G01.01 Students should be able to determine differences between two given presorted sets of 2-D shapes or 3-D objects. Sorting rules for 2-D shapes may include colour but ideally should have an emphasis on geometric attributes such as

- straight sides / curved sides
- large shapes / small shapes
- shapes with / without points
- shapes with / without square corners
- 4-sided shapes / 3-sided shapes
- squares / rectangles

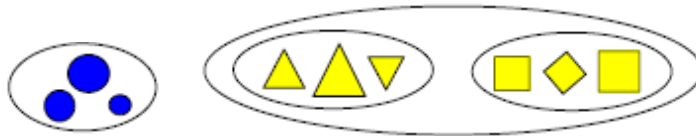
To help focus attention on these attributes, careful consideration should be given to the materials that are used in creating the presorted set. For instance, if a set is comprised of 2-D attribute blocks, students may automatically focus on colour and shape as attributes as opposed to observing the sides or corners of the shape. Another learning material to use could be paper cutouts all of the same colour.

G01.02 If students are presented with a set of shapes or objects that have been sorted by two attributes, they need to be able to analyze the set to identify the two common attributes and describe how the shapes or objects were sorted. For example, if they are given a single set with both blue squares and red rectangles, they should realize that this set of shapes could not have been sorted by colour (two colours in the set) and type of shape (both squares and rectangle), but may have been sorted by their number of sides (4) and by their corners being square. Their understanding could be further checked by asking them to place other shapes in the set and explain why the new shapes are members of the set.

G01.03 Regular 2-D shapes are those that have all equal sides and all equal corners, such as the triangle in the pattern blocks and squares. Irregular shapes are those that have sides that are not all equal and corners that are not all equal, such as the triangles in a set of tangrams. Students are not expected to know the definitions of regular and irregular shapes but should be exposed to sorting and identifying attributes of these shapes. Students have already had many experiences with sorting shapes and naming

common attributes. To further solidify their conceptual understanding of sorting and naming common attributes, it is important to include shapes that are not as common, such as hearts, arrows, and stars.

In Mathematics 2, students should build on their prior experiences to sort objects and shapes using two attributes. If students find it difficult to sort by two attributes at the same time, they may find it helpful to sort by one attribute and combine two of the sorted groups. For example, in the diagram below, students may have first sorted the shapes by number of sides and then combined the last two groups to make a single group that includes yellow shapes and straight sides. It is important that students are able to explain their sorting rule to others.



G01.04 When presented with a set of 3-D objects, students should be challenged to identify different sorting rules for the same set of objects. For example, the sorting rule for a set of spheres and cylinders could be those that roll and have curved edges or those that don't stack and have no vertex. By exposure to many different ways to sort 3-D objects, students will expand their repertoires of sorting rules.

SCO G02 Students will be expected to recognize, name, describe, compare, and build 3-D objects, including cubes and other prisms, spheres, cones, cylinders, and pyramids.

[C, CN, R, V]

[C] Communication

[PS] Problem Solving

[CN] Connections

[ME] Mental Mathematics and Estimation

[T] Technology

[V] Visualization

[R] Reasoning

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- G02.01** Sort a given set of 3-D objects and explain the sorting rule.
- G02.02** Identify common attributes of cubes and other prisms, spheres, cones, cylinders, and pyramids from given sets of the same 3-D objects.
- G02.03** Identify and describe given 3-D objects with different dimensions.
- G02.04** Identify and describe given 3-D objects with different positions.
- G02.05** Create and describe a representation of a given 3-D object using materials such as modelling clay.
- G02.06** Identify and name examples of cubes and other prisms, spheres, cones, cylinders, and pyramids found in the environment.

Performance Indicator Background

G02.01 This performance indicator is directly linked to outcome G06; however, students' sorting rules should extend to include the names of the objects. For example, they may sort classroom objects into sets of cubes or sets of spheres.

G02.02 Attributes that students may observe in 3-D objects are

- square, circle, or triangle faces
- number of faces or edges
- identical faces or faces that are the same shape
- number of vertices
- rounded or curved surfaces
- more vertices than faces
- ability to slide/roll/stack

G02.03 and **G02.04** Students should identify and describe 3-D objects with different dimensions and in different positions. Dimensions refer to the length, width, and height of a 3-D object, each of which could be measured. It is not necessary to engage students in measuring these dimensions; they can do direct comparisons. The point of these performance indicators is to be sure that students recognize cubes and other prisms, spheres, cones, cylinders, and pyramids of all sizes and in all positions. Concrete and pictorial 3-D objects should not be considered as interchangeable when assessing students because pictorial representations are a great deal more difficult for students to process.

G02.05 By making models of 3-D objects, students focus on the shape attributes of different solids.

Describing shapes allows students to focus on their basic characteristics. Use questioning to focus student thinking; for example,

- What other shapes are like this one? In what way are they alike?
- What does this object look like?
- Does your shape have vertices or corners?

-
- Does the object have flat faces or curved surfaces?
 - Pick two of the shapes and tell how they are alike and how they differ.

Students should be given many opportunities to build 3-D objects using a variety of materials as it increases their visualization skills. It is also important to provide students with these experiences and focus less on the accuracy of the end product. Students should also be encouraged to make constructions using a combination of 3-D objects.

G02.06 Observing objects in their environment that look like a cube, prism, sphere, cone, cylinder, or pyramid is a critical aspect in learning their geometric names. Caution will have to be exercised because so many of those objects will not be exactly one of these named objects. Students should be encouraged to recognize why an object may be, for example, almost a cube. It is difficult to find pyramids in the classroom or at home, so examples will likely have to be constructed. Looking for pictures of these 3-D objects in storybooks or on the computer should also be encouraged.

SCO G03 Students will be expected to recognize, name, describe, compare, and build 2-D shapes, including triangles, squares, rectangles, and circles.

[C, CN, R, V]

[C] Communication

[PS] Problem Solving

[CN] Connections

[ME] Mental Mathematics and Estimation

[T] Technology

[V] Visualization

[R] Reasoning

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- G03.01** Sort a given set of 2-D shapes and explain the sorting rule.
- G03.02** Identify common attributes of triangles, squares, rectangles, and circles from given sets of the same type of 2-D shapes.
- G03.03** Identify given 2-D shapes with different dimensions.
- G03.04** Identify given 2-D shapes with different positions.
- G03.05** Identify and name examples of triangles, squares, rectangles, and circles found in the environment.
- G03.06** Create a model to represent a given 2-D shape.
- G03.07** Create a pictorial representation of a given 2-D shape.

Performance Indicator Background

G03.01 This performance indicator is related to outcome G06; however, students' sorting rules should include sorts by shape names, as well as by number of sides or corners, and square corners.

G03.02 Most students will be familiar with many of the 2-D shapes from their experiences, but they may not necessarily know the attributes. While at this level of development they need to be encouraged to focus on the common attributes of any shape for which they have concrete examples, they are not likely yet identifying these attributes as properties of the class of that shape, nor are they likely able to talk about the attribute of a shape without seeing an example of that shape. For example, through explorations of, and focussed questions about, a set of squares, students may state that all these squares have four sides and four square corners.

G03.03 Students in Mathematics 2 will need experiences exploring shapes of different sizes and dimensions. Students usually easily identify squares and circles of different sizes because these two 2-D shapes maintain their overall appearance; however, students may find it difficult to recognize triangles and rectangles with dimensions that are different from typical examples. For example, students may not readily recognize a 1 cm x 30 cm rectangle as a rectangle or a triangle with a 170 degree angle as a triangle.

G03.04 It is important to be aware of how shapes are presented to students. Many times they are positioned in stereotypical ways that can lead students to think that the way a shape is positioned is part of what defines it. For example, triangles, squares, and rectangles are often only presented sitting on their bases. If these 2-D shapes are rotated so they are not sitting on their bases, students may think they are different shapes. The ability to recognize a shape regardless of how it is positioned is called position-in-space perception. Activities where students trace a shape, such as a rectangle from a set of attribute blocks, on a piece of paper, move it in a variety of ways by sliding, flipping, and turning it and retracing it until the paper is full of images of that rectangle, help students develop this ability.

G03.05 Associating a 2-D shape with examples in their home and school environments is a critical part of the development of this outcome. While many examples they select may come from pictures on the walls or in books, there will also be examples that are faces of 3-D objects (see SCO G09). Therefore, students should explain which part of the 3-D object is the square, rectangle, triangle, or circle.

G03.06 and **G03.07** Students may use a variety of media to create models of 2-D shapes. Suggested materials would include yarn, pipe cleaners, stir sticks, toothpicks, straws, and elastics on geoboards. Freehand drawings should be discouraged because students should produce exemplars of squares, triangles, rectangles, and circles that truly represent these shapes. Templates of these shapes could be provided, so students can trace the shapes as needed. Triangular and square dot paper used with straight edges and pencils will enable students to produce good exemplars of triangles, squares, and rectangles.

SCO G04 Students will be expected to identify 2-D shapes as part of 3-D objects in the environment.

[C, CN, R, V]

[C] Communication

[PS] Problem Solving

[CN] Connections

[ME] Mental Mathematics and Estimation

[T] Technology

[V] Visualization

[R] Reasoning

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

G04.01 Compare and match a given 2-D shape, such as a triangle, square, rectangle, or circle, to the faces of 3-D objects in the environment.

G04.02 Name the 2-D faces of a given 3-D object.

Performance Indicator Background

G04.01 A number of different 3-D objects, such as aluminum cans, balls, die, and boxes, should be collected to be available for students to identify faces of a particular named 2-D shape. They should also get experience locating pictures of 3-D objects on charts and in picture books that have faces that are triangles, rectangles, squares, or circles. In addition to a number of visual experiences, students should also have some tactile experiences. For example, if a 3-D object is placed in a bag, students should be asked to reach in and find by touching a face of a particular type, or if three 3-D objects, two of which are the same and one different, are placed in a bag, students should feel the objects inside the bag to identify the one that is different and to explain how it is different.

G04.02 Through many explorations and focused questions, students will be able to recognize and name the 2-D faces and/or surfaces of a given 3-D object. If pictures of 3-D objects are used, it is recommended that students initially also have concrete examples available to examine. Students will not necessarily see the 3-D object that is intended in a picture without experiences making pictorial connections to concrete objects. For example, if students work with a spinner containing pictures of the geometric solids, they could take turns spinning, selecting the correct concrete object, and naming and describing the faces of that solid. After many such opportunities, students may be able to name and describe faces of an object when just given its picture.



SCO SP01 Students will be expected to gather and record data about self and others to answer questions.

[C, CN, PS, V]

[C] Communication

[PS] Problem Solving

[CN] Connections

[ME] Mental Mathematics and Estimation

[T] Technology

[V] Visualization

[R] Reasoning

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

SP01.01 Formulate a question that can be answered by gathering information about self and others.

SP01.02 Organize data as it is collected using concrete objects, tallies, checkmarks, charts, or lists.

SP01.03 Answer questions using collected data.

Performance Indicator Background

SP01.01 When students formulate questions of interest to themselves and about their communities, the data they gather will be more meaningful. Encourage students to start with questions that require a yes or no answer as this data is much easier to collect and organize. For example, Do you have a pet? It is suggested to go beyond classroom limits when formulating questions and gathering data, such as surveying neighbouring classes, parents, and siblings.

SP01.02 Students should be encouraged to organize the data they collect in the course of classroom investigations. Tally marks are one of the simplest ways to record and organize data and it also promotes skip counting by 5s and 10s; however, students may also use checkmarks or objects to record their data. When creating charts and lists, it is important to provide tables to ensure proper alignment of data. This will guard against misrepresentation of data caused by varying penmanship and size of student printing.

SP01.03 At this point, not only may the organized data answer the original question, it can also provide answers to other questions and provide stimulus for the collection of additional data. For example, after students collect the data related to “Do you have a pet?” they could answer questions such as, How many more students have pets than do not?, and they may want to investigate the types of pets. Encourage students to formulate questions that can be answered from the data collected.

SCO SP02 Students will be expected to construct and interpret concrete graphs and pictographs to solve problems.

[C, CN, PS, R, V]

[C] Communication

[PS] Problem Solving

[CN] Connections

[ME] Mental Mathematics and Estimation

[T] Technology

[V] Visualization

[R] Reasoning

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- SP02.01** Determine the common attributes of concrete graphs by comparing a given set of concrete graphs.
- SP02.02** Determine the common attributes of pictographs by comparing a given set of pictographs.
- SP02.03** Answer questions pertaining to a given concrete graph or pictograph.
- SP02.04** Create a concrete graph to display a given set of data and draw conclusions.
- SP02.05** Create a pictograph to represent a given set of data using one-to-one correspondence.
- SP02.06** Solve a given problem by constructing and interpreting a concrete graph or pictograph.

Performance Indicator Background

SP02.01 and SP02.02 The common attributes of graphs, such as titles, labels, columns, and rows should be explored. Once a graph is completed, students need to discuss what the graph shows and what other questions it may answer. Encourage students to discuss the similarities and differences that they can see, between two concrete graphs, a concrete graph and a pictograph, or two pictographs.

SP02.03 When answering questions about data, encourage students to provide support for their answers by referring to the data. For example, they can ask, How do you know? When interpreting concrete graphs and pictographs, model good questioning that requires students to think on both a literal level (How many students have a dog? How many students have a cat?) and on interpretive or evaluative levels (How many more cats than dogs are there? Or if dog treats come in bags of ten, how many bags would we need to buy? Would we have any treats left over?).

SP02.04 The most basic type of concrete graph is a people graph, in which the students themselves form the graph. It is important initially to allow students to form lines on their own so they learn the importance of all lines starting at the same level and of matching students in the lines in some form of one-to-one correspondence. Eventually, it is helpful to use some form of graphing mat on which each student stands in one square. If the students tape their name tags to their boxes before stepping out, they will be able to view the graph as a whole. With these initial experiences with concrete graphs involving themselves, students should further develop using concrete real objects and representative objects.

SP02.05 Pictographs are bar graphs that use a drawing of some type to represent what is being graphed. The pictographs should be based on a one-to-one correspondence (i.e., a picture represents one item, not a group of items). Students should create and interpret graphs that run horizontally and vertically using magnetic board and pictures, felt board and pictures, cutting pictures from magazines or using students' photos.

SP02.06 Students should be able to solve a given problem by constructing and interpreting a concrete graph or a pictograph. Discuss the data displayed in a random, unorganized manner. This discussion should lead students to suggest organizing the data more clearly. Suggestions may include displaying

data on a chart with headings, groups of ten circled on a chart, horizontal pictograph with data shown on sticky notes, vertical graph using cubes to represent the data, or vertical object graph with clothespins to represent the data or tally charts. Students further organize this data by constructing their own pictographs. In their journals, students may write all that they know about the graph they constructed.

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**Atlantic Canada Mathematics
Curriculum Guide:
Grades Primary–3**

Atlantic Canada Mathematics Curriculum: Grades Primary-3

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Background and Rationale

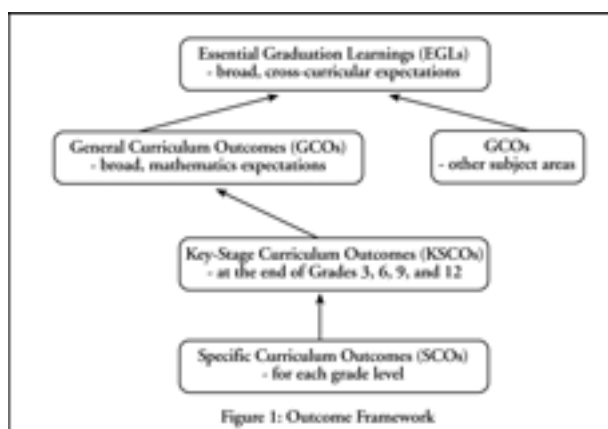
Background

Mathematics curriculum reform in Atlantic Canada is shaped by a vision that fosters the development of mathematically literate students who can extend and apply their learning, and who are effective participants in an increasingly technological society. Curriculum reform has been motivated by a desire to ensure that students in Atlantic Canada benefit from world-class curriculum and instruction in mathematics as a significant part of their total school-learning experience.

The *Foundation for the Atlantic Canada Mathematics Curriculum* used the *Curriculum and Evaluation Standards for School Mathematics* (National Council of Teachers of Mathematics, 1989) as a guiding beacon for pursuing this vision. These publications endorse the principles of students learning to value mathematics and of their being active “doers”; they advocate a meaningful curriculum focussing on four unifying ideas—mathematical problem solving, communication, reasoning and connections. The foundation document subsequently establishes a framework for the development of detailed grade-level guides describing mathematics curriculum, assessment, and instructional techniques.

Mathematics curriculum development has taken place under the auspices of the Atlantic Provinces Education Foundation (APEF), an organization sponsored and managed by the governments of the four Atlantic provinces. APEF brought together teachers and provincial department of education officials to plan and develop co-operatively the curricula in mathematics, science, and language arts in both official languages.

Each of these subject initiatives has produced a curriculum using a learning-outcomes framework, outlined in Figure 1, that supports the



regionally developed essential graduation learnings (EGLs) in aesthetic expression, citizenship, communication, personal development, problem solving, and technological competence. (See the *Foundation for the Atlantic Canada Mathematics Curriculum* for a detailed presentation of these EGLs and the contribution of the mathematics curriculum to their achievement.)

Rationale

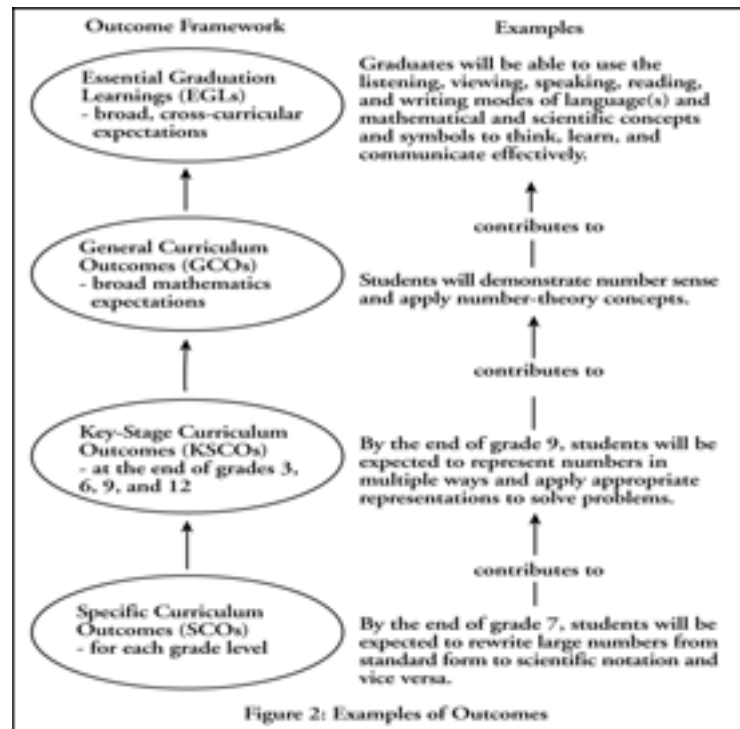
The *Foundation for the Atlantic Canada Mathematics Curriculum* provides an overview of the philosophy and goals of the mathematics curriculum, presenting broad curriculum outcomes and addressing a variety of issues with respect to the learning and teaching of mathematics. The curriculum is based upon several beliefs about mathematics learning that have grown out of research and practice. These beliefs include: mathematics learning is an active and constructive process; learners are individuals who bring a wide range of prior knowledge and experiences, and who learn via various styles and at different rates; learning is most likely to occur when placed in meaningful contexts and in an environment that supports exploration, risk-taking, and critical thinking and that nurtures positive attitudes and sustained effort; and learning is most effective when standards of expectation are made clear with ongoing assessment and feedback.

Program Design and Components

Program Organization

While the mathematics curriculum does contribute to students' achievement of each EGL, the communication and problem solving EGLs are particularly well addressed by the curriculum's unifying ideas—mathematical problem solving, reasoning, communication, and connections. The *Foundation for the Atlantic Canada Mathematics Curriculum* describes the mathematics curriculum in terms of a series of outcomes—general curriculum outcomes (GCOs) that relate to subject strands and key-stage curriculum outcomes (KSCOs) that further articulate the GCOs for the end of grades 3, 6, 9, and 12.

This curriculum guide and those for other grade levels provide greater specificity and clarity for the classroom teacher by relating grade-level specific curriculum outcomes (SCOs) to each KSCO. As illustrated in Figure 2, these outcomes represent the means by which students work toward accomplishing the KSCOs, the GCOs and, ultimately, the EGLs. It is important to emphasize, however, that the presentation of the SCOs follows the outcomes structure established in the *Foundation for the Atlantic Canada Mathematics Curriculum* and does not represent a suggested teaching sequence. While some outcomes will need to be addressed before others, a great deal of flexibility exists as to the structuring of the program. It is expected that teachers will make decisions regarding the sequencing of outcomes. Many outcomes across a number of strand may very well be addressed in one lesson or series of lessons; some outcomes such as those in patterns and data management may be addressed on an ongoing basis throughout a number of strands.



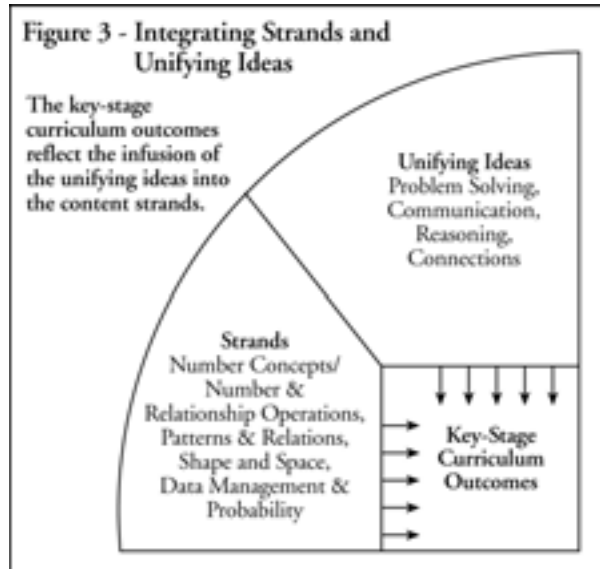
Decisions on sequencing of outcomes will depend on a number of factors, including the nature and interests of the students themselves. For instance, what might serve well as a “kickoff” strand for one group of students might be less effective in that role with a second group. Another consideration will be coordinating the mathematics program with other aspects of the students’ school experience. For example, they could study facets of measurement in connection with topics in science, data management with social studies issues, and aspects of geometry with physical education. Sequencing could be also influenced by other factors such as major events in the community or province like elections, exhibitions, or fairs.

Unifying Ideas

While the GCOs are organized around content strands, every opportunity has been taken to infuse the KSCOs with one or more of the unifying ideas—mathematical problem solving, communication, reasoning, and connections. (See Figure 3.)

These unifying ideas serve to link the content to methodology. They make it clear that mathematics is to be taught in a problem-solving manner and that classroom activities and student assignments must be structured to provide opportunities for students to communicate mathematically. Furthermore, through teacher encouragement and questioning, students must explain and clarify their mathematical reasoning and the mathematics with which students are involved on any given day must be connected to other mathematics, other disciplines, and/or the world around them.

Students will be expected to solve routine and/or non-routine mathematical problems on a daily basis. Over time, numerous problem-solving strategies should be modelled and discussed.



Students should be encouraged to employ various strategies in many different problem-solving situations. While choices with respect to the timing of the introduction of any given strategy will vary, strategies such as try-and-adjust, look for a pattern, draw a picture, act it out, use models, make a table or chart, and make an organized list should all become familiar to students during their early years of schooling. Other strategies such as working backward, logical reasoning, trying a simpler problem, changing point of view, and writing an open sentence or equation would be part of a student's repertoire upon leaving elementary school.

Learning and Teaching Mathematics

The unifying ideas of the mathematics curriculum suggest quite clearly that the mathematics classroom needs to be one in which students are actively engaged each day in the “doing of mathematics.” Often students have come to view mathematics as a set of concepts and algorithms that the teacher transmits for them to practise. Instead, students must come to see mathematics as a vibrant and useful tool for helping them understand the world, and as a discipline that lends itself to multiple strategies, student innovation, and, quite often, multiple solutions. (See Contexts for Learning and Teaching Mathematics in the *Foundation for the Atlantic Canada Mathematics Curriculum*.)

The learning environment will be one in which students and teachers make regular use of manipulative materials and technology, actively participate in discourse, conjecture, verify reasoning, and share solutions. This environment will be one in which respect is given to

Adapting to the Needs of All Learners

all ideas and in which reasoning and sense-making are valued above *getting the right answer*. Students will have access to a variety of learning resources, will balance the acquisition of procedural skills and the attainment of conceptual understanding, will estimate routinely to verify the reasonableness of their work, will compute in a variety of ways while continuing to place emphasis on basic mental computation skills, and will engage in homework as a useful extension of their classroom experience.

The *Foundation for the Atlantic Canada Mathematics Curriculum* stresses the need to deal successfully with a wide variety of equity and diversity issues. Not only must teachers adapt instruction to accommodate differences in student readiness, but also they must avoid gender and cultural biases. Ideally, every student should find his/her learning opportunities maximized in the mathematics classroom.

The reality of individual student differences must not be ignored when making instructional decisions. While this curriculum guide presents grade-level SCOs, it must be acknowledged that all students will neither progress at the same pace nor will attain a given outcome at a given time. The SCOs represent, at best, a reasonable framework for assisting students to ultimately achieve the KSCOs and GCOs.

As well, teachers should design instruction to accommodate differences in student learning styles. Different instructional modes are clearly appropriate, for example, for those students who are primarily visual learners versus those learners who learn best by doing. Designing classroom activities to support a variety of learning styles must also be reflected in assessment strategies.

Support Resources

This curriculum guide for grades primary to 3 represents the principal reference for teachers of mathematics. This guide is central to daily, unit, and yearly planning, as well as a reference point to determine the extent to which the outcomes have been met.

Texts and other resources will have significant roles in the mathematics classroom in as much as they support the SCOs. Many manipulative materials need to be readily at hand, and technological resources (e.g., software and videos) should be available. Calculators will be an integral part of many learning activities. Also, professional resources will need to be available to teachers as they seek to broaden their instructional and mathematical understandings. Key among these are the *Curriculum and Evaluation Standards for School Mathematics* (NCTM), *Addenda Series* (NCTM), various *Yearbooks* (NCTM), *Elementary School Mathematics: Teaching Developmentally* (Van de Walle 1994), *Elementary and Middle School Mathematics: Teaching Developmentally* (Van de Walle 1998), *Developing Number*

Concepts Using Unifix Cubes (Richardson), and *About Teaching Mathematics: A K-8 Resource* (Burns 1992).

Role of Parents

Societal change dictates that students' mathematical needs today are in many ways different from those of their parents. These differences are both in respect to mathematical content and in instructional approach. As a consequence, it is important that educators take every opportunity to discuss with parents changes in mathematical pedagogy and why these changes are significant. Parents who understand the reasons for changes in instruction and assessment will be better able to support their children in mathematical endeavours. They can foster positive attitudes towards mathematics, stress the importance of mathematics in their children's lives, assist their children with mathematical activities at home, and, ultimately, help their children to become confident, independent learners of mathematics.

Assessment and Evaluation

Assessing Student Learning

Assessment and evaluation are integral to learning and teaching. Ongoing assessment and evaluation not only are critical for clarifying student achievement and thereby motivating student performance, but also for providing a basis upon which teachers can make meaningful instructional decisions. (See the Assessment and Evaluating Student Learning section in the *Foundation for the Atlantic Canada Mathematics Curriculum*.)

Characteristics of good student assessment include the use of a wide variety of assessment strategies and tools, the alignment of assessment strategies and tools with the curriculum and instructional techniques, and the assurance of fairness both in application and scoring. The *Principles for Fair Student Assessment Practices for Education in Canada* (University of Alberta 1993) which elaborates good assessment practices served as a guide for student assessment in the *Foundation for the Atlantic Canada Mathematics Curriculum*.

Program Assessment

Program assessment will serve to provide information to educators on the relative success of the mathematics curriculum and its implementation. It will address whether or not students are meeting the curriculum outcomes, whether or not the curriculum is being equitably applied across the region, whether or not the curriculum reflects a proper balance between procedural knowledge and conceptual understanding, and whether or not technology is fulfilling its intended role.

Curriculum Outcomes

This guide provides details regarding SCOs for each grade. As indicated earlier, the order of presentation neither prescribes a preferred order of presentation nor suggests an isolated treatment of each outcome; rather, it organizes the SCOs in terms of the broad framework of GCOs and KSCOs.

The SCOs are presented on two-page spreads (see Figure 4). At the top of each page the overarching GCO is presented, while the appropriate KSCO and SCO(s) are displayed in the left-hand column. As well, the bottom of many left-hand columns contains a relevant quotation. The second column of the layout, entitled Elaboration–Instructional Strategies/Suggestions, provides a clarification of the SCO(s) with suggestions for possible teaching strategies and activities that could be used to help students achieve the outcome(s). While these strategies and activities are only suggestions and are not intended to be rigidly applied, they should help to establish the nature and extent of the development of the SCO(s). Background theory based on research findings is also often included in this column.

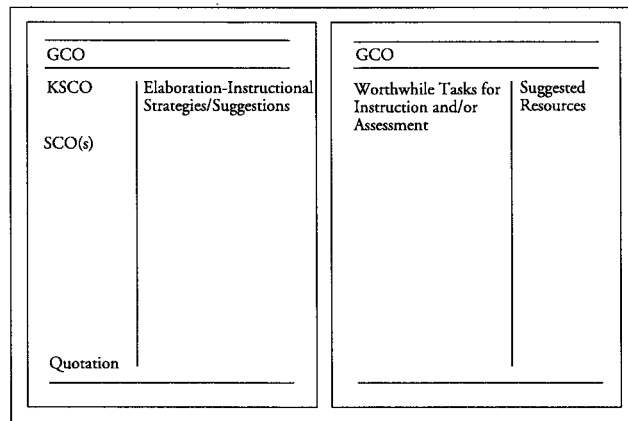


Figure 4: Layout of a 2-Page Spread

The third column of the two-page spread, entitled Worthwhile Tasks for Instruction and/or Assessment, serves several purposes. While the sample tasks presented may be used for assessment, they will also further clarify the SCOs and will often represent useful instructional activities. As well, they regularly incorporate one or more of the four unifying ideas of the curriculum. While these tasks have assessment headings—performance, paper and pencil, interview, observation, presentation, and portfolio—, teachers should treat these headings

only as suggestions. These sample tasks are intended as examples only: teachers will want to tailor them to meet the needs and interests of their students.

The fourth column of each display, entitled Suggested Resources, is provided for teachers to use to collect useful references to resources that are particularly valuable in achieving the outcome(s).

Primary

**Number Concepts/
Number and Relationship
Operations:
Number Sense and
Number Concepts**

GCO A: Students will demonstrate number sense and apply number theory concepts.

Elaboration–Instructional Strategies/Suggestions

KSCO

- (i) By the end of grade 3, students will be expected to construct and communicate number meanings and to explore and apply estimation strategies, with respect to whole numbers.

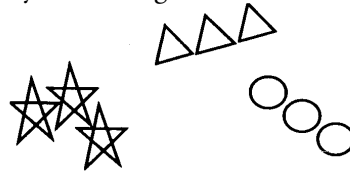
SCO

By the end of grade primary, students will be expected to

- A1** sort sets on the basis of number
- A2** count to determine the number in a group

Students develop a solid understanding of number when teachers call attention to the many ways numbers are used in everyday life, provide opportunities for children to explore number relationships with different objects, and encourage conversations about these concepts. (NCTM 1993, 1)

A1 Sorting sets on the basis of number helps students understand that a concept such as “3” has meaning in relation to the quantity of items in a variety of sets, e.g.,



Given the concrete focus of children at this stage, sorting activities should frequently involve the use of concrete materials, as well as pictorial representations of sets.

- Some possible activities include sorting
 - animals by the number of legs
 - buttons by the number of holes
 - clothes by the number of pockets
 - vehicles by the number of wheels
 - Have students consider situations in which they usually see
 - one of something, e.g., one clock in a room
 - two of something, e.g., a pair of shoes
 - three of something, e.g., the wheels on a tricycle
 - four of something, e.g., the legs on a chair
- A2** As contrasted with rote counting, meaningful counting involves an understanding of the following principles: (a) one number is said for each item in the group, (b) counting begins with the number 1, (c) no item is counted twice, (d) the arrangement of objects is irrelevant, and (e) the number in the set is the last number said.
- Have children play a variety of games such as
 - bowling (counting both the pins knocked down and the pins left standing)
 - board games (counting the number of spaces to be moved based on a spin)
 - throwing bean bags (counting how many land in the target box)

GCO A: Students will demonstrate number sense and apply number theory concepts.

Worthwhile Tasks for Instruction and/or Assessment
Suggested Resources
Performance

A2.1 Have students draw pictures of their favourite toys. Then ask them to count the number of toys in their pictures.

A2.2 Provide a variety of interesting photographs involving multiple people, animals, or objects. Ask students to tell how many are in the photographs.

A2.3 Allow students to count the number of napkins, cups, and plates that are present or that are needed for snack time or a special party.

Observation and Interview

A1.1 As children sort based on number, observe whether or not they

- verbalize their sorting rules
- use number names to describe their sorting rules
- attend to small details in looking for ways to sort
- sometimes re-sort the same items using different number ideas

A1.2 Provide a selection of buttons. Ask students to sort them and explain the sorting selection. Have students sort them in another way, and then another, each time asking for the sorting rule.

A2.4 Observe children to determine their understanding of each of the principles underlying meaningful counting.

A2.5 Note the way in which students count.

- Do they touch each object as they count?
- Do they set items aside as they count them?
- Do they show confidence in their count or feel the need to check?
- Do they check their counting in the same order as the first count or in a different order?

A2.6 Ask students, individually, to count out six counters into your hand. Shake them up in both hands and then open your hands to display a “6” combination, e.g., 4 in one hand, 2 in the other. Ask students how many you have altogether. Repeat this exercise using different combinations. Observe whether students need to count.

GCO A: Students will demonstrate number sense and apply number theory concepts.

KSCO

- (i) By the end of grade 3, students will be expected to construct and communicate number meanings and to explore and apply estimation strategies, with respect to whole numbers.

SCO

By the end of grade primary, students will be expected to

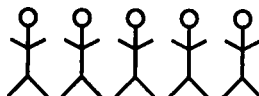
- A3** create sets of a given number
- A4** explore a variety of physical representations of numbers

In kindergarten, number sense evolves from the total classroom experience as well as through specific activities. If we want children to develop a solid understanding of numerical concepts, we must call attention to the many ways numbers are used in everyday life, provide opportunities for children to explore number relationships with different objects, and encourage conversations about these concepts. Many materials are useful for counting and grouping: beans, toothpicks, buttons, shells, and pebbles. (NCTM 1991, 7)

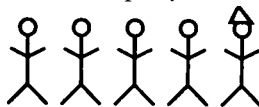
Elaboration–Instructional Strategies/Suggestions

A3 Once students are able to determine the number in a group by counting, the next step is to be able to create a group of that number. This can best be accomplished in context.

- Have students take turns dealing cards.
- In a dramatic play centre, ask children to set the table for five, or put birthday candles on a cake for someone turning 6.
- Have students tell stories involving numbers that are then dramatized, modelled, or illustrated, e.g., there were 5 children going to Sarah’s birthday party.

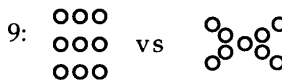
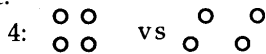


One of the children wore a party hat. The other four did not.



- Ask students to count
 - the number of doors in the hallway
 - the number of steps to the top of the slide
 - the number of grey cars in the parking lot

A4 Students should recognize that there are many ways to arrange a set of objects, and that some arrangements are easier to recognize than others. Observe whether students are able to immediately say how many objects are displayed in familiar arrangements without doing a 1-to-1 count.



Particularly useful materials for this outcome are dice, board games, dominoes, dot cards, and playing cards.

- Ask children to
 - create their own dice
 - arrange raisins in cookies to make patterns
 - create personal counting books in which each spread shows a particular number of items, but in a variety of physical arrangements

GCO A: Students will demonstrate number sense and apply number theory concepts.

Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

Observation and Interview

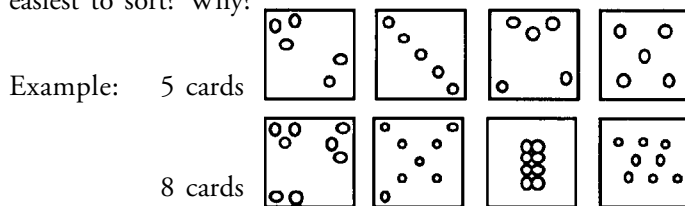
A3.1 Note student confidence and the strategies used when creating sets: (a) if a model set of that number is visible; (b) if no model set of that number is available; (c) with a given number of items; (d) that must have more items than a given amount, e.g., sets with more than 5; and (e) that must have exactly 1 more or 1 less than a given amount.

A4.1 Observe which physical representations of numbers are easier for various children to identify quickly. Also note

- whether children count the dots on a die or simply recognize the pattern
- whether a linear representation of a larger number makes it harder to identify than other representations, e.g., paired
- what other things children discover about representing numbers

A4.2 Have students roll dice and state the numbers rolled. Notice which numbers are not counted by the students but are recognized by their configurations.

A4.3 Have students work in pairs. Provide a set of cards and ask them to sort them according to number. Ask: Which cards were easiest to sort? Why?



Portfolio

A3.2 Have students select their favourite numbers and create booklets showing those numbers in many different ways.

A4.4 Have students choose their favourite numbers. Have them create and save displays that show many ways to physically model the numbers.

GCO A: Students will demonstrate number sense and apply number theory concepts.

KSCO

- (i) By the end of grade 3, students will be expected to construct and communicate number meanings and to explore and apply estimation strategies, with respect to whole numbers.

SCO

By the end of grade primary, students will be expected to

- A5** count in a variety of ways
- A6** interpret ordinal numbers

Counting skills, which are essential for ordering and comparing numbers, are an important component of the development of number ideas. Counting on, counting back, and skip counting mark advances in children’s development of number ideas. (NCTM 1989, 39)

Elaboration–Instructional Strategies/Suggestions

A5 Students should experience situations in which they count (a) from 1 onward, i.e., 1, 2, 3, ... ; and (b) from 10 backward, i.e., 10, 9, 8, ...

Some students may be ready to begin counting onward from a number, e.g., 4, 5, 6, ...

Counting situations should occur naturally in the course of daily tasks, such as those that occur at work centres.

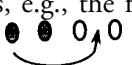
To encourage *counting on* rather than always counting from 1, children might play a game in which they cover the starting quantity with a cloth and other items are then added.

- Have children
 - count backward while taking items out of the water table
 - count down to special days
 - count while performing finger plays or exercises
 - count on while determining the total on a pair of dice
- Have students count on and count back while simultaneously creating the count on a calculator.

Press ... Stop at 10

Press ... Stop at zero

A6 Terms such as first, second, and third should be regularly used in everyday contexts, e.g., in retelling a story a student might tell what happened using these terms. The symbolic forms of ordinals need not be used. Most students should use the terms first, second, and third; some students will be able to extend the ordinals.

The most important ideas for students to realize are (a) position matters, e.g., the first item in a group loses its designation if it moves  ; (b) for every cardinal number, there is an ordinal number, e.g., for the cardinal number 10, the ordinal is 10th; and (c) first is not always fixed but may depend on point of view, e.g., When a child is considering the arrangement of books on a ledge, the first (or second or third) will depend upon the end at which he/she starts counting.

- Use songs and rhymes to practise counting forward and backward, e.g., *Ten Little Monkeys*.

GCO A: Students will demonstrate number sense and apply number theory concepts.

Worthwhile Tasks for Instruction and/or Assessment
Suggested Resources
Performance

A5.1 Begin counting to 10, but omit some numbers. Allow students to fill in the numbers you skip.

A6.1 Ask students to place a set of objects so that a particular object is third. Observe whether students believe that there should be only three objects or realize that there can be more.

A6.2 Ask students to line up five 2-colour counters in a row, same colour facing up. Direct them to turn over the first and third counters. Invite students to provide similar directions for their classmates.

Observation and Interview

A5.2 Note whether or not students

- recognize natural counting situations
- need to line objects up to count them
- need to touch objects to count them
- need to start from the beginning when counting additional objects
- correct one another as they count together

A6.3 Observe whether or not students

- use terminology like *first* and *second* instead of *we did this, then we ... , then we ...* when discussing events
- realize that first from one point of view is last from another
- are comfortable using ordinals when taking turns, lining up, describing days of the week or months of the year, and in other situations

A6.4 Tell students that John said, “I came in last.” Then tell them Mary said to John, “You came in third.” Ask, How can they both be right?

GCO A: Students will demonstrate number sense and apply number theory concepts.

KSCO

- (ii) By the end of grade 3, students will be expected to concretely explore common fractions and decimals in meaningful situations.

SCO

By the end of grade primary, students will be expected to

- A7 recognize the meaning of halves when used in context

Elaboration–Instructional Strategies/Suggestions

A7 Although no formal work with fractions is expected, students should become familiar with the concept of one-half and recognize the term as it is commonly used. The goal should be to stress the concept of “fair shares.” Explorations should be provided to add to their previous experiences of sharing with another person.

Part of a whole (area model) The sharing of a brownie, a piece of licorice, or a piece of paper is commonplace to students. The more opportunities they have to partition fairly, the better their visual concept will be for one-half.

Part of a set The students’ experiences with sharing often involve finding fair shares of a set of objects. This generally involves partitioning one at a time to each person to ensure that the sharing is fair. The number each gets is irrelevant; e.g., when sharing a package of candies, children will give “one for you, one for me” without determining how many each gets in total. As with the part of a whole, it is important that the children develop a mental image of what a *half* of a set looks like.

Students should be aware when talking about one-half that (a) two shares are involved; (b) the size of the half is dependent on the size of the whole; (c) each sharer gets a part of something, whether it is a group of objects or an individual item that is cut up; and (d) the sharing is fair.

- Invite two students to share a box of raisins, offering each one-half. Suggest that they decide what to do to accomplish this. Ask why you could not invite three students over to each get one-half.
- Provide many opportunities for students to estimate one-half, e.g., one-half of a pile of baseball cards, a jar of pencils, or a chocolate brownie.

Young children find estimating difficult. Provide tasks using the word *about*, e.g., “Show me about one-half of the blocks.”

- Place a number of sets of objects and single items on a table. Tell students a story about twins who always share fairly. Ask them how the twins would share these objects. Encourage students to verbalize as they divide up the objects.

GCO A: Students will demonstrate number sense and apply number theory concepts.**Worthwhile Tasks for Instruction and/or Assessment****Suggested Resources***Performance*

A7.1 Break a muffin into two equal pieces for two children. Ask if they think that each has half.

A7.2 Ask students, individually, to create a picture, half of which is red and half of which is blue.

A7.3 Provide a collection of 20 counters or blocks. Ask children to show you about one-half without sharing them out. Have them explain how they could check to see how close to a half it is.

A7.4 Ask children to use colour tiles to make designs in which half of the tiles are one colour and the other half a different colour.

Observation and Interview

A7.5 Note whether or not students use the term half to mean any sort of sharing situation, regardless of the number involved.

A7.6 Ask students to tell you when they have received half of something.

A7.7 Show the student a set of six pencils, a glass of juice, and a package of raisins. Ask, Which is the easiest to find one-half of and which would you find most difficult to divide in half? Why?

A7.8 Provide examples of wholes that are divided to represent halves and wholes that have divided into two parts that are not equal. Ask students to identify the wholes that have been separated into halves and to explain their reasoning.

Presentation

A7.9 Provide a picture of an irregular shape. Have students work in pairs to estimate where to draw a line to divide it in half. Invite them to explain their reasoning.

GCO A: Students will demonstrate number sense and apply number theory concepts.

Elaboration–Instructional Strategies/Suggestions

KSCO

(iii) By the end of grade 3, students will be expected to read and write whole numbers and to demonstrate an understanding of place value (to four places).

SCO

By the end of grade primary, students will be expected to

A8 use symbols to represent numbers

A8 Numeral writing should be taught as needed and requested, i.e., as students are ready to record information and need to write the appropriate numeral. Observe students as they write their numerals both when copying from a model and when forming them from memory. Because it is important that they develop an efficient means of recording numerals, direction and practice are required. One suggestion for practice is to use their index fingers to form the numerals on their desks, in the air, or in the sandbox.

Students may initially use their own number symbols, if they wish, but should soon realize that a conventional symbol makes communication easier.

- Have students
 - write their phone numbers in a class book
 - use a tally or invented system to record changing amounts
 - record numbers on a hopscotch grid or number line
 - copy numbers from the environment
- Have students sort the numerals in terms of characteristics, e.g., those with rounded parts (like 8 and 0), those with only straight parts (like 1 and 7), and those with both (like 5 and 2).
- Provide a calculator for each student. Direct students to show a particular number on the display. To confirm their responses, write the symbol on the board, display it on an overhead calculator, or show a large number card. Repeat the process by having students clear their displays and then calling out (or having a student call out) another number.

Variation: Ask students to press the key to show

- the number of windows in the room
- the number of lights in the ceiling
- the number of students wearing jeans

Having a set of digit cards for each student to use for many number activities is also recommended. These cards not only help students with identification and association, but also make it easier for them to record when the focus is on aspects of counting and not recording.

GCO A: Students will demonstrate number sense and apply number theory concepts.

Worthwhile Tasks for Instruction and/or Assessment**Suggested Resources***Performance*

A8.1 Ask students to paint pictures that include at least one copy of the symbols for the numbers they like.

A8.2 Have students draw original symbols that they think might be used to describe five of something.

A8.3 Provide each of the students with cards on which the digits 1 to 5 are written. Make number arrangements on an overhead using bingo chips. Ask students to select, and raise in the air, the digit card that represents the arrangement.

A8.4 Have students sort the digits in terms of the way they start to write them, e.g., those that start at top left and go right, that start at top and go down, or that start at top right and go left.

Note: Left-handed students may form their numbers differently.

Observation and Interview

A8.5 As children record numerical information, observe whether or not

- they recognize the standard number symbols, even when they are stylized
- they seek out opportunities to practise their numeral writing, either in sand, in the air, or on paper
- they create their own symbols
- their invented symbols bear any relationship to the standard symbols
- their invented symbols somehow reflect the actual number being recorded

GCO A: Students will demonstrate number sense and apply number theory concepts.

KSCO

- (iv) By the end of grade 3, students will be expected to order whole numbers and represent them in multiple ways.

SCO

By the end of grade primary, students will be expected to

- A9** determine which group has more, which has less/fewer, or whether groups are equivalent

While the concept of “less” is very similar to the concept of “more,” the word “less” proves to be more difficult for children than does “more.” ... To help children with the concept of “less,” frequently pair it with the word “more” and make a conscious effort to ask “which is less” questions as well as “which is more” questions. (*Van de Walle 1994, 89*)

Elaboration–Instructional Strategies/Suggestions

A9 Most students have an intuitive idea that to decide if one group has more than another, it is possible to match up items in one-to-one correspondence to see if one group has any leftovers.

- Provide students with many opportunities to pose or answer questions such as
 - Who has less? Who has more?
 - Are there as many napkins as plates?
 - Are there more boys or girls here today?
 - Is there a child here for every name tag?
 - Are there the same numbers of chairs and students?
- Provide students with experiences matching sets in which
 - the items go together, e.g., forks and spoons
 - the items are unrelated, e.g., glue containers and pieces of paper for the children at the art table
- Have children vote for an idea. To clarify the results, the *yes* line and the *no* line can hold hands to find which line has more (or fewer) votes.
- Ask students to shake and spill a handful of 2-colour counters and then spin a *more/less/same* spinner. Based on the spinner results, the students will make a *more/less/same* prediction such as “I think there are more red counters” or “I think there are fewer yellow counters.” Ask students to verify their predictions.
- Invite students to play *Dot Challenge*. Provide pairs of students with decks of dot cards (1 to 10 dots each). Each student turns up a card; the one with the greater number of dots gets both cards. The winner is the one finishing with the greater total number of cards.

Making comparisons can be a natural part of early graphing activities and daily calendar activities.

There is no reason to force children to engage in one-to-one matching activities if they can successfully distinguish more, less, and equivalence by counting.

GCO A: Students will demonstrate number sense and apply number theory concepts.

Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

Performance

A9.1 Ask students to put out fewer of one item than another, e.g., fewer pencils than crayons.

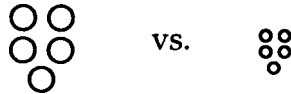
A9.2 Have students check in the toy bin to see if there are more of one item than another, e.g., more trucks than cars.

A9.3 Ask students to build a building with more of one type of block than another.

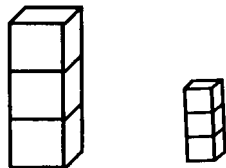
Observation and Interview

A9.4 Observe whether or not the following extraneous factors influence children as they compare:

- arrangements that take up different amounts of space, e.g.,

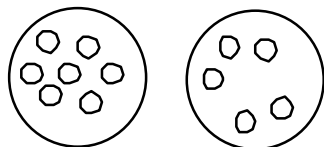


- arrangements that involve different heights, e.g.,



A9.5 Note whether or not children regularly use matching to answer questions such as, Are there enough napkins for everyone in the group? Are there enough skipping ropes for those who want them?

A9.6 Place counters on each of two paper plates or jar lids.



Ask students which has fewer. Encourage them to talk about it, not just indicate by pointing, e.g., This one has fewer and this one has more. Then ask students to make the sets the same size.

**Number Concepts/
Number and Relationship
Operations:
Operation Sense and
Number Operations**

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

KSCO

(iii) By the end of grade 3, students will be expected to create and model problem situations involving whole numbers using one or more of the four basic operations.

SCO

By the end of grade primary, students will be expected to

B1 count the results when small groups are combined

B2 count the results when small groups are separated

... “Operation sense,” a highly integrated understanding of the four operations and the many different but related meanings these operations take on in real contexts. (*Van de Walle* 1994, 109)

Elaboration–Instructional Strategies/Suggestions

B1 Many students are capable of adding informally. It is important to provide many experiences in which students will be combining and counting sets using a variety of manipulative materials. The intent at this level is to treat addition as counting situations in contexts without any formal attempt to symbolize it.

Students should become involved in situations for which (a) action is involved, e.g., some objects are added to an existing group; (b) no action is involved, e.g., two existing groups considered together for some purpose.

Students should come to understand that the act of combining groups increases the count of both groups.

- Have students work with play/real coins in a pretend/real yard sale. They can price items using small penny amounts and determine the cost if two or more items are purchased.
- Have students consider amounts combined in recipes, e.g., a cup of sugar added to two cups of flour.
- Ask students to make 2-colour bars using connecting cubes, e.g., 5 blue and 3 red. Ask students to tell you about the number of cubes. (This may also be an opportunity to look at patterns, e.g., 2 red, 1 blue, 2 red, 1 blue ... See SCO, C1.)
- Ask students to work in pairs. The first student rolls a die, counts out the number of blocks indicated by the die, and places them on a mat. The second student does the same, adding his/her blocks to those of the partner. Have them count to determine the results of the combination.

B2 Many students deal with experiences that involve removing or separating items from a group to determine how many are left. This form of subtraction would be dealt with only informally at this level.

- Have students count how many are left when 4 green speckled frogs are sitting on a log and 1 leaves, or when 6 raisins are on a plate and 6 are eaten.
- Invite students to make up their own story problems and present them to their classmates for them to solve using materials, e.g., Sue had 10 blocks. She gave 3 to her friend, Brian. How many are left?

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

Performance

B1.1 Ask students to combine two groups of objects and tell how many objects there are altogether.

B1.2 Ask students to divide sets of toys into two groups based on a criterion of their choice. They can then tell how they separated the groups, or ask you to guess the criterion. Have them describe how many are in each group, and how many there are in total.

B1.3 Model for the class a number of story problems involving the combination of sets. Invite students to make up their own story problems and present them to their classmates for them to solve using materials, e.g., Margo has 4 crayons. Mario gave her 2 more. How many does Margo now have?

B2.1 Ask students to remove items from a group and tell how many are left.

B2.2 Tell students a subtraction story and ask them to model it using materials, e.g., You have 8 individual packets of crackers. Five students want crackers for snack time. How many packets will you have left?

Observation and Interview

B1.4 Observe children as they find totals to determine whether or not they

- count on from the original amount
- count on from the greater amount
- count each individual item
- can keep track only when the numbers are quite small
- can find totals above ten

B2.3 Observe how students determine the number remaining, to see whether they

- start with the first number and count backwards
- count forward those items that are left
- sometimes *take away* mentally rather than laying out the items
- find certain combinations more quickly than others, e.g., those in which only one is subtracted

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Elaboration–Instructional Strategies/Suggestions

KSCO

(iii) By the end of grade 3, students will be expected to create and model problem situations involving whole numbers using one or more of the four basic operations.

SCO

By the end of grade primary, students will be expected to

B3 determine how many more one group has than another

B3 An important meaning of subtraction that might begin informally is the comparison meaning. The student is trying to determine how many more items one group contains than another. These situations arise every day, e.g., How many more cookies do you have than I? How many more pennies does my brother have than I have?

Students should be dealing with relatively small-group sizes. There is no intention that students be even aware that this is one of the meanings of subtraction.

Some students will realize that an easy way to compare two groups of objects is to use one-to-one correspondence, i.e., if they line up both groups in a one-to-one way, they can just count how many extras there are in the larger group. Another approach might be to set up a grid for this, e.g., If I have 6 pencils, and you have 4, I have 2 extra.

X	X	X	X	X	X				
0	0	0	0						

Some students might approach this question in other ways, e.g., they might put out 4 counters and count up to 6 to see how many extra they need.

- Invite pairs of students to take turns throwing a die and making towers with the corresponding numbers of cubes. Have them tell which tower is taller and by how much.
- Give students some pennies. Use the store corner where items are individually priced (in pennies). Ask, How much more does ____ cost than ____? Which items do you not have enough pennies to buy? How many more pennies would you need?

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Worthwhile Tasks for Instruction and/or Assessment**Suggested Resources***Performance*

B3.1 Put out two sets of counters. Ask students to show how many more one set contains than the other.

B3.2 Tell students that in an animal reserve there are 5 bears and 3 giraffes. Ask them to choose materials to model how many more bears there are than giraffes.

Observation and Interview

B3.3 Observe how students determine *how many more*, to see whether they

- set up a one-to-one correspondence and count extras
- count up from the lesser number to the greater

B3.4 Ask students to look at the coins in each of your hands and determine which hand is holding more coins and how many more.

B3.5 Observe to see whether students who are setting a table for snacks count up to figure out how many more napkins are needed.

Portfolio/Presentation

B3.6 Ask students to draw pictures to show situations in which they may want to know how many more there are in one group than another.

Patterns and Relations

GCO C: Students will explore, recognize, represent, and apply patterns and relationships, both informally and formally.

KSCO

- (i) By the end of grade 3, students will be expected to recognize, describe, extend, and create patterns and sequences in a variety of mathematical and real-world contexts (e.g., geometric, numeric, and measurement).

SCO

By the end of grade primary, students will be expected to

- C1 copy and extend patterns including those involving number, shape, size, and colour

Children first explore patterns with their own bodies, actions, and words. They see and hear patterns in the world around them. Before creating pictorial representations and patterns at symbolic levels, children need objects with which to make patterns. (*NCTM* 1991, 1)

Elaboration–Instructional Strategies/Suggestions

C1 Children notice and hear patterns in many contexts. It is important to help them recognize the mathematical aspects of these patterns. Grade primary children enjoy being part of action and word patterns. Before they can create patterns, children need a great deal of experience with given patterns, e.g., use children to model a *stand-sit* pattern and then have small groups copy the pattern. Expand it to more components when the children are able to handle them, e.g., stand, sit, stand, lie down (repeat) and other combinations. Have the children chant the patterns as they perform the actions.

Action patterns can include, among others, clap hands, slap knees (repeat), touch toes, pat head, turn around (repeat), and clap, clap, snap (repeat).

Patterns can be explored through skipping rope chants and performing the actions for such songs as the “Hokey-Pokey.” Children also benefit from hearing predictable books and repeating the patterns. Children will require many patterning experiences with concrete materials prior to recording patterns on paper and/or working from patterns of pictures.

- Have students reproduce and extend patterns that focus on
 - shape, e.g., ○□△○□△ ...
 - size, e.g., □ □ □ □ ...
 - colour, e.g., red block, blue block, red block, blue block ...
 - position, e.g., □ ◇ □ ◇ ...
 - number, e.g., 2, 1, 3, 2, 1, 3 ...
 - auditory patterns, e.g., clap, clap, clap (pause), clap, snap, (pause), clap, clap, clap, (pause), clap, snap ...

Encourage the children to talk about patterns and include pattern activities across the curriculum, particularly in art and music classes.

It is important that students recognize that there is not a pattern until there is a repetition, e.g., ABC is not a pattern but ABCABC ... or ABCCBAABCCBA ... are patterns. Repeat a pattern two or three times before asking students to continue it. Also, include some examples that are not patterns.

GCO C: Students will explore, recognize, represent, and apply patterns and relationships, both informally and formally.

Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

Performance

C1.1 Ask students to copy a set of actions that you model and to make a pattern, e.g., clap, clap, snap, snap or snap, pause, snap, pause, snap, snap, snap.

C1.2 Ask students to use concrete materials to reproduce a given pattern and to continue it, e.g.,



C1.3 Show a pictorial representation of a pattern and ask students to use materials to reproduce and extend it, e.g.,



C1.4 Observe students as they act out such songs as “Head and Shoulders” or “The Hokey-Pokey” for their abilities to discern and repeat patterns.

Observation and Interview

C1.5 Show a colour pattern and ask students to describe it, e.g., red cube, red cube, yellow cube, red cube, red cube, yellow cube ...

C1.6 Read a story that has a pattern and observe when students recognize and can repeat the pattern.

C1.7 Note the ease with which students

- recognize patterns
- copy patterns with materials
- copy patterns involving body movements
- describe patterns verbally

C1.8 Using digit cards, show students a series of numbers and ask them to continue the pattern, e.g., 5, 4, 7, 5, 4, 7 ...

C1.9 Ask students to start with ABC and continue in two different ways to make two different patterns.

GCO C: Students will explore, recognize, represent, and apply patterns and relationships, both informally and formally.

KSCO

- (i) By the end of grade 3, students will be expected to recognize, describe, extend, and create patterns and sequences in a variety of mathematical and real-world contexts (e.g., geometric, numeric, and measurement).

SCO

By the end of grade primary, students will be expected to

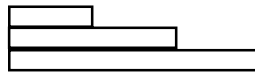
- C2** copy patterns based on measurement attributes
- C3** create patterns

Pattern is a unifying theme that weaves mathematical topics together. The study of patterns supports children in learning to see relationships, to find connections, and to make generalizations and predictions. Understanding patterns nurtures the kind of mathematical thinking that helps children become problem solvers and abstract thinkers. It is a problem-solving tool. (*NCTM* 1991, 1)

Elaboration–Instructional Strategies/Suggestions

C2 Measurement attributes provide another alternative for students engaged in pattern making. Provide students with patterns that include variations in (a) size, (b) mass, (c) capacity, and (d) time. This type of patterning can be done in conjunction with the measurement strand.

- Ask students to order Cuisenaire Rods based on length.



- Provide Plasticine balls of different sizes placed in tubs with tops. Have students create patterns based on mass.
- Provide multiple samples of containers of different sizes. Have students create and explain capacity patterns.
- Explore patterns found in nature, e.g., the progression of the seasons. These are patterns that never change. Point out to the children the established patterns that make up their daily schedules. Have them share patterns that they regularly follow at home.

C3 It is important to remember that grade primary children need to be exposed to many forms of patterning. They should copy and reproduce given patterns before being expected to create their own.

- Have groups of students form small circles, each student having a different musical instrument. Ask one student in the group to create a pattern that each student, in turn, will copy with his/her instrument. Repeat this activity with a different student creating the pattern. (Instruments would include the traditional rhythm band student instruments, as well as those made from classroom materials such as spoons, a block and a stick, two chalkboard erasers, or a tin can with pebbles inside. A creative assignment would be for students to design their own instruments from materials in the classroom or from home.)
- Provide students with different kinds of pasta and have them create patterns. These pasta pieces could be glued on to paper for display.

GCO C: Students will explore, recognize, represent, and apply patterns and relationships, both informally and formally.

Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

Performance

C3.1 Provide multiple samples of different lengths of string. Ask students to select some, arrange them in patterns, and describe the patterns.

C3.2 Provide a number of pencils of different lengths and ask students to use some to create patterns. Ask them to explain the patterns.

C3.3 Provide students with a number of cookie cutters and ask them to create patterns in rolled out Plasticine dough.

C3.4 Ask students to create patterns with finger paints.

C3.5 If appropriate for your students, provide pictures representing various holidays (Kwansa, Halloween, Easter, Valentine's Day, Christmas, Canada Day, Hanukkah) and ask students to arrange them in a sequential pattern.

C3.6 Have students collect objects of nature from the playground, e.g., pebbles, leaves, pine needles, buds, nuts. Model a pattern using the materials, asking students to tell what will come next.

C2.1 Have pairs of children take turns creating and extending patterns using collected materials like those in C3.6. They may wish to glue their patterns on construction paper and display for others to see.

Interview

C2.2 Ask students to describe patterns that you have shown them.

C2.3 Show students a geoboard on which a shape pattern is displayed, e.g., a one-unit square, a two-unit square, a three-unit square. Ask them to describe the pattern and to describe the next shape in the pattern.

Presentation

C3.5 Ask students to work in pairs to design action patterns and then present them to their classmates.

GCO C: Students will explore, recognize, represent, and apply patterns and relationships, both informally and formally.

KSCO

- (i) By the end of grade 3, students will be expected to recognize, describe, extend, and create patterns and sequences in a variety of mathematical and real-world contexts (e.g., geometric, numeric, and measurement).

SCO

By the end of grade primary, students will be expected to

- C4** represent the same pattern in multiple ways

Elaboration–Instructional Strategies/Suggestions

C4 In addition to developing the abilities to recognize, describe, and continue patterns, grade primary children should also be learning to translate from one representation of a pattern to another representation and to represent the same pattern in a variety of ways. These are more complex tasks.

- Provide a simple pattern such as snap, clap, snap, clap ... and ask students to use unifix blocks to represent the pattern, e.g., white, red, white, red ... Have them share their patterns and discuss the various responses. Ask students to continue their pattern trains of blocks. Encourage them to use other representations for the same pattern, e.g., boy, girl, boy, girl ... or opened book, closed book, opened book, closed book ... or crayon, pencil, crayon, pencil ... or red block, green block, red block, green block ... Change the original clapping pattern on subsequent days, encouraging more challenging representations, e.g., AABABAABAB ...
- Have students present their own patterns that their classmates will represent in some form. Conversely, have students represent patterns using materials and ask their classmates to describe the patterns.
- Describe this pattern: 2 boys, 1 girl, 2 boys, 1 girl ... Have students represent this pattern by using snaps and claps and by using two different coloured cubes.

Some materials that can be collected to be used for patterning include buttons, shells, macaroni, cereal, paper clips, plastic forks and spoons, bottle caps, and jar lids.

GCO C: Students will explore, recognize, represent, and apply patterns and relationships, both informally and formally.

Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

Performance

C4.1 Show a *clap, snap* pattern. Provide a variety of materials for students and ask them to represent the pattern in two different ways.

C4.2 Provide a pattern and ask students to represent it with *clap, snap* patterns, e.g.,



C4.3 Invite students to select a musical instrument. Ask them to use it to present patterns to their classmates for them to represent with different coloured cubes.

C4.4 Provide students with a collection of colour tiles. After they have developed and described patterns with the tiles, ask that the patterns be represented in another way, using a different material.

Interview

C4.5 Present students with two different action patterns and ask them to indicate which of the two they prefer and why, e.g., sit down, stand up, turn around, sit down, stand up, turn around ... and snap fingers, clap hands, pat knees, snap fingers, clap hands, pat knees ...

Shape and Space: Measurement

GCO D: Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

KSCO

- (i) By the end of grade 3, students will be expected to measure and understand basic concepts and attributes of length, capacity, mass, area, and time.

SCO

By the end of grade primary, students will be expected to

- D1** compare and order objects based on length, capacity, and mass

When students compare objects on the basis of some measurable attribute, that attribute becomes the focus of the activity. (*Van de Walle 1994, 293*)

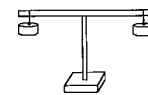
Elaboration–Instructional Strategies/Suggestions

D1 It is important that students focus on what it means to measure, rather than how to measure with non-standard or standard units. Many students’ first experiences with measurement involve length because they are often vitally interested in such things as who is taller or which toy car went farther.

Have students become familiar with the various terms associated with length such as long, short, wide, tall, high, and far. Provide opportunities to compare lengths by measuring directly and indirectly. It is important that students have a significant amount of comparison experience prior to using non-standard and standard units for describing length.

- Have two children stand back to back to determine who is taller/shorter.
- Cut a string the length of one object and then have children lay the string along a second.
- Have students fill containers with sand and water to determine their capacities. Students can explore the effect of pouring the same amount of water into containers of different shapes and sizes. The focus should be on comparison, rather than on describing the capacities of individual containers. It is important to provide a wide range of materials including many sizes and shapes of containers.

Because students are often interested in the heaviness of objects, initial activities can include objects that are quite large where only gross comparisons can be made. However, this interest can be transferred to smaller objects that are amenable to comparison on a pan balance. If manufactured balances are not available, a variety of homemade pan balances can be created, e.g., cottage cheese containers hung from the ends of strips attached to a base.



- Have students compare structures in terms of height/width; compare their own heights or arm lengths; compare the capacities of different types of cups, spoons, pails, boxes, and bowls; fill containers of the same size and shape with different amounts of water to produce different pitches of sound; and compare the masses of different rocks.

GCO D: Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

Performance

D1.1 Provide a coffee mug and a thermos and ask students to predict and test to find which holds more.

D1.2 Ask students to show you something that is long, something that is wide, and something that is high.

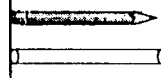
D1.3 Ask students to estimate which of two toys is heavier and then check by using pan balances.

D1.4 Ask students to find the longest carrot in a bunch.

Observation

D1.5 As students compare the lengths of objects, note

- whether or not they recognize the importance of a base line, i.e., comparing by starting both objects at the same place
- whether or not they understand that if A is longer than B and B is longer than C, then A must be longer than C
- how they handle situations in which direct measurement of length is difficult



D1.6 As students predict the capacities of containers, observe whether their decisions are affected by

- the height of the containers only
- the width of the containers only
- both height and width of the containers
- the shape of the containers
- the familiarity of the containers
- the function of the containers

D1.7 Observe students as they consider the masses of objects.

Note whether or not they

- accurately use terms like heavier and heaviest
- consider the shape of an object in predicting its mass
- consider the size of an object in predicting its mass
- consider the material out of which the item is made when they predict its mass
- are familiar with the types of materials that are generally light or heavy

GCO D: Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

KSCO

- (i) By the end of grade 3, students will be expected to measure and understand basic concepts and attributes of length, capacity, mass, area, and time.

SCO

By the end of grade primary, students will be expected to

- D2 sequence events
- D3 sort items based on measurement attributes

Problem solving should be the central focus of the mathematics curriculum. As such, it is a primary goal of all mathematics activity. Problem solving is not a distinct topic but a process that should permeate the entire program and provide the context in which concepts and skills can be learned. (*NCTM* 1989, 23)

Elaboration–Instructional Strategies/Suggestions

D2 Students’ experiences with sequencing events can include reconstructing prior events or anticipating future ones.

- Have students sequence events such as
 - parts of the day
 - classroom routines
 - home routines, e.g., for going to bed or for starting the day
 - seasons
 - days of the week
 - months of the year
 - food preparation
 - parts of a story

While some students may be interested in measuring time using sand or water clocks, the primary focus is on sequencing, not measuring duration. Students should recognize that sometimes sequence is important and sometimes it is not.

D3 Students can sort items based on a variety of measurements including length, capacity, mass, and time.

- Have students sort
 - clothing by size under the headings for babies or dolls, for children, for adults
 - kitchen items by capacity under the headings holds enough for one person, holds enough for a small group, holds enough for a large group
 - “fantasy” items made by the children under the headings for giants, for regular people, for elves
 - same size canisters under the headings light, medium, heavy
 - “time” pictures such as brushing teeth, reading a book, eating lunch under the headings a few moments, a little while, a long time

As well as planning directed sorting activities, you should also provide open-ended sorting tasks that allow the students to choose how many groups to make and what attribute(s) to use.

GCO D: Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

Performance

D2.1 Ask students to describe the sequence of events in a particular story that was recently read.

D2.2 Ask students to describe the steps, in order, that one would take to decide which of two objects is longer.

D3.1 Provide a variety of tools to a group of students. Ask them to determine which they would label small, which medium, and which large. Ask how they decided.

D3.2 Read the story *The Three Bears*. Ask students to find appropriate bowls for Mama Bear, Papa Bear, and Baby Bear.

D3.3 Ask students to choose a cup that they would call large and one they would call small.

Observation and Interview

D2.3 The contexts in which students are comfortable sequencing events may vary. Determine how the following variables affect students' abilities to sequence events:

- the degree of familiarity of the children with the situation
- the interest level of the children in that particular situation or activity
- the recency of a related experience for the children

D2.4 Ask students, (a) Does it matter in which order you brush your teeth, eat your breakfast, wash your hands? (b) Does it matter in which order you swing on a swing, play on the jungle gym, slide down a slide?, and (c) Does it matter in which order you put on your shirt, put on your shoes, put on your socks?

D3.4 Observe students as they sort objects based on measurement. Note whether or not they

- can make fine distinctions
- always sort into two groups or sort into a number of groups
- can re-sort the same items based on different measurement characteristics

Shape and Space: Geometry

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

KSCO

- (i) By the end of grade 3, students will be expected to explore and experiment with geometric shapes and relationships (including the orientation and perspectives of objects).

SCO

By the end of grade primary, students will be expected to

- E1** develop spatial sense, including position-in-space and the language associated with it

Informal geometry is aimed at development of “spatial sense” ... This spatial sense grows and develops over the entire time children are in school. Students should have several in-depth geometric experiences every year. (*Van de Walle* 1994, 324)

Elaboration–Instructional Strategies/Suggestions

E1 Position-in-space perception is the ability to determine the relationship of one object to another and to the observer. Because children perceive themselves to be the centre of their universe, they naturally perceive objects in relation to themselves. Objects are above, below, beside, in front of, and behind them. Children who have difficulty with this perception are likely to experience reversals in reading and writing, and struggle with mathematics as well.

Children at this age are developing mental maps of the areas familiar to them, e.g., their homes and schools. Their knowledge is limited by their experiences; therefore, it is important to provide opportunities for exploration of space—going over, under, and between objects—and moving objects to observe from different perspectives. The gymnasium provides a perfect venue for children to explore space.

Language required to describe positions in space must be developed. Encourage the use of spatial language in connection with daily routines. Provide students with opportunities to follow, and to give, directions using spatial language such as below, above, between, beside, behind, through, to the right, and to the left. (It is not unusual for some students to continue to have a problem differentiating between right and left at this age.)

- Have students construct a “town” with building blocks. Ask them to direct a fellow student through the town using spatial language. This process should first be modelled by the teacher. With practice students will use more terms.

Extend self-object relationships to object-object-self relationships.

- Give directions to students using spatial language terms, e.g., put one blue block beside the red one, or place another blue block behind the red one and a green one on top of each of the blue ones.

Students should be encouraged to work regularly with puzzles. These provide practice for all the spatial skills. It is important to provide puzzles that span a wide range of complexity; some children come to grade primary with a great deal of experience while others have had no experience at all. Selected puzzles should challenge all students.

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

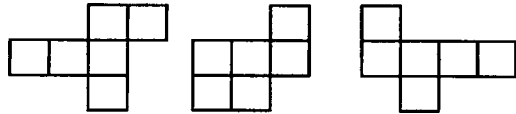
Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

Performance

E1.1 Set up an obstacle course and have students follow directions provided by you or classmates to complete the course. Include a variety of appropriate spatial language.

E1.2 Make several different shapes by fastening (up to seven) linking blocks together. Ask students to duplicate the shape, e.g.,



E1.3 Show a 3-D construction of different-shaped building blocks and ask students to duplicate it for you.

E1.4 Have students build as you tell them: One blue triangle is beside a red circle. Another blue triangle is above the circle. There is a yellow rectangle between the red circle and a green square. Ask, What does the shape look like? Can there be more than one shape? (Repeat the directions and encourage dialogue.)

E1.5 Provide students with yarn loops. Ask them to put four counters in the loop, five outside the loop, and two on the loop.

Interview

E1.6 Ask students to tell you what things in the room they could hide in or behind; stand on, in, or under; jump over or climb under. Extend this to a discussion of where house or classroom pets or larger animals could go within the classroom.

Presentation

E1.7 Have children create maps in sand or with blocks, paints, or crayons. They might map the classroom, the school, playground, familiar buildings, or routes to school. Observe global accuracy, attention to detail, consideration of relative distances, and positions in their maps.

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

KSCO

- (i) By the end of grade 3, students will be expected to explore and experiment with geometric shapes and relationships (including the orientation and perspectives of objects).

SCO

By the end of grade primary, students will be expected to

- E2 develop spatial sense, including eye-motor co-ordination

Elaboration–Instructional Strategies/Suggestions

E2 Children’s early experiences within their environments are mostly spatial. They need experiences to develop visual memory, discrimination, and perception of spatial relations.

Eye-motor co-ordination is the ability to co-ordinate vision with body movements during various activities. Students are unable to fully concentrate on learning experiences if their eye-motor co-ordination has not become proficient and automatic. Children’s eyes direct the movement of their feet when they run, jump, kick balls, or step over obstacles. The eyes work with the body to perform such tasks as getting dressed, cutting out a figure, or packing a lunchbox. It is through practice that skills in eye-motor co-ordination develop to the point that the child is not concentrating on the motor skills; the skill is habitual and concentration can be directed to new skills. The many hands-on activities found in all grade primary classes help to develop eye-motor skills for use in all areas of students’ lives.

- Plan geometric activities involving fine-motor spatial skills, e.g.,
 - tracing over dots
 - joining broken lines
 - making shapes on geoboards
 - joining dots on geopaper, including slanted and curved lines as well as vertical and horizontal ones
 - refining a sense of direction through tracing figures and filling in regions, including colouring
 - drawing within guidelines, including narrow, straight, curved, and angled paths
 - constructing 2- and 3-D shapes, both from oral directions and from models

Activities such as these strengthen position-in-space as well as eye-motor abilities.

Eye-motor co-ordination skills vary in any incoming group of grade primary students. Their developmental level should be assessed and activities modified accordingly, e.g., Some students will be able to stretch elastics around pegs with relative ease while others will find it to be a difficult task and require more practice.

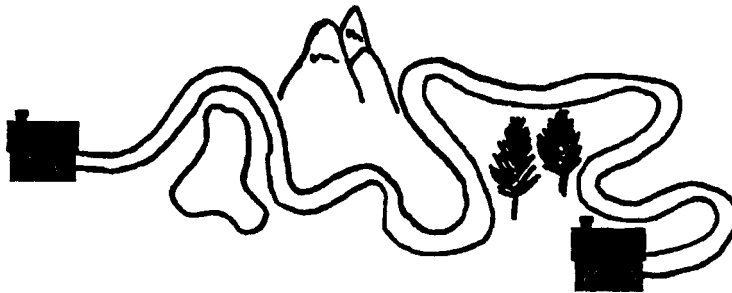
GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

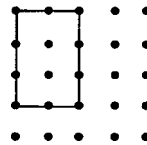
Performance

E2.1 Ask students to use crayons to trace along paths that have been provided, e.g.,



E2.2 Have students trace along designated paths in sand and ask them to describe the paths being followed.

E2.3 Show students a simple shape that has been drawn on geopaper and ask them to duplicate it on geoboards, e.g.,



E2.4 Have students place connecting cubes on the top left-hand peg of geoboards and one on the bottom right-hand peg. Have them imagine that the geoboards are bodies of water and the pegs are stones on which they can hop. Ask them to use yarn to illustrate paths from one “house” (cube) to the other, hopping on the stones. Encourage students to display their geoboards and to find as many different paths as they can. As an extension, students may wish to record paths on 5 x 5 geopaper.

E2.5 Ask students to fill outlined shapes with the fewest pattern blocks that they can, e.g.,



(Not drawn to scale.)

E2.6 Ask students to describe how to make a shape on a geoboard, e.g., a triangle. Follow the student’s directions exactly as you build a shape on an overhead geoboard.

Observation

E2.7 Observe students as they colour in a given shape.

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

KSCO

- (i) By the end of grade 3, students will be expected to explore and experiment with geometric shapes and relationships (including the orientation and perspectives of objects).

SCO

By the end of grade primary, students will be expected to

- E3** sort and build with 2-D and 3-D shapes

Sorting or classifying shapes using models is a good way to introduce geometric ideas ... When students omit shapes from a category they have identified or fail to create a category you hoped they would discover, it is a clue to their perceptual thinking. (*Van de Walle 1994, 328*)

Elaboration–Instructional Strategies/Suggestions

E3 The block centre, or corner, is an important place for students to explore and experiment regularly. As they make their constructions, they begin to learn, often through trial and error, about the attributes of the various shapes. They can identify those that stack, those that roll, those that stack and roll, and those that are stable and make sturdy foundations for constructions. It is through these investigations that students are able to learn the characteristics and properties of shapes.

Children should have many opportunities to sort shapes in the context of everyday activities, e.g., when putting blocks away in the proper places. With experience, they learn that each shape has many attributes and one shape may fit in more than one category, e.g., the ramp-shaped block has both rectangular and triangular faces, so it would fit with either shapes with triangular faces or shapes with rectangular ones.

- Have students sort 3-D shapes using criteria such as number of faces, thickness, and whether it rolls and/or stacks, as well as non-geometric criteria such as colour, function, and texture.
- Give pairs of students a mixed set of attribute or logic blocks. Ask them to sort the blocks and to explain their sorting rule. Encourage them to resort them in other ways. (Some may want to begin sorting by placing them properly in their containers that have indentations for each shape.) This activity could be extended by sorting under more than one heading, e.g., red and thick.

As children engage in building activities, they have many opportunities to explore geometric concepts such as the same blocks can be arranged in different ways, some blocks make better bases for structures, some blocks are more stable, some arrangements of blocks are stronger than others, and some blocks can be substituted for others. These are some examples of things that could be observed as children engage in various building activities.

Children’s exploration and development are enhanced by your interest and questions, space and time to build and elaborate, opportunities to discuss their constructions, and permission to leave them standing for free play or additions.

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

Performance

E3.1 In pairs, have one student create a structure and ask his/her partner to create one that is quite different and to describe the differences.

E3.2 Sort some attribute blocks by a mystery rule. Ask students to determine your rule and choose other blocks that would belong.

E3.3 Give students a collection of 3-D shapes and ask them to sort the shapes in some way and to explain the sorting rule.

E3.4 Have students build imaginary buildings, monsters, or machines from 2-D shapes or 3-D materials such as boxes, cans, balls, paper cylinders, and cones.

Observation and Interview

E3.5 As children participate in various activities, note whether or not they

- show awareness of the shapes used in structures like bridges, garages, roads, and road signs, and use them in their own constructions
- use geometric terminology to describe the shapes
- consider the function of particular shapes in particular situations, e.g., a curved block for a bend in the road
- comment on the similarities and differences within a set of geometric materials
- discover which shapes contain other shapes within them

E3.6 As students sort shapes, observe whether or not they

- can re-sort the same group of shapes in different ways, e.g., recyclables by brand, shape, size, and materials
- use both traditional and unique sorting approaches, e.g., by shape, by “fanciness” of the shape, or by function
- use a variety of attributes of shapes for sorting
- are affected by the orientation or position of the shapes

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

KSCO

- (i) By the end of grade 3, students will be expected to explore and experiment with geometric shapes and relationships (including the orientation and perspectives of objects).

SCO

By the end of grade primary, students will be expected to

- E4 pattern with 2-D and 3-D shapes

Elaboration–Instructional Strategies/Suggestions

E4 Early patterning activities should include students copying patterns that have been set up for them. (See also SCO, C1.)

- Lay out on a table a pattern such as that shown below. Ask students to make the same pattern underneath. Note: This will not ensure that they are able to discern the pattern; they may be only imitating the arrangement. Encourage students to say “two triangles, one square, two triangles, one square ...”



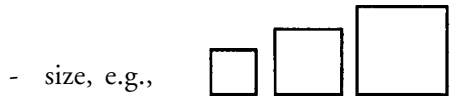
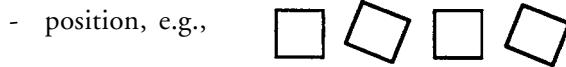
- Begin a pattern such as that shown below and ask students to continue it.



- Have students continue given patterns.



- Include patterns involving



- Have students add to this in some way to create a pattern.



GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

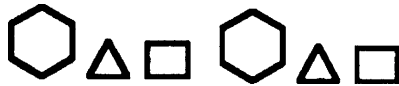
Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

Performance

E4.1 Give students a collection of 3-D shapes and ask them to use some or all of them to create patterns, then describe them.

E4.2 Ask students to use materials to reproduce a given pattern and to continue it, e.g.,



E4.3 Have students individually use coloured 2-D shapes to create a pattern on the overhead. Ask them to describe the pattern to the class.

E4.4 Have students continue the pattern below using rectangles.



Interview

E4.5 Show students the pattern below and them to describe it.



Note: Some talented students might be able to determine and construct more abstract patterns with shapes, e.g., something that rolls, something that slides, something that rolls, something that slides ...

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

KSCO

- (ii) By the end of grade 3, students will be expected to describe, model, draw, and classify 2-D and 3-D figures and shapes.

SCO

By the end of grade primary, students will be expected to

- E5 recognize, name, describe, and compare 3-D shapes (including sphere, cylinder, cone, and cube) and 2-D shapes (including square, triangle, circle, and rectangle)

Elaboration–Instructional Strategies/Suggestions

E5 Children have a natural curiosity about shapes. Many come to school already knowing the names of some common shapes in the world and like to know and use the shapes' names. Initially they learn to recognize and name a shape by its overall appearance and its association with familiar things in their world. Hence, it is recommended that teachers isolate each shape for in-depth treatment.

- For each shape, teachers include activities such as
 - providing various examples and non-examples
 - modelling its name orally and in writing
 - having students look for examples of the shape used in the real world of the classroom, home, and community (see E9) to make posters or bulletin board displays
 - having students make examples using structured materials
 - having students make their own little books of shapes and read other books about shapes
 - sorting, patterning, and building so that students can use the correct names of the shapes
 - having students compare the shape to others

Students will use informal language at this stage rather than precise mathematical language, e.g., Students might say corners rather than angles, edges rather than sides for 2-D shapes, and sides rather than faces for 3-D shapes. While you should use precise language yourself, you should not expect it from the students.

Throughout these activities, students will notice the attributes of shapes; however, most students will not be formulating these as properties of all other shapes with the same name. At this stage in their development, students should be forming vivid mental images of the shapes. It is important, therefore, that positive exemplars are used and that objects that are *almost* a particular shape be discussed as such, e.g., Explain that a grapefruit is *almost a sphere* and a paper cup is *almost a cylinder*. As well, if students present 3-D shapes as examples of 2-D shapes, it should be discussed so that no false impressions are conveyed, e.g., If a filing cabinet is suggested as an example of rectangle, it should be pointed out that a side (face) of the cabinet is a rectangle.

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

Performance

- E5.1 After one student has created a structure, ask another to name the shapes that were used.
- E5.2 Provide a number of grocery store product containers. Ask students to describe what shapes they see.
- E5.3 Have students make “footprints” in the sandbox using a cube and a cylinder. Ask, What are the shapes of the footprints? Have them make a footprint pattern.
- E5.4 Cut out a number of 2-D shapes from cardboard. Place them in a “feely bag” and provide a sample of each on a table for students to use as guides. Reach into the bag to touch a shape and give students the first clue to identify it. Encourage them to develop a plan for guessing what shape has been selected. Continue to provide clues until students are able to accurately determine what shape is being described.
- E5.5 Ask each member of a group of students to make a triangle on a geoboard. Have them compare their triangles.
- E5.6 Use masking tape to form large outlines of 2-D shapes on the floor. Invite students to walk, hop, crawl, skip, or jump on the tape around the shapes and to talk about the sides and corners, comparing them with others.
- E5.7 Show students a shape on the overhead for 5–10 seconds and then have them reproduce it on plain paper. Compare their shapes with the original. (Use triangle, square, and rectangle shapes.)

Interview

- E5.8 For each of the shapes below, ask students, Is this a rectangle? Why or why not? How can it be made into a rectangle?



Presentation

- E5.9 Provide students with a collection of toys that they might find in a toy box. Have them find examples of 2-D and 3-D shapes among these toys and present their findings to the class.

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

KSCO

(iii) By the end of grade 3, students will be expected to investigate and predict the results of combining, subdividing, and transforming shapes.

SCO

By the end of grade primary, students will be expected to

E6 build 2-D shapes using structured materials

E7 subdivide and change shapes

Elaboration–Instructional Strategies/Suggestions

E6 Have students play with 2-D materials such as pattern, attribute, and logic blocks. They should be encouraged to put shapes together to make larger shapes and also to take apart creations that are already made.

- Ask students to see what animals they can make using a group of pattern blocks. Have them count the number of blocks that were used, if appropriate.
- Have students reach into a bag to select eight pieces from a collection of tangram pieces. Ask students to create figures from these pieces and encourage them to draw background features to complete their pictures.
- Ask students to each work with a partner. Before beginning, have them agree on the number of attribute blocks they will use to create a design. One partner makes the design; the other replicates it. Have them switch roles and repeat the task.
- Have students pick a shape, e.g., a triangle, and ask them to build a short one, a tall one, a fat one, and a thin one using straws and pipe cleaners or toothpicks and marshmallows. Have them present their shape families to their classmates.

E7 Provide opportunities for students to divide shapes into two or more parts. A triangle might be cut into smaller triangles; a rectangle can be cut into many smaller rectangles; a circle can be cut into two semi-circles, but not two circles. Students can construct a bigger rectangle by combining rectangular-shaped attribute blocks. They can investigate to see if it is possible to make a bigger square, a bigger triangle, and a bigger circle.

When possible, draw students' attention to the fact that they might start with one shape but end up with a totally different shape, e.g., two triangles can become a rectangle.



- Challenge students to make a shape that they can identify from two or more different shapes. For those who are able, have them trace around their shape parts. Provide students with cut-out shapes of various sizes and ask them to explore different ways of putting them together to make a new shape.

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

Performance

E6.1 Ask students to put together four pattern blocks to see what arrangements they can make. Have them draw or trace pictures of their creations.

E6.2 Show students a 2-D shape made of pattern, attribute, or logic blocks and ask them to replicate it.

E6.3 Ask students to select from the pattern blocks a number of pieces of the same shape. Challenge them to use the blocks to make a larger version of the same shape, e.g., to make a larger square using the smaller orange squares or to make a larger triangle using the small green triangles.

E7.1 Give students an outline and ask them to fill it in with pattern blocks. Challenge them to find additional ways of filling in the outline.

Observation and Interview

E6.4 Using tangram pieces, have students individually find a square and build two other squares using triangles. Have each student build a rectangle using three tangram pieces.

E6.5 Give students sets of cut-out shapes and ask them to combine two or more to make different shapes. As they work, observe whether

- they are persistent
- they are willing to take risks
- they recognize when a shape does not fit and make adjustments
- they attempt to assemble particular shapes or simply explore to see what comes together

Encourage them to talk about the assignment.

E7.2 Make one-unit squares on geoboards. Ask students to move the elastics to make (a) a rectangle, (b) a larger square, and (c) a triangle.

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

KSCO

(iii) By the end of grade 3, students will be expected to investigate and predict the results of combining, subdividing, and transforming shapes.

SCO

By the end of grade primary, students will be expected to

E8 make transformations of figures and shapes

Elaboration–Instructional Strategies/Suggestions

E8 Students will begin to explore shapes that have been moved to new positions. It is not necessary for the students to use the terms flip, turn, and slide, but some may model your language when you give such directions as “Slide your cookie cutter up” or “Flip your cutout over and trace the new shape.” It is important that students recognize that the shape stays constant regardless of the flipping, sliding, or turning. Some students have difficulty drawing or tracing a shape repeatedly so that it clearly looks the same, simply repositioned. These students might use stickers to create their turn and slide patterns.

- Using wallpaper and/or wrapping paper, ask students to find patterns that involve shapes that are repeated, either by sliding or by flipping.
- Have students make potato pattern prints with paint, or cookie cutter patterns in playdough or cookie dough.
- Ask students to find samples of patterns in clothing and encourage them to discuss the pattern formation.
- Provide templates, or have students make their own, and ask them to create slide and flip patterns to decorate areas in the classroom such as bulletin boards.
- Have students make their own wrapping bags by decorating paper bags with paint patterns using a combination of slides and turns.
- Prepare a design using six of the same pattern block. Trace around the blocks on paper. Provide students with this traced design and one of the pattern blocks. Have them investigate and describe how the block moves from one part of the design to another.

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

Worthwhile Tasks for Instruction and/or Assessment**Suggested Resources***Performance*

E8.1 Ask students to create and describe a flip or slide pattern in the sand.

E8.2 Ask students to create and describe a flip or slide pattern using attribute blocks.

E8.3 Have students create slide and turn patterns in the paint corner using a template of a pumpkin, a poppy, a Valentine heart, or other seasonal objects, if appropriate.

E8.4 Provide stickers with designs such as happy faces. Ask students to create patterns with the stickers illustrating slides or turns.

E8.5 Provide a piece of checked fabric. Have students look for slides and flips.

Interview

E8.6 Show students a number of triangles, all drawn from the same template, representing slides, flips, and turns. Ask, What can you tell me about these triangles? Are they the same or are they different? For students who are not convinced that they represent the same triangle in different orientations or positions, provide them with the template and ask that it be used to test whether the triangles are the same or different.

E8.7 Use two-colour counters. Ask students to create patterns by flipping every other counter. Have them explain the colour patterns.

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

KSCO

(iv) By the end of grade 3, students will be expected to relate geometric ideas to number and measurement ideas and to recognize and apply geometric principles in real-world situations.

SCO

By the end of grade primary, students will be expected to

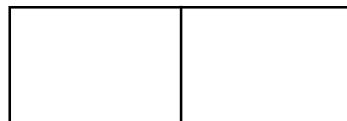
E9 recognize familiar shapes occurring in the environment

Geometry helps people have more complete appreciation of the world in which they live. (*Van de Walle* 1994, 325)

Elaboration–Instructional Strategies/Suggestions

E9 As children work with 2-D and 3-D shapes, it is crucial that they relate the shapes to things in the real world. (See SCO, E5.) Children often fail to discern the parts that make up the whole. It is important to design activities that will direct the students’ attention to detail so they actually *see* the shapes within other shapes.

- Invite students to collect objects, to cut out pictures, or to make drawings of familiar shapes that they find in their environment.
- Have students report on and draw examples of both 2-D and 3-D shapes that they find on buildings.
- Ask students to find containers of different shapes in their kitchen cupboards, and to conjecture why they selected those shapes over other shapes.
- Encourage students to look for odd-shaped containers that they could share with the class.
- Ask students to collect objects of different shapes that they find on the playground.
- Give students a set amount of time to walk around the school in search of four examples of a particular shape. Ask them to record their findings in writing or with pictures.
- Ask students who live near a bridge to make a sketch of it. For those who do not, have them find a picture. Encourage discussion on the construction. This could lead to a more in-depth study of bridges, e.g., Where do they start to build a bridge? What about covered bridges?
- Provide the class with a large grid divided into four columns with the headings square, rectangle, triangle, and circle. Divide the class into four groups. Give students small pieces of paper, all the same size, and ask them to find and sketch an object that would fit under the column designated to them. As a class, have them create a pictograph of these shapes.
- Have the class discuss how many rectangles they can find in this figure.



GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

Performance

E9.1 Ask pairs of students to work together to select and cut out pictures of 3-D shapes. Have them paste their pictures on a class graph that has the following column headings: spheres, cylinders, cones, and cubes.

Interview

E9.2 Ask students, What blocks would you select to build a wall around a building? Explain why you selected these blocks. What shapes would you select to make a tree house? What kind of tree would you need?

E9.3 Ask students, Why is the shape of a footprint made by a shoe closer to a rectangle than a circle?

E9.4 Ask students, Why do some people think of mountains as cones?

Discussion

E9.5 Ask pairs of students to discuss why they think classroom walls are in the shape of rectangles. Ask, What other shape might they be? Could they be triangles or circles? Why or why not?

E9.6 Provide a catalogue of building materials for students. Have them explore a section, e.g., the section on windows to discover what shapes of windows are sold.

Portfolio

E9.7 Ask students to make shape books in which they draw and/or paste pictures of objects that are a particular shape, e.g., a circle.

Presentation

E9.8 Have students select shapes from the block corner to construct models of dog houses and dog runs. When they have completed this task, encourage them to talk about their creations and to provide reasons for their designs, e.g., the kind/size of the dog, the sleeping room required, and the kind of weather conditions. Have them talk about size of the model in relation to an actual dog house.

**Data Management
and Probability:
Data Management**

GCO F: Students will solve problems involving the collection, display, and analysis of data.

KSCO

- (i) By the end of grade 3, students will be expected to collect, record, organize, and describe relevant data.

SCO

By the end of grade primary, students will be expected to

- F1** collect and organize data about issues of personal interest

Elaboration–Instructional Strategies/Suggestions

F1 To make sense of the world, young students often find themselves collecting and organizing data, either verbally, pictorially, or in charts.

Students often imitate the pictorial data organization schemes that they observe adults use, e.g., some form of tallying; however, they should be encouraged to develop their own schemes as well.

- Plan a variety of activities for students such as
 - keeping track of birthdays in the class
 - keeping track of classroom tasks that they have performed
 - creating an inventory list in one of the centres
 - surveying classmates for eye colour
 - using hula hoops to sort footwear into various categories, e.g., those with laces, buckles, velcro
 - organizing a set of pumpkins using various criteria
- Ask a student to plan snack time for the next day. Have him/her report his/her classmates' selections of two different snacks.
- Ask students to find out whether their classmates would prefer to be older, younger, or their present age.
- Ask students to keep track of the weather for the week/month and report their findings.
- Ask students to collect washed empty juice and milk cartons from the class over a two-day period. Have them use the cartons to construct a real graph.

Try to incorporate these collecting and displaying data activities in problem-solving contexts, e.g., collecting information about students' favourite drink might help make decisions when planning a class trip.

Many of these activities should not only deal with collecting and organizing data, but also involve displaying and interpreting data.

One of the most important rules to follow in conducting graphing and statistics activities is to let students gather their own data. (*Van de Walle* 1994, 391)

GCO F: Students will solve problems involving the collection, display, and analysis of data.

Worthwhile Tasks for Instruction and/or Assessment**Suggested Resources***Performance*

F1.1 Ask students to find out how many of them want apple juice at snack time and how many of them want orange juice. Note how they collect and keep track of the information.

F1.2 Ask students to keep a record of how many pictures they have painted.

F1.3 Ask students to decide how to organize a list that will tell the types of shoes the children in the class wear.

Observation and Interview

F1.4 Because organizing data is such a natural part of the classroom experience, there will be many opportunities to observe students engaged in such tasks. You may wish to note whether or not students

- prefer verbal or pictorial forms of organization
- use a traditional type of tally scheme
- attempt to predict the results before data is organized
- choose to tape record data that is gathered orally
- keep track of their data sources to ensure that they do not duplicate or omit information

F1.5 Ask students to tell you the number of sheets of paper that will be needed in order for each person in their small groups to get two.

GCO F: Students will solve problems involving the collection, display, and analysis of data.

Elaboration–Instructional Strategies/Suggestions

KSCO

- (ii) By the end of grade 3, students will be expected to construct concrete and pictorial displays of relevant data.
- (iii) Students will be expected to read and interpret displays of relevant data.

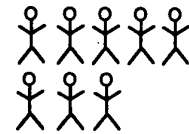
SCO

By the end of grade primary, students will be expected to

- F2 form and interpret “people” graphs
- F3 interpret and create real and picture graphs

Bar graphs are one of the first ways to group and present data and are especially useful in grades K to 3. At this early level, bar graphs should be made so that each bar is made of countable parts such as squares, objects, or pictures of objects. No numeric scale is necessary. (*Van de Walle 1994, 392*)

F2 The most basic type of real graph is a “people” graph, in which the children themselves form the graph.



It is important initially to allow children to form lines on their own. Only then will they learn the importance of all line-ups starting at the same level and of matching students in the line-ups in some form of one-to-one correspondence.

Eventually, it is helpful to use some form of graphing mat, e.g., a masking tape grid, on which each student stands in one box. If the children tape their name tags to their boxes before stepping out, they will be able to view the graph as a whole.

- Have students stand to make a graph showing the various colors of T-shirts that they are wearing.
- Have students with missing teeth and those with no missing teeth form a people graph.

F3 Real and picture graphs are early forms of bar graphs made of actual materials such as sneakers, apples, or caps; and pictures/drawings/cut-outs of objects such as footprints, cars, or ice cream cones. The people graph noted above (SCO, F2) is another example of a real graph. At this early stage, since each bar is made of countable parts, no numeric scale is needed.

- Have students create or interpret graphs regarding
 - the number of each type of block they used in a tower
 - their favourite books
 - the buses on which they travel to school
 - the colours of their houses
 - their favourite kinds of apples
 - the colours of their hair or eyes
 - their first or last initial
 - the weather

It is important for students to create graphs with actual objects and pictures before they move on to bar graphs in grade 1.

When students make picture graphs, it is important that they use the same size of paper for each item that will appear on the graph.

GCO F: Students will solve problems involving the collection, display, and analysis of data.

Worthwhile Tasks for Instruction and/or Assessment
Suggested Resources
Performance

F2.1 Ask students to form lines in a way that will show how many more children are wearing high top sneakers than other types of sneakers.

F3.1 Ask students to use pictures to show the different numbers of toy vehicles in a class collection.

Observation and Interview

F2.2 Observe children to see if they naturally consider people graphs as a way to determine which of two groups has more people.

F2.3 As the children use people graphs, note whether or not they

- pay attention to lining up children one-to-one
- consider their own placement in the graph, if they are the ones outside of the graph doing any counting or observing
- can handle graphs involving more than two categories

F3.2 As students create picture graphs, note whether or not they

- use appropriate illustrations
- ensure that the different items in the graphs illustrate a one-to-one correspondence

F3.3 As students interpret graphs, observe whether or not they

- are comfortable with both horizontal and vertical arrangements
- note a variety of relationships suggested by the graph, where appropriate
- question the validity of the data, where appropriate

Grade 1

**Number Concepts/
Number and Relationship
Operations:
Number Sense and
Number Concepts**

GCO A: Students will demonstrate number sense and apply number theory concepts.

Elaboration–Instructional Strategies/Suggestions

KSCO

- (i) By the end of grade 3, students will be expected to construct and communicate number meanings and to explore and apply estimation strategies with respect to whole numbers.

SCO

By the end of grade 1, students will be expected to

- A1** compare two sets for size in a variety of ways
- A2** create equivalent sets and sets that differ by small amounts

Children who have number sense understand how numbers relate to each other and furnish information about the world. Number sense evolves from a child's total experiences as well as through specific activities. (*NCTM 1991b, 6*)

A1 Students should compare the sizes of sets in many different contexts. Include situations in which the sizes of the sets are the same, and ones in which the sizes of the sets are different. This will lead to exploring number relationships such as *one more than*, *one less than*, and *two more than*.

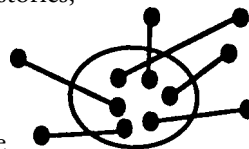
When students compare sets, ensure that sometimes the two sets are lined up side by side with the students pairing the items, and also grouped in clusters with students moving the items to match them one-to-one and comparing the size of the sets. It is desirable, at times, that the items in the sets go together naturally, e.g., left gloves–right gloves, and that at other times the items are unrelated, e.g., people–oranges. It is a natural extension for students to graph these comparisons.

Students should be encouraged to compare amounts to benchmarks like 0 or 1, 5 or 10, in order to get a feel for the relative size of quantities.

- Ask students to sort a collection of buttons by various criteria, e.g., number of holes, size, shape, texture, or colour, and to compare the sizes of the sets.

A2 Students should be able to create a set equal in number to a given set by matching one-to-one. Include situations in which students sometimes need to add items to one set, and sometimes need take items away from one set.

- Invite students to make up story problems to solve, e.g., If the tooth fairy gives me a quarter for each tooth, and I have 4 quarters so far, how many teeth have I lost?
- Invite students to create their own “dot” stories, e.g., The dots inside the circle are seats on a bus and the dots outside the circle are children. The student might say, “There are just enough children to fill the seats.”



Students should be able to create sets which are (a) one more than a given set, (b) one less than a given set, (c) about 5, (d) a little less than 10, and (e) close to 0.

GCO A: Students will demonstrate number sense and apply number theory concepts.

Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

Performance

A1.1 Ask students to compare the number of letters in their first names to the number in their last names to see which has the greater number.

A1.2 Divide a group into teams, unequal in number. Ask students to fix the teams so that they are equal.

A1.3 Put out a set of 10 photos, each of which includes a person or people. Ask students to sort the photos into two groups and to decide which group has more photos. Students should explain their sorting rules and graph the two groups.

A1.4 Have the students play *Dot Bingo*.

Rules

- (a) Take turns rolling a die,
- (b) cover any one square that is one more than the number on the die, and (c) the player who first covers three in a row is the winner.

Game: Dot Bingo

2	4			7
	3	3		
7	5	Free		6
		4		
6		3		4

A2.1 Show students several right hand prints and a smaller number of left hand ones. Ask them to dip their hands in finger paint and end up with as many left-hand prints as right-hand prints.

A2.2 Ask students to make two shapes on geoboards, one with almost five sides and one with one more side than the first.

Interview

A1.5 Ask students, How might you find out if more people like or dislike peanut butter?

A1.6 Place 3 red counters and 2 blue counters in one group and 3 blue and 2 red in another, as shown: (RRR BB) (BBB RR) Ask students, How do you know there are the same number of each colour?

A1.7 Ask students, Where might you see more adults than children? Why?

GCO A: Students will demonstrate number sense and apply number theory concepts.

Elaboration–Instructional Strategies/Suggestions

KSCO

- (i) By the end of grade 3, students will be expected to construct and communicate number meanings and to explore and apply estimation strategies with respect to whole numbers.

SCO

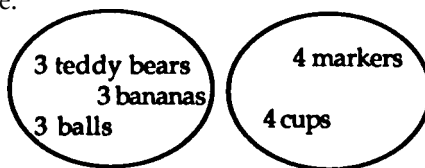
By the end of grade 1, students will be expected to

- A3** count in a variety of ways
- A4** sort sets based on number

A3 Students should be encouraged to count items in natural situations that arise in the classroom. They should also continue to practise rote counting. Include situations which require (a) counting forwards and backwards, (b) counting on by ones from a given number, and (c) skip counting, e.g., 2, 4, 6, 8, ... and 1, 3, 5, 7 ...

- Ask students to count items which occur naturally in twos, e.g., shoes, hands, and eyes.
- Place 5 counters under a cup and tell the students how many are there. Show 3 more beside the cup. Ask them, How many counters are there altogether?
- Invite students to sing songs and recite poems which involve counting backwards and forwards, e.g., “Ten in a Bed,” “One, Two, Buckle My Shoe,” and “This Old Man.”
- Read students some children’s books that deal with counting. Ask them to demonstrate various ways to count.
- Invite students to use calculators to count, e.g., as some students place cookies into a bag and count aloud, others may repeatedly add one on their calculators to keep track.

A4 To demonstrate an understanding of the concept of a number, students must be able to distinguish between those sets that have that number of items from those sets that do not. They should be presented with situations and asked why they think the sets are grouped as they are.



Students should be presented, in addition to sets of items of different sizes, with sets that have the same number of items but differ in the amount of physical space they cover.

- Have students look around the room to find as many sets as possible containing a given number of items, e.g., have them find sets that contain exactly 2 items.

There is a difference between being able to count ... and knowing what counting tells. When we count a set, the last number word used is the name of the "many-ness" of the set or the "cardinality" of the set. (*Van de Walle* 1994, 88)

GCO A: Students will demonstrate number sense and apply number theory concepts.

Worthwhile Tasks for Instruction and/or Assessment
Suggested Resources
Performance

A3.1 Ask students to count backwards starting at 8.

A3.2 Ask students to count aloud to 50 by fives while using the counting constant on their calculators.

(The keystrokes might be $\boxed{5} \boxed{+} \boxed{5} \boxed{=} \boxed{=} \boxed{=} \dots$)

A3.3 Ask students to count a large number of items in a photo. Observe how they count.

A4.1 Ask students to find a number of pictures, each of which shows the same amount.

Paper and Pencil

A4.2 Have students work in pairs to make lists of words or drawings of things they can think of that come in twos.

Interview

A3.4 Ask students, If you count by twos, starting at 0, will you say 7? Why or why not?

A4.3 Show students a set of 3 small items, a second set of 4 small items, and a third set of 3 large items. Ask, What set might not belong? Why?

Presentation

A4.4 Ask students to work in pairs to decide if there are more items in the room that come in 3s or in 4s. Have them share their findings with other groups.

A4.5 Ask students to describe and act out parts of a story in which there are several groups with the same number of items in them, e.g., 3 bowls, 3 chairs, 3 beds in *Goldilocks and the Three Bears*.

GCO A: Students will demonstrate number sense and apply number theory concepts.

Elaboration–Instructional Strategies/Suggestions

KSCO

- (i) By the end of grade 3, students will be expected to construct and communicate number meanings and to explore and apply estimation strategies with respect to whole numbers.

SCO

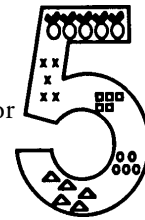
By the end of grade 1, students will be expected to

- A5** match quantities with numerals
- A6** count beyond 10 in a variety of ways
- A7** estimate amounts between 10 and 100

A5 Using numerals is society’s way of communicating about number size. It is important, therefore, that students become familiar with these standard symbols. Students should be able to create sets for given numerals and to assign numerals to given sets.

Providing students with sets of numeral cards for some activities will be useful. Some students will need to practise recording numerals. Tactile experiences such as tracing numerals and copying them are useful for this practice.

- Provide large cutouts or drawings of a numeral, e.g., 5. Invite students to create collages of sets representing that numeral by gluing on pictures or drawings.



A6 Students should experience a wide variety of situations that require counting beyond 10; however, students will be expected to deal only with 2-digit numbers. Include (a) skip counting by 2s, 5s, and 10s, starting from 0, as well as from other numbers; (b) counting using coins (pennies, nickels, dimes); (c) counting on from a given number: and (d) counting back from a given number. This outcome is an extension of SCO, A3.

Hundreds charts that show the numbers from 1 to 100 in lines of 10 are an excellent tool for these activities, e.g., When skip counting by 5s, students could put a counter on every 5th number, reading the number as the counter is placed on it. This activity helps students see patterns and to visualize the number system.

- Ask students to use the constant function on the calculator to skip count to a target number, e.g., If you start at 0 and want to end on 40, by which number(s) could you skip count? What if you started at a different point? What if you wanted to end at a different number?

A7 Students should be provided with collections of objects, e.g., marbles, bread tags, counters, and pop bottle tops, and should be asked to estimate the sizes of the groups. For small groups, ask, Is it closer to 5 or 10? For larger collections, ask whether the group is closer to 20, 50, or 100. The ability to estimate should develop with regular practice over the course of the year.

GCO A: Students will demonstrate number sense and apply number theory concepts.

Worthwhile Tasks for Instruction and/or Assessment

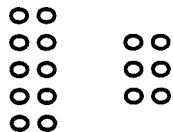
Suggested Resources

Performance

- A5.1** Use attribute blocks. Ask students to make sets of (a) 8 different shapes, (b) 4 blue triangles, and (c) 5 yellow shapes.
- A5.2** Provide several sets of varying quantities and sets of numeral cards. Ask students to match the appropriate cards with the sets.
- A5.3** Provide string and pasta. Ask each student to make a “6” bracelet using 6 pasta pieces.
- A6.1** Provide each student with approximately 40 counters. Ask students to find ways to determine how many there are without saying each number, 1, 2, 3, ...
- A7.1** Show a collection of about 30 counters and ask students to estimate the number in the group.

Interview

- A5.4** Ask students to tell three things about the number 3.
- A6.2** Present 4 items to each student. Ask, How many would there be if there were 2 more? How did you find the answer?
- A6.3** Show 32 scattered objects and then another 32 objects grouped into 3 groups of ten, plus two. Ask students, How do the quantities compare? Which is easier to count? Why?
- A6.4** Ask students to begin counting at 13 and to stop at 25.
- A6.5** Provide hundreds charts. Tell students, I skip counted from 10 to 50 and said only 5 numbers. What do you think I said?
- A6.6** Arrange counters as shown and observe students as they count them.



- A6.7** Tell students, I said 10, 20, 25 when I was counting some coins. What coins do you think I had?
- A7.2** Ask students why it might be easier to estimate the size of a group of 13 counters than a group of 49 counters.

GCO A: Students will demonstrate number sense and apply number theory concepts.

KSCO

- (ii) By the end of grade 3, students will be expected to concretely explore common fractions and decimals in meaningful situations.

SCO

By the end of grade 1, students will be expected to

- A8** demonstrate an understanding of simple fractional parts

Elaboration–Instructional Strategies/Suggestions

A8 Fractions are students' first experiences in which a number represents something more than a count. They will need experiences with a variety of materials, including, among others, geoboards, counters, pattern blocks, egg cartons, grid paper, folded papers, and circle pieces.

Provide many and varied opportunities for students to explore the idea of one-half in meaningful situations. It is important that students understand the *fair-share* concept of one-half in which common objects are used, e.g., sharing a popsicle, a cookie, or an apple. The emphasis should be on equal parts. This *part-of-a-whole* meaning can be extended to the *part-of-a-set* meaning in certain situations, e.g., When sharing a pizza that has been cut into 8 equal pieces, students can see that one-half also means 4 of the 8 pieces.

The writing of symbols should be delayed until the fraction concept has been thoroughly explored. To record the amount in writing, it is preferable to write 3-fourths rather than $\frac{3}{4}$ because this more clearly shows which fraction family is being counted (fourths) and how many of these parts there are (3).

- Invite pairs of students to make square construction paper pizzas and to cut them into various numbers of equal pieces. Ask them to determine how many pieces each of them would get if they shared the pizza fairly and to present their findings.

Explore one-fourth, one-third, and other fractional parts such as fifths, sixths, eighths, or tenths where they arise in context, e.g., pizzas cut into slices, and hexagons split into six equilateral triangles. Pie, pizza, and chocolate bar models work well.

Continue to present the fair-share concept, stressing a whole being divided into equal shares. Emphasize fraction families, e.g., If a whole is divided into four equal shares, we might discuss 1, 2, 3, or 4 of those shares, and note that all of these shares belong to the same family—fourths. (Using fourths rather than quarters is recommended at this level.)

Informal experiences will help students see that when wholes are divided into a greater number of fair shares, the shares are smaller; however, no formal comparison work should be undertaken.

GCO A: Students will demonstrate number sense and apply number theory concepts.

Worthwhile Tasks for Instruction and/or Assessment

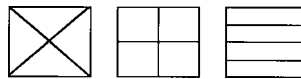
Suggested Resources

Performance

A8.1 Ask students to show one-half of a group of objects.

A8.2 Ask students how to split a symmetric building (made of multilink cubes) to show halves.

A8.3 Give students a square piece of paper and ask them to show one-fourth by folding. Have students compare their fourths. Are they the same shape? Are they all really fourths?



Interview

A8.4 Ask students, Why doesn't it make sense to talk about the bigger half?

A8.5 Ask students, When might you hear someone talk about one-half?

A8.6 Ask students, What is meant when we talk about 2 equal parts?

A8.7 Tell students that sometimes we say, "She was third." Other times we say, "She gets one-third." What is the difference in the meanings of the word third?

A8.8 Ask students to give examples when getting one-fourth of something means getting a lot and when getting one-fourth means getting a very small amount.

A8.9 Ask a pair of students to describe ways to organize their classmates so that one-half are at the front of the room and the other half are at the back. Encourage students to suggest different ways of doing it. How can they be sure that they have one-half?

Portfolio

A8.10 Ask students to examine this picture and describe the fraction family shown, naming some members of that family.



A8.11 Have students use pattern blocks to show the different members of the *sixths* family, recording each family member with a drawing and notation, e.g., 5-sixths.

GCO A: Students will demonstrate number sense and apply number theory concepts.

KSCO

- (iii) By the end of grade 3, students will be expected to read and write whole numbers and to demonstrate an understanding of place value to four places.
- (iv) Students will be expected to order whole numbers and to represent them in multiple ways.

SCO


By the end of grade 1, students will be expected to

- A9** order numbers and use ordinal language
- A10** explore the meaning of the numbers between 10 and 20

Children need to be encouraged to talk and write about what they have learned. ... Researchers have found that most children learn best if they discuss their work. Other studies also indicate that paper-and-pencil activities should follow extensive exploration of numerical relationships with manipulatives. (*NCTM* 1991b, 6)

Elaboration–Instructional Strategies/Suggestions

A9 Students should use ordinal numbers to identify position, e.g., 1st, 2nd, 3rd, and should relate ordinal words and symbols, e.g., third with 3rd. Students should use ordinal numbers to 10th.

Students should also recognize the relative aspect of ordinals, e.g., In the diagram below, the triangle is second if we count from the left, but third if we count from the right. 

- Have students stand in a line. Ask them, Who is eighth?
- Have ten children standing in a line. Ask, If ____ is fifth in line, where did I start counting?
- Invite children to use coloured counters to create a pattern such as



Ask them to predict what colour a particular one will be, e.g., the 10th.

A10 Before students are introduced to place-value concepts, they need opportunities to explore the numbers between ten and twenty. The uniqueness of the *teen* numbers must not be overlooked. When dealing with numbers such as 28 or 46, we hear the tens number first; that is, we say the twenty and the forty first. This is not the case with eleven, twelve, or the *teen* numbers. Children need to understand that eleven is 10 and 1 more, twelve is 10 and 2 more and 16 is 10 and 6 more.

Development should not be rushed and place-value concepts should not be discussed at this exploratory stage, e.g., do not expect students to tell what the 1 in 16 represents. The *ten set*, however, figures prominently in all explorations when developing number meanings for 11 to 19. Provide students with a variety of counting activities in which sets of items numbering 11 through 19 are counted. Students will be developing number sense and recognizing that certain groupings, such as a group of ten and 7 more, make it easier to determine the size of the set.

- Have students count out 10 counters on one side of a 2-part mat. Have them place 5 counters on the other side. Together count all the counters by ones. Say, Ten and five is fifteen. Turn the mat around and say, Five and ten is fifteen. Repeat with other numbers without changing the 10 side of the mat.

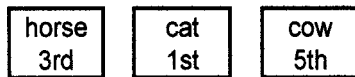
GCO A: Students will demonstrate number sense and apply number theory concepts.

Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

Performance

A9.1 Ask students to place sets of farm animals in lines by following directions given on cards, e.g.,

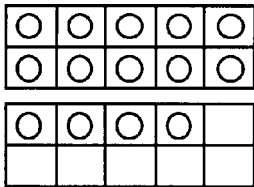


A9.2 Ask students to use counters to create patterns in which the amount in the 4th position is less than the amount in the 3rd.

A9.3 Ask students to make linking cube trains in which the 3rd and 5th cars of the train are different colours from the rest of the train.

A9.4 Ask students to draw rows of shapes in which the first shape is round and the third is square. How many shapes might there be?

A10.1 Provide 2 ten-frames and counters for each student.



Ask students to model 14 with the counters. (Note: For numbers greater than 10, one ten-frame must be completely filled; for 5 and under, use the top row only.) Have them say aloud, Ten and four is fourteen. Practise with other numbers.

Observe the students as they model additional numbers.

- Do they remove all the counters?
- Do they remove all the counters on the bottom frame?
- Do they add to/remove counters on the bottom frame?
- Are they able to verbalize appropriately?

Interview

A9.5 Have students make patterns using counters in which every 4th and 6th counter are yellow.

A9.6 Ask students, If there are 8 people standing in line, and you are counting 1st, 2nd, 3rd, ... , what will you say last?

A9.7 Show a sequence of pattern blocks. Describe one of the blocks and ask students to identify its position.

GCO A: Students will demonstrate number sense and apply number theory concepts.

Elaboration–Instructional Strategies/Suggestions

KSCO

- (iii) By the end of grade 3, students will be expected to read and write whole numbers and to demonstrate an understanding of place value to four places.
- (iv) Students will be expected to order whole numbers and to represent them in multiple ways.

SCO

By the end of grade 1, students will be expected to

A11 model numbers grouped in tens and ones

A11 Encourage students to think about items that come in natural groups, e.g., 5 fingers on a hand and 2 wheels on a bicycle. Emphasize that we could say 20 fingers or 4 hands, using the grouped number. This grouping idea naturally leads into grouping by 10, the value upon which our place-value system is based.

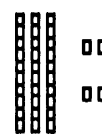
After exploring a variety of grouping sizes, students should begin to concentrate on grouping by tens. Proportional, groupable models should be used first, i.e., materials that can be put together or taken apart to make or unmake tens. Suggested materials are popsicle sticks that can be placed together by rubber bands in groups of 10, linking cubes that can be connected to make 10, or beans that can be placed in cups in groups of 10.

It is important that this stage not be rushed. Many problems that children later encounter with place-value concepts are believed to stem from inadequate attention to early place-value activities. The major objective here is helping the children make the important connection between all that they know about counting by ones and the concept of grouping by tens.

- Give each student a different number of counters. Ask, How many do you have? Have students combine their counters in groups to make it easier for a classmate to count them, e.g., 36 might be grouped as 7 groups of 5 and 1 more or as 3 groups of 10 and 6 more. Have half the class go from desk to desk counting the number of counters classmates have. The other half of the class then takes its turn. Encourage a variety of oral counting, e.g., by ones, by tens and ones, or by fives and ones.

Students should proceed from the groupable proportional model to a pre-grouped proportional model. The size of the ten continues to be equivalent to 10 of the ones; however, the ten cannot be separated into individual ones. Examples of such models are 10 beans glued to a stick, Cuisenaire rods, and base-10 blocks.

- Ask students to model numbers using pre-grouped materials, to record numbers from an existing model, and to show how to count an amount that is modelled.



GCO A: Students will demonstrate number sense and apply number theory concepts.

Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

Performance

A11.1 Give students 45 coffee stir sticks. Ask, How many sticks are there? Observe how students respond.

- Are they counting by groups of ten or other groupings?
- Are they considering grouping easier than counting by ones?

A11.2 Ask students to record the names of some of their classmates, in ten-frames without leaving any spaces, e.g.,

S	A	N	D	Y	L	I	V	I	A	N	S	T	E	V
M	A	R	K	O	A	L	Y	S	O	E	N	L	A	N

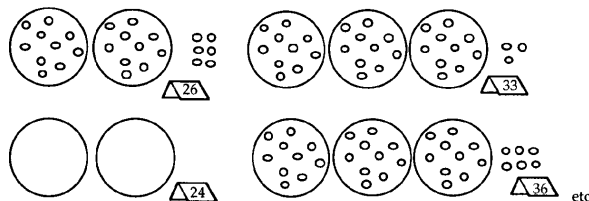
C	E	A	M	Y	N	N	J	E	F
L	I	S	A	A	F				

Ask, How many letters are there in all?

A11.3 Have students play a game in which the roll of a die tells how many pennies they can accumulate. Once they have 5 pennies, they must exchange them for a nickel. The first player with 5 nickels wins the game. As the students play, ask questions such as, I see 2 nickels and 3 pennies. How many pennies is that worth?

A11.4 Show a number of base-10 unit cubes, e.g., 34. Beside these show 5 rods and 6 unit cubes. Ask students, Which has more? Is one easier to count than the other? Explain.

A11.5 Math Centre: In stations, set out a number of arrangements of beans grouped by tens and individual units. Also have some empty stations. Provide the students with a set of prepared number cards. Invite them to place the matching card in front of the appropriate display and to build the arrangements for the other cards at the empty stations.



This activity could also be done using pre-grouped proportional models.

GCO A: Students will demonstrate number sense and apply number theory concepts.

KSCO

- (iii) By the end of grade 3, students will be expected to read and write whole numbers and to demonstrate an understanding of place value to four places.
- (iv) Students will be expected to order whole numbers and to represent them in multiple ways.

SCO

By the end of grade 1, students will be expected to

A12 compare 2-digit numbers

Elaboration–Instructional Strategies/Suggestions

A12 Students should encounter many numbers in context which help them develop an understanding of number size, e.g., Students can be asked, Which of the following numbers probably tells how old your classmate’s dad is: 5, 35, or 85? How do you know?

When comparing two numbers, students should be encouraged to make use of benchmarks, e.g., $48 < 95$ since 95 is closer to 100, and $37 > 27$ since 37 is more than 30 and 27 is less than 30. This reasoning process is part of having number sense.

Frequently, students will refer to the number of tens in a number in order to compare it to another, e.g., $47 > 21$ since 47 is more than 4 tens, but 21 is only a bit more than 2 tens. This type of language is preferable to 4 is more than 2 so 47 is greater, particularly since students should focus on the fact that the 4 in 47 is 40, not 4, and the 2 in 21 is 20, not 2.

The hundreds chart is a particularly valuable reference device for number comparison since students easily learn that down and right means greater; however, it is also important for students to see concrete models of numbers, e.g., using base-10 blocks to get a visual sense of the difference in size of the numbers.

Students should recognize that (a) every 2-digit number is greater than every 1-digit number; (b) when they compare 2-digit numbers, the tens digit is the more vital element of the number; and (c) when they compare two numbers with the same tens digit, they should compare the ones digits.

- Provide cards with 2-digit numbers, such as



Have students, in pairs, each select a card and model the chosen number with centimetre cubes. After each counts to verify the other’s model, each student joins his/her cubes together and cuts a piece of string equal in length to the cube train. They can then compare the size of their numbers by comparing the lengths of the pieces of string.

A variety of concrete materials should be used to model numbers and to group into tens and ones, e.g., Beans and other small objects can be collected and grouped 10 to a yogurt cup.

GCO A: Students will demonstrate number sense and apply number theory concepts.

Worthwhile Tasks for Instruction and/or Assessment
Suggested Resources
Performance

A12.1 Ask students to use models to show why 42 is greater than 29. Have them verify their answers using a hundreds chart.



A12.2 Provide nine base-10 rods and nine unit cubes. Have students create two different amounts, each using exactly five of the blocks provided. Ask, “Which amount is greater than the other? How is it possible to use the same number of blocks, but still have one amount greater than the other?”

Paper and Pencil

A12.3 Provide teacher-made flyers in which prices are less than a dollar. Ask students to identify the item that costs the most, the least, more than 50 cents, and less than 25 cents.

Interview

A12.4 Ask students to use hundreds charts to explain the relative size of the numbers 36 and 52.

A12.5 Show students papers on which the units digits of two numbers are smudged, e.g., 3  4 

Ask, “Can you tell which number is greater? Why?”

A12.6 Ask students, “Is a number with a 7 in it always greater than a number containing only digits less than 7? Explain.”

Presentation

A12.7 Have pairs of students discuss when they might want to compare the sizes of two numbers and invite them to present their ideas.

A12.8 Have students work in small groups to list some situations in which they would rather have 22 than 28 and vice versa.

**Number Concepts/
Number and Relationship
Operations:
Operation Sense and
Number Operations**

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

KSCO

- (i) By the end of grade 3, students will be expected to demonstrate an understanding of the connection between relevant, concrete experiences and the mathematical language and symbolism of the four basic operations.

SCO

By the end of grade 1, students will be expected to

- B1** recognize that addition is used to represent the joining of two groups

... “operation sense,” a highly integrated understanding of the four operations and the many different but related meanings these operations take on in real contexts. (Van de Walle 1994, 109)

Elaboration–Instructional Strategies/Suggestions

B1 As with many early concepts, the development of the meaning of addition should not be rushed. It is important that students have extensive investigative experiences in which they use concrete materials to model addition story problems. They also need experience interpreting how addition situations are portrayed in print. Addition and subtraction situations should be presented together, emphasizing the relationship between the two concepts.

Include both *join* problems—active situations which involve the physical joining of sets, e.g., I had 4 pencils and I bought 3 more. How many do I now have?—and *part-part-whole* problems—static situations involving the implied joining of sets, e.g., There are 3 boys and 4 girls. How many are there altogether?

For the join problems, present the following types of situations:

Result Unknown: I had 4 marbles and I won 3 more. How many do I now have? ($4 + 3 = \underline{\quad}$)

Change Unknown: I had 4 marbles and now I have 7. How many did I win? (Missing addend: $4 + \underline{\quad} = 7$)

Initial Unknown: I had some marbles. I won 3 and now I have 7. How many did I have in the beginning? (Missing addend: $\underline{\quad} + 3 = 7$)

Some students might begin to realize that subtraction could be used to find missing-addend situations.

Instructional decisions should include personalizing word problems for students, using concrete materials, providing situations that sometimes state the lesser number first and sometimes the greater number first, and modelling the advantage of counting on from the larger of two addends to get the result.

- Manipulate materials on an overhead or chalk/flannel board as you relate word problems. Verbalize as you manipulate, e.g., 6 buttons and 3 more buttons is 9 buttons, then 6 buttons plus 3 buttons is 9 buttons.
- Have students model solutions to word problems on the overhead.

Observe whether the children (a) count all objects to find sums, (b) count on from one group to find sums, (c) use the term “add” and/or “plus,” and (d) verbalize the problems as they model them.

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

Performance

B1.1 Choose a book, or make up a story, that tells about an addition situation and ask students to model the situation with counters as you read the book or tell the story.

B1.2 Model this problem, using manipulatives, for pairs of students: I had 5 pennies and now I have 9. How many pennies did I earn? Ask the pair to make up similar problems using objects of their choice and to model and describe them.

B1.3 Give students a card (such as the one shown), a pair of dice, and some bingo chips. Ask them to roll the dice, find the total, and try to fill the card.

/ / / / /	11	7
6	4	8
10	2	3
9	5	12

Paper and Pencil

B1.4 Ask students to make drawings to show how many pencils Celia has if she has 2 blue ones, 6 yellow ones, and 3 red ones.

Interview

B1.5 Ask students to think of situations in restaurants when someone might add.

B1.6 Tell students that Carlos had 2 pennies and he earned 10 more pennies. Ask, How many pennies does Carlos now have? Explain how you found your answer.

Presentation

B1.7 Present a number of shapes worth various amounts, e.g.,



Have students create designs using the shapes, in each case describing how much the design would be worth.

Portfolio/Presentation

B1.8 Ask students to interview people at home to find situations in which they use addition. Have them make lists of these and share them with their class.

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

KSCO

- (i) By the end of grade 3, students will be expected to demonstrate an understanding of the connection between relevant, concrete experiences and the mathematical language and symbolism of the four basic operations.

SCO

By the end of grade 1, students will be expected to

- B2** recognize that subtraction is used to represent separating situations

Elaboration–Instructional Strategies/Suggestions

B2 One of the easiest meanings of subtraction for students to access is that of *taking away*. When addition and subtraction situations are considered together, students realize the close connection of the two concepts.

The following types of *separation* problems should be presented:
Result Unknown: There are 5 people at the table; 2 finish and leave. How many are left at the table? ($5 - 2 = \underline{\quad}$)

Change Unknown: There were 5 people at the table. Two are still there. How many left? ($5 - \underline{\quad} = 2$)

Initial Unknown: Two people left the table. There were 3 remaining. How many were at the table originally? ($\underline{\quad} - 2 = 3$)

Also include static situations where the whole and one part are known, e.g. There are 12 students at the table. If 3 of them are girls, how many are boys?

These three types of separation problems have the common feature of having a sense of loss that might not be seen as subtraction by all students—some students might begin to realize that *initial unknown* type problems are most efficiently solved by addition. Students should be encouraged to use whatever operation symbols make sense to them, looping the number that they had to find.

When modelling solutions to problems, include as much descriptive language as possible, e.g., If the problem were that there had been 6 cookies, and 3 had been eaten, you might say, 6 cookies and 3 are eaten. How many are left? 6 take away 3, and 6 subtract 3.

It is desirable to combine joining and separating situations in the same context, e.g., You have 3 pencils. Your mom gives you 2 more. Then your brother borrows 1. How many do you have left?

It is important that separation situations be personalized and that students use materials to model problems.

Students might observe that it is sometimes easier to count back in situations in which only a few are removed, e.g., for $8 - 2$, count 7, 6; it is sometimes easier to count up from the number removed when most of the total is removed, keeping track of how many numbers are counted, e.g., for $8 - 6$, count 7, 8 to get 2.

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Worthwhile Tasks for Instruction and/or Assessment**Suggested Resources***Performance*

B2.1 Provide students with given numbers of counters. Ask them to remove 3 counters and tell how many are left.

B2.2 Ask student to tell a take-away story involving 8 and 5.

Paper and Pencil

B2.3 Ask students to make drawings to model this problem: Robert had 10 baseball cards. His sister convinced him to give her 2 of the cards. How many cards did Robert have left?

Interview

B2.4 Tell students that Jane had 9 pencils and lost 3, while Michael had 7 pencils and lost 2. Ask, Who has more pencils left? Explain how you know.

B2.5 Tell students that you have a nickel and 4 pennies. You want to buy a candy that costs 3¢. Ask, How much money will be left? Tell how you know.

B2.6 Ask students to discuss why it might be difficult to model take-away situations when they are illustrated in books.

B2.7 Tell students that you had 9 marbles, but lost some. There are only 4 marbles left. Ask, How many did I lose? Show how you know.

Presentation

B2.8 Have students role play take-away stories.

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Elaboration–Instructional Strategies/Suggestions

KSCO

(ii) By the end of grade 3, students will be expected to recognize and explain the relationships among the four basic operations.

SCO

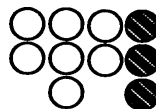
By the end of grade 1, students will be expected to

B3 recognize the relationship between addition and subtraction

B4 recognize that subtraction can be used to solve missing addend problems

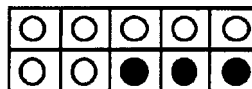
B3 Students should be aware that every time they encounter an addition or a subtraction situation, the other operation is implicit. In fact, different students may see the same situation in these different ways, e.g., There are 9 children. Three are boys. How many are girls? Some students see this as an addition. (3 + how many is 9.) Others see it as a subtraction. (Remove the 3 boys from the 9 total to get how many girls.)

- Have students spill 10 two-coloured counters from a cup. Ask them to describe the addition and subtraction situations that are shown, e.g.,



7 and 3 are 10; 3 and 7 are 10; if 3 are removed from 10, 7 are left; if 7 are removed from 10, 3 are left.

- Have students show a variety of combinations of numbers using ten-frames, e.g., For the 7 and 3 combination, they might show



This provides a very clear visual representation for $7 + 3$, $3 + 7$, $10 - 3$, and $10 - 7$.

Students should also understand that subtraction and addition *undo* each other, i.e., If 3 is subtracted and then added, or vice versa, it is as if nothing happened.

B4 Missing-addend situations are those in which a total and one part are known and the other part is sought, e.g., There are 8 pink and purple flowers. Four are pink. How many are purple? or Sue had 5 toys. She got some new ones for her birthday. Then she had 8 toys. How many did she get for her birthday? See SCO, B1.

Students might solve these problems directly by adding or they might think of these problems in terms of subtraction. They should be encouraged to model these problems with materials and to verbalize their thinking processes. Through this modelling, you can help students understand how subtraction will find missing addends.

Addition and subtraction concepts are very closely related. Both can be derived from the same basic relationships between sets: either a part-part-whole relationship or a comparison relationship. (*Van de Walle 1994, 110*)

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

Performance

B3.1 Ask students to use counters or felt board pieces to make up addition or subtraction stories involving the numbers 7 and 3.

B3.2 Show 5 red counters and 3 blue ones. Have students tell an addition story and a subtraction story while manipulating the counters.

B4.1 Have students model the following situation to find the answer: I have to read a total of 7 pages to finish my work. I have already read 3 pages. How many more pages have I to read?

Paper and Pencil

B3.3 Ask them to create a story problem that would be solved by adding 4 and 3.

B3.4 Have students individually draw a ten-frame containing 2 counters. Ask them to write four number sentences represented by this ten-frame.

Interview

B3.5 Display a chart showing item prices, all under 10 cents. Ask students, What could you buy with 10 pennies? What does $10 - 4$ represent if one item costs 4 cents?

B3.6 Ask students, How does knowing that $5 + 3 = 8$ help explain how much 8 take away 5 is?

B3.7 Ask students, If you have 10¢, how much change would you get from a 7¢ purchase? Explain how you found the answer.

B3.8 Tell students, I have \$5 and I need \$13. Ask them, How much more do I need? Then ask, Why do I get the same answer for the following problem: I had \$13. I spent \$5. How much do I have left?

B4.2 Tell students that after Jim bought 5 new comic books, he had 8 altogether. Ask them, How many comic books did Jim have before his purchase?

Portfolio

B3.9 Have students use the numbers 3, 6, and 9 to create a variety of addition and subtraction stories.

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Elaboration–Instructional Strategies/Suggestions

KSCO

- (ii) By the end of grade 3, students will be expected to recognize and explain the relationships among the four basic operations.

SCO

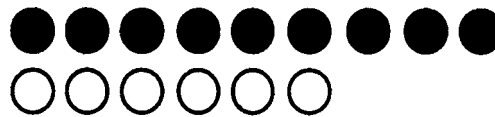
By the end of grade 1, students will be expected to

- B5 recognize how to use addition or subtraction to solve comparison problems

B5 Even young students are inclined to ask, Who has more? They naturally find *how many more* by doing a one-to-one match and counting on, especially when making comparisons on graphs. Finding how many more by using subtraction is something which develops later.

There are many comparison situations which arise in their everyday life, e.g., How many more people chose this centre than that one? or How many more dolls do I have than you?

To find how many more, students should model problems, e.g., Suppose they have 9 dark counters and 6 light ones. If they want to find how many more dark counters there are than light ones, they might model this by lining up counters.



They might count up from the 6 white counters to see how many extras are in the top row, or they might take away 6 dark counters to match the 6 white counters and count how many are left.

These two strategies help students relate the *difference* meaning of subtraction to the *take-away* and *missing-addend* meanings. These different terms need not be specifically discussed; however, the different situations should all be dealt with and students helped to see the connections among them.

Examples of the types of comparison problems that should be included and modelled include the following:

- (a) Elaine has \$12 and Jason has \$8. How much more money does Elaine have than Jason?
- (b) Elaine has \$12. Jason has \$4 less than Elaine. How much money does Jason have?
- (c) Elaine has \$12. This is \$4 more than Jason has. How much money does Jason have?

You might want to use two-section mats to model these types of problems in order to keep the two sets apart. Students will find problems like (c) the most difficult because of the indirect nature of the comparison.

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Worthwhile Tasks for Instruction and/or Assessment**Suggested Resources***Performance*

B5.1 Prepare a mixed group of items. Ask students to sort the items into two groups using any rules they choose. Ask, What was your rule for sorting? Which of the two groups has more items? How many more?

B5.2 Ask students to create and tell comparing stories that could be solved using the expression $8 - 5$.

Paper and Pencil

B5.3 Tell students that one number is 3 more than another. Ask what the numbers could be.

B5.4 Ask students to make drawings to model the following problem: Jake collected 8 bottle tops and Samuel collected 5. How many more did Jake collect than Samuel?

Interview

B5.5 Show a group of 4 boys and 2 girls, and another group of 3 boys and 3 girls. Ask students to compare the number of children in each group.

B5.6 Ask students, Why is taking 3 away from 5 like finding out how much more 5 is than 3?

B5.7 Display a bar graph showing the number of houses of different colours in a neighbourhood. Ask students a variety of questions, e.g., How many more white houses are there than grey ones?

Presentation

B5.8 Have students make up skits in which one student has more of something than another student has and he/she brags about how much more.

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Elaboration–Instructional Strategies/Suggestions

KSCO

(iii) By the end of grade 3, students will be expected to create and model problem situations involving whole numbers, using one or more of the four basic operations.

SCO

By the end of grade 1, students will be expected to

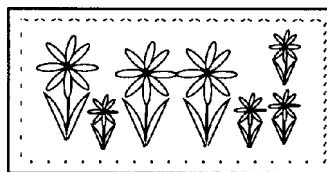
B6 move freely among representing an addition or subtraction situation with a picture, a model, or a number sentence

B6 When students are comfortable with many addition and subtraction contexts as well as with their concrete and pictorial representations, you should introduce the corresponding symbols, helping students connect these symbols to their context, concrete, and pictorial representations. The connections among contexts, models, pictures, and symbols should be explicitly discussed. Students should be able to start with any one of these representations and translate it to the other representations. Students should understand how each part of the number sentence relates to the situation, e.g., There are 9 crayons. Only one is broken. How many are not? To solve this problem, students might write $9 - 1 = 8$ or $9 = 1 + 8$. In either case, it might be advisable to suggest that the 8 be circled since it is the quantity that is being sought.

- Invite small groups of students to act out situations. Other students should watch and choose correct number sentences to match the situations. There may be several interpretations of a given situation, e.g., Consider a problem in which two children are comparing the number of stickers when one has 7 and the other, 4. They want to know how many more one has than the other. Some students might think of the subtraction sentence $7 - 4 = 3$, while other students might think of the addition sentence $4 + 3 = 7$.

Materials that may be used for modelling are numerous, including common classroom materials such as cubes, counters, and beans.

- Give students picture cards like the one below and ask them to tell or write a story about the pictures. Have them write number sentences that also tell the story.



For all four of the operations, models (usually sets of counters and number lines) and word stories or word problems are the two basic tools the teacher has to help students develop operation concepts. (*Van de Walle* 1994, 109)

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

Performance

B6.1 Provide each student with one of the cards shown.



Ask students to tell number stories for which the card might be used.

B6.2 Ask students to use toy vehicles to act out stories involving addition or subtraction, recording the number sentences that go with their stories.

Interview

B6.3 Ask students to create number stories for each of the following: (a) $5 - 2$ (b) $8 + 4$ (c) $5 + 3 - 2$

B6.4 Have students describe situations in which they could subtract.

B6.5 Ask students, Is it possible for a number such as 3 to be a sum and a difference? Explain.

Presentation

B6.6 Have students create story books involving both addition and subtraction stories. As well, you could facilitate the creation of a big book for the class or a story board display.

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Elaboration–Instructional Strategies/Suggestions

KSCO

- (iv) By the end of grade 3, students will be expected to demonstrate proficiency with addition and subtraction facts.
- (v) Students will be expected to apply computational facts and strategies with respect to the four basic operations and to model addition and subtraction in situations involving whole numbers.

SCO

By the end of grade 1, students will be expected to

- B7** use mental strategies to find sums to 18 and differences from 18 or less
- B8** know simple addition facts from among those for which the total is 10 or less and know the corresponding subtraction facts

For those children who do not spontaneously develop efficient fact strategies, it is our job to help them do so by engaging them in activities that will encourage construction of these helpful relationships. (*Van de Walle 1994, 134*)

B7 When students’ thinking has developed at least to the point where they are counting on from the larger number, strategy learning should begin. Children should be encouraged to use the relationships between facts to learn new facts rather than using counting to find sums or differences. Possible useful strategies include (a) *doubles* by associating each double with a real-life context, e.g., $5 + 5$ is the number of fingers on two hands; (b) *plus 1s* by associating any fact involving $+1$ with asking for the next number, e.g., $7 + 1$ or $1 + 7$ is asking for the number after 7; (c) *1-aparts or near doubles* for facts that differ by 1 which will be 1 more than the double of the smaller, e.g., $4 + 3$ is the same as $3 + 3 + 1$; d) *2-aparts* for facts that differ by 2 which will be the same as the double of the number between, e.g., $4 + 6$ is the same as $5 + 5$ because 1 can be removed from the 6 and added to the 4; (e) *make-10* for facts involving 9 and 8, e.g., $9 + 3$ is the same as $10 + 2$ where 1 is borrowed from the 3 to make the 9 a 10; (f) *plus 2s* by associating any fact involving $+2$ with skip-counting by 2 or asking for the next even or odd number.

An addition table is useful to help students visualize relationships, e.g., All the sums for 4 can be found by taking a known fact like $4 + 0 = 4$ and successively reducing the 4 by 1 and increasing the 0 by 1. Notice that all of these combinations are along a diagonal of the table.

+	0	1	2	3	4	5...
0	0	1	2	3	4	5...
1	1	2	3	4	5	6...
2	2	3	4	5	6	7...
3	3	4	5	6	7	8...
4	4	5	6	7	8	9...

B8 As well as using strategies, many students will find visualising ten-frames useful for fact learning. You should provide a variety of opportunities for students to practise facts including games and oral drills. Visualization prompts and strategies should be continually discussed in practice sessions. You should commit 3 to 5 minutes per day over several months in order to introduce and practise strategies; the goal is to get most students to give 3-second responses to the facts that total 10, thus indicating that they “know” their facts.

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

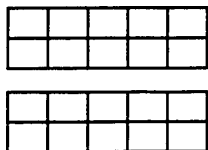
Performance

B7.1 Ask students to choose any number, add 10, and take away 1. Have them repeat this task using other starting numbers and to observe what happens, describing any pattern that they see.

B8.1 Have students play a form of bingo where you call out a number and students cover any number sentences on their cards that have this number as an answer, e.g., If you call out 5, students might cover $3 + 2$, $6 - 1$, or $10 - 5$.

Paper and Pencil

B7.2 Ask students to use ten-frames to help them solve $8 + 5$.



B7.3 Ask students to list three other facts that would be easier to remember if they know that $6 + 5 = 11$.

Interview

B8.2 Two addends that combine to make 10 are sometimes called “special-10 numbers,” e.g., $8 + 2$, $4 + 6$, ... These can serve as good reference points for students and are easily visualized on ten-frames. Develop a game situation in which they identify pairs of special-10 numbers. Note: These can provide a basis upon which strategies may be applied, e.g., If students know that $7 + 3$ is 10, they can use it to find $7 + 4$ because it will be 1 more, or 11.

B7.4 Ask students how they could use $6 - 4 = 2$ to find $6 - 3$.

B7.5 Tell students, I am using a dime to pay for 4¢ worth of candy. To find the change I think, “1 makes 5 and 5 makes 10. The change is 6¢.” Have students explain this process.

Presentation

B8.3 Have students create and present games that involve knowing and using addition and/or subtraction facts. The games could be presented to other students or to parents for them to play.

Patterns and Relations

GCO C: Students will explore, recognize, represent, and apply patterns and relationships, both informally and formally.

KSCO

- (i) By the end of grade 3, students will be expected to recognize, describe, extend, and create patterns and sequences in a variety of mathematical and real-world contexts (e.g., geometric, numeric, and measurement).

SCO

By the end of grade 1, students will be expected to


- C1 create and recognize physical configurations for numbers
- C2 reproduce, extend, and create simple patterns based on number


Patterns weave mathematical topics together. Through the study of patterns, children learn to see relationships and make connections, generalizations, and predictions about the world around them. Working with patterns nurtures the kind of mathematical thinking that empowers children to solve problems confidently and relate new situations to previous experiences. (*NCTM* 1991b, 1)

Elaboration—Instructional Strategies/Suggestions

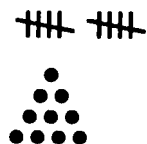
C1 Pattern is a topic that emerges in all curricular areas, not just mathematics. However, the detection of patterns is the very essence of what it means to do mathematics.

Students need to be able to recognize, without counting, various configurations—spatial patterns—for small numbers of items. This ability to recognize patterns will be useful with respect to

(a) addition, e.g., $5 = 4 + 1$ or $2 + 1 + 2$ is apparent from 

and $6 = 3 + 3$ or $2 + 2 + 2$ is apparent from 

and (b) place value, e.g., groups of 10 can be easily observed in these configurations




Playing games that use dice helps strengthen students' recognition of several configurations; various patterns for numbers made with dots on paper plates and used for a variety of activities reinforce set recognition; numbers made with dots on ten-frames provide opportunities for set recognition as well as for visualization of the relationships of these numbers to 5 and 10. Provide opportunities for students to discover which configurations are easiest to recognize, e.g., Ask students to show 7 in several ways, then to decide which configuration(s) is (are) easiest to identify. Possible configurations might include



Continue to provide opportunities for students to discover, extend, and translate color, size, sound, tactile, and other patterns. They should view all mathematics as “looking for patterns.”

C2 Number patterns that students reproduce and extend may be based on numerals or actual objects which must be counted in order to interpret the pattern.

- Provide students with a variety of numeral patterns that they can extend, e.g., 3, 1, 3, 1, ... or 1, 2, 3, 1, 2, 3, ...
- Provide students with a variety of shape patterns based on number, e.g., 
- Have students reproduce patterns using objects or sounds, e.g., clap, clap, whistle, clap, clap, whistle, ...
- Have students create patterns for other students to extend.

GCO C: Students will explore, recognize, represent, and apply patterns and relationships, both informally and formally.

Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

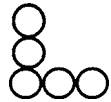
Performance

C1.1 Ask students to arrange 8 counters in ways that will make it possible to detect at a glance that there are 8.

C1.2 Ask students to make sketches that show how they visualize the number 9.

C2.1 Have students create audio tapes of clapping patterns, and have them use stickers or coloured counters to make their favourite visual patterns.

C2.2 Show students 5 counters arranged in an L-shape with equal sides. Ask, What other numbers of counters can be arranged to form L shapes?

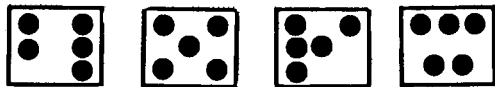


C2.3 Ask students to colour on hundreds charts all the numbers that they say when counting by threes. Ask them to describe the pattern(s) they observe.

C1.3 Arrange counters on an overhead projector. Turn the projector on for a few seconds—not long enough for students to count the counters. Ask, What number was represented? Repeat this task several times, using different configurations of the same number. Ask, Which configuration was easiest to recognize? Why?

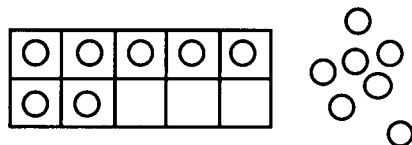
Interview

C1.4 Briefly show a series of dot cards, e.g.,



Ask students what they notice.

C1.5 Ask students to explain why it might be easier to count the number of counters on the left than the number on the right.



C2.4 Ask students to continue the following patterns:

(a) 4, 3, 2, 4, 3, 2, ... (b) 2, 6, 8, ... (c) 1, 1, 2, 1, 3, 1, 4, 1, ...

GCO C: Students will explore, recognize, represent, and apply patterns and relationships, both informally and formally.

KSCO

- (i) By the end of grade 3, students will be expected to recognize, describe, extend, and create patterns and sequences in a variety of mathematical and real-world contexts (e.g., geometric, numeric, and measurement).

SCO

By the end of grade 1, students will be expected to

- C3 sequence events
- C4 create patterns with 3-D solids and 2-D shapes

First-grade patterning activities are often extensions of those used earlier. The focus in kindergarten is on repetition of events or designs using sound, motion, shape, and quantity. In the first grade there is continued emphasis on extending patterns and transferring them from one medium to another. (*NCTM* 1991b, 1)

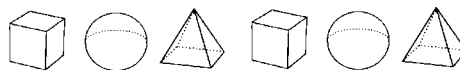
Elaboration–Instructional Strategies/Suggestions

C3 Sequencing events is based on the temporal patterns of students’ experiences. They should have opportunities to sequence events, using language such as *before/after* and *first, then, last*.

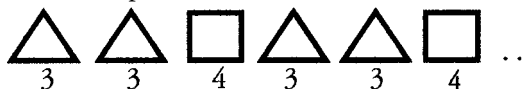
You could have students consider the order of events in a day, days of the week, months of the year, and seasons of the year as well as sequences for performing tasks and for ordering events in stories.

C4 Students should observe many spatial patterns in their environment. In creating patterns with shapes, students need to focus on the attributes of the shapes they have chosen.

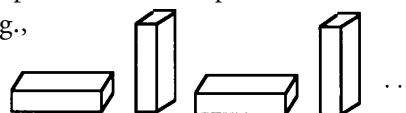
- Have students create patterns for which the pattern rule depends on the name of the shape, e.g.,



- Have students create patterns based on the number of sides, e.g.,



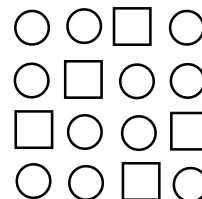
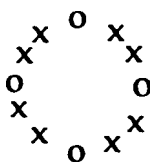
- Have students create patterns with shapes based on their position-in-space, e.g.,



- Have students create patterns based on the type of side (straight or curved), e.g.,



- Encourage students to create circular, multi-directional, and surface patterns, e.g.,




GCO C: Students will explore, recognize, represent, and apply patterns and relationships, both informally and formally.

Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

Performance

C4.1 Have students use pattern blocks or attribute blocks to construct simple patterns. Ask them to explain their patterns.

C4.2 Ask students to continue the shapes below in two different ways to make patterns: 

C4.3 Ask students to make patterns so that a triangle is the third item.

Paper and Pencil

C3.1 Ask students to draw simple pictures, using one item per picture, to represent, in order, the first five things they do when they arrive at school.

Interview

C3.2 Provide calendars. Ask students to solve the following problem: Your mom, your sister, and you take turns tidying up the living room floor at the end of the day. If your next turn is on Friday, on which days next week will your turns be?

C3.3 Tell students that you think that there is a pattern to the days in a week. Ask them to explain the pattern.

C3.4 Ask students to explain, in order, the steps that they follow to find $8 + 5$ using counters.

Presentation

C3.5 Have students ask someone at home for situations that show the importance of doing things in order. Ask them to share their findings.

GCO C: Students will explore, recognize, represent, and apply patterns and relationships, both informally and formally.

Elaboration–Instructional Strategies/Suggestions

KSCO

- (ii) By the end of grade 3, students will be expected to use patterns to solve problems.

SCO

By the end of grade 1, students will be expected to

- C5 use number patterns to help solve addition and subtraction questions

C5 Students should work with the pattern formed by the three numbers in a traditional fact family, e.g., $3 + 5 = 8$, $5 + 3 = 8$, $8 - 3 = 5$, and $8 - 5 = 3$. They should be encouraged to use this pattern when faced with a subtraction such as $9 - 6$ to which they may not have an immediate answer. If they recognize $9 - 6$ is part of the family that includes $6 + 3 = 9$, they will easily be able to get $9 - 6 = 3$.

Students should also be provided with experiences with other fact patterns, e.g., $4 + 5 = 9$, $5 + 5 = 10$, and $6 + 5 = 11$ might help a student determine that $7 + 5 = 12$.

Using concrete materials such as counters to model fact families and addition/subtraction situations will be very advantageous to most students.

A hundreds chart provides a good model to develop patterns, e.g., By coloring every fifth number in a hundreds chart, students will practise skip counting by 5s and see the pattern made. They should see the pattern that adding 10 is like going down one row.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	00

+	0	1	2	3	4	5	...
0	0	1	2	3	4	5	...
1	1	2	3	4	5	6	...
2	2	3	4	5	6	7	...
3	3	4	5	6	7	8	...
4	4	5	6	7	8	9	...
⋮							
⋮							

An addition table is yet another model to develop patterns, e.g., Students can observe that the number in any line is one more than in the preceding line since one addend is increased by one and the other is not changed.

It is also helpful for students to see the connection between number sentences involving +2 and skip counting by 2.

Seeing many different representations of the same pattern helps children learn to generalize and recognize patterns in broader contexts. Noting similarities and differences in several examples of the same pattern develops skills in using pattern recognition to solve problems. (NCTM 1991b, 1)

GCO C: Students will explore, recognize, represent, and apply patterns and relationships, both informally and formally.

Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

Performance

C5.1 Ask students to use Cuisenaire rods, or another suitable manipulative material such as multi-link cubes, to show the pattern for all of the facts for 8, e.g.,

		$1 + 7 = 8$
		$2 + 6 = 8$
		$3 + 5 = 8$

Paper and Pencil

C5.2 Ask students to continue the following pattern:

$$4 + 3 = 7$$

$$5 + 3 = 8$$

$$\square + 3 = \square$$

$$\square + 3 = \square$$

C5.3 Ask students to list all the pairs of numbers that can be added for a total of 4. Repeat the process for totals of 5 and of 6. Have them arrange the lists so that the first number increases by one each time. Ask them to describe the patterns.

Interview

C5.4 Ask students to state how many facts are in a fact family and to explain the pattern in fact families.

Shape and Space: Measurement

GCO D: Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

Elaboration–Instructional Strategies/Suggestions

KSCO

- (i) By the end of grade 3, students will be expected to measure and understand basic concepts and attributes of length, capacity, mass, area, and time.

SCO

By the end of grade 1, students will be expected to

- D1 identify procedures (not involving units) to compare and/or order lengths, capacities, and areas

D1 Students should recognize that length tells about the extent of an object along one dimension. Initially they should compare lengths informally by simply viewing the two lengths. Then they should investigate strategies to compare the lengths of two or more objects in situations involving both *direct* and *indirect* measurement. *Direct* measurement consists of comparing lengths by lining up items side by side beginning at a common starting point with students appreciating why a common starting point is important. *Indirect* measurement consists of comparing lengths of two objects by comparing them both to a third object.

Students should order objects from longest to shortest, first by sight and then by measurement. Teachers should include situations with an extraneous variable, e.g., using objects that are not straight and objects that are wide or thick.

Students should recognize that capacity tells how much something will hold. They should investigate strategies to compare the capacities of two or more containers. Initially, however, they would be expected to compare informally by simply viewing the containers. *Direct* measurement involves filling one container and then pouring the contents into another to find which holds more. *Indirect* measurement involves comparing the capacities of each of two containers to that of a third container, e.g., B holds more than A, C holds more than B, so C holds more than A.



Provide opportunities for students to order the capacities of a number of containers, as well as opportunities for students to select the container most appropriate to hold a particular item.

In comparing areas, students are examining the amount of space taken up by an object, e.g., One placemat might take up more of the table than another. *Direct* measurement involves placing one surface on top of another to see which “sticks out.” *Indirect* measurement involves comparing the areas to a third area.

Students should be exposed to a variety of words involving these measurements in real situations. To help students develop measurement sense, all measurement activities should begin with an estimate.

A developmentally appropriate curriculum ... incorporates real-world contexts, children’s experiences, and children’s language in developing ideas. It recognizes that children need considerable time to construct sound understandings and develop the ability to reason and communicate mathematically. (*NCTM* 1989, 16)

GCO D: Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

Performance

D1.1 Provide students with three differently shaped containers and with filling material, e.g., beans or styrofoam packing material. Ask them to order the containers based on how much they hold, initially by sight and then by using the filling material.

D1.2 Provide students with trains of various lengths made from interlocking cubes. Ask them to order the trains from shortest to longest.

D1.3 Provide students with pictures of two differently shaped corrals. Ask them which corral would give a horse more room. Have them share how they made their decisions.

D1.4 Ask two students to perform standing long jumps. Encourage them to find a way to determine who jumped farther. Ask, How important was having a common starting point?

Interview

D1.5 Place a longer, curved piece of string beside a shorter, straight piece. Ask, Which is longer? Why?



D1.6 Ask students to show, without using rulers, how they could find out which is wider, the door or the window.

D1.7 Ask students, How would you decide whether your thumb is shorter than your other fingers?

D1.8 Ask students, What does “holds more” mean? Have them explain their thinking.

Presentation

D1.9 Have students prepare sets of ribbons for first, second, and third places in a race, so that the fastest runner gets a longest ribbon.

D1.10 Show students a coffee mug and a drinking glass. Ask them to decide whether the coffee mug holds more liquid than the drinking glass.

GCO D: Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

KSCO

- (i) By the end of grade 3, students will be expected to measure and understand basic concepts and attributes of length, capacity, mass, area, and time.

SCO

By the end of grade 1, students will be expected to

- D2 identify procedures (not involving units) to compare and/or order masses and durations of time

The purpose of an interview is to uncover how students think about mathematics, so provide opportunities for contradictions in students' beliefs about mathematical concepts to emerge. (*NCTM* 1991c, 29)

Elaboration–Instructional Strategies/Suggestions

D2 Students should recognize that mass tells about the heaviness of an object. They should explore methods to compare and order masses, including situations involving both *direct* and *indirect* measurement. *Direct* measurement involves simultaneously placing two objects, one on each of the two pans of a balance, to see which object has the greater mass. *Indirect* measurement involves comparing the masses of two objects by comparing them both with the mass of a third object.

Students should explore methods to compare and order lengths of time. This aspect of measuring time should be distinguished from the act of reading a clock, which is an important, but different, skill. Include situations involving both direct and indirect measurement. *Direct* measurement would involve beginning two or more actions at the same time to see which takes longer. *Indirect* measurement occurs when two actions are independently compared to a third action, e.g., It takes longer to tie my shoes than to say my complete name and it takes longer to say my complete name than to clap twice; so, it takes longer to tie my shoes than to clap twice.

Students should be exposed to, and encouraged to use, a broad measurement vocabulary, e.g., They should hear and use words such as heavier, lighter, has the same mass, takes longer, and lasts the same amount of time.

Students should recognize that objects of different volume can have the same mass, e.g., a large piece of styrofoam and a small plastic item, and that the mass of an object can be visually deceiving, e.g., A small iron ball can be heavy.

GCO D: Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

Worthwhile Tasks for Instruction and/or Assessment**Suggested Resources***Performance*

D2.1 Have students individually make a Playdoh ball, then make two others with an estimated mass one-half as much as the first. Ask them to check for the accuracy of their estimates using pan balances.

D2.2 Ask students to decide if it takes longer to clean out a school desk or eat an apple. Carry out an experiment to check.

D2.3 Ask students to predict whether it takes more time to print their first names or their last names, and then to show how they could find out for certain.

D2.4 Display a set of five objects of similar size and a sixth target object. Ask students to sort the five objects into two groups—one with masses less than the target and one with masses greater than the target.

Interview

D2.5 Ask students to explain how they could find out which of three activities (which you select) takes the longest time to do.

D2.6 Display a pan balance and two items. Ask students to lift the items to decide which one has the greater mass and to explain how to use the pan balance to check their decision.

D2.7 Ask students to explain how to compare the masses of an item at home and one at school, if they cannot transport either item.

Presentation

D2.8 Have students participate in skits in which someone measures incorrectly. The other students have to figure out what is wrong.

GCO D: Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

KSCO

- (ii) By the end of grade 3, students will be expected to identify and use non-standard and standard units of measurement and to appreciate their role in communication.
- (iii) Students will be expected to estimate and determine measurements in everyday problem situations and to develop a sense of the relative size of units.

SCO

By the end of grade 1, students will be expected to

- D3** identify and use non-standard units to estimate and measure length, capacity, time, mass, and area
- D4** read hours on an analog clock

Informal [non-standard] units make it easier to focus directly on the attribute being measured. (*Van de Walle 1994, 293*)

Elaboration–Instructional Strategies/Suggestions

D3 Provide activities for students to (a) estimate and measure objects in non-standard units for the purpose of answering relevant and practical questions, (b) determine what unit might be appropriate in a given situation, (c) use the same non-standard unit to measure a variety of items in order to compare them, and (d) measure the same object with a variety of non-standard units to see how the quantities vary.

- Have students estimate and measure the lengths of various objects using finger widths, eraser lengths, hand spans, foot lengths, paces, and paper clips. Have them compare the results.
- Ask students to search for items that are a given number of paper clips long.
- Have students measure the capacities of various containers using cups of rice, spoonfuls of sand, or boxes of raisins.
- Have groups of students decide how many cans of apple juice are required for each student in the class to have one glass.
- Have students measure the time lapses for various activities using sand timers, number of claps, counting, or water clocks.
- Ask students to determine how many times one can sing “Row, Row, Row Your Boat” while doing 50 jumping jacks.
- Have students measure the masses of a variety of items using pan balances and pennies, marbles, or linking cubes.
- Ask students to measure the areas of a variety of objects using pattern blocks, postage stamps, and pieces of paper.

Students should become accustomed to estimating before measuring, and subsequently comparing their estimates with the actual measurements.

They should realize that, when they measure, the smaller the unit they use, the more units that will be needed, and vice versa.

As illustrated in the suggested activities above, measurements should be performed in relevant situations.

D4 Analog clock reading should begin with a focus on hours only. Having students work with one-handed clocks is a useful way to begin this. Times should be stated in terms of hours only, but using approximate language such as “a little past seven o’clock” or “between eight and nine o’clock.”

GCO D: Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

Performance

D3.1 Show students a small jar with 10 marbles in it and tell them how many marbles there are. Ask students to estimate how many marbles it would take to fill the jar. Collect their estimates. Have a pair of students fill the jar with marbles to check the estimates.

D3.2 Ask students to use linking cubes to measure the distances toy cars travel when rolled down a ramp.

D3.3 Provide students with cookie cutters and dough. Ask them to roll out the dough and estimate how many cookies could be cut from it.

D4.1 Present a number of routine daily events, e.g., waking up, recess, supper, and bus departure from school. Ask students to position the hand of one-handed clocks appropriately for each event, and to state these times to the nearest hour.

Paper and Pencil

D3.4 Ask students to write or draw pictures that would explain why a book is not a good unit to use for length.

D3.5 Provide students with pattern block puzzles. Ask them to select a number of one type of pattern block and to cover the puzzles with them in order to find their areas. Ask them to explain why different students might have different answers.

Interview

D3.6 Ask students to explain how they could find out whose dog is the fastest runner.

D4.2 Present students with times shown on a one-handed analog clock. Ask them to read and record the times to the nearest hour and to name events/activities that often occur at about these times of day.

Portfolio

D3.7 Ask students to choose a particular mass, e.g. the mass of 10 pennies or 5 marbles. Have them identify a variety of items with approximately that mass and organize their findings.

Shape and Space: Geometry

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

KSCO

- (i) By the end of grade 3, students will be expected to explore and experiment with geometric shapes and relationships (including the orientation and perspectives of objects).

SCO

By the end of grade 1, students will be expected to

- E1 develop aspects of spatial sense, including visual memory

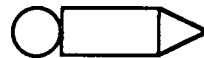
Spatial sense involves perceptual abilities that are important for early school success. (*Curriculum and Evaluation Standards, Addenda Series, First-Grade Book, p. 19*)

Elaboration–Instructional Strategies/Suggestions

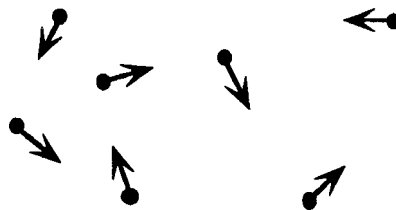
E1 While this outcome particularly focusses on *visual memory*, instruction must provide for the continuation of the development of all spatial skills including *eye-motor co-ordination* and *position-in-space perception*, which were emphasized in grade primary.

Visual memory is the ability to recall objects no longer in view, e.g., asking students to draw triangles to see if they remember what triangles look like. Other times students might be shown a shape for a short time and then asked to reproduce it. Seeing a simple pattern and continuing this pattern from memory also requires visual memory skills.

- Ask students to make a shape on their geoboards, e.g. a triangle. Have them compare their shapes with those of others in their groups. Then ask them to turn their geoboards, viewing their shapes from different perspectives. Have them discuss these turned shapes. Ask, Are they new shapes or still triangles? Is there any way you could turn the geoboards so the shapes would not be triangles?
- Make a shape on an overhead geoboard, e.g. a right triangle. Show students the shape for 10 seconds, or less. Ask them to make the shape, as they remember it, on their own geoboards. As with many of these activities, it is important for the students to share their shapes and to justify their answers, particularly if they differ from those of others. Early work in this area would be with simple shapes and would become more complex as the students’ abilities develop.
- Show designs on a card or overhead for a few seconds and ask the students to draw them from memory, e.g.,



- To provide practice in eye-motor coordination have the students follow directional arrows to create a shape, e.g.,



GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

Performance

E1.1 Make a triangle on a geoboard. Make a very different triangle on the same board. Ask students, Are they the same shape? Why or why not? How are they alike? How do they differ? Invite them to make on their geoboards other triangles that look different from both the other two. Encourage them to talk about the similarities and differences among the triangles.

E1.2 Provide students with three shapes. Use a second set of the same three shapes, e.g., a rectangle, a hexagon, and a triangle, and create a design. Have students reproduce the design. Then move one of the shapes to a new position relative to the others and ask students to reproduce this new design. Observe which task students find easier and whether the relative positions of the shapes makes a difference.

Paper and Pencil

E1.3 Show a drawn shape for 10 seconds, e.g., the one opposite. Ask students to copy the shape they remember seeing.



E1.4 Show students a design made from a combination of objects, e.g., pencils, erasers, buttons, and counters as in the design below. Allow sufficient time for them to get a visual representation of the design. Ask students to reproduce the design when it is hidden from view.



Presentation

E1.5 Invite pairs of students to use four building blocks to make shapes behind barriers. Ask them to remove the barriers for a specified number of seconds before asking other students to make the same shape. Have students check their classmates' work.

E1.6 Have students reproduce structures built from materials, e.g., wooden block shapes. Many students may also be able to reproduce simple structures from pictorial models.

E1.7 Give directions to students to build pattern block designs using spatial language, e.g., Place a yellow shape to the left of a red one. Put another yellow shape above the red one. Place a green shape below the red one.”

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

KSCO

- (i) By the end of grade 3, students will be expected to explore and experiment with geometric shapes and relationships (including the orientation and perspectives of objects).

SCO

By the end of grade 1, students will be expected to

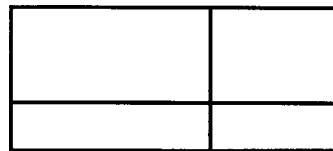
- E2 develop aspects of spatial sense, including figure ground perception

Informal geometry is aimed at development of 'spatial sense' ... This spatial sense grows and develops over the entire time children are in school. Students should have several in-depth geometric experiences every year. (*Van de Walle 1992, 324*)

Elaboration–Instructional Strategies/Suggestions

E2 *Figure-ground perception* is the visual act of identifying a specific figure or focus in a picture. In order to focus attention on a specific figure, a student must disregard its surroundings. This skill, like all spatial skills, will develop through experiences and over time.

- Provide a large posterboard-sized laminated picture made of a number of overlapping shapes. Ask students to find and outline, with a washable marker, a particular shape whenever it appears in the picture.
- Use different coloured elastics to make several overlapping shapes, e.g., triangles, squares, rectangles, and hexagons, on a geoboard. Ask students to find specific figures and to trace over them with their fingers. As an extension, have students make overlapping shapes and exchange their boards with classmates to see if they can find particular shapes.
- Ask students to reassemble rectangles that have been cut along one or both diagonals.
- Ask students how many rectangles they can see in a given diagram, e.g.,



It is highly recommended that students use 5 x 5 rather than 10 x 10 geoboards—locating specific pegs on geoboards is a figure-ground task more easily undertaken with fewer pegs.

Students should be encouraged to work with puzzles regularly. These provide practice for all the spatial skills. It is important to provide puzzles that span a wide range of complexity; some students will have had a great deal of experience while others have had little experience. Selected puzzles should challenge all students.

Having students copy from the overhead or blackboard provides practice in most of the spatial skills.

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

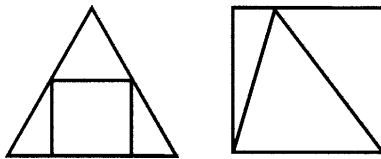
Performance

E2.1 Provide a laminated picture of a number of shapes. Ask students individually to trace over the rectangles with a washable marker. Change marker colours and ask them to trace other shapes.

E2.2 Show students pictures that include objects of many different shapes. Ask them to point out and trace with their fingers examples of one type of shape, e.g., triangle.

Interview

E2.3 Show students pictures and ask them what shape(s) they see, e.g.,



Observation

E2.4 Provide multi-coloured shape drawings. Have students look for a particular shape, e.g., squares. Observe whether the shape's colour and/or background colours affect their abilities to recognize shapes.

Paper and Pencil

E2.5 Give students a prepared collage of overlapping shapes. Ask them to follow, or create, colour codes for tracing over particular shapes, e.g., Trace over all triangles in red and rectangles in blue.

Portfolio

E2.6 Ask students to each make a picture of overlapping shapes on plain paper. Some may wish to trace around commercial materials, such as attribute blocks, while others may be able to use straight edges to connect dots on geopaper to make the shapes—this would require more developed eye-motor coordination. Before filing the pictures in their portfolios, have them exchange and take part in shape searches.

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

KSCO

- (i) By the end of grade 3, students will be expected to explore and experiment with geometric shapes and relationships (including the orientation and perspectives of objects).

SCO

By the end of grade 1, students will be expected to

- E3 sort, build, and pattern with 2-D and 3-D shapes

Sorting or classifying shapes using models is a good way to introduce geometric ideas... When students omit shapes from a category they have identified or fail to create a category you hoped they would discover, it is a clue to their perceptual thinking. (*Van de Walle 1992, 328*)

Elaboration–Instructional Strategies/Suggestions

E3 Students need many varied opportunities to manipulate both 2-D and 3-D shapes. Activities in which they sort, make, and recognize patterns, build shapes, and talk about what is happening help to develop essential geometric skills. Such activities reinforce the names of 2-D and 3-D shapes and help students to begin recognizing the characteristics of these shapes.

Sorting activities help to develop *visual discrimination*. It is important to encourage students to look for alternate ways of sorting—this necessitates the further investigation of objects.

- Ask students to bring to school one object that is round. Provide some time for students to talk in groups about what they have brought, discussing how they would sort the objects. The common sorting rule that “some are round like a cylinder, others are round like a sphere” should capture most of the shapes collected; however, if a student has brought a 2-D shape such as a happy face sticker, it would be part of a third group in the sorting.
- Provide an assortment of 2-D shapes cut from tagboard. Ask students to work in small groups to sort the shapes. Encourage discussion and have the groups share their sorting rules.
- Have students make patterns with breakfast cereal, e.g., There are dry cereals that have square, rectangular, circular, and other shapes.
- Have students create and continue patterns using attribute blocks. It is important that children are encouraged to verbalize the attributes, e.g., small, red, thick triangle.

Provide students with many opportunities to work together building using different materials. It is important that they discuss their constructions and learn from each other.

Provide opportunities for students to build 2-D and 3-D shapes by following oral directions, e.g., Place the cube between the small cylinder and the cone and place the large cylinder behind the cube. Invite students to make their own designs and then challenge their classmates to build the same structure following their oral directions.

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

Performance

E3.1 Provide several different 3-D shapes. Ask students to sort them and to explain their sorting criteria. Ask them to sort them again using different criteria.

E3.2 Make assortments of 2-D shapes from posterboard. Ask students to sort them and to give the sorting rules.

E3.3 Ask students to build as many different structures as possible using four cubes.

E3.4 Have students build structures with about the same widths as heights.

E3.5 Have students cut out many different sizes and shapes of one figure, e.g., triangles. Ask them to use these shapes to design aliens. Encourage them to describe their aliens.

E3.6 Provide students with a collection of solids and pictures of constructions. Ask, Which of the pictures show constructions that could be built with the available solids?

E3.7 Ask students to work in pairs to sort the shapes below into two categories and to give their sorting rules.

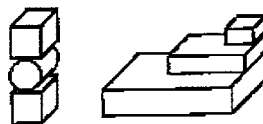


E3.8 Place a collection of tagboard shapes in a baggie to send home with students. Include the directions that ask them to sort the shapes and to write down the sorting rules.

E3.9 Ask students to contribute to class books of pictures of objects shaped like a triangle, a square, and a circle.

Interview

E3.10 Provide 3-D shapes and display pictures of possible constructions. Ask students, Which of these will topple? Build ones that will not topple.



Portfolio

E3.11 Ask students to cut pictures from magazines of things that they can sort according to shape. Have them display their sorted and labelled collections before placing them in their portfolios.

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

KSCO




- (i) By the end of grade 3, students will be expected to explore and experiment with geometric shapes and relationships (including the orientation and perspectives of objects).

SCO

By the end of grade 1, students will be expected to

- E4 recognize and represent angles

Elaboration—Instructional Strategies/Suggestions

E4 It is important for students to be able to recognize shapes in their world. To do this they will need to identify the attributes of those shapes. Through investigation, students will note that the sides of shapes come together to form *corners*—it is these *corners* in shapes that students should explore. There is no intent at this level for students to identify angles by naming them or discussing their degree measure. Some students may be comfortable using the term *angle*, others may prefer *corners*, while still others may use the two terms interchangeably. It is expected that students at this age will investigate angles only as parts of shapes in which they could compare sharp corners like  and square corners like  to wide corners like .

- Have students compare the corners of large cutout shapes, e.g., Ask them to work in pairs comparing the angles of a square to those of a rhombus.
- Ask students to find angles in the classroom that are different from the square angles that are most common.
- Provide a square corner or square angle template made from sturdy paper. Have students work with the templates in pairs to make a list of things in the classroom that have these corners, as well as a list of things that have corners that are not square. Ask them to compile their data, to compare the two lists, and to discuss why they think the square corner list is longer.
- Have students make many-sided shapes on geoboards. Ask them to describe all the angles in their shapes.
- Have students bend pipe cleaners to make a variety of shapes. Ask them to compare the angles in their shapes with those of a partner.

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

Worthwhile Tasks for Instruction and/or Assessment**Suggested Resources***Performance*

E4.1 Show students an angle in a particular 2-D shape. Ask them to select other shapes that have similar angles and to justify the selection.

E4.2 Ask students to draw shapes with lots of sharp angles or shapes with some wide angles.

E4.3 Ask students to make shapes on geoboards that have 2 square corners, 2 wide corners, and 1 sharp corner.

E4.4 Have students compare the angles in the six blocks in the pattern block set.

Interview

E4.5 Ask students to choose blocks from the pattern block set and to tell you about the angles in the shapes.

Portfolio

E4.6 Have students select particular 2-D shapes from among familiar geometric shapes. Ask them to write about the different angles in these shapes and to compare their findings with those of other students who selected the same shapes. Encourage them to represent the angles in different ways such as sketching, tracing, making a chart, or graphing.

E4.7 Have pairs of students use angle templates to go on an angle search around the school. Ask them to report their findings to the class and prepare written reports.

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

KSCO

- (ii) By the end of grade 3, students will be expected to describe, model, draw, and classify 2-D and 3-D figures and shapes.

SCO

By the end of grade 1, students will be expected to

- E5 recognize, name, describe, and represent a variety of 2-D and 3-D shapes

Elaboration–Instructional Strategies/Suggestions

E5 Students’ previous experiences with 2-D shapes would include, but might not be limited to, squares, triangles, circles, and rectangles. Because pattern blocks are regularly used for geometric inquiry, it would seem reasonable that students become familiar with the terms rhombus, trapezoid, and hexagon as well.



Through experiences with their building blocks, students should be comfortable naming 3-D shapes such as cylinder, sphere, cone, and cube. Students should be encouraged to use accurate geometric terminology. You should help students develop the concepts of square prism, rectangular prism, triangular prism, triangular pyramid, and square pyramid. These are common shapes in commercial 3-D sets. These shapes individually should be investigated by finding objects in the environment shaped like them, by assembling pictures of objects shaped like them, by building models of them, and by making picture books about them. Students will probably be more comfortable using the word *side* to describe what is actually a *face* of these 3-D shapes.

Prisms are special shapes that have identical polygons as bases and rectangular faces; the type of base is the name given to the prism, e.g., A rectangular prism has rectangles as bases, a triangular prism has triangles as bases.

- Ask students to stack a number of pattern, logic, or attribute block triangles to make a triangular prism, stack rectangles to make a rectangular prism, and stack hexagons to make a hexagonal prism. Have them find the matching 3-D blocks in commercial sets.

Pyramids have a polygon as a base with the other triangular faces coming together at a point. Pyramids are named by the shape of their bases, e.g., The Egyptian pyramids would be named *square pyramids* because their bases are squares.

- Place a variety of 3-D shapes in a bag. Invite students to feel the shapes to identify particular ones, e.g., triangular pyramid.
- Ask students to make a trapezoid on a geoboard that has 2 pegs inside.
- Ask students to make a rhombus using toothpicks.

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

Performance

- E5.1 Have students select from a collection of 3-D shapes ones that have faces of only one shape.
- E5.2 Place three 3-D shapes in bags, two of which shapes are the same. Ask students to feel the shapes inside the bags to identify the odd one and tell why it does not belong.
- E5.3 Place a number of different 2-D or 3-D shapes in bags. Name a particular shape for students to find by feeling the shapes within the bags.
- E5.4 Show students a shape and ask them to reproduce the shape on geoboards. Ask, What is the shape? Explain how you know.
- E5.5 Ask students to make shapes on geoboards that have six sides and touch 8 pegs. Ask them to make the smallest trapezoid and the biggest rectangle that they can. Ask them to make four triangles that overlap.
- E5.6 Play a game of “Attribute Block Riddles.” Divide sets of attribute blocks among students working in groups. Give them sets of cards with riddles on them, e.g., My block is blue. It has three sides. It is large. It is thin. As each block is identified, the player places it back in the box. The winner is the first person in the group to put back all his/her blocks.
- E5.7 Have students work in pairs with geoboards to make a large squares with smaller squares inside them.

Paper and Pencil

- E5.8 Ask students to circle all the triangles that they can find in the following set of shapes. Ask them to give reasons for the selections and to explain why some did not belong.



Interview

- E5.9 Provide a cylinder and a cone. Ask students to describe how they are alike and how they are different.
- E5.10 Ask students to explain how cubes and square prisms are the same and how they are different.

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

KSCO

- (i) By the end of grade 3, students will be expected to describe, model, draw, and classify 2-D and 3-D figures and shapes.

SCO

By the end of grade 1, students will be expected to

- E6** describe attributes of and sort and compare 2-D and 3-D shapes

Elaboration–Instructional Strategies/Suggestions

E6 Students should be encouraged to observe the attributes of many shapes. Describing shapes allows students to focus on their basic characteristics. Use questioning to focus on student thinking, e.g., Ask, What other shapes are similar to this one? In what way are they alike? What does this object look like? Is the object long? Tall? Does the object roll or stack? Pick two of the shapes and tell how they are alike and how they differ.

It is important for you to model geometric language and to encourage students to use the words they have learned; however, terms such as *cylinder* and *sphere* could be interspersed with everyday language such as can and ball.

Considering the attributes of various 2-D and 3-D shapes allows students to compare them and sort them, using first one, then another attribute as the sorting criterion. It is not intended that students remember these attributes as properties of the shapes, but these activities will prepare them for future development when such analysis of the shapes will be expected.

- Provide student partners with a large triangle, square, and rectangle from logic block sets. Ask them to compare the three shapes, and to list all the ways in which they are the same and are different. They should be able to identify the shapes and to compare such things as sides and angles.
- Collect cylinders of different sizes. Ask students to order them according to different criteria, e.g., height, width, and capacity. (Capacity could be checked by using beans, rice, or pasta.) The same investigation could involve a variety of boxes.
- Have students make mobiles of 2-D and 3-D shapes based on some prescribed attributes.
- Ask students to paint and decorate boxes, cans, balls and cones to be used for storage, decorations, gifts, and party hats. Ask them to determine how much wrapping paper they should cut to wrap boxes so as not to waste too much paper.
- Provide students with a set of shapes, e.g., prisms and pyramids placed randomly and some of them toppled. Ask them to sort the shapes into two groups. Observe whether or not separating prisms from pyramids is their natural sorting criterion.

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

Performance

E6.1 Ask students to compare 3-D shapes to determine which of the following rolls best: a sphere, a cylinder, or a cone.

E6.2 Provide students with a collection of 3-D shapes. Give clues, asking them to pick the shape, e.g., Explain, It will roll. It will stack. Explain, One side is flat. It won't stack. Have students create their own set of clues to give to their classmates.

E6.3 Provide students with large paper triangles. Ask them to fold them so they look like cones.

E6.4 Show students a brown and a blue block from the pattern block set. Ask them to name the blocks and to describe how they are the same and how they are different.

E6.5 Ask students to make 4-sided figures on geoboards. Have them work as partners to describe how their shapes are the same and how they are different.

Interview

E6.6 Display a cylinder and a cone, or a cube and a rectangular prism. Ask students to describe how they are alike and how they are different.

E6.7 Show students two rectangular prisms, one of which is a lot longer than the other. Ask, Would these shapes have the same name? Why?

E6.8 Ask students to name solids that have only flat faces and some that do not.

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

KSCO

- (ii) By the end of grade 3, students will be expected to describe, model, draw, and classify 2-D and 3-D figures and shapes.

SCO

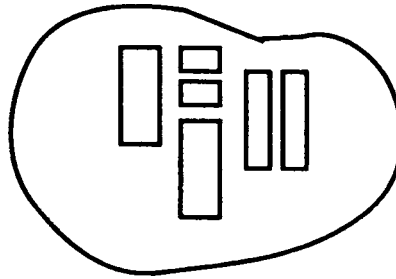
By the end of grade 1, students will be expected to

- E7 recognize 2-D figures in 3-D shapes

Elaboration–Instructional Strategies/Suggestions

E7 Students should become familiar with the 2-D shapes of the faces of 3-D shapes. They should name all the solids that have a particular shaped face and name all the shapes of the faces of a particular 3-D solid.

- Have students press each face of a solid into Plasticine to get a “map” of the solid.



- Place a 3-D solid on the overhead projector and ask students to predict what shape they will see when the projector is turned on. Have students project the shape onto paper and trace around the image.
- Give students a combination of 2-D and 3-D shapes. Ask them to investigate which of the 2-D shapes appears most often as a face of 3-D shapes. Have them make a graph to display their findings.
- Have students discuss why a circle is not a face of a sphere.


3-D solids have surfaces; these surfaces are called faces if they are flat, e.g., A cube has six faces that are squares while a cylinder has two faces that are circles and one curved surface. Students at this age might use the word “side” when referring to a face.

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

Performance

E7.1 Provide students with several 3-D solids and show them a sets of faces, e.g., 

Ask them to match these sets of faces to the appropriate 3-D solid.

E7.2 Make a “map” of a 3-D shape in Plasticine or the sand box. Ask students to select the matching shape.



E7.3 Give students sets of clues involving the shapes of the faces of 3-D solids, e.g., I have one square face. My other faces are triangles. What shape am I?

Paper and Pencil

E7.4 Ask students to draw all the shapes that are the faces of a square prism.

E7.5 Ask students to draw the shape that they think most often appears as a face of 3-D shapes and to explain their choice.

Interview

E7.6 Ask students to name 3-D shapes that have circles as faces.

E7.7 Ask students to select from a collection of 3-D shapes one that is made up of only one shape of face.

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

KSCO

(iii) By the end of grade 3, students will be expected to investigate and predict results of combining, subdividing, and transforming shapes.


SCO

By the end of grade 1, students will be expected to

E8 build, divide, and change 2-D shapes

Elaboration–Instructional Strategies/Suggestions

E8 Activities selected in geometry should provide students with the opportunity to explore. They need to see, to feel, to build, and to take apart shapes, as well as to share their observations with their classmates. They must feel free to experiment and to take risks.

- Have students individually make a large square on a geoboard. Ask them to use another elastic to divide the square into two equal parts. Ask them to find many different ways to do this. They should be able to divide horizontally, vertically, and on both diagonals. Some will be able to discover many other ways.
- Provide students with square pieces of paper. Ask them to fold the paper so that there are two equal parts. Have them share their work.
- Show students a collection of half-squares and ask them to match each with its other half. Some pairs might be shapes other than triangles or rectangles. 
- Ask students to cut a square, rectangle, or triangle into three parts. Have them exchange their pieces with partners, asking each to rearrange the other's pieces to make the original shapes. Activities such as these, in which students are required to assemble a figure from its parts, help them develop many spatial perception skills.

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

Performance

E8.1 Divide each of two triangles into two parts. Give the four mixed-up parts to students, asking them to make two triangles. Encourage students to explain how they knew which pieces to put together.

E8.2 Have students watch as you cut rectangles along one of their diagonals. Ask students to reassemble them. Have them explore to see if there are other shapes that can be made with the two pieces.

E8.3 Ask students to cut squares along one of their diagonals and to see if they can make different shapes with the pieces.

E8.4 Provide students with a set of tangrams. Ask them to build squares with the two small triangles. Ask, Can you build other tangram shapes using these two small triangles?

E8.5 Have students investigate which pattern blocks can be used to make larger versions of their same shape, e.g., have them try to make large triangles using only the green triangles.

E8.6 Have students individually make a small square in the middle of a geoboard. Ask them to describe how they could change these squares to rhombi.

Interview

E8.7 Show students an isosceles triangle that has been folded in half. Ask them to tell what shape it will be when it is unfolded.

E8.8 Show students a rectangle. Ask, If I were to cut this rectangle along the diagonal, from one corner to another, what shapes would I have?

E8.9 Show students a square that has been folded along the diagonal. Ask, What shape will this be when I unfold it?

Portfolio

E8.10 Have students tape record their own story about shapes that keeps changing.

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

KSCO

- (iii) By the end of grade 3, students will be expected to investigate and predict the results of combining, subdividing, and transforming shapes.

SCO

By the end of grade 1, students will be expected to

- E9** recognize, name, describe, and represent slides and reflections of 2-D shapes

Elaboration–Instructional Strategies/Suggestions

E9 Much of students' geometric development depends on understanding how shapes do and do not change when they are transformed in different ways. Two important transformations are slides and reflections (flips). Students should develop intuitive notions that a slide moves a shape up or down, right or left without changing its orientation. A flip, on the other hand, does change the orientation of a shape; the effect is one of reversing a shape, i.e., right becomes left or up becomes down.

To describe slides, students might use informal language such as, *up and to the right* or *over a lot and up a little*. To describe flips, students should be able to use language such as, *flipped up* or *flipped to the left*.

- Have students follow a set of instructions for creating pattern block pictures, e.g., Put down a square. Trace around it. Slide it a little to the right and a little down. Trace around it. Slide it down a little more and a little to the left. Trace around it. As they observe each other's designs, they might notice certain things are the same and other things are different.
- Place two strips of masking tape about 10 cm apart on a table top. Give a pair of students some pattern blocks to make designs. One student makes a design on one side; the partner makes the mirror image on his/her side. The opportunity to use a Mira or mirror would be worthwhile. You may wish for them to place one block at a time and then its reflected image. Encourage students to use position-in-space language such as right, left, on top of, in front of, and behind.
- Place a barrier on a table between two students. One student makes a construction with three 3-D shapes. His/Her task is to communicate to the other how the construction can be replicated as a mirror image. This is difficult which is why students should use only 2 or 3 shapes.
- Provide students with sheets of paper filled with a triangle in different positions as well as some that are different triangles altogether. Also provide cut-outs of the triangle in the upper left corner. Have the students flip and slide the cut-out to see what matches they can find and describe each match as a flip or slide of the one in the upper left corner.

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

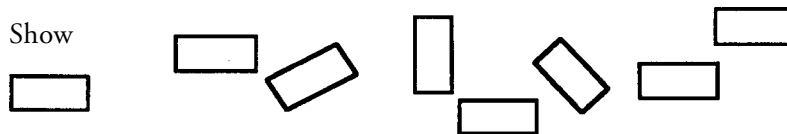
Performance

E9.1 Give each of a pair of students the same set of pattern blocks. Ask one of the pair to create a design. Have both students look at the image of the design in a mirror. Then remove the mirror and ask the students to recreate the mirror image. Ask, How might a mirror be used to help you see if the designs are mirror images of each other?

E9.2 Give students Miras. Ask them to place two pattern blocks on one side of a line segment and to use the Mira to help place identical blocks on the reflected image. Observe how adept students are at finding the placements.

E9.3 Give students pattern block hexagons. Have them use them to show and describe slides.

E9.4 Show students a shape. Provide pictures of a number of shapes and ask students to identify all those that are slides of the original shape, e.g.,

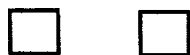


Paper and Pencil

E9.5 Place two pattern blocks together, e.g., a hexagon attached to a trapezoid, on one side of miras and ask students to trace the reflections on the other sides.

Interview

E9.6 Place two squares as shown below:



Ask, Do you think a slide or flip happened? Explain your thinking. Why is it hard to tell?

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

KSCO

- (iv) By the end of grade 3, students will be expected to relate geometric ideas to number and measurement ideas and to recognize and apply geometric principles in real-world situations.

SCO

By the end of grade 1, students will be expected to

- E10** recognize and identify 2-D and 3-D shapes in the environment
- E11** cover figures and fill shapes with countable non-standard units

... Informality should be the keystone of early geometry experiences. Children have observed geometry in their world since birth and have already acquired some strong ideas that need to be explored for validity. Hence a variety of experiences investigating and discussing geometric concepts in different contexts is needed. (*NCTM* 1991b, 19)

Elaboration–Instructional Strategies/Suggestions

E10 Real-world associations are very important in the development of geometric concepts. Students should become familiar with the variety of sizes and proportions possible for circles, rectangles, triangles, squares, rhombuses, trapezoids, and hexagons by observing these 2-D shapes in their surroundings. Students should also recognize 3-D shapes such as cylinders, cones, spheres, rectangular prisms, square prisms, triangular prisms, triangular pyramids, and square pyramids in their environment and in pictures.

It is important that students recognize the difference between objects that are true examples of a shape and those that are almost a certain shape, e.g., a computer diskette is almost a square because one of its corners is cut off. You should always encourage students to notice such variations from the true shapes.

- Invite students to hunt around the school to find various 2-D and 3-D shapes. Have them share their findings and speculate on why certain shapes are more common than others.

While students will be having experiences with many 2-D shapes, they should be comfortable recognizing and naming a rhombus (diamond), a trapezoid, and a hexagon, as well as rectangle, square, triangle, and circle.

E11 Students will enjoy filling in picture outlines with various 2-D shapes. These assignments can be challenging and serve as ideal problem-solving activities. Many commercial materials include pictures with instructions for the students, e.g., Fill the picture in with as many blocks as you can. Can you fill the picture using just one type of block? How many different ways can you fill in this shape? Can you fill in this shape with exactly seven blocks?

When these tasks involve using one type of block, these are early *area* investigations with the block being a countable non-standard unit of area. The focus should be on estimation, e.g., looking at the non-standard units, the picture to be filled in, and attempting to name a reasonable number of units that would cover the picture.

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

Worthwhile Tasks for Instruction and/or Assessment**Suggested Resources***Performance*

E10.1 Ask students to sort by shape a collection of pictures of common objects, and to describe the reasons for their sortings.

E10.2 Ask students to examine containers such as ones used for yogurt or cottage cheese. Ask, Are these cylinders? How do you know?

E11.1 Give students outlines of an animal. Ask them to find the fewest number of pattern blocks that will fill the picture.

E11.2 Ask students to make a shape using eight of the same pattern block. Trace around the shape or have students do the tracing. Challenge other students to fill in the shape using eight blocks.

Paper and Pencil

E10.3 Ask students to describe or draw some of the different kinds of triangles that they see.

E10.4 Have students draw pictures of things in the world that includes lots of squares and circles.

Interview

E10.5 Show a triangle. Ask students to find three things in the classroom that make them think of that shape.

E10.6 Ask students, Where might you see a hexagon?

E10.7 Ask students, What makes a circle a special shape? Can you think of things that are circular that would work just as well if they were a different shape?

E10.8 Find and display pictures of bridge supports. Ask students, What shapes do you see more of?

**Data Management
and Probability:
Data Management**

GCO F: Students will solve problems involving the collection, display, and analysis of data.

Elaboration–Instructional Strategies/Suggestions

KSCO

- (i) By the end of grade 3, the students will be expected to collect, record, organize, and describe relevant data.

SCO

By the end of grade 1, students will be expected to

F1 collect and organize data

F1 Students should recognize that data is collected and organized in order to answer questions and/or to make decisions. This could be as simple as asking them how parents would know how much milk and juice they need to take on their class trip. They will realize that when information is organized, questions about it, as well as decisions based on it, can more easily be answered.

Students should be encouraged to organize and interpret data discovered in the course of classroom investigations. Data might be tallied or organized in charts. This is a good time for students to apply the traditional tallying process, e.g., $\text{||||} \text{||||} \text{||}$ to represent 11 and it provides a context for skip counting. (See SCO, A3).

Pertinent data could be collected in the different strands of mathematics and in other subject areas such as social studies and science. This might include information about solids and shapes; addition by groups, e.g., sums < 5 , sums > 5 ; capacities of various containers; results of probability experiments, e.g., results of flipping coins; classroom demographics, e.g., hair colour; personal likes and dislikes; types of pets; types of dwellings; and objects that sink or float.

- Have students conduct small surveys to collect data.
- Ask students to collect data to determine whether more addition facts have a sum less than 10 or a sum greater than 10. (Addition facts refer to those additions for which both addends are less than 10.)
- Have students collect data to determine their classmates' most commonly preferred fruit or vegetable.

Collecting and organizing data techniques should rarely be taught in isolation; rather, they should be taught in real contexts that lend themselves also to displaying and interpreting data. (See SCOs, F2 and F3.)

First grade is a wonderful time to help students make sense of data. Often more new information is presented at this grade level than at any other. As students learn to sort and organize, they develop extremely useful skills that will help them deal with the vast amount of information they will learn during their school years. (NCTM 1991b, 13)

GCO F: Students will solve problems involving the collection, display, and analysis of data.

Worthwhile Tasks for Instruction and/or Assessment
Suggested Resources
Performance

F1.1 Have students make charts to show which colours the students in their classes are wearing and how many of them are wearing each of these colours.

F1.2 Ask students to sort the addition facts into two groups. They should explain their sorting rule and tell how many facts are in each category.

F1.3 Have students skip count to 50 by 2s, then by 5s, and finally by 10s. For each count, have them tally how many numbers they said.

F1.4 Provide students with 12 to 15 objects, some of which are smooth and some of which are rough. Have students sort and tally these items under the two headings, smooth and rough.

Interview

F1.5 Give students some red, yellow, and blue pattern blocks randomly organized. Ask how they could reorganize them to tell very quickly how many blocks of each colour there are.

F1.6 Have students explain what the following display might mean:

RED HAIR	-	
BROWN HAIR	-	
BLONDE HAIR	-	
BLACK HAIR	-	

F1.7 Ask students how they might find out whether more students wore gloves than wore mittens to school.

Presentation

F1.8 Ask small groups of students to devise a way to find out the favourite colour of each teacher in the school. Have them carry out their plans and invite them to present their findings to the class, explaining how they organized their information.

F1.9 Have students write on slips of paper whether they live in a house or in an apartment; collect the slips. Ask small groups of students to suggest what could be done with the information on the slips and what question(s) could be answered.

GCO F: Students will solve problems involving the collection, display, and analysis of data.

KSCO

- (ii) By the end of grade 3, students will be expected to construct concrete and pictorial displays of relevant data.
- (iii) Students will be expected to read and interpret displays of relevant data.

SCO

By the end of grade 1, students will be expected to

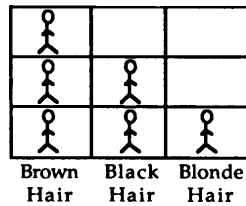
- F2** interpret and create concrete and picture graphs
- F3** interpret and create pictographs and symbolic graphs

Bar graphs are one of the first ways to group and present data and are especially useful in grades K to 3. At this early level, bar graphs should be made so that each bar is made of countable parts such as squares, objects, or pictures of objects. No numeric scale is necessary. (*Van de Walle 1994, 392*)

Elaboration–Instructional Strategies/Suggestions

F2 Students can create real and picture graphs using objects such as candies, books, and themselves. It is essential that experiences are provided to ensure that students understand the importance of having a common base line, and a one-to-one matching of objects in the various categories.

- Use a piece of vinyl and masking tape to create a floor mat on which students can stand to form a graph.



If a picture graph is going to be created, it is essential that students draw their pictures on pieces of paper that are the same size.

- Have students vote for favourite items or activities, e.g., favourite books or team logos, by drawing pictures of them and making a graph under the appropriate headings.

F3 Pictographs are picture graphs that make use of stylized drawings instead of pictures. At this stages pictographs should be based on a one-to-one correspondence; i.e., one picture represents one item, not a group of items. Students should begin creating symbolic bar graphs by making towers with unifix cubes and by using pre-made squares of equal size. Later, students can colour in a grid. Students should interpret and create symbolic graphs that run both horizontally and vertically.

Graphs should be created primarily in the context of other investigations, rather than as isolated activities.

- Invite students to create a pictograph that shows birth months of class members using pre-cut pictures of birthday cakes. Have students discuss what kinds of information they can get from reading the graph, e.g., They could find out which months have the most/fewest birthdays and how many students are in the class.

GCO F: Students will solve problems involving the collection, display, and analysis of data.

Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

Performance

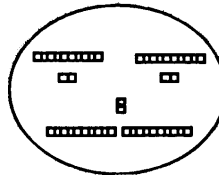
F2.1 Ask students to create real graphs to show the number of students in the class who play various games or who play various musical instruments.

F2.2 Provide data, as shown below, and the corresponding colour of linking cubes. Ask students to use the cubes to make a graph to represent the information.

- blue eyes - 8
- brown eyes - 12
- green eyes - 4

Paper and Pencil

F3.1 Ask students to create bar graphs to show the values of the left eye, right eye, nose, and mouth of the face in this picture.

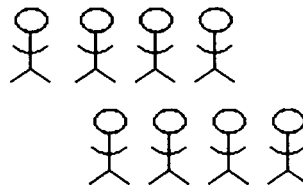


Interview

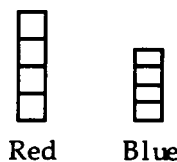
F3.2 Ask students what is wrong with the following conclusion drawn from the graph below: As many kids like pizza as burgers.



F3.3 Present the model as shown at right. The first row represents students with one-piece snowsuits. The second row shows those with two-piece snowsuits. Ask students, Why is it difficult to tell which line has more students in it?



F3.4 Present bar graphs such as the one shown at right. Ask students, What is wrong with this graph?



Presentation

F2.3 Invite small groups of students to demonstrate, using class members, why it is important for students to stand next to each other when deciding if more students are wearing red or blue.

GCO F: Students will solve problems involving the collection, display, and analysis of data.

Elaboration–Instructional Strategies/Suggestions

KSCO

(iv) By the end of grade 3, students will be expected to generate questions, develop and modify predictions, and implement plans with respect to data analysis.

SCO

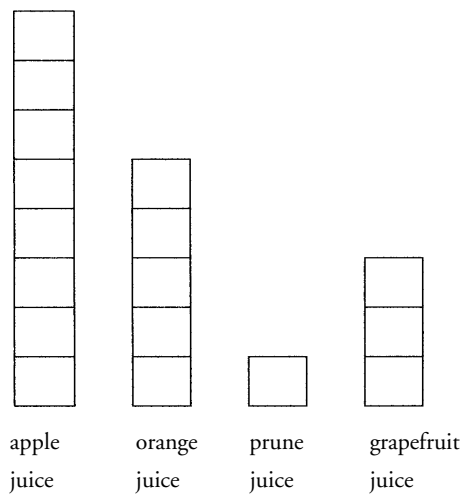
By the end of grade 1, students will be expected to

F4 pose oral questions in relation to conducting surveys and/or interpreting data

F4 Students need to practise formulating and asking questions as well as answering them. You should provide opportunities for them to formulate questions that could be used in conducting surveys as well as those that could be used in interpreting results.

- Have students brainstorm a list of possible investigation questions for surveys. Record their ideas on chart paper. Choose one or more of their suggestions, or have them vote, and proceed to collect and organize the appropriate data. Create, or have them create, questions that could be answered from the graph. If appropriate, discuss how the information could be used to make decisions.
- Working with a bar graph that shows what class members had for breakfast, supply students with answers for which they have to infer the questions. Have pairs of students create answers and ask other students to supply the questions.

Help students to see that there are many questions that could be answered by looking at graphs, e.g.,



- How many students like apple juice?
- How many more students like apple juice than prune juice?
- How many students answered the questions about their favourite juice?
- Order the juices from most popular to least popular.
- Why might someone want to know this information? What would it tell him/her?

GCO F: Students will solve problems involving the collection, display, and analysis of data.

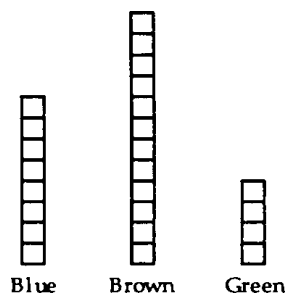
Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

Performance

F4.1 Ask students, If we are allowed to make one class trip this year, how will we decide what that trip will be. Carry out their plan.

F4.2 Display a graph showing the eye colours of students in the class, e.g.,



Ask students why the data might be interesting to know.

F4.3 Show students a graph indicating how many students in the class have cats, dogs, birds, and other pets. If, for example, there were 6 more students with cats than birds, explain, I looked at the graph and thought of a question it answers. The answer is 6. What do you think my question was?

F4.4 Ask students to pose questions, about food packages, for which they might investigate and collect data. Have them discuss how they would collect the information.

Presentation

F4.5 Have groups of students generate some questions they might ask to find out more about animals.

**Data Management
and Probability:
Probability**

GCO G: Students will represent and solve problems involving uncertainty.

KSCO

- (i) By the end of grade 3, students will be expected to conduct informal investigations of chance and to estimate probabilities with respect to games and other simple, everyday situations.

SCO

By the end of grade 1, students will be expected to

- G1** predict whether an event can never occur, must always occur, or simply might occur sometimes

Elaboration–Instructional Strategies/Suggestions

G1 Everyday situations should be used as contexts for students to make predictions. You should ask them whether events will always, sometimes, or never occur.

- Have students watch as you place 10 red cubes in an empty bag. You pick a cube out of the bag. Ask them, Will the cube I pick be red always? sometimes? never? Will it be green always? sometimes? never?
- Have students watch as you place five red cubes and five blue cubes in a bag. You pick a cube out of this bag. Ask them, Will the cube I pick be red always? sometimes? never? Will it be yellow always? sometimes? never?
- A totally red spinner is spun. Ask students, Will the pointer land on red always? sometimes? never?
- A spinner with a one-fourth red section and a three-fourths yellow section is spun. Ask students, Will the pointer land on yellow always? sometimes? never?
- Ask students, Does the bell ring at the end of the day always? sometimes? never? Is a teacher taller than students always? sometimes? never? Are students in a class all the same age always? sometimes? never?
- As part of the daily calendar activities, ask questions about the days of the week or the weather, e.g., If today is a Tuesday, will tomorrow always be a Wednesday? Will it rain today for sure?
- Ask students, If a coin is flipped, will it never, always, or sometimes turn up heads?

GCO G: Students will represent and solve problems involving uncertainty.

Worthwhile Tasks for Instruction and/or Assessment**Suggested Resources***Performance*

G1.1 Provide opaque bags and coloured cubes for students. Ask them to put 10 cubes in the bags so they will be sure that red will never be chosen. Have students repeat this task, this time putting in cubes so they will be sure that red will always be chosen. Finally, have them repeat this task so they can only be certain that red will sometimes be chosen.

G1.2 Have students create designs for spinners so that the pointer will never land on green.

Interview

G1.3 Ask students to describe events that will never happen in the classroom.

G1.4 Ask students to describe three events that will always happen.

G1.5 Ask students to describe events that happen sometimes, but not a lot.

Grade 2

**Number Concepts/
Number and Relationship
Operations:
Number Sense and
Number Concepts**

GCO A: Students will demonstrate number sense and apply number theory concepts.

Elaboration–Instructional Strategies/Suggestions

KSCO

- (i) By the end of grade 3, students will be expected to construct and communicate number meanings, and to explore and apply estimation strategies, with respect to whole numbers.

SCO

By the end of grade 2, students will be expected to

- A1 order numbers and use ordinal language
- A2 count in a variety of ways

A1 By the end of this grade, students should be comfortable using ordinal language up to *thirty-first*.

- Continue using the calendar, asking questions such as:
 - Today is the eighth. What date will it be a week from Monday?
 - This is the seventeenth of April. Marc’s birthday was two weeks ago today. When was his birthday?
 - Ten days from yesterday will be the _____.
 - This is the third of the month. Explain why it is easy to find two weeks from today using the calendar.
 - What is the sixth month of the year?
 - January is the first month. Say the months in order and stop at the eleventh month.
- Ask pairs of students to create their own set of calendar questions involving ordinal language.

A2 Students should count (a) backwards and forward; (b) by 2s, 3s, 4s, 5s, 10s, 25s, and 100s; and (c) from various starting points, e.g., 30, 25, 20 ... or 60, 59, 58, 57 ...

It is through a wide range of activities presented regularly throughout the school year that students begin to develop number sense and number concepts, which form the foundation required for understanding more advanced mathematical concepts.

- Provide students with rhythmic skip counting activities. These include clapping, marching, drumming with hands on the desk, and striking instruments.
- Have students use their fingers to separate counters as they count by 2s or 3s.

Students should recognize how to count coins, e.g., for 3 quarters, 2 dimes, and a penny—25, 50, 75, 85, 95, 96. Before students are able to do this, they must practise counting nickels, dimes, and quarters separately. As well, they should come to recognize that to find the total for a collection of coins, sorting them and counting the larger coins first is easiest way.

Many number patterns can be used to encourage skip counting,

- e.g., 25, 50, _ , _ , 125, _ , _ , ...
 450, _ , 350, _ , 250, _ , ...
 95, 90, _ , _ , 75, _ , _ , _ , ...

GCO A: Students will demonstrate number sense and apply number theory concepts.

Worthwhile Tasks for Instruction and/or Assessment
Suggested Resources
Performance

A2.1 Begin to count, 25, 50, 75, 100, 125, 150. Ask students to continue to count this way to 500.

A2.2 Give students 4 quarters, 3 dimes, 2 nickels, and 6 pennies. Ask them to count the coins to find if an item costing \$1.50 can be bought.

A2.3 Provide coins for students. Ask, Can you use six of these coins to make 43 cents? Can you make a total of \$2.00 with six of these coins?

A2.4 Provide 60 beans for each student. Ask them to separate the beans from their piles as they count them by 2s, then by 3s.

A2.5 Provide students with play coins. Tell them that you have 5 coins in your hand that total 81 cents. Ask, What coins am I holding?

A2.6 Use the calculator counting constant to count by 2s, 5s, 10s, 25s, and 100s. Ask students to say each number as it appears on the display.

A2.7 Have students count the beans that you provided in jars. Observe or ask them how they grouped the beans to count them.

Interview

A1.1 Tell students that the 24 students in a class are standing in a row. Ask, If you were next to the last, in what position would you be? Which position would be the middle of the line?

A1.2 Show students calendars. Ask, Why is it easy to find the date which is two weeks from the sixth of the month?

A2.8 Ask students, Why do we say fewer numbers when counting to 100 by 10s than when counting by 5s?

A2.9 Ask students to decide which starting point, 3 or 6, is easier when counting by 2s to 30. Have them explain their choices.

A2.10 Ask students, When I say 25, 50, 51, 52, 53, what coins am I counting?

A2.11 Play *What's in the Can?* Tell students that you are going to drop nickels into a can. Have them listen as the coins drop and count to find the totals.

GCO A: Students will demonstrate number sense and apply number theory concepts.

Elaboration–Instructional Strategies/Suggestions

KSCO

- (i) By the end of grade 3, students will be expected to construct and communicate number meanings, and to explore and apply estimation strategies, with respect to whole numbers.

SCO

By the end of grade 2, students will be expected to

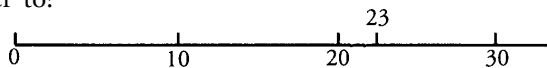
A3 estimate the size of numbers to the nearest multiple of 10

A3

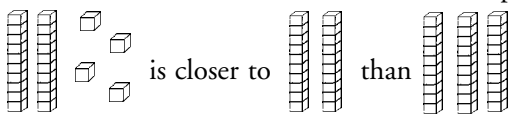
- Have students explore situations that require them to estimate in terms of the nearest multiple of 10, e.g.,
 - About how much money will you need to ... ?
 - About how many packages will you need to buy if ... ?
 - About how much time will it take to ... ?
 - About how many popsicles will we need to buy so everyone in the school will get at least two?

Note: Skill in rounding off is but one of the useful strategies for estimation.

- Use a number line as a visual model and ask, Which number is 23 closer to?



- Use base-10 material to model relationships such as



- Have students estimate the number of dots on a page, or the number of objects in pictures, e.g., use the strategy of dividing into sections of 25 to estimate the number of dots below.



- Present students with a situation in which they would get one dollar for every 10 flyers, or major portion thereof, that they deliver. Have them use a hundreds chart to calculate how much money they would earn in different neighbourhood, e.g., one in which there are 56 houses, one in which there are 34 houses, and one in which there are 119 houses.

GCO A: Students will demonstrate number sense and apply number theory concepts.

Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

Performance

A3.1 Using a number line to 100 with pictures of gas stations at the multiples of 10, place a car at various points on the line. Ask students to identify the location of the closest gas station.

A3.2 Ask students to use base-10 materials to show why 143 is closer to 140 than to 150.

A3.3 Tell students that you are trying to model the number 57 with base-10 blocks but have no unit cubes. Ask them to use the blocks to explain what closest number they would show.

A3.4 Show students a picture of 40 to 70 items, e.g., dots or hearts. Ask them to estimate the total and to explain how they arrived at their estimates.

A3.5 Display 40 to 70 beans, randomly placed on the table. Ask students to estimate how many there are and to explain their strategies.

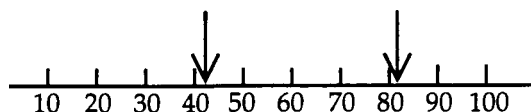
Interview

A3.6 Ask students to name five numbers that are about 100 and that are about 50.

A3.7 Explain to students that to estimate $48 + 33$ Jeff said, “I’ll use 50 and 30.” Ask them to explain Jeff’s reasoning.

A3.8 Ask students to name all the numbers that they could round to 80.

A3.9 Using a number line, put counters at various points. Ask students which mark on the number line is nearer each counter.



A3.10 Explain to students that Celia collected 262 autographs from students in her school. Her friend June said that Celia had about 300 autographs but her brother said that she had about 250. Ask, Which estimate do you think is better? Why?

GCO A: Students will demonstrate number sense and apply number theory concepts.

KSCO

- (ii) By the end of grade 3, students will be expected to concretely explore common fractions and decimals in meaningful situations.

SCO

By the end of grade 2, students will be expected to

- A4 identify simple fractions using models

Models must be used at all grade levels to develop fraction concepts adequately. Further ... children should have experiences with a wide assortment of models. (*Van de Walle 1994, 222–23*)

Elaboration–Instructional Strategies/Suggestions

A4 Students should investigate the more common fraction families such as halves, thirds, fourths, fifths, sixths, eighths, and tenths. Provide opportunities to explore other fractions in problem situations and in literature, e.g., *Eating Fractions* (MacMillan 2000).

It is important that the representation of *the whole*, *one whole* or *one* is clear so students understand what they are taking a part of. As well, the meaning of *fair share*, or *equal share*, should be stressed throughout. Students should explore various ways of finding fair shares of the same whole, e.g., various ways of fairly sharing a chocolate bar among four students.

Include situations that have different models: (a) *part of a whole*, e.g., $\frac{1}{2}$ of a chocolate bar or $\frac{2}{3}$ of a strip of paper; (b) *part of a set*, e.g., $\frac{3}{4}$ of the students in the class. Students should explore these models in a variety of contexts. Both meanings can be explored using pizzas, e.g., they eat $\frac{1}{4}$ of a pizza and $\frac{3}{8}$ of the slices have pepperoni.

Provide many opportunities to explore and discuss fractions orally before introducing the symbols. Continue to use language such as “1 of 3 equal parts” and help students connect the language with its symbol. This is the first time that fractions are presented symbolically. Fractions such as $\frac{1}{3}$ and $\frac{2}{5}$ are relatively easy for students to read since familiar ordinal language third and fifth is used for the denominator of each. Point out to students, however, that $\frac{1}{2}$ is read one-half (not one-second) and that $\frac{1}{4}$ may be read either one-fourth or one-quarter. The money application of four quarters make a whole dollar can be connected to this use of the word one-quarter.

To assist with clarity of meaning, always write fractions with a horizontal bar.

- Provide copies of pizza outlines divided into 10 pieces. Describe this problem to the students: You invite six friends to your pizza party. While you are making the pizza, you find out that two people do not like mushrooms, another two do not like pepperoni, and yet another person hates onions. Challenge pairs of students to draw and describe how they are going to distribute the ingredients on their pizzas. Have them express their ideas using fractions.

GCO A: Students will demonstrate number sense and apply number theory concepts.

Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

Performance

A4.1 Show students 15 marbles. Ask, If you were given one-third of them, how many would you have? Can you find one-half of the marbles? Why or why not?

A4.2 Ask students to show how to divide chocolate bars so that each of four people gets a fair share. Ask, What part of the whole bar would each person get? Encourage students to find alternative ways to divide the bar so that each person gets $\frac{1}{4}$.

A4.3 Provide paper shapes. Ask students to cut or fold them to show $\frac{1}{2}$ and $\frac{1}{4}$.



A4.4 Provide a number of different pictures and ask students to sort them into groups illustrating halves, thirds, fourths and fifths. Be certain to provide numerous examples of both part of a whole and part of a set, e.g., for thirds you might have pictures such as



Include some pictures that incorrectly represent the same fractions, e.g., a picture with three unequal parts.

Paper and Pencil

A4.5 Provide students with strips of circles.



Ask then to draw happy faces on $\frac{1}{4}$ of the circles.

Interview

A4.6 Ask students what fraction would suggest a fair share if four students were sharing a piece of cake.

A4.7 Ask students when they might talk about $\frac{1}{3}$.

A4.8 Ask students, Is $\frac{1}{2}$ a lot or a little? Why?

A4.9 Ask students, If you are really hungry and want a large piece of cake, would you cut the cake into thirds, fourths or tenths?

Portfolio/Presentation

A4.10 Have students ask family members when they use fractions, make lists, and share them with the class.

A4.11 Ask students to find and copy recipes that contain some fractions and that they could easily make.

GCO A: Students will demonstrate number sense and apply number theory concepts.

Elaboration–Instructional Strategies/Suggestions

KSCO

(iii) By the end of grade 3, students will be expected to read and write whole numbers and to demonstrate an understanding of place value (to four places).

SCO

By the end of grade 2, students will be expected to

A5 describe numbers in a variety of ways

A5 The development of number sense must be a focus of mathematics instruction. Number sense cannot be properly developed, however, without a sound understanding of place value. This develops slowly, so it is important that students experience a wide variety of worthwhile activities in order to build a firm foundation.

Many students who can state that 36 is 3 tens and 6 ones may simply be following a pattern that has been presented to them without a clear understanding of the concept of place-value, i.e., that it is convenient to group numbers consistently since we use only ten symbols, and the placement of a digit in a numeral determines its meaning.

It is important that students have many and varied experiences with materials that they can count and group in a variety of ways. These purposeful activities should be presented throughout the year as students develop their sense of number.

- Have students work in pairs with a given number of counters. Ask them to group the counters in several different ways and to record each grouping using charts, e.g., for 24 counters

Number of groups of five	Number left	Number of groups of eight	Number left	Number of groups of ten	Number left
4	4	3	0	2	4

- Plan a valuable daily activity called *Number of the Day*. Beginning with the first day of school in September, have students express the number in as many ways as they can, e.g., school day 26 may be expressed as 5 + 5 + 5 + 5 + 5 + 1, 20 + 6, 10 + 10 + 6, 10 + 16, 26 ones, 2 tens and 6 ones. By spring students will have had their *Day 100* celebration and will describe numbers like 139 as 25 + 25 + 25 + 25 + 25 + 10 + 4; five 25s and 14 more; 5 quarters, 1 dime and 4 pennies; 100 + 39; 139 ones; 13 tens and 9 ones; 1 hundred, 3 tens and 9 ones. It is not unreasonable to expect many students to express a number in 20 or 30 different ways. Activities such as these help students become comfortable with number and provide necessary practice towards the development of both number sense and the place-value concept.

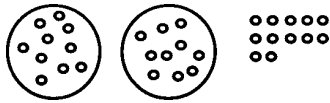
GCO A: Students will demonstrate number sense and apply number theory concepts.

Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

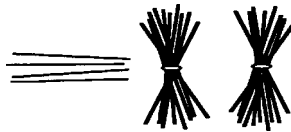
Performance

A5.1 Ask students, How many does this picture represent? How else might it be modelled?

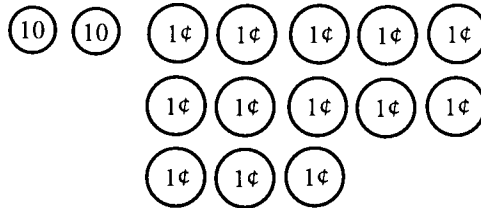


Encourage them to talk about the groupings. Do they recognize that each grouping represents the same number? Ask them to identify the grouping that they find the easiest to read and to explain why.

A5.2 Show students this picture. Ask questions similar to those in A5.1.



A5.3 Show students this money picture. Provide additional dimes and pennies for them. Ask them to represent this in other ways?



Paper and Pencil

A5.4 Ask students to write the number 43 in at least six different ways.

Interview

A5.5 Provide 26 counters and ask students to count them, recording the amount. Point at the digit 2 and ask them to show you that part using the counters. Do the same with the digit 6.

Portfolio

A5.6 Have students design a page on which they have expressed their favourite 2-digit number in a number of different and creative ways.

GCO A: Students will demonstrate number sense and apply number theory concepts.

KSCO

(iii) By the end of grade 3, students will be expected to read and write whole numbers and to demonstrate an understanding of place value (to four places).

SCO

By the end of grade 2, students will be expected to

A6 demonstrate an understanding of base-10 groupings

Elaboration–Instructional Strategies/Suggestions

A6 The early place-value focus must be on the development of an understanding of base-10 groupings and how these groupings are recorded. Students will come to understand how much easier it is to count when the numbers are grouped. It is important that this development not be rushed because later problems with number can often be traced back to a poor development of place value.

The foundation for the development of place-value centers around grouping activities. Students should have experiences with groupings of different sizes in preparation for the standard base-10 place-value system.

Activities must begin with proportional models so that students can see that the piece that represents the 10 is actually 10 times the size of the unit piece. These can be groupable materials such as unifix cubes or pregrouped materials such as bean sticks (ten beans glued to a popsicle stick or tongue depressor) and base-10 blocks.

- Have students use a die to play *Race for a Ten*. They roll the die and count out the number of unit cubes. When they get exactly ten, they trade them in for a base-10 rod. Play *Race For A Fifty* or *Race for a Hundred* using the same rules.
- Direct students to count the children in the class, represent the number symbolically, and model the number using base-10 blocks.
- Have students model 46 and other 2-digit numbers in various ways, e.g., 46 as 46 ones, 4 tens and 6 ones, or 3 tens and 16 ones. Note: The last way is useful for regrouping in the subtraction algorithm.

Once students understand the grouping concept for 2-digit numbers, grouping for 3-digit numbers should not be difficult as they learn to trade 10 rods for a flat to make 100. Students should realize that both 10 tens (10 rods), 100 ones (100 unit cubes), and one hundred (1 flat) are three representations of 100.

Column headings such as Ones, Tens, and Hundreds on place-value mats and recording sheets should not be used because these headings can be misleading.

GCO A: Students will demonstrate number sense and apply number theory concepts.

Worthwhile Tasks for Instruction and/or Assessment

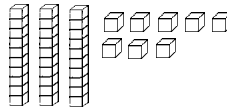
Suggested Resources

Performance

A6.1 Observe students as they play *Race for a Hundred*. Question them to assess their understandings of the trading concept.

A6.2 Give students a number of counters. Ask them to make groupings of 10 counters and to represent this number both symbolically and with base-10 materials.

A6.3 Show students a number using base-10 blocks. Ask them to show a list of numbers that might be rolled on a die to add this number to reach a total of 50. Have them show and explain the trading.



A6.4 Show students numbers using base-10 blocks. Ask them to show at least one other way of representing the numbers with base-10 blocks.

A6.5 Provide a place-value mat and base-10 blocks. Have students work in pairs rolling a die, counting out the ones, and trading for tens. Observe the students as they work.

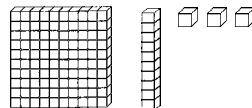
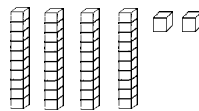
Number of groups of ten	Number left

- Do they use counting-on or counting-from-the-start?
- Do they arrange the unit cubes for easy recognition?
- Have they progressed to using mental strategies?

With continued practice, more sophisticated strategies surface, e.g., If 5 is on the ones side and a 6 is rolled, they might clear all but one of the units and add the rod right away without making the 6. Not all students are able to do this; however, those who can should not be discouraged from proceeding in this way as they have clearly grasped the trading concept.

Interview

A6.6 Show students some numbers modelled with base-10 materials. Ask, What do the models represent? How do you know the second one has a greater value than the first one?



GCO A: Students will demonstrate number sense and apply number theory concepts.

Elaboration–Instructional Strategies/Suggestions

KSCO

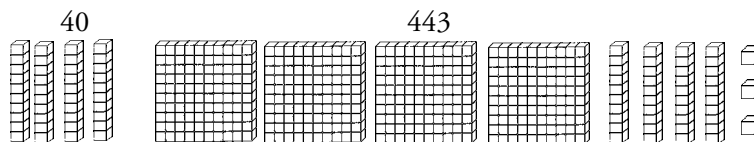
(iii) By the end of grade 3, students will be expected to read and write whole numbers and to demonstrate an understanding of place value (to four places).

SCO

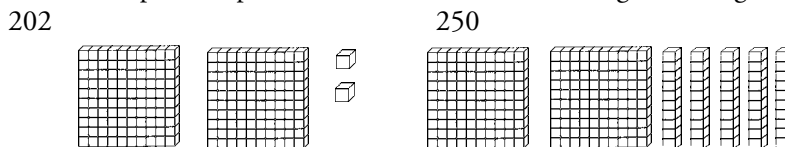
By the end of grade 2, students will be expected to

A7 model numbers to three places

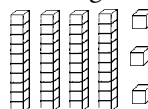
A7 Students should be able to create and interpret models representing either 2-digit or 3-digit numbers, using proportional materials, e.g.,



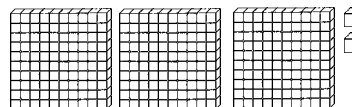
Be sure to provide practice with numbers containing zeros, e.g.,



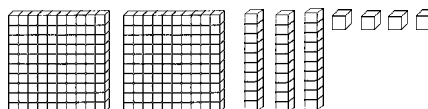
Students should express numbers symbolically, verbally, and with models. They should be expected to translate from any one to both of the other representations, e.g., 43 is the symbol for forty-three and is modelled as



When shown a model like the one to the right, students should be expected to write 302 and say the number as *three hundred two*.



When hearing two hundred thirty-four, students should be able to write 234 and model it as



It is important that students realize a number can be expressed in different ways, e.g., 234 can mean 2 hundreds, 3 tens and 4 ones; 23 tens and 4 ones; or 234 ones.

- Have students explore all the numbers that can be modelled given a fixed number of base-10 blocks, e.g., six base-10 blocks can represent 600, 501, 510, 420, 402, 411, ...

The key instructional tool for developing the conceptual knowledge of place value and also for connecting these concepts to symbolism is the use of base-10 models. (*Van de Walle 1994, 157*)

GCO A: Students will demonstrate number sense and apply number theory concepts.

Worthwhile Tasks for Instruction and/or Assessment

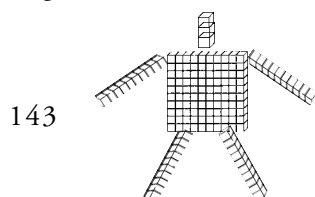
Suggested Resources

Performance

A7.1 Ask students to use base-10 materials to show 314 in three different ways.

A7.2 Ask students to model numbers that contains more hundreds than tens but more ones than hundreds. Once the models are complete, have students write the numbers that the models represent.

A7.3 Invite students to make robots with base-10 blocks and to symbolize its value, e.g.,



A7.4 Ask students to model five hundred one and five hundred ten, write the numbers symbolically, and discuss the similarities and differences between these two numbers.

A7.5 Show students numbers written symbolically. Ask them to read the numbers and model them with base-10 materials.

Paper and Pencil

A7.6 Ask students to write numbers symbolically as you read them. Include examples that have zeros in their symbolic representation.

Interview

A7.7 Ask students to describe how much 100 is.

A7.8 Show four base-10 flats, 15 rods, and six units. Ask, What number is this showing? Can you model it in another way?

A7.9 Explain to students that Ann said that there are 3 tens in 135 while Jeff said that there are 13 tens. Ask, Do you agree with Ann or Jeff? Why?

A7.10 Ask students, How is 500 like 50 and how is it different?

A7.11 Ask students, Why is 234 not the same as 324?

GCO A: Students will demonstrate number sense and apply number theory concepts.

KSCO

- (iv) By the end of grade 3, students will be expected to order whole numbers and represent them in multiple ways.
- (v) Students will be expected to apply number theory concepts (e.g., place-value pattern) in meaningful contexts with respect to whole numbers and commonly used fractions and decimals.

SCO

By the end of grade 2, students will be expected to

- A8** compare and order numbers by size
- A9** recognize, extend, and create simple place-value patterns

Number sense takes a long time to develop; even adults continue to grow in this ability. To establish an environment that nurtures the development of number sense in children, you will need to provide interesting questions to explore, suitable materials with which to investigate those questions, and a classroom climate that encourages the discussions and the display of the results of mathematical investigations. (*NCTM* 1992, 10)

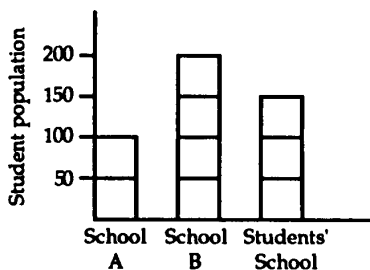
Elaboration–Instructional Strategies/Suggestions

A8 Students should be able to compare two or more numbers, each less than 1000, to determine relative sizes. Include situations in which numbers are located on hundreds charts and on number lines.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Students should be shown how to use the symbols $<$ and $>$ to describe the relative sizes of numbers. They should be able to name numbers greater than, less than, or between given numbers.

- Provide groups of students with small pumpkins. Ask them first to estimate the number of seeds in their pumpkins, then to remove and count the seeds. Have the groups arrange their data on a single graph for easy comparisons.
- Have students compare the numbers of students in several schools to see which have populations greater or less than their school. The data can be arranged on a graph for easy comparisons.



A9 Provide students with a variety of number patterns based on place-value relationships, e.g., 28, 38, 48, 58, ... ; 47, 147, 247, ... ; 258, 248, 238, ... ; 312, 412, 512, ... ; 512, 502, 492, ... Have them extend these patterns and provide opportunities for them to create similar patterns.

GCO A: Students will demonstrate number sense and apply number theory concepts.

Worthwhile Tasks for Instruction and/or Assessment
Suggested Resources
Performance

A8.1 Ask students to use base-10 materials to show why 243 is less than 324.

A8.2 Ask students to find numbers between 312 and 387 that can be represented using eight base-10 blocks.

Paper and Pencil

A8.3 Ask students to use the symbols 5, 2, 4, 3, <, and > to create two true number sentences.

A8.4 Ask students to fill in the box in different ways to make the statement true. $3 \square 4 < 352$

A8.5 Ask students, How many different ways can the following blanks be filled to make a true statement?

$$1 \square 4 < 17 \square$$

A9.1 Have students continue each of the patterns below by adding three or four numbers to the list.

32, 42, 52, ...

378, 388, 398, ...

Interview

A8.6 Ask students, What do you do to compare the size of two numbers?

A8.7 Ask students, Why are there more numbers greater than 123 than less than 123?

A9.2 Ask students, What number do you think comes tenth in each pattern below? twentieth?

141, 142, 143, ...

141, 151, 161, ...

**Number Concepts/
Number and Relationship
Operations:
Operation Sense and
Number Operations**

GCO B: Students will demonstrate number sense and apply operation principles and procedures in both numeric and algebraic situations.

KSCO

- (i) By the end of grade 3, students will be expected to demonstrate an understanding of the connection between relevant, concrete experiences and the mathematical language and symbolism of the four basic operations.

SCO

By the end of grade 2, students will be expected to

- B1** recognize that multiplication can be used to determine the total amount in groups of equal size
- B2** recognize that division can mean determining how many groups of a fixed size are in a larger group or fair sharing

For all four of the operations, models (usually sets of counters and number lines) and word stories or word problems are the two basic tools the teacher has to help students develop operation concepts. (*Van de Walle* 1994, 109)

Elaboration–Instructional Strategies/Suggestions

B1 Multiplication should be treated informally, dealing strictly with the concept of putting together groups of equal size, not the memorization of facts. Encourage students to use language that makes sense to them, e.g., 3 groups of 2, 3 sets of 2, 3 piles of 2 as opposed to three times two. Teachers should model such language, e.g., I have four groups of five; that’s twenty.

These early experiences with the multiplication concept should include contexts that naturally lend themselves to forming sets, e.g., Jose had to put 5 chocolate chip cookies on each plate. There were 4 plates. How many cookies did he need? The concrete and pictorial representations of such story problems should be connected to skip counting, e.g., 5, 10, 15, 20, and to repeated addition, e.g., five plus five plus five plus five.

Regular practice with skip counting, including using the hundreds chart and the calculator constant, will help students efficiently find totals in multiplication situations.

- Ask students to arrange themselves into groups of four. Have them skip count the number of children in the class. Try groups of three, five, etc.
- Tell the students that each member of their group requires four pieces of paper. Have them distribute the paper and tell how many pieces they needed in all. Observe how they count.

B2 Similarly, it is important to focus on the meaning of division rather than its symbolization. Contexts for division should include (a) ones where the size of the groups is unknown—*fair sharing*, e.g., If 24 cookies are equally distributed on 4 plates, how many are on each plate? (b) ones where the number of groups is unknown, e.g., If you start with 24 cookies and put 4 on each plate, how many plates will you need? Situations might or might not involve leftovers/reminders, e.g., How many packs of 2 cookies can you make out of a group of 24? 23?

While discussing solutions to story problems, help students make the connection between multiplication and division. As well, often present situations involving both concepts.

- Provide students with a certain number of stickers and have them share them equally with each member of their group. Ask, How many stickers did each person get?

GCO B: Students will demonstrate number sense and apply operation principles and procedures in both numeric and algebraic situations.

Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

Performance

B1.1 Ask students to count out eight raisins for each member in their groups. How many raisins are there altogether? Encourage students to use a hundreds chart if they have difficulty adding eights or skip counting by eight.

B1.2 Tell students that you have bought a number of packages, each containing 3 juice boxes. Ask them to model the problem and tell you how many boxes you might have.

B1/B2.1 Simulate a bake sale by providing trays and cutout squares and cookies. Have students group the sweets on the trays for the sale, maybe 4 squares per tray and half a dozen cookies per tray. Encourage students to talk about what they are doing and to use informal multiplication and division language.

B2.1 Provide each student with 12 counters. Ask students to think of them as a dozen eggs. Have them model different ways to divide up the eggs so that all the groups formed will have the same size.

Paper and Pencil

B1.3 Ask students, Which of the following situations could be described using multiplication? Why?



B1.4 Ask students to draw pictures showing why the number of wheels on 4 bicycles is the same as on 2 cars.

Interview

B1.5 Explain to students that you might say 2, 4, 6, 8, ... when counting boots. Ask, How high would you have to count if there were 8 pairs of boots? Why?

B2.2 Present a situation in which there are 15 pencils to be put into piles with the same number in each pile. Ask, How many piles might there be and how many pencils would be in each pile?

Portfolio

B1/B2.2 Ask students to create a series of problems about bicycle and tricycle wheels, identifying which ones involve multiplication and which ones involve division.

GCO B: Students will demonstrate number sense and apply operation principles and procedures in both numeric and algebraic situations.

KSCO

- (ii) By the end of grade 3, students will be expected to recognize and explain the relationships among the four basic operations.
- (iii) Students will be expected to create and model problem situations involving whole numbers, using one or more of the four basic operations.

SCO

By the end of grade 2, students will be expected to

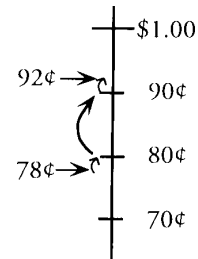
- B3** demonstrate an understanding that addition can be used to solve subtraction problems and vice versa
- B4** create word problems involving addition and subtraction

[A] very important way to help children construct operation meanings is from word problems or word stories. The word problem provides an opportunity for examining a much more diverse set of meanings for each operation. Models remain a critical part of the development. (Van de Walle 1994, 110)

Elaboration–Instructional Strategies/Suggestions

B3 While students have had previous experiences with many types of addition and subtraction situations, many will still need to be convinced that subtraction is a way to solve *comparison* and *missing-addend* problems.

- Present this problem: Paul has 78 cents and his brother has 92 cents. How much more money does Paul’s brother have? Provide a visual aid for students. Many students naturally count up from 78, thinking $78 + \underline{\quad} = 92$. Through problems like this one, they will come to see that subtraction would efficiently find the solution, e.g., $92 - 78 = \underline{\quad}$.



Using the missing-addend meaning, students see the relationship between addition and subtraction and how subtraction problems can be solved by adding. Teachers can model this connection by reuniting parts in an addition sentence after solving a subtraction sentence and vice-versa.

B4 Creating word problems based on contexts of interest requires students to think about the meaning of the operations more thoroughly than does simply solving problems. Although creating word problems may seem difficult to some students at first, facility will increase with experience. Students should be encouraged to dramatize the situations because this is very helpful in clarifying their thinking. When dealing with subtraction, encourage students to create problems involving all three meanings: take away, comparison, and missing addend.

- Invite groups of students to choose books which have addition or subtraction contexts that they especially like. Give them time to prepare dramatizations of the books, changing whatever variables they like, and to present them. Their presentations should include a number of problems for others to solve.
- Provide activities involving the use of money to purchase items, e.g., Fran had 25 cents. She spent 16 cents. How much change does she get back? Encourage students to explain how they go about solving the problem, e.g., A student might say, “16 and 4 more is 20, plus 5 is 25. She gets 9 cents change. or 16 cents and 10 cents is 26 cents, so she gets only 9 cents back.”

GCO B: Students will demonstrate number sense and apply operation principles and procedures in both numeric and algebraic situations.

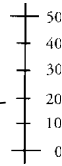
Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

Performance

B3.1 Explain to students that Eric had two quarters and he bought a package of baseball cards for 39 cents. Ask them to explain, using coins provided, how to determine how much change Eric got back.

B3.2 Tell students that Janet read 18 books and Frederica read 42. Ask them to use the number chart shown to explain how to find the difference. Ask them for addition and subtraction number sentences that represent this problem.



B3.3 Present this story to students: Jake had some marbles to play with at recess. During recess, he lost 28 of them, returning to class with only 17 marbles. How many marbles did he have at the start of recess? Ask students to write the number sentence that they think would describe this story. Have them share their number sentences to see if both addition and subtraction sentences were used. If so, lead a discussion of the connection between them.

Paper and Pencil

B3.4 Ask students to write the addition fact that would help them solve $18 - 9 = \square$ and $50 - \square = 20$

B3.5 Tell students that $15 + 25 = 40$. Ask, What subtraction questions might this information be used to answer?

Interview

B3.6 Tell students that someone told you that you do not have to learn to subtract if you know how to add. Ask, Do you agree? Why or why not?

B4.1 Ask students to create subtraction problems involving the numbers 23 and 12.

B4.2 Ask students to create word problems about the heights of two students.

Presentation

B4.3 Have groups of students create and present skits involving a number of addition and subtraction situations.

GCO B: Students will demonstrate number sense and apply operation principles and procedures in both numeric and algebraic situations.

KSCO

- (iv) By the end of grade 3, students will be expected to demonstrate proficiency with addition and subtraction facts.

SCO

By the end of grade 2, students will be expected to

- B5** develop and apply strategies to learn addition and subtraction facts
- B6** recall addition facts involving two addends, each less than 10, and the related subtraction facts

The widespread availability of calculators in no way diminishes the importance of basic fact mastery; quite the contrary. With the shift in emphasis from pencil-and-paper computation to mental computation and estimation skills, command of the basic facts is more important than ever. (*Van de Walle 1998, 140*)

Elaboration–Instructional Strategies/Suggestions

B5 Help students develop and apply a variety of strategies that will help them learn their basic facts. Each of these strategies should be highlighted, focussing on its concrete representation to help students understand the strategy. Each strategy should be practised in isolation and then combined with other strategies. Some facts can be found by using more than one strategy. The following are some of the possible strategies that could be used: (a) *Doubles* These facts from $0 + 0$ to $9 + 9$ seem to come quite naturally to most students and are the foundation for strategies to learn other facts. (b) *Near doubles* These facts involve addends that are one apart, e.g., $5 + 6$, $6 + 7$, $8 + 7$, and can easily be found by doubling the smaller and adding 1, e.g., $4 + 5$ is $4 + 4 + 1 + 9$. (c) *Two apart* These facts involve addends that are two apart and can be found by doubling the number between them, e.g., $5 + 7 = 6 + 6 = 12$. (d) *Make 10* These facts involve addends of 8 or 9 and can be found by using part of the second addend to build up 8 or 9 to make 10, then adding on the rest of the second addend, e.g., $8 + 5 = (8 + 2) + 3 = 10 + 3 = 13$. (See *Van de Walle 1994.*)

Similarly for subtraction facts, students should be exposed to a variety of strategies. The following are some of the possible strategies: (a) *Think addition* This strategy makes use of the connection between addition and subtraction, e.g., For $16 - 9$, start with 9 and think what you must add to get to 16. (b) *Bridging to 10* This strategy involves subtracting in stages by going back to 10 first and then subtracting the rest, e.g., For $17 - 9$, start by thinking that $17 - 7$ is 10 and then subtract the 2 left from the 9 to get 8. (c) *Subtract a 10* This strategy is useful when subtracting 8 or 9 by subtracting 10 and compensating, e.g., For $17 - 9$, start by thinking that $17 - 10$ is 7 and then 10 is 1 more than needed to be subtracted, so the answer is 8. (d) *Constant difference* This strategy makes use of the relationship that if the same amount is added to both numbers in a subtraction situation, the difference between them is preserved, e.g., For $16 - 9$, start by adding 1 to both 16 and 9 to make $17 - 10$ which is easy to subtract to get 7.

B6 Facts with a sum to 10 should be easily recalled by midyear. By year-end, most students should be able to recall most addition facts to 18. It is important to provide opportunities for practice using games and meaningful contexts as much as possible, rather than only requiring fact learning in isolation, e.g., board games in which students find the sum of 2 dice.

GCO B: Students will demonstrate number sense and apply operation principles and procedures in both numeric and algebraic situations.

Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

Performance

B5.1 Have students play a game of concentration. Provide 18 cards with 9 different sums—2 different facts for each sum—that will have to be matched, e.g., $9 + 5$ and $7 + 7$; $6 + 1$ and $0 + 7$. Have students turn over cards to find matches. If they match, they stay face up; if they don't match, they are turned back over. Students try to remember the ones they've seen for future matches.

B6.1 Have students work in pairs to sort addition facts into groups of facts that are related. Give them time to share their sorting rules.

Paper and Pencil

B5.2 Provide price lists with items costing 18 cents or less. Ask students, Which items could you buy with two dimes? How much would be left over after each purchase?

Interview

B5.3 Ask students, How does knowing $4 + 4 = 8$ help you to know other facts?

B5.4 Ask students for the answers to several facts and get them to explain any strategies they might have used.

B6.2 Ask students 15 addition facts allowing about 3 seconds for each response.

Portfolio

B5.4 Have students describe in as many ways as they can why $15 - 8 = 7$.

Note: Fact learning is a mental exercise with an oral and/or visual prompt; therefore, the focus is on oral and not paper-pencil practice. Also, drills should be short with immediate feedback, e.g., 3 - 5 minutes per day, over an extended period of time. Ultimately, each student may be “doing their facts” in unique ways; this could be a combination of visualization, quick strategy application, and memory. It really does not matter how students recall the facts as long as recall is immediate, i.e., within about three seconds.

GCO B: Students will demonstrate number sense and apply operation principles and procedures in both numeric and algebraic situations.

Elaboration–Instructional Strategies/Suggestions

KSCO

- (v) By the end of grade 3, students will be expected to apply computational facts and strategies with respect to the four basic operations and to model addition and subtraction in situations involving whole numbers.

SCO

By the end of grade 2, students will be expected to

- B7** demonstrate an understanding of basic principles of addition
- B8** add 3 single-digit numbers

B7 Understanding the basic principles of addition allows students greater flexibility in their thinking. Recognizing and applying these principles also reduces the number of facts that students need to learn, e.g., understanding *commutativity* of addition reduces 100 facts to 55 facts. When dealing with this principle, e.g., $3 + 4 = 4 + 3$, it is helpful for students to use visual models to physically reverse the order of the two groups.

Students need to understand the patterns of adding 1 and 10 to numbers which are greatly enhanced by using a hundreds chart. As well, students should recognize that adding 0 to a number has no effect.

- Invite students to participate in a game of mental math that involves adding 1 or 10 each time, e.g., begin with 3, add 10, 10, 1, 10, 1, 1. Although some students may be able to do the computation entirely in their heads, others may require a hundreds chart to follow the pattern of numbers. It is through this practice that students will begin to visualize the position of numbers in the base-10 system, developing their number sense.

Understanding *associativity* of addition (arranging 3 or more addends in a different order) is useful when students are adding three single-digit numbers, e.g., $5 + 3 + 5$ is easier when thought of as $5 + 5 + 3$. It is not intended that students learn these terms (*commutativity* and *associativity*); rather, that they understand and apply the principles.

B8 When finding totals for 3 addends, regrouping into 5s or 10s is often useful for easier handling, e.g.,

could become

$$\begin{array}{c}
 3 \quad + \quad 2 \quad + \quad 7 \\
 | \quad / \quad \backslash \\
 3 + 2 + 5 + 2 \\
 \vee \quad / \quad \backslash \\
 5 + 5 + 2
 \end{array}
 \quad \text{OR} \quad
 \begin{array}{c}
 3 \quad + \quad 2 \quad + \quad 7 \\
 \backslash \quad / \quad / \\
 10 + 2
 \end{array}$$

Encourage students to make decisions about which sequence to use when adding a series of digits, e.g., To add $3 + 8 + 7$, many students might add 3 and 7 together, then add on 8.

Students should investigate real-life situations that require them to add 3 numbers (each less than 10) to get a total, e.g., the sum of 3 dice, the number of members in 3 students' families, the number of points scored in 3 games.

GCO B: Students will demonstrate number sense and apply operation principles and procedures in both numeric and algebraic situations.

Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

Performance

B7.1 Ask students to use materials to show why the units digit does not change when 10 is added to a number.

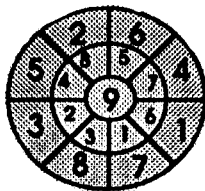
B7.2 Ask students to explain, using models, why they know that $3 + 4$ has to equal $4 + 3$ even before finding the sums.

B8.1 Ask students to use models to find as many different ways as they can to make a total of 10 using 3 numbers.

B8.2 Have students work in pairs. Present a series of questions involving the sum of single digit numbers. Have one student determine the sums using a calculator, while the other calculates mentally. The object is to determine who gets the answer first. Have students reverse roles at some point. Include sums that encourage students to look for combinations that lend themselves to efficient calculation, e.g., $8 + 6 + 2$ and $5 + 5 + 4$.

Pencil and Paper

B8.3 Explain to students that someone throws three darts and they all land on the board. Ask them to find five possible total scores.



B8.4 Ask students to find, in as many ways as possible, three numbers that total 6.

Interview

B7.3 Ask students, Why is it easy to add 0 and 1 to numbers?

B8.5 Ask students, Why is it easy to add the numbers $5 + 5 + 6 + 4 + 8 + 2$?

Presentation

B8.6 Ask students to explain to classmates the way they would add $5 + 9 + 1 + 5$.

GCO B: Students will demonstrate number sense and apply operation principles and procedures in both numeric and algebraic situations.

Elaboration–Instructional Strategies/Suggestions

KSCO

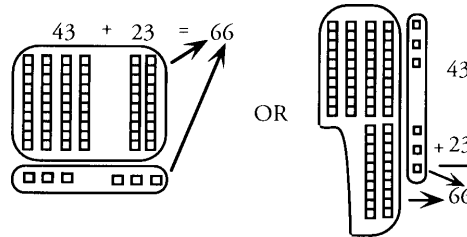
- (v) By the end of grade 3, students will be expected to apply computational facts and strategies with respect to the four basic operations and to model addition and subtraction in situations involving whole numbers.

SCO

By the end of grade 2, students will be expected to

- B9** model and perform the addition of two 2-digit numbers, with and without regrouping

B9 Using models is helpful for students to relate the physical action of joining two groups to the symbols, e.g.,



Students should be encouraged to estimate first, starting with the tens. Adding in this way also strengthens place-value concepts, e.g., When adding 43 and 23, a student might start by saying, “40 plus 20 is 60.”

By writing number sentences horizontally, you can encourage more divergent thinking, e.g., a student might think of $31 + 52$ as

$31, 41, 51, 61, 71, 81, 82, 83$	or	$31, 32, 33, 43, 53, 63, 73, 83$
$(31 + 5 \text{ tens} + 2)$		$(31 + 2 + 5 \text{ tens})$
$52, 62, 72, 82, 83$		$50, 60, 70, 80, 82, 83$
$(52 + 3 \text{ tens} + 1)$		$(5 \text{ tens} + 3 \text{ tens} + 2 + 1)$

- Have students use hundreds charts to perform 2-digit additions, e.g., For $32 + 45$, they would start at 32 and move 4 squares down and 5 right.
- Provide examples of situations in which students will have to devise methods for further regrouping, e.g., There were 55 M & Ms in one bag and 58 in another. How many were there in the two bags? Ask them to model and explain their solutions.

In the activity above, some students will combine the 2 fifties to make 10 tens, or a hundred flat, then add the 5 and 8 together to get 13, for a total of 113. Others may add the units digits first and recognize that they can make a ten to add to their tens digits and have 3 extras. Encourage and support the use of a variety of strategies.

By looking at addition conceptually, rather than only procedurally, you will find that many students will have no more difficulty with computations involving regrouping than with those that do not. Once students choose to use a procedure, it is important for you to accept the different choices, e.g., In an addition, some students might add from the left, others from the right.

GCO B: Students will demonstrate number sense and apply operation principles and procedures in both numeric and algebraic situations.

Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

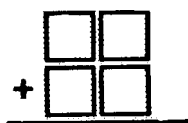
Performance

B9.1 Ask students to use bundles of ten Popsicle sticks and single sticks to solve this problem: Donna had 42 marbles before recess. She was lucky and won 26 marbles during recess. How many marbles did she have after recess?

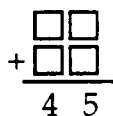
B9.2 Ask students to select six base-10 blocks (any combination of units and rods). Have them add four more blocks. Ask students, What different answers can you get and what addition sentence goes with each one?

Paper and Pencil

B9.3 Ask students to arrange the digits 2, 3, 4, and 5 in the boxes to have the greatest possible sum.

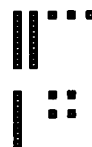


B9.4 Have students fill in the boxes below in at least four ways so that the addition will be true.



Interview

B9.5 Using base-10 blocks, model for students the addition shown on the right. Ask, What addition question is being modelled?



B9.6 Tell students that to add 36 and 29, Jake said, “30 and 20 is 50 and 10 more is 60, plus 5 is 65.” Ask them to explain where Jake got the numbers in this description of his thinking.

B9.7 Tell students that to add 36 and 29, Sara said, “36, 46, 56, 66, 65. The answer is 65.” Ask them to explain what Sara was doing.

B9.8 Show students a variety of addition questions and ask them to indicate the ones that require regrouping.

Portfolio

B9.9 Have students prepare displays showing, as many ways as they can think of, how to perform an addition of their choice.

GCO B: Students will demonstrate number sense and apply operation principles and procedures in both numeric and algebraic situations.

Elaboration–Instructional Strategies/Suggestions

KSCO

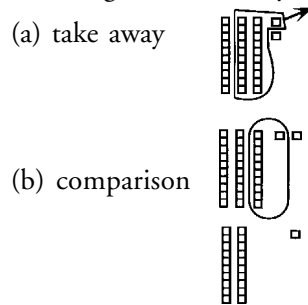
- (v) By the end of grade 3, students will be expected to apply computational facts and strategies with respect to the four basic operations and to model addition and subtraction in situations involving whole numbers.

SCO

By the end of grade 2, students will be expected to

- B10** model and perform the subtraction of two 2-digit numbers, with and without regrouping

B10 Use models to relate the physical action of separating or comparing two groups to the symbols in subtraction computations, e.g., $32 - 21$ may be modelled as



It is important for students to estimate first. As with addition, when using the *front-end* method to estimate, it is often just as easy to get the actual answer as it is to estimate.

When students are recording their work, they may use any one of a variety of methods. All of these may be acceptable general procedures if they reflect understanding of the subtraction concept. It is important to honour all expressions of correct reasoning. Some of these procedures include:

A. $\begin{array}{r} 45 \\ -27 \\ \hline 18 \end{array}$	B. $\begin{array}{r} 45 \quad 25 \\ -27 \quad \rightarrow -7 \\ \hline 18 \end{array}$	C. $\begin{array}{r} 31 \\ \cancel{45} \\ -27 \\ \hline 18 \end{array}$	D. $\begin{array}{r} 45 \\ -27 \\ \hline 18 \end{array}$
--	--	---	--

Student A thinks, “Well, $40 - 20$ is 20 . I’ll take 1 of the leftover tens and add it to the 5 ones, so there’s 1 ten left and $15 - 7 = 8$.”

Student B thinks, “ $45 - 20 = 25$. I still have to subtract 7. $25 - 7 = 25 - 5 - 2$, which is 18.”

Student C thinks, “I can’t take 7 from 5. So I’ll take 1 of the 4 tens, leaving 3, and subtract 7 from that 10. That’s 3 to add to the 5 already there, to make 8 ones. Then 2 tens from 3 tens is 1 ten.”

Student D thinks, “What do I add to 7 to end in a 5? It’s 8. That gives 15; write down the 1; regroup the 10 ones to 1 ten. What do I add to 2 tens and 1 ten to get 4 tens? 1 ten.”

This computation might also have been done by subtracting 30 from 45 and then adding on the extra 3 or it might have been done by taking 30 from 48, using the *constant-difference* strategy. In short, there are many ways to approach a subtraction question.

GCO B: Students will demonstrate number sense and apply operation principles and procedures in both numeric and algebraic situations.

Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

Performance

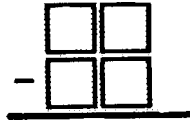
B10.1 Have students conduct class yard sales for which they bring in items priced under a dollar. Have every student start with a dollar and choose a banker who will make change. Have students record balances left after purchases using subtraction sentences.

B10.2 Ask students to use bundles of 10 Popsicle sticks and single sticks to solve this problem: Elaine has 46 marbles and Donna has 22 marbles. How many fewer marbles does Donna have?

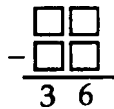
B10.3 Tell students to begin with six base-10 blocks (any combination of units and rods). Have them remove any four blocks. Ask, What answers are possible and what subtraction goes with each possibility?

Paper and Pencil

B10.4 Ask students to arrange the digits 2, 3, 4, and 5 in the boxes to have the greatest possible difference.



B10.5 Have students fill in the boxes below in at least four different ways so that the subtraction will be true.



Interview

B10.6 Ask students, How are the answers to the following questions related?

43 - 21 53 - 21 63 - 21 63 - 31

B10.7 Explain to students that Caroline was subtracting two numbers and said, “48, 38, 35.” Ask, What might the numbers have been that she was subtracting? Why?

B10.8 Have students explain two different ways to solve 53 - 24.

Portfolio

B10.9 Have students prepare a display showing, as many ways as they can think of, how to perform subtraction questions of their choice.

GCO B: Students will demonstrate number sense and apply operation principles and procedures in both numeric and algebraic situations.

Elaboration–Instructional Strategies/Suggestions

KSCO

(vi) By the end of grade 3, students will be expected to apply estimation techniques to predict, and justify the reasonableness of, results in relevant problem situations involving whole numbers.

SCO

By the end of grade 2, students will be expected to

B11 estimate the sum or difference of two 2-digit numbers

B11 Students should view estimation as a way to get a sense of the size of a quantity, rather than as just another set of rules. You should provide the needed practice so that students intuitively and automatically estimate, not just doing so when you or textbooks require it. Regular questioning will help students to focus on this skill, e.g. Is your answer reasonable? How do you know?

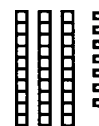
Rounding each number to the nearest multiple of 10 is just one way to estimate, e.g., For $46 + 35$, students just using a rounding rule might estimate the sum as $50 + 40 = 90$, whereas those who increase one number and decrease the other ($50 + 30 = 80$) have a better estimate.

When estimating $45 + 36$, students might choose to use the *front-end* method and think, “ $40 + 30$ is 70 , the units make about another 10 , for an estimate of 80 .”

Sometimes students may select a combination of strategies, e.g., For $48 + 27$, they might combine *rounding* and *front-end* strategies and think, “ $50 + 27$ for a total of 77 .”

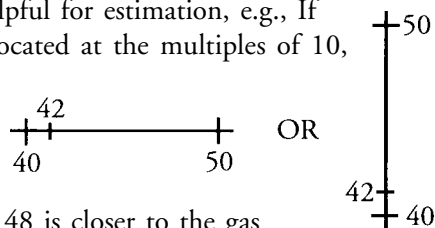
Eventually, with sufficient practice, the various estimation skills will become automatic.

Using base-10 blocks is helpful for students as they begin estimating, e.g., If students visualize the estimate of a 2-digit number as a set of rods only, they might see that 37 (3 rods and 7 ones) is closer to 4 rods than to 3 rods.

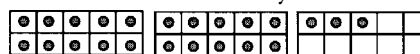


Similarly, visualizing the position of number on hundreds charts can help some students see the nearest multiple of 10.

Using a number line is also helpful for estimation, e.g., If students imagine gas stations located at the multiples of 10, they can put an imaginary car on any given number and determine the closer gas station. 42 is closer to the gas station at 40 than 50, whereas 48 is closer to the gas station at 50. 45 can be estimated by either 40 or 50.



Using ten-frames for relatively small numbers may also be used for estimation, e.g., Displaying 23 on ten-frames clearly shows that it is closer to 20 than to 30.



GCO B: Students will demonstrate number sense and apply operation principles and procedures in both numeric and algebraic situations.

Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

Performance

B11.1 Ask students to use toy cars on number lines to estimate the sum of $48 + 37$.

B11.2 Provide counters, ten frames, number lines, and base-10 blocks. Ask students to choose models to explain how they would show a grade 1 student that 27 is closer to 30 than to 20.

Paper and Pencil

B11.3 Tell students that two numbers have been added and the sum is about 40. Ask them to list 4 possible pairs of numbers that might have been added.

B11.4 Ask students to estimate each of the following. Have them explain, without actually finding the answer, if their estimates are probably too high or too low.

$$48 + 57 \qquad 39 - 18 \qquad 60 - 38$$

Interview

B11.5 Tell students that a number between 30 and 40 is added to a number between 40 and 50. Ask, What might be a good estimate for the answer? Why?

B11.7 Ask students to explain why a good estimate for a subtraction might be greater than the actual answer sometimes, but less other times. Have students use examples to help them explain.

B11.8 Tell students that José thinks she will need about 33 hot dogs for her party. Ask, If the hot dogs come in packages of 10, how many packages should José buy? Explain your answer.

B11.9 Tell students that Jason knew there were 35 members in his Campers' Club and about 28 in the club in the neighbouring town. When asked to estimate the number of name tags to make for members of both clubs, Jason said, "I think I should make 65." Ask, How do you think Jason estimated? Was it a good estimate?

Note: Estimation is usually a mental process. Consequently, students need to be able to mentally calculate sums involving numbers that are multiples of 10, e.g., they should be able to determine $30 + 50$ by thinking "3 tens plus 5 tens makes 8 tens, or 80."

GCO B: Students will demonstrate number sense and apply operation principles and procedures in both numeric and algebraic situations.

KSCO

(vii) By the end of grade 3, students will be expected to select and use appropriate computational techniques (including mental, paper-and-pencil, and technological) in given situations.

SCO

By the end of grade 2, students will be expected to

B12 use technology to solve problems involving sums or differences of larger numbers

Elaboration–Instructional Strategies/Suggestions

B12 When students solve authentic problems, they may encounter numbers beyond their capabilities to add or subtract, e.g., using a catalogue to find the total price for a set of furniture for their dream bedroom. In these situations, rather than abandoning the problems, students should use calculators or computers to assist them. Provide opportunities for students to become familiar with the various functions on their calculators. Setting up a play store using play money opens up a range of opportunities. The shopper can estimate when selecting items, while the cashier practises both estimation and calculator use, as he/she checks the reasonableness of the calculator result.

Using a calculator should also be part of the development of other outcomes (see SCOs, A2, B5, and B7) as illustrated below.

- Have students show 69 on their calculator. Ask them to change this to 49 in one step.
- Explain to students that Mary wanted to add 24 and 37 on her calculator, but the “7” button was not working. Ask them what she might do to calculate the answer accurately.
- Have students use the calculator constant to skip count by 8s, i.e., $8 + 8 = =$.

Provide students with problem situations for which it makes sense to select a calculator. Discuss the technology and stress the importance of estimating to check reasonableness. As well, provide situations for which it is appropriate to estimate only, and an exact answer is not required.

Although students will routinely use calculators in computational situations involving numbers with more than two digits, they should work with multiples of 100 mentally, e.g., If students see 3 hundreds + 2 hundreds as no different from joining 3 and 2 of any other unit, they will easily mentally calculate $300 + 200$ as 500. Similarly, students should mentally calculate sums such as $50 + 30$ or differences like $80 - 20$ rather than using paper and pencil or a calculator.

GCO B: Students will demonstrate number sense and apply operation principles and procedures in both numeric and algebraic situations.

Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

Performance

B12.1 Have students use calculators to find the sums of 3-digit numbers that you provide orally.

B12.2 Explain to students that they cannot use the “clear” buttons on their calculators. Ask them to show 443 and then change it to 543. Have them explain how they did it. Ask them to change 543 to 523 and explain how they did it.

B12.3 Have students solve this problem: The school sold chocolate bars to raise money for field trips. A teacher reported that grade 1 sold 86, grade 2 sold 118, grade 3 sold 74 and grade 4 sold 98. How many chocolate bars were sold by the four grades?

B12.4 Simulate a store by providing a wide selection of items, each priced under a dollar. Ask student “shoppers” to use estimation to select as many items as possible without exceeding 200 cents. Have other students take turns being the “cashier”, using a calculator and giving change for each selection of purchases.

Interview

B12.5 Tell students that you were using your calculator to find $48 + 37$. You accidentally pressed $48 + 27$. Ask, What should I do now?

B12.6 Tell students that you need to know the answer to $70 - 40$. Ask, Would you use a calculator, do it mentally, or do it on paper? Why?

B12.7 Ask students why it might be easier to do a question like $52 + 20$ mentally rather than using a paper and pencil.

B12.8 Ask students to tell you computations that they would use a calculator to solve.

B12.9 Tell students that Jacob used a calculator to add $18 + 36$. He got an answer of 64 and knew immediately that he had made a mistake. Ask, How do you think that Jacob knew this?

Portfolio

B12.10 Have students make charts that would help other students decide whether to do a computation mentally, work it out on paper, or use a calculator.

Patterns and Relations

GCO C: Students will explore, recognize, represent and apply patterns and relationships, both informally and formally.

Elaboration–Instructional Strategies/Suggestions

KSCO

- (i) By the end of grade 3, students will be expected to recognize, describe, extend, and create patterns and sequences in a variety of mathematical and real-world contexts (e.g. geometric, numeric and measurement).

SCO

By the end of grade 2, students will be expected to

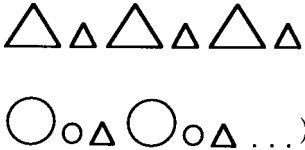
- C1 compare and contrast patterns
- C2 demonstrate an understanding that there are often many ways to continue a pattern, unless a pattern rule is provided

Students at this grade vary greatly in their intuitive recognition of patterns. Some students immediately see patterns, whereas others recognize them only after their attention is focused on a pattern by the teacher or another student. Because seeing relationships and making generalizations are abilities to be nurtured, pattern activities should continue throughout the year. (NCTM 1992a, 1)

C1 Many patterns share certain characteristics. Students should be able to identify the ways in which pairs of patterns are similar and different, e.g.,

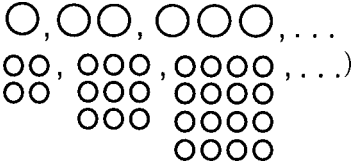
- (a) Consider these patterns: 1, 2, 1, 2, 1, 2, ...

3, 5, 7, 3, 5, 7, 3, 5, 7, ...


 The pairs of patterns are similar in that they are *repeating* patterns.

- (b) Consider these patterns: 2, 4, 6, 8, 10, ...

1, 4, 7, 10, 13, ...

The pairs of patterns are similar in that they are *growing* patterns.
 

- (c) Consider some patterns that share some of the same numbers or shapes but have different rules, e.g., 1, 1, 2, 3, 5, 8, 13, ... and 2, 3, 5, 8, 13, ...

C2 Explain to students that if they are told that a given series of numbers or shapes is a pattern but are not told the rule, there may be many ways to continue the pattern depending upon what they think the rule is, e.g., If they are told that 1, 2, 3, 5, ... is a pattern, without being given the rule, it could be continued in a variety of ways:

- (a) 1, 2, 3, 5, 1, 2, 3, 5, ... (repeating pattern)
- (b) 1, 2, 3, 5, 8, 13, 21, ... (growing pattern—each number after 2 is the sum of the 2 preceding ones)
- (c) 1, 2, 3, 5, 3, 2, 1, ... (going up and down)

Provide opportunities for students to complete the beginnings of patterns started with numbers or shapes in a variety of ways.

GCO C: Students will explore, recognize, represent and apply patterns and relationships, both informally and formally.

Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

Performance

C1.1 Ask students to create two different patterns using pattern blocks, both of which focus on the number of sides in the shapes.

C1.2 Ask students to create two different patterns using counters. Ask them to describe two ways in which the patterns are the same and two ways in which they are different.

C1.3 Provide green pattern blocks (triangles) for students working in pairs. Ask them to work together to produce growing patterns of triangles.

C2.1 Provide students with the starts of patterns using base-10 blocks. Ask them to continue the patterns in more than one way and to describe the pattern rules they used.

Interview

C1.4 Ask students to create two patterns which are similar, but not exactly the same. Ask them to describe the similarities and differences.

C2.2 Tell students that the first two numbers in a pattern are 5 and 10. Ask them to continue this pattern in several different ways.

C2.3 Tell students that one of their classmates continued the pattern 1, 2, 3, 4, ... by saying 2, 3. Ask, Do you think that this student is incorrect or is there a rule that might explain continuing the pattern in this way?

Portfolio

C1.5 Have students collect photographs or drawings of patterns they observe in the environment. Have them write about these patterns, explaining what makes them patterns.

GCO C: Students will explore, recognize, represent and apply patterns and relationships, both informally and formally.

KSCO

- (ii) By the end of grade 3, students will be expected to use patterns to solve problems.

SCO

By the end of grade 2, students will be expected to

- C3** identify and use patterns in an addition table
- C4** identify and extend place-value patterns

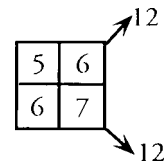
Mathematics empowers students. Because mathematics is in part the study of patterns, recognizing patterns and predicting what comes next are crucial problem-solving skills that students must develop early if they are to succeed. (NCTM 1992, 1)

Elaboration–Instructional Strategies/Suggestions

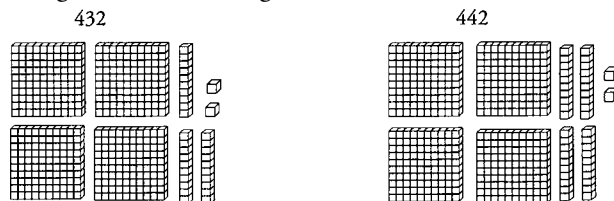
C3 Provide students with addition tables like the one shown. There are many patterns in these tables, many of which students can use to determine an unknown sum or difference, e.g.,

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

- (a) Only even numbers are located on the main diagonal (upper left to lower right), so the sum of a number with itself is always even.
- (b) The numbers increase by 1 across a row, since 1 more is added for each step right.
- (c) All of the 8s are on one diagonal line, since each time an addend is 1 greater, the other must be 1 less.
- (d) There are three 2s, four 3s, five 4s, ...
- (e) The diagonals of any four numbers that form a square will have the same sum.



C4 Students should be able to explain how to transform the model of a 3-digit number to that of another number in which only one digit is different, e.g.,



Students can then extend given place-value patterns and identify their own, e.g., 512, 502, 492, ... and 214, 314, _ , _ , ...

- Have students convert hundreds charts so they can see from 100–199, 200–299, ... ,900–999. On these charts, have them use coloured counters to cover numbers forming a pattern and have them explore the place-value representation of the covered numbers, e.g., The pattern 13, 23, 33, 43, ... , is depicted as a vertical column of counters, representing increases of 10.

GCO C: Students will explore, recognize, represent and apply patterns and relationships, both informally and formally.

Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

Performance

C3.1 Have students describe patterns on the addition table, then explain how these patterns could help them remember forgotten facts.

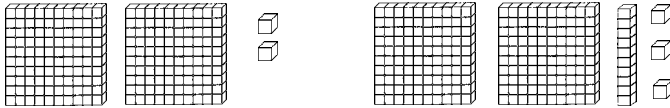
C4.1 Ask students to use base-10 materials to construct place-value patterns in which the second term is 302.

C4.2 Have students count by 20s to 200, recording each number. Ask, How often do you say a number that has a 0 in the tens place?

Paper and Pencil

C3.2 Cover all the 9s within an addition table. Ask students to describe the pattern in the addends for these 9s.

C4.3 Show the first and second terms of a pattern using base-10 blocks.



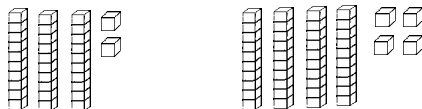
Ask students, What do you think the fifth term is? Explain why.

C4.4 Have students continue the patterns below for 3 or 4 terms:

- (a) 132, 142, 152, ...
- (b) 378, 388, 398, ...
- (c) 213, 313, 413, ...

Interview

C4.5 Present these base-10 blocks:



Ask students what would come next if it is a pattern. Explain their patterns.

GCO C: Students will explore, recognize, represent and apply patterns and relationships, both informally and formally.

KSCO

(iii) By the end of grade 3, students will be expected to represent mathematical patterns and relationships in informal ways, including via open sentences (i.e., statements with missing addends).

SCO

By the end of grade 2, students will be expected to

C5 represent patterns using their own notation or symbolism

C6 solve simple open sentences involving addition and subtraction facts

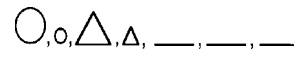
Elaboration–Instructional Strategies/Suggestions

C5 To explain a pattern such as o x o x x o x x x ... , students might use their own notational systems, e.g., 1, 1, 1, 2, 1, 3 ...

In order to continue a numerical pattern, students might want to make notations between numbers, e.g., 2, ⁺¹ 3, ⁺² 5, ⁺³ 8, ...

It is important for students to verbalize

what they see, e.g., When using attribute blocks, a student might say, “Big blue circle, small red circle, big blue triangle, small red triangle. Well, I guess that next I could show a big blue square and then a small red square.”



C6 Students should solve open sentences of the following six forms:

$$a + b = \square \quad (\text{e.g., } 6 + 3 = \square)$$

$$a + \square = c \quad (\text{e.g., } 2 + \square = 8)$$

$$\square + b = c \quad (\text{e.g., } \square + 4 = 5)$$

$$c - a = \square \quad (\text{e.g., } 7 - 2 = \square)$$

$$c - \square = b \quad (\text{e.g., } 4 - \square = 2)$$

$$\square - a = b \quad (\text{e.g., } \square - 8 = 1)$$

Generally, students find the first form of sentence easiest to solve. Many students find the last form of sentence, one without an obvious starting place, the most difficult. Students should have manipulatives available to them when solving these open sentences—cubes and balances are very effective.

The solving of open sentences should be connected to problem situations. Students should see it as a useful process to find solutions to a broad range of applications.

Emphasizing the link between addition sentences and subtraction sentences will also help students to solve open sentences, e.g.,

$\square - 3 = 7$ and $7 + 3 = \square$ are really two expressions of the same relationship.

GCO C: Students will explore, recognize, represent and apply patterns and relationships, both informally and formally.

Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

Paper and Pencil

C6.1 Ask students to find the number for each sentence that makes it true.

(a) $5 + \square = 13$

(b) $16 - \square = 7$

(c) $6 + 4 = \square$

Ask, Can there be more than one answer for each sentence? Why or why not?







Interview

C5.1 Clap this pattern: * ** * ** * ** * **. Ask students, How might this pattern be described by someone who can't clap it?

C5.2 Have students continue these patterns and explain how they knew what to do.

(a) small yellow , big red , small blue , big red 

(b) big thick yellow , small thin yellow , ...

(c)      

C6.2 Ask students, How might you use counters to find the number to make this sentence true?

$$\square + 8 = 15$$

C6.3 Have students create story problems for the number sentence in C6.2.

Shape and Space: Measurement

GCO D: Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

Elaboration–Instructional Strategies/Suggestions

KSCO

- (i) By the end of grade 3, students will be expected to measure and understand basic concepts and attributes of length, capacity, mass, area, and time.

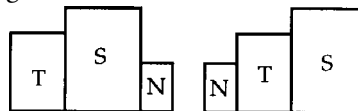
SCO

By the end of grade 2, students will be expected to

- D1 identify procedures not involving units to be used to compare areas

D1 Students should understand that *area* tells about the space an object takes up on a flat surface. Students’ first comparison activities should be by perception, i.e., deciding just by sight that one shape has a larger area than another. Subsequently, students should investigate strategies to compare the areas of two or more objects. These strategies should include *direct measurement*—comparing areas by covering one item with another—and *indirect measurement*—comparing areas by reshaping one item so that it fits on top of the other or by comparing both items to a third referent.

Students should come to understand that the area of a shape is preserved, i.e., the area of a shape does not change if it is cut up and rearranged to make a different shape. Students might think of how a space might be covered by a textbook, a scribbler, and a small notebook. Even though the objects may switch their positions, the same space is being covered.



Provide opportunities for students to order the areas of a number of shapes or flat objects from largest to smallest, as well as opportunities for students to select the item most appropriate to cover a particular space.

One of the purposes of early comparison activities with areas is to help students distinguish between size (or area) and shape, lengths, and other dimensions. (Van de Walle 1994, 298)

GCO D: Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

Worthwhile Tasks for Instruction and/or Assessment

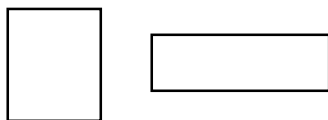
Suggested Resources

Performance

D1.1 Provide students with five identical squares. Have them create single shapes with them, trace around them, then cut them out. Continue by having the students create different single shapes using the squares, trace around them, and cut them out. Ask how they know that different looking shapes can have the same area.

D1.2 Show students an envelope and have them estimate where to cut pieces of adding machine tape so that they have the same area as this envelope. Suggest to students that they cut up and rearrange the tape on top of the envelope to check their estimates.

D1.3 Provide students with two sheets of paper as below. Ask them to decide which has the greater area and to provide proof.



Interview

D1.4 Ask students whether or not an item that is long and thin can have the same amount of area as one that is short and fat.

D1.5 Ask students to name or draw items which are long but do not have much area.

D1.6 Ask students, Why is a tablecloth's area usually greater than the area of the table it is supposed to cover?

GCO D: Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

KSCO

- (ii) By the end of grade 3, students will be expected to identify and use non-standard and standard units of measurement and to appreciate their roles in communication.

SCO

By the end of grade 2, students will be expected to

- D2 demonstrate a sense of how long 1cm and 1m are
- D3 estimate and measure length in non-standard and standard units
- D4 recognize and explain why standard units are used

A developmentally appropriate curriculum encourages the exploration of a wide variety of mathematical ideas in such a way that students retain their enjoyment of, and curiosity about, mathematics. It incorporates real-world contexts, students's experiences, and students's language in developing ideas. It recognizes that students need considerable time to construct sound understandings and develop the ability to reason and communicate mathematically. (*NCTM* 1989, 16)

Elaboration–Instructional Strategies/Suggestions

D2 Students should develop a sense of the length of a metre and a centimetre through a variety of activities.

- Have students relate centimetres and metres to parts of their own bodies, e.g., A student might say, “My legs are about half a metre long, my nose is 4 cm long, and 8 of my footprints would make a metre.”
- Frequently ask students to show different centimetre lengths with their fingers or hands and different metre lengths with their arms.

Focusing on estimation as an essential part of the measurement process helps students develop a better sense of the sizes of the units.

D3 Students should estimate and measure lengths in metres and centimetres, as well as using non-standard units. Students should establish a personal *referent* or *benchmark* for each unit to use for estimation, e.g., Students might use the distance from the floor to the door handle as a referent for 1 metre and the width of one finger as a referent for 1 centimetre.

- Provide situations that are of interest to the students and that provide useful information, e.g., buying carpet for the reading corner, measuring book heights for a new bookcase, or determining if a large piece of furniture can fit through the door.

D4 Students should understand that standard units are used in order to ensure that everyone understands each other when describing measurements.

- Provide students with experiences in which they measure an item's length with a pencil of their own choosing and tell others how many pencils long the item is. Inevitably, different numbers will arise, as different pencils have different lengths. Have students discuss how this might cause a problem if they had to call someone on the telephone and tell them how long the item was. Introduce the centimetre cube and have everyone measure the item again with a line of centimetre cubes or rods. This time everyone should get the same number; thus, communication of that number to others would be meaningful.

It would be appropriate to read some children's books that dramatize situations where non-standard units lead to confusion.

GCO D: Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

Performance

D3.1 Ask the students to cut lengths of about 1m from balls of string. Have them verify their estimates.

D3.2 Have students make paper clip chains that fit just around their wrists. Ask them to estimate how many of these chains would fit along the length of their arms. Have them check their estimates.

Interview

D2.1 Ask students to hold apart their two index fingers to show 5cm.

D4.1 Ask students, Why does it make more sense to measure the length of a running track for a meet in metres than in foot paces?

D4.2 Ask students, Why might it not be a good idea to tell someone how long a table is by using pieces of paper as a measurement unit?

Presentation

D3.3 Ask groups of students to develop appropriate plans to compare the height of the wastepaper basket to the distance around it, and to explain their plans to the class.

Portfolio

D3.4 Have students develop measuring books that they can add to throughout the year.

D3.5 Give students metre-long pieces of twine and invite them to use them to measure objects in their homes. Have them make lists of items that are almost a metre, 1 metre, or a little more than 1 metre. Have the students enter their findings in a table such as the one shown below:

almost a metre	one metre	more than a metre

GCO D: Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

KSCO

- (ii) By the end of grade 3, students will be expected to identify and use non-standard and standard units of measurement and to appreciate their roles in communication.

SCO

By the end of grade 2, students will be expected to

- D5** demonstrate a sense of how much 1L is
- D6** estimate and measure capacity in non-standard and standard units
- D7** demonstrate a sense of how much 1kg is
- D8** estimate and measure mass using non-standard and standard units

Elaboration–Instructional Strategies/Suggestions

D5 Through exposure to a variety of 1-litre containers, students should develop a sense of how much a litre is.

- Have students fill an empty 1-litre milk container with sand. Have them estimate which other containers that you have supplied would hold the same amount. Have them check by pouring the 1-litre container of sand into it.

D6 Students should estimate and measure capacity in litres, as well as using non-standard units. Their estimates should be made using personal *referents*, or *benchmarks*. To provide a common personal referent for 1 litre, the large base-10 cube (1000 block) which holds 1 litre of liquid can be used.

- Ask students to estimate how many litres of trash the wastebasket will hold. Have them discuss how they arrived at their estimates.
- Provide five or six large capacity items, e.g., fish tank, pail, jug, baby bath, roasting pan. Tell students the actual number of litres that one of these containers holds. Have them estimate and vote for the container they think has this capacity. Encourage students to talk about their reasoning, then confirm their estimates.

D7 Through lifting and holding a variety of objects that have a mass of 1 kilogram, students should develop a sense of what a kilogram feels like.

- Have students compare a variety of objects to an established 1kg mass, e.g., They might determine which objects are more than, less than, or about the same as 1kg of sugar.
- Have students fill containers with various materials until they think a mass of 1kg is reached. Have them check their estimates.

D8 Students should continue to use non-standard units to measure mass, e.g., marbles, ball bearings, multi-link cubes, base-10 materials, and erasers. Mass should be measured on a balance scale, as opposed to something like a bathroom scale for which the source of the number appearing is less clear to students.

Students should subsequently estimate and measure mass using the kilogram as the unit.

- Have students investigate such things as the number of kilograms they could comfortably carry in their backpacks and the total number of kilograms of a group of books on a shelf.

GCO D: Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

Performance

D5.1 Ask students to mark 1-litre estimates on plastic containers. Have them compare their estimates with the standard litre measurement.

D6.1 Ask students to determine the capacities of several containers, graph them, and discuss the results.

D7.1 Ask students to predict, from a collection of objects, which ones have a mass of about 1 kilogram.

D7.2 Ask students to choose small items. Have them estimate and determine how many of the items would be required to make masses of 1 kilogram.

D8.1 Ask students to find some things that have the same mass as two bags of marbles.

D8.2 Ask students to find out the number of potatoes in 2kg. Ask, Will the number always be the same? Why or why not?

Interview

D5.2 Ask students, Would 1L of juice be enough for the whole class? Why or why not?

D6.2 Ask students to describe the sizes of containers that would hold 25L of water.

D7.3 Ask students to predict, from a collection of objects, the one that has a mass of about 1kg.

D8.3 Ask students, What unit would you use to measure the mass of a watermelon?

Portfolio

D6.3/8.4 Have students develop measuring books that they can add to throughout the year.

GCO D: Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

KSCO

- (ii) By the end of grade 3, students will be expected to identify and use non-standard and standard units of measurement and to appreciate their roles in communication.

SCO

By the end of grade 2, students will be expected to

- D9** estimate and measure time using non-standard units
- D10** read hours and half-hours on a clock
- D11** explore properties of the calendar

Elaboration–Instructional Strategies/Suggestions

D9 Students should use non-standard units to measure time, e.g., They might determine how many times they can clap their hands while a particular song is being played or while a race is run.

D10 Students should be able to read time in hours and half-hours on an analog clock and on a digital clock.

They should be aware that the minute hand and hour hand on an analog clock are different lengths, and that the minute hand is at 6 for the half-hour and at 12 for the hour. Some students will be aware that the hour hand moves during the course of the hour, and that at the half-hour, it is halfway between two numbers.

While using an analog clock, students can be introduced to the term *half past* because half of the clock has been swept by the minute hand since the corresponding “o’clock time.” Some students may be confused since there is not a half-clock spread between the two hands.

An analog clock can serve as a useful model to reinforce the number concept of one-half. Students can visualize one-half either in terms of the hour hand moving halfway from one number to the next or the minute hand moving halfway around the circular face of the clock.

D11 The calendar provides rich opportunities to explore not only knowledge of time, but also number concepts, e.g., ordering numbers, using ordinal language, and counting in a variety of ways.

Students should know the days of the week, the months of the year, and the four seasons. Students may have more difficulty with the months for which they have less experience in their own lives. Ask questions regularly, e.g., It is March. Which month comes next? As well, they will be developing a sense of the arrangement of our year in relation to the months and seasons, e.g., January is the first month of a new year and is early in our winter season.

- Use calendars throughout the school year to help strengthen the students’ sense of time. Each month will bring new explorations.
- Have students learn the jingle, Thirty days has September, April, June and November. All the rest have 31 ...
- Have students explore calendar patterns, e.g., months that need only four lines, months with the same number of Mondays, and adding 6 by going down one week and back one day.

GCO D: Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

Performance

D10.1 Ask students to show, on an analog clock, the times (to the nearest half-hour) at which they do various activities.

D11.1 Ask pairs of students to predict how many weeks there are in a year. Have them use calendars for the year to check.

Interview

D11.2 Show students calendars for the year. Ask them to point out today's date and to find out what date it will be in six weeks.

D11.3 Show students calendars for the year and ask them to identify ways in which months are the same and ways in which they are different.

D11.4 Referring to calendars for the year, ask students to show when they think spring starts.

D11.5 Ask students, How might a calendar be used to help subtract 14 from a number?

D11.6 Tell students that it is the 11th of the month. Ask, How might a calendar be used to add 16 and what is the date?

Presentation

D11.7 Tell students that Stacy was born February 29, 1992. Ask them, working in pairs, to determine how many birthdays Stacy has had and when she would celebrate her birthday. Ask them to write a letter outlining how unfair it is to be born on February 29th and what they suggest be done to change the situation.

Portfolio

D11.8 Provide calendars for the year. Have students figure out and write about how many school days each month will have, how many Friday the 13ths there are in the year, and on what days the birthdays of friends and family fall.

Note: Students might enjoy the “Knuckle Method” for remembering the number of days in each month: Make a fist showing four knuckles; start by pointing to the first knuckle and saying, “January.” The space between knuckles is February, the second knuckle is March, and so on. After July, go back to the beginning, making August land on the first knuckle, and continuing until year end. The months that land on the knuckles each have 31 days.

GCO D: Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

KSCO

- (iii) By the end of grade 3, students will be expected to estimate and determine measurements in everyday problem situations and develop a sense of the relative size of units.

SCO

By the end of grade 2, students will be expected to

- D12** choose appropriate units with which to estimate and measure, and perform the measurements
- D13** demonstrate an understanding that the size of unit used affects the number describing the measurement
- D14** demonstrate an understanding that 100cm make up 1m

A portfolio is a showcase for student work, a place where many types of assignments, projects, reports, and writings can be collected. Progress in, attitudes toward, and understanding of mathematics can be seen in a comprehensive way. The collection exemplifies the goals of the NCTM Evaluation Standards and shows much more than will a single test. (*NCTM 1991c, 35*)

Elaboration–Instructional Strategies/Suggestions

D12/D13 It is important to incorporate estimation in all measurement activities, not treat it as a separate topic. Estimating and measuring in non-standard units help students focus on principles of measurement such as (a) the bigger the unit used, the fewer the number of units it takes, e.g., your arm might be 4 pencils—a longer unit—or 24 paper clips—a shorter unit—in length; (b) some units are more appropriate than others for measuring a certain object, e.g., measuring a large container with a large unit rather than a small unit.

- Have students use body units, e.g., arm lengths, foot lengths, or strides, to estimate and measure lengths. This might provide a permanent referent when estimating and measuring, e.g., a centimetre referent might be the width of a finger. Have students estimate, explore, and record the lengths of various classroom objects, using their own body units, e.g., They might find that the door is almost 10 hands wide.
- Have students estimate and measure in many everyday situations, e.g., Have them find the number of kilograms they could comfortably carry in a back pack, have them find the number of kilograms of food they eat in a normal week, have them compare the masses of various types of balls, have them find the size of bag needed to hold 1kg of flour, have them estimate the capacity of the sink in litres, and have them estimate the height of a young tree.

Students should recognize which linear unit—metres or centimetres—is appropriate in a given situation., e.g., Using centimetres would be appropriate when measuring the length of your foot while using metres would be appropriate when determining the distance from your classroom to the school library. Also, provide students with some situations in which something is measured in centimetres sometimes and in metres other times.

D14 Students should understand that a metre is 100 cm long. Although many metre sticks are marked up to 100, it is often still not clear to students that lining up 100 centimetres really produces a metre. It is worthwhile to actually interlock 100 centimetre cubes and measure them against the metre stick.

As new standard units are introduced, it is important to take time to teach the format of recording measurements with that unit, e.g., five centimetres as 5cm.

GCO D: Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

Performance

D12.1 Have students locate a number of items in the room that would be appropriately measured with the length of a base-10 rod.

D13.1 Have students determine how many teaspoons make up a tablespoon.

D13.2 Have students find out how many grapes usually balance the mass of one plum.

D12.2 Provide students with a number of measuring situations, asking them to choose the appropriate units that should be used to measure. Have them estimate the measurements using these units.

Interview

D12.3 Ask students to explain why it is more appropriate to describe the length of a pencil in centimetres than in metres.

D13.3 Explain that two students have each correctly measured an item, but one says it is 18 pencils long while the other says it is 7 pencils long. Ask, How could this be true?

D13.4 Tell students that Sue measured an item and found it to be 3 pencils long. Ask, How many erasers long might it be?

D12.4 Have students name an appropriate unit for measuring the height of a brown bear.

D12.5 Ask students, Why might a long piece of rope not be a good unit for measuring the thickness of a book?

D14.1 Explain to students that someone measured an object and said that it was more than 1m long. Ask, How many centimetres might you expect the object to be?

Shape and Space: Geometry

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

KSCO

- (i) By the end of grade 3, students will be expected to explore and experiment with geometric shapes and relationships.

SCO

By the end of grade 2, students will be expected to

- E1 develop aspects of spatial sense, including perceptual constancy, perception of spatial relationships, and visual discrimination

Elaboration-Instructional Strategies/Suggestions

E1 *Perceptual constancy* is the ability to recognize a shape when it is seen from a different viewpoint or from a different distance (enlargement/reduction), e.g., When a rectangular tabletop is viewed, we usually actually see a parallelogram but experience tells us that it should be a rectangle and therefore we perceive a rectangle. While our eyes may get one message, our minds make the necessary adjustments to have it fit with our preconceived ideas; therefore, perceptual constancy is dependent to a large degree on experiences. The development of perceptual constancy helps students adjust to the environment and gives stability to their surroundings.

Perception of spatial relationships is the ability to see the relationship between two or more objects, e.g., If students are presented with a figure made up of four different pattern blocks, they are able to make an exact replica. For some tasks, this skill is similar to *position-in-space perception*, e.g., being able to recognize congruency in slides, flips and turns.

- Show students a number of arrangements of blocks, such as



Ask students to make these same arrangements and place them beside the ones modelled.

Visual discrimination is the ability to identify the similarities and differences between or among objects, e.g., an activity which shows two pictures and asks students to find all, or a specific number, of differences between them. It should not be dependent upon position.

- Put a different triangle on each of two geoboards. Ask students to change one of the triangles to be the same as the other.

Students should be encouraged to work with various puzzles regularly, e.g., pattern block, tangram, and jigsaw puzzles. They provide practice for all the spatial skills. It is important to provide puzzles that span a wide range of complexity since some students will have had a great deal of experience while others may have had limited experience.

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

Performance

E1.1 Make a shape that is a composite of more than one shape on a geoboard. Ask students to make the same shape on their geoboards and observe how the task is performed.

E1.2 Set up a geoboard as for E1.1. Ask students to make the reflection of this shape (with the right side of the geoboard as the mirror line) on their geoboards.

E1.3 Give students outlines of a three-piece tangram puzzle and seven tangram pieces. Ask them to fill in the outlines. Observe how they solve the problem, e.g., See if they select pieces that are reasonable given the shape of the outline, if they are persistent, and if they have a plan for shape selection.

Interview

E1.4 Make a display of triangles on geoboards or on 5 x 5 geopaper. Ensure that some of the triangles are the same but in different orientations. Point out one triangle to students and ask them to find other triangles that are the same as it, explaining how they know.

E1.5 Some students can be challenged to an activity similar to the one in E1.4, except that all of the shapes are in different positions and orientations.

E1.6 Give students pictures of a number of triangles, all of which are the same except for one that differs slightly. Ask them to identify the one that is different.

E1.7 Provide the seven-piece tangram sets. Ask students to find any shapes that are enlargements of other shapes among the pieces.

E1.8 Show students an isometric picture of a cube and ask them what they see.

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

Elaboration-Instructional Strategies/Suggestions

KSCO

- (i) By the end of grade 3, students will be expected to explore and experiment with geometric shapes and relationships.

SCO

By the end of grade 2, students will be expected to

- E2 recognize 3-D shapes from drawings and from alternative perspectives

E2 Students should be able to recognize common 3-D shapes when they are viewed from various perspectives. Students should learn that the appearance of solids changes with change of view-point. As well, they should recognize drawings of these shapes which, for some students, is quite difficult because drawings are 2-dimensional representations of 3-dimensional objects.

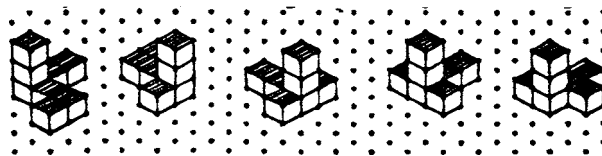
- Pose a number of questions such as
 - What does a triangular prism look like to a bird flying over it?
 - If a worm saw a square pyramid, what would it see?
 - What does a square prism look like from different sides?
 - How does the view of a cone from the top compare to the view from the bottom?

- Build a structure out of multi-link cubes such as



Have students describe what it looks like from the front or from the side. Discuss whether or not it would look different from the right side than from the left side.

- Use a variety of picture books that include pictures of cubes, prisms, pyramids and spheres to allow students to see what artists or illustrators do to make 3-D pictures look real when drawn on a sheet of paper.
- Provide isometric drawings, such as those below, and ask the students to build the structures with cubes. It must be recognized that this is developmental; some students will find this activity difficult and will therefore require much practice.



GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

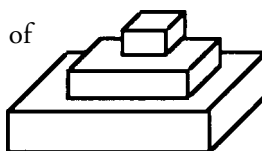
Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

Performance

E2.1 Present drawings of a variety of 3-D shapes from different perspectives. Ask students to match these with the actual shapes which you display.

E2.2 Ask students to build structures out of multi-link cubes that might look like this from the back.

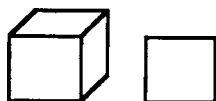


E2.3 Ask students to build structures that look the same from above as from below.

Interview

E2.4 Ask students to build structures like the one shown in E2.2. Ask, What does the structure look like from above?

E2.5 Ask students why it is more informative to draw a cube as shown on the left than as shown on the right.



E2.6 Have students describe two different shapes that look the same from the top.

E2.7 Ask students how a prism could be placed so that it looks the same from above and below, but a pyramid could not.

Presentation

E2.8 Show students two different drawings of a cube. Ask, How many faces can be seen in each drawing? Which drawing do you think is the better drawing? Explain your choice.



GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

KSCO

- (i) By the end of grade 3, students will be expected to explore and experiment with geometric shapes and relationships.

SCO

By the end of grade 2, students will be expected to

- E3 sort, build, and pattern with 2-D and 3-D shapes

Young students should explore geometric concepts informally and intuitively. The use of manipulatives is crucial. Sorting and classifying geometric figures in many ways provides students with informal analyses of the properties of these figures before the more formal work in later grades. (*NCTM 1992a, 25*)

Elaboration-Instructional Strategies/Suggestions

E3 Students will have had many opportunities to explore shapes through sorting, patterning, and building activities. Activities that will further develop these skills should be provided. Students will begin at different levels of development so it will be necessary to design a range in the complexity of the activities.

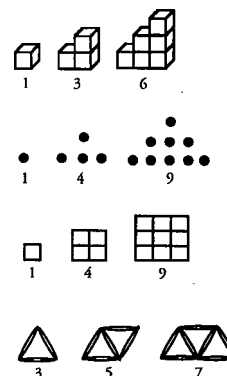
- For sorting purposes, prepare a collection of 2-D shapes cut from posterboard. These should include not only shapes with which students are familiar, but also some less common shapes that would fit into potential sorting categories, e.g., a semi-circle or a cashew shape could be classified under *things with curved edges*. Because it is important for students to share their thinking with others, it is recommended that they work in small groups, talking about their ideas and strategies.

Students benefit from using hardwood blocks for construction. They will build increasingly more complex structures over the year. Encourage them to be creative, experimenting with different constructions, e.g., Challenge them to explore constructions of increasingly greater height and to talk about these constructions with their classmates.

Students will have had numerous experiences with patterning. It is important that they continue to use concrete materials in patterning activities, such activities becoming increasingly more complex.

- Have students work in pairs to examine growing patterns and to discuss the number pattern associated with them.

It is not expected that square or triangular numbers be addressed at this time; the focus here is simply on patterning.



GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

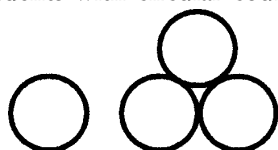
Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

Performance

E3.1 Provide students with toothpicks, asking them each to build a square. Ask, How many more toothpicks do you need to make two squares? Three squares? Have students make a chart to record the number pattern and see if they are able to describe it.

E3.2 Provide students with circular counters. Begin a pattern such as



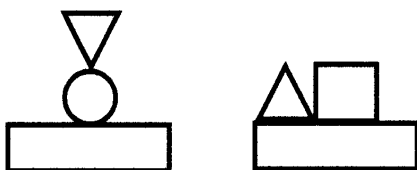
Ask students how many counters must be added to continue the pattern and have them record it in some way. Ask, Can you predict how many counters are needed to make the next part of the pattern?

Paper and Pencil

E3.3 Provide students with 10 green triangular pattern blocks. Ask them to make as many different-size triangles as they can and to record their answers by making a drawing of each.

Interview

E3.4 Give students a picture of two constructions like those below. Ask students to talk about them, inviting them to use hardwood blocks to convince you as to whether or not they can be constructed.



Presentation

E3.5 Ask students who have created a special construction to present it to the class, outlining any problems they encountered and how they were able to overcome them.

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

KSCO

- (i) By the end of grade 3, students will be expected to explore and experiment with geometric shapes and relationships.

SCO

By the end of grade 2, students will be expected to

- E4 recognize, name, and represent parallel lines and right angles

Elaboration-Instructional Strategies/Suggestions

E4 Students should be introduced to the concept of *parallel* as a descriptor for sides of shapes contained in real-world objects, e.g., In a picture of a verandah, the spindles are parallel; in the classroom, windows, doorways, ceilings, picture frames, and a variety of other objects provide examples of parallel lines/segments. Many students associate parallel lines with train tracks; however, the lines in their notebooks would provide a readily available referent for parallel lines. At the same time, students should examine other lines and objects that are not examples, e.g.,



Some students might perceive parallel as meaning “the lines go in the same direction or at the same slant”; others might perceive it as “the lines never meeting”; still others might notice that “the lines stay the same distance apart.” There should be no attempt to learn a definition; rather, students should recognize examples and non-examples in contexts and should be able to represent parallel lines/segments using toothpicks, straws, elastics on geoboards, drawings, and other materials.

- Have students draw examples of objects or shapes that contain parallel lines/segments and make a collage with their drawings.
- Have students place the various pattern blocks on lined scribbler pages to help them determine whether or not the shapes have parallel sides.

Similarly, students should be introduced to the concept of *right angle* as a descriptor for what they have probably referred to as “square corners” in shapes. Again, classroom objects provide many examples of right angles and these should be explored, drawn, and discussed. Logic blocks provide examples of shapes that have right angles and shapes that do not. Since most notebook pages are rectangular, the corners of these pages provide a referent for right angles.

Examples of right angles should be presented in a variety of positions so students do not associate the word right in any way with how the angle points. There probably should be some discussion of the meanings they now know for the one word, including right meaning correct as well as right as a direction.

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

Worthwhile Tasks for Instruction and/or Assessment**Suggested Resources***Performance*

E4.1 Ask students to make drawings of table tops with no sides parallel or with no sides forming right angles.

E4.2 Draw a right triangle for students. Ask, Is there a right angle in this shape? Where is it?

E4.3 Present students with large printed capital letters for E, F, H, L, M, N, T, V, W, and Z. Have them examine these letters for examples of parallel lines/segments and of right angles.

E4.4 Have students arrange pairs of toothpicks to make parallel lines and right angles, as well as lines that are not parallel and angles that are not right angles.

Paper and Pencil

E4.5 Ask students to draw shapes with parallel lines and right angles on geopaper.

E4.6 Ask students to draw pictures of shapes with parallel sides, but no right angles.

Interview

E4.7 Ask students to study the corners of objects in the classroom to see how common right angles actually are. Have them report their findings to you.

E4.8 Ask students to find pattern blocks that have parallel sides and no right angles, and blocks with parallel sides and right angles.

E4.9 Ask students to make shapes with lots of right angles on their geoboards.

E4.10 Ask students to point out or draw three items in the room that have parallel sides.

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

KSCO

- (ii) By the end of grade 3, students will be expected to describe, model, draw, and classify 2- and 3-D figures and shapes.

SCO

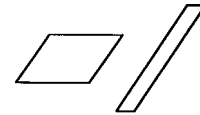
By the end of grade 2, students will be expected to

- E5 recognize, name, describe, and represent parallelograms

Elaboration-Instructional Strategies/Suggestions

E5 Students should be introduced to the 2-D shape *parallelogram* through explorations with a variety of shapes, by giving many different examples of parallelograms, and by examining real-world objects that have parallelogram shapes. Parallelograms should be the focus of several lessons, treated first in isolation where they are named and explored, then in extended activities such as sorting and patterning, and finally integrated with other already familiar shapes. As with all other shapes, once the name is known, students should be encouraged always to use its name and should be given many opportunities to use its name.

The purpose of the variety of activities is to help students construct a strong visual image of typical parallelograms.



- *A possible introduction* Prepare 15–20 cut-out shapes including squares, rectangles, triangles, circles, and 5 parallelograms. Have students sort the shapes under the headings: *squares, rectangles, triangles, and circles*. They will be left with the 5 shapes that will not fit under these headings. Have them examine these shapes for the ways in which they are all the same and the ways they are different. If they do not know the name, introduce it, spell it for them, and help them learn to say it. Provide a few more cutouts, some of which are parallelograms, and ask students which ones could join the others under the *parallelogram* heading.

It is sometimes difficult for students to draw parallelograms so that both pairs of sides are parallel; they could be encouraged to use lined or dot paper to help them. Students might use the two sides of their rulers to form a pair of parallel sides.

- Encourage students to bring in empty paper towel rolls and ask them to carefully unfold them along the seam. Most students are surprised to see that the shape formed is a long, thin parallelogram.
- Have students form rectangles using geostrips and paper fasteners. When the rectangles are picked up, students will notice that they readily make conventional parallelograms.

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

Performance

E5.1 Provide drawings which incorporate many shapes. Ask students to point out the parallelograms among the shapes.

E5.2 Use masking tape to make a parallelogram and a rectangle on the floor. Ask students to walk along the shapes, count the number of steps, and describe how they move to walk all the way around. Ask, How does the walk around the parallelogram differ from the walk around the rectangle?

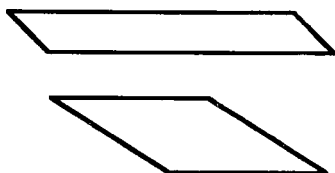
E5.3 Ask students to draw pictures of shapes that are almost, but not quite, parallelograms. Ask why they think they are not quite parallelograms.

Pencil and Paper

E5.4 Provide looseleaf. Ask students to draw pictures of long, thin parallelograms and short, fat ones.

Interview

E5.5 Show students two parallelograms such as the ones shown below:



Ask, How are the shapes alike and how are they different?

E5.6 Ask students to tell how they would describe a parallelogram to someone who does not know what one is.

E5.7 Ask students to make three different parallelograms on geoboards.

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

KSCO

- (ii) By the end of grade 3, students will be expected to describe, model, draw, and classify 2-D and 3-D figures and shapes.

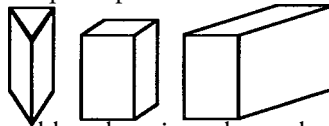
SCO

By the end of grade 2, students will be expected to

- E6 recognize, name, describe, and represent triangular, square, and rectangular prisms and pyramids
- E7 cut and assemble nets of cubes and triangular, square, and rectangular prisms and pyramids

Elaboration-Instructional Strategies/Suggestions

E6 While students have worked with prisms and pyramids in general, they have not been expected to name them by the shape of their bases, e.g., The pictures of the shapes below are of a triangular prism, a square prism, and a rectangular prism.



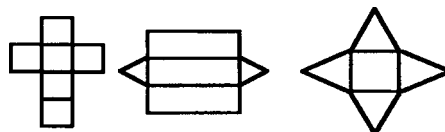
Students could make triangular and square prisms of various heights by stacking the green and orange pieces from the pattern blocks. Similarly, they could make these prisms as well as rectangular prisms by stacking the appropriate pieces from the logic blocks. This stacking to build these shapes helps students understand the uniform nature of prisms.

Showing models of triangular pyramids, square pyramids, and rectangular pyramids next to the corresponding prisms will help students see the similar way in which prisms and pyramids are named, i.e., by their bases.

Looking for real-world examples of these prisms and pyramids would also help students construct strong visual images of these shapes. Some stores and speciality shops have products that come in a variety of shaped packages, or they sell boxes in a variety of shapes, all of which make good examples for these 3-D shapes. Specialty candles often have these shapes.

- Have students make the “footprints” for a triangular prism and a triangular pyramid. Ask them to compare these “footprints” (faces). Have them share their comparisons.

E7 One way to become familiar with prisms and pyramids is to observe, touch, and manipulate 3-D models of these shapes. Another way is to build them from 2-D plans, called *nets*. Students should be provided with nets for triangular, square and rectangular prisms and pyramids. They should then cut them out and fold and tape them into the 3-D models. Sample nets:



GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

Worthwhile Tasks for Instruction and/or Assessment**Suggested Resources***Performance*

E6.1 Provide a variety of prism and pyramid models. Have students name the models, observing that they use the correct names.

E6.2 Provide drawings of a variety of 3-D shapes. Ask students to match these with the actual solid models and to name them.

Observation

E7.1 Provide square pyramid nets that have two pairs of the triangles joined rather than all triangles joined to the square. Observe students as they cut out and assemble these nets to see whether they can easily visualize which piece must be the base.

Interview

E6.3 Ask students to describe how a square pyramid and square prism are alike and how they are different.

E6.4 Provide a Toblerone box. Ask, What name would you give this shape? Why?

E6.5 Ask, Why might you suggest that there are two bases on a triangular prism? Could someone call one of the rectangles the base? Explain.

E6.6 Have students use a set of prisms and pyramids to place the correct pyramid “hat” on top of each prism.

E7.2 Ask, Why might it be more difficult to identify which part of the net of a triangular pyramid should be called the base than it would be for a square pyramid?

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

Elaboration-Instructional Strategies/Suggestions

KSCO

(ii) By the end of grade 3, students will be expected to describe, model, draw, and classify 2-D and 3-D figures and shapes.

SCO

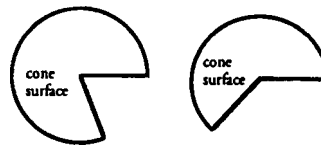
By the end of grade 2, students will be expected to

E8 recognize surfaces and faces of 3-D shapes

E8 Many 3-D shapes that students see or explore have flat surfaces called faces, but others, like cylinders, spheres and cones, include curved surfaces. Students should begin to explicitly relate the description of 3-D shapes to descriptions of their surfaces and faces (flat surfaces), e.g., A cylinder has 2 faces and a curved surface; a sphere has one curved surface; and a prism has all faces.

Students might observe that most of the prisms familiar to them have many rectangular faces, while pyramids have many triangular faces. When dealing with shapes like cylinders and cones which have flat faces as well as curved surfaces, many students tend to focus on the curved surface since it provides a better sense of the whole shape.

Geoblocks provide good sources of 3-D shapes with different faces. The faces of prisms made by stacking identical pattern blocks would be interesting to examine. Sections can be cut out of cardboard circles to produce nets which can be folded into curved surfaces to explore a variety of cones.



GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

Worthwhile Tasks for Instruction and/or Assessment**Suggested Resources***Performance*

E8.1 Provide a variety of 3-D models. Ask students to find all the rectangular and square faces.

E8.2 Have students count the number of faces of various prisms. Ask, Can different types of prisms have the same number of faces?

E8.3 Provide a variety of 3-D models including a triangular pyramid (with all equal faces), a cube, and other solids. Ask students to pick out the shapes that have all the same faces.

E8.4 Ask students to select shapes with exactly five faces from among a variety of models (including some triangular prisms and square pyramids).

Interview

E8.5 Have students describe how the curved surface of a cylinder feels compared to the curved surface of a cone.

E8.6 Display a variety of different cylinders. Prepare a paper rectangle that could be rolled to cover the curved surface of one of the cylinders. After rolling the paper, ask students to find the cylinder whose curved surface the paper cylinder would fit.

E8.7 Have students compare the curved surfaces of a sphere, a cone, and a cylinder.

E8.8 Ask students to decide whether knowing the number of curved surfaces of an object helps tell them how well it might roll.

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

KSCO

(iii) By the end of grade 3, students will be expected to investigate and predict the results of combining, subdividing, and transforming shapes.

SCO

By the end of grade 2, students will be expected to

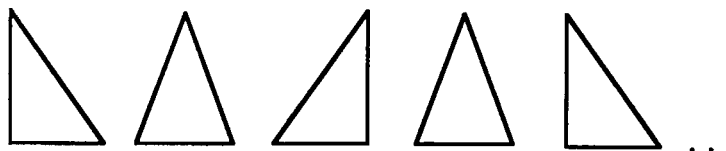
E9 sort, build, and pattern with 2-D and 3-D shapes

Elaboration-Instructional Strategies/Suggestions

E9 Students should continue their experiences sorting, building with, and creating patterns with 2-D and 3-D shapes. These experiences should be more sophisticated than perhaps in previous years, e.g., Sorting and patterning criteria might include

- circle or not
- parallelogram or not
- parallel sides or not
- right angles or not
- curved surfaces or not

Ask students to continue the following patterns and explain their pattern rules:



- Provide a variety of shapes including parallelograms, trapezoids, and triangles. Ask students to describe various ways to sort the shapes.
- Ask students to combine a variety of pattern blocks to build a single prism.

See also SCO, E3.

Constructing two-dimensional and three-dimensional figures with a variety of materials (blocks, geoboards, straws and pipe cleaners, tagboard) helps students identify specific characteristics of each figure, including the symmetries of figures and some relationships between two-dimensional and three-dimensional figures. (*NCTM 1992a, 25*)

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

Worthwhile Tasks for Instruction and/or Assessment**Suggested Resources***Performance*

E9.1 Make a pattern in which every second shape has a pair of parallel sides and the other shapes do not. Ask students to examine your pattern and describe the pattern rule.

E9.2 Provide an assortment of 2-D shapes. Ask students to find several ways to sort the shapes.

E9.3 Ask students to use different-sized square and rectangular prisms to build a pyramid-like building with steps.

Interview

E9.4 Show a set of shapes, all but one of which contain right angles. Ask students to identify the shape that is different from the rest and to tell how it is different.

E9.5 Provide a set of shapes that include only squares and equilateral triangles. Ask students to sort the shapes. Have them describe the sorting rule they used. Ask students if the rule could have been *has right angles* or *has parallel sides*.

E9.6 Ask students why it might be easier for them to build a prism than to build a sphere.

Portfolio

E9.7 Have students create pictures that include a parallelogram, a rectangle, and a triangle.

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

Elaboration-Instructional Strategies/Suggestions

KSCO

(iii) By the end of grade 3, students will be expected to investigate and predict the results of combining, subdividing, and transforming shapes.

SCO

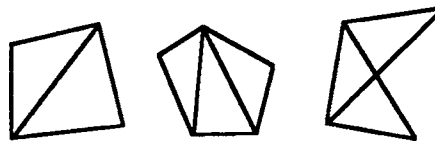
By the end of grade 2, students will be expected to

E10 subdivide and change 2-D figures

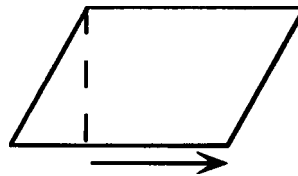
E10 As an extension of grade 1 activities, in which students experimented with cutting familiar shapes into two or three parts, students should work with more complicated constructions. It is through these explorations that they are able to recognize the attributes of various shapes and further develop their spatial sense. Many mathematical concepts can be explored as students subdivide and change 2-D figures, e.g., Students can explore simple fractional ideas by dividing a shape in different ways.



Students should recognize that any polygon can be subdivided into triangles.



Students can see the similarities and differences between figures by subdividing and reforming them, e.g., students can see how parallelograms are like rectangles by cutting one end off and placing it at the other end.



They can see that any triangle can always be divided into two triangles with right angles.



- Provide each student with two identical parallelograms. Have them cut one along the shorter diagonal, the other along the longer one. Compare the parts produced. Ask students to make all the possible shapes using the four pieces.

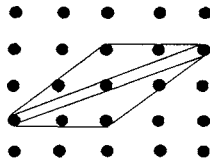
GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

Performance

E10.1 Have students show different ways to cut geoboard parallelograms in half, e.g.,



E10.2 Ask students to rearrange these four-fourths of a square to make other shapes.



E10.3 Ask students to find four different ways to cut a rectangle into equal pieces.

E10.4 Ask students how to cut a trapezoid into two pieces to show a parallelogram and a triangle.



Interview

E10.5 Have students draw triangles and try to divide them into smaller triangles. Ask if it would always be possible. Then ask the same question about dividing a square into smaller squares.

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

KSCO

(iii) By the end of grade 3, students will be expected to investigate and predict the results of combining, subdividing, and transforming shapes.

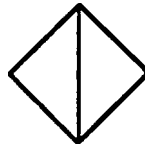
SCO

By the end of grade 2, students will be expected to

E11 recognize, identify, describe, and represent reflective symmetry in 2-D shapes

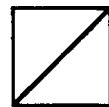
Elaboration-Instructional Strategies/Suggestions

E11 Figures with reflective symmetry are those in which one-half of the shape is the mirror image of the other half.



To test for symmetry, students can use a mira. By placing the mira on the line that splits the two symmetric halves (the flip or mirror line), students can see if the flip image of one half falls exactly on top of the other half of the shape. Another way to test for symmetry is by folding to see if the two halves match perfectly.

Many students mistake symmetry for *halving*. Figures with reflective symmetry can easily be split in half, but not every object with two congruent halves has reflective symmetry.



has reflective symmetry



does not have reflective symmetry

Students should realize that the line of symmetry can be vertical, horizontal, or slanted.

Mira Math Activities for Elementary School (1973) provides a number of very interesting activities for students to explore symmetry.

- Have each student fold a piece of paper in half, cut out a shape and unfold to examine the result. This can become an art activity by folding more than once and by creative cutting.

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

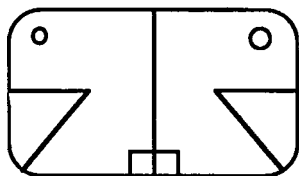
Performance

E11.1 Provide a variety of shapes and ask students to sort them, grouping those with mirror symmetry and those without mirror symmetry.

E11.2 Have students draw pictures to show how a shape can have symmetry, even if the right side does not match the left side.

E11.3 Provide a butterfly design. Ask students to use a mira to test the symmetry.

E11.4 Provide designs like the one below in which it would appear that one-half of the design is a mirror image of the other half, but there is one small difference. Ask students to find the difference.



Interview

E11.5 Ask, Why might someone say that a circle has a lot of symmetry?

Paper and Pencil

E11.6 Ask students to draw pictures of shapes with symmetry.

E11.7 Ask students to draw examples of triangles with symmetry and triangles without symmetry.

E11.8 Have students draw pictures to show what creatures that are not symmetric might look like.

E11.9 Have students create crayon or marker designs that exhibit symmetry.

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

KSCO

- (iv) By the end of grade 3, students will be expected to relate geometric ideas to number and measurement ideas and to recognize and apply geometric principles in real-world situations.

SCO

By the end of grade 2, students will be expected to

- E12 recognize and identify reflective symmetry in the environment

Elaboration-Instructional Strategies/Suggestions

E12 Many of the objects (both living and inanimate) in the world are symmetric. Students could explore symmetry in man-made objects such as houses, furniture, food containers, tools, and dinner plates. Books with pictures of interesting buildings such as the CN Tower, or interesting structures such as the Confederation Bridge, would be good sources of objects with symmetry for students to explore.

- Have students study animal books to find symmetry. They might debate whether giraffes, whose spots are not symmetric, should be called symmetric or not.
- Many fabric designs such as found on clothes, tablecloths, and linens exhibit symmetry. Bring in samples of fabric and discontinued wallpaper books for students to explore for reflective symmetry.

Students will differ with respect to the level of precision they use in determining whether or not a shape has reflective symmetry, e.g., Some students will say that the number of buttons on a telephone form a symmetric design since the placements do, whereas others will notice that the numbers are not the same on both sides of the design, so the phone pad is not totally symmetric.

- Send students on a “shape hunt” to look for 10 objects that display reflective symmetry. Encourage them to look for less obvious objects to include in their examples.
- Ask students how deciding which mitten goes on which hand concerns symmetry.

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

Worthwhile Tasks for Instruction and/or Assessment**Suggested Resources***Performance*

E12.1 If the school is within walking distance of a number of houses, take students on a walking tour to look for architectural examples of reflective symmetry.

E12.2 Have students check to see if their feet are exactly the same length. Ask, Would they be considered symmetric if they were not the same length?

Observation

E12.3 Observe whether students spontaneously point out the symmetry of various objects.

Interview

E12.4 Ask students to name pieces of clothing that display symmetry.

E12.5 Ask, Does any object which is circular automatically display reflective symmetry? Why or why not?

E12.6 Ask students to identify some number combinations, e.g., 818, that show symmetry.

Portfolio

E12.7 Have students write a paragraph describing whether they think animals are symmetric.

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

Elaboration-Instructional Strategies/Suggestions

KSCO

(iv) By the end of grade 3, students will be expected to relate geometric ideas to number and measurement ideas and to recognize and apply geometric principles in real-world situations.

SCO

By the end of grade 2, students will be expected to

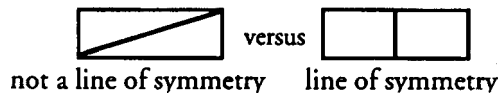
- E13 make the connection between reflective symmetry and one-half using squares, rectangles, and circles
- E14 make the connection between even/odd numbers and rectangles

E13 Students have explored simple fractions as parts of the whole. It is particularly easy for students to show halves of symmetric shapes by dividing them on the lines of symmetry.

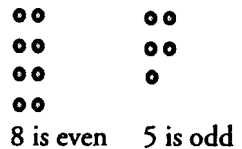
It would be useful to ensure that symmetry lines are oriented in different ways, e.g., horizontally, vertically, and slanted.



Some students will observe that rectangles can be cut in half in a variety of ways, not all of which divide them along lines of symmetry. They could fold along the lines to test, e.g.,



E14 Many students learn that even numbers are numbers that end in 0, 2, 4, 6, or 8 (and odd numbers are the others) without really understanding that an even number of objects can be paired up. If numbers of objects are displayed as rectangles in which one dimension is 2, students can easily see the difference between even and odd numbers. The odd numbers will always have one object without a partner. This can be done using square tiles or counters arranged in rectangular formations



GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

Worthwhile Tasks for Instruction and/or Assessment**Suggested Resources***Performance*

E13.1 Ask students to trace around small rectangular objects, such as attribute blocks, to make several copies. Then ask them to show $\frac{1}{2}$ of the rectangle in as many ways as they can. Observe whether students choose to look for novel ways to do this.

E13.2 Provide students with pictures of circles and mirrors. Ask them to find different ways to show $\frac{1}{2}$.

E13.3 Have students fold squares on lines of symmetry to show halves.

E14.1 Ask students to arrange 10 squares to show that 10 is an even number.

Interview

E14.2 Provide students with copies of several rectangles that have 2 units as one of their dimensions and that are subdivided into unit squares. Ask students to explain what numbers are represented by these rectangles and how they know if these numbers are even.

**Data Management
and Probability:
Data Management**

GCO F: Students will solve problems involving the collection, display, and analysis of data.

Elaboration - Instructional Strategies/Suggestions

KSCO

- (i) By the end of grade 3, students will be expected to collect, record, organize, and describe relevant data.

SCO

By the end of grade 2, students will be expected to

- F1 conduct simple surveys and record data

F1 Students should develop strategies to collect and record information. This information should concern the students themselves, their schools, their communities, and other topics that are meaningful to them. Attention should be paid to how best to ask questions, if necessary, to gather the information.

Students should be encouraged to use a tally system for recording data, e.g.,



- Ask students to compare recording systems and discuss why it is easier to compare the numbers in groups A & B in the second situation below.



- Have students predict what they think the most popular pet is among the students in grade 1 and discuss how they could find out. This should lead to collecting pet data from the students in grade 1, recording the information they get using tallies with headings like *dog, cat, bird, ...* , and examining the results.

One of the most important rules to follow in conducting graphing and statistics activities is to let students gather their own data. (*Van de Walle 1994, 391*)


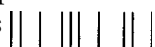
GCO F: Students will solve problems involving the collection, display, and analysis of data.

Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

Interview

F1.1 Tell students that they might be asked to conduct a survey to find out bedtimes of 7- and 8-year olds. Ask them what things they would have to think about when actually asking fellow students so that the question would not be confusing, e.g., whether they want them to respond for weekends and/or for weekdays.

F1.2 Ask students why it is easier to count the *yes* responses when they are shown like this , rather than like this .

Presentation

F1.3 Have students plan and conduct an in-class survey about favourite toys. Have them present the results of the survey in an organized form.

GCO F: Students will solve problems involving the collection, display, and analysis of data.

Elaboration - Instructional Strategies/Suggestions

KSCO

- (ii) By the end of grade 3, students will be expected to construct concrete and pictorial displays of relevant data.
- (iii) Students will be expected to read and interpret relevant data.

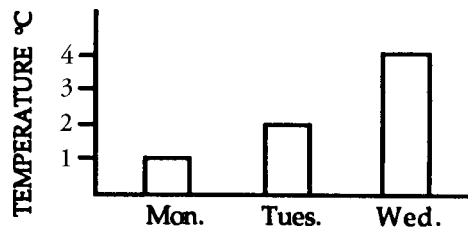
SCO

By the end of grade 2, students will be expected to

F2 create and interpret pictographs and symbolic bar graphs

F2 Students should create graphs in a variety of situations as a means of organizing collected data. While students have been introduced to pictographs and symbolic bar graphs, they should now study these two types of graphs more in-depth. It is appropriate, however, that symbols in pictographs and grid blocks on bar graphs continue to represent only one unit. As well, students should be exposed to both vertical and horizontal forms of bar graphs.

- Have students roll a die 20 times, record each number, and then create a graph showing frequencies.
- Have students collect and graph data in connection with activities in other subject areas, e.g.,



Students should interpret numerous graphs (both those created in class and those found elsewhere) to gain information about a variety of topics.

GCO F: Students will solve problems involving the collection, display, and analysis of data.

Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

Performance

F2.1 Ask students to create pictographs to show the favourite books of other students in the class.

Paper and Pencil

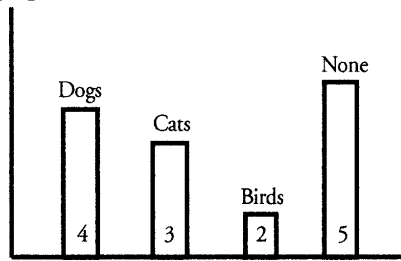
F2.2 Provide a paragraph of text. Ask students to draw graphs to show the number of words on each line.

F2.3 Ask students to draw graphs showing the number of addition facts with sums of 0, 1, 2, ... ,10. Ask, What do you notice?

F2.4 Provide students with a bar graph and ask them to interpret the graph by explaining various things it tells the viewer.

Interview

F2.5 Provide a graph such as



Ask students to tell you all they can about the information displayed.

GCO F: Students will solve problems involving the collection, display, and analysis of data.

KSCO

(iv) By the end of grade 3, students will be expected to generate questions, develop and modify predictions, and implement plans with respect to data analysis.

SCO

By the end of grade 2, students will be expected to

F3 develop and modify predictions with respect to data collected or presented to them

Elaboration - Instructional Strategies/Suggestions

F3 While students should continue to generate questions about data, increased attention should be paid to making predictions based on data collected or presented to them. They might state their expectations before data is collected in a certain situation and then verify or modify their thinking as a result of seeing the actual data.

- Have students make predictions and collect data about
 - the number of right-handed students for each left-handed student
 - whether more students get to school by riding buses, walking, or getting a drive
 - whether more students swim in the winter or do not

Predictions might be based on population extensions or time extensions, e.g., If students observe that more students in their class walk to school than take a bus, they might predict that the same would be true for another class in their school, but might not be true for a class in another school. Similarly, if students observe that fewer people take music lessons than do not, they might predict that this could be different for older students.

Predictions could be based on information shown in tally or chart formats as well as on information shown in graphical formats.

GCO F: Students will solve problems involving the collection, display, and analysis of data.

Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

Performance

F3.1 Have students predict which vowel will occur most often in a piece of text. Have them tally and graph the number of each vowel that occurred to test their predictions. If their predictions are inaccurate, ask them to describe why this might have been.

F3.2 Ask students to decide whose name they expect to see most often when students name their favourite sports hero. Have each student write the name of his/her hero on a slip of paper and post it on the board. Ask students to organize and display the data on the board to determine the validity of their predictions.

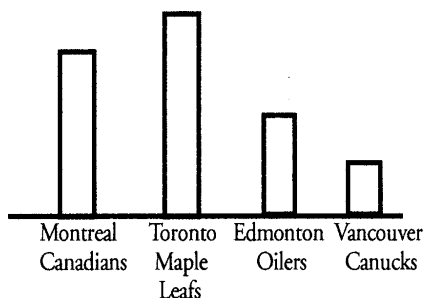
Interview

F3.3 Ask students, Do you think that more students in the class would choose skiing or skating as the activity for a class party? What do you think would be a good way to find out if you are correct?

F3.4 Show students a bar graph of ice cream sales for a store during the months of January through May. Ask them to predict what the graph would look like for the next three months.

Presentation

F3.5 Show students a graph of favourite hockey teams for students in a school in Atlantic Canada. Have them work in groups to predict what hockey graphs might look like for students in British Columbia. Ask them to create graphs and to present them to the class, explaining their reasoning.



**Data Management
and Probability:
Probability**

GCO G: Students will represent and solve problems involving uncertainty.

KSCO

- (i) By the end of grade 3, students will be expected to conduct informal investigations of chance and estimate probabilities with respect to games and other simple, everyday situations.

SCO

By the end of grade 2, students will be expected to

- G1** demonstrate an understanding that some events are more likely than others
- G2** demonstrate an understanding that probability predictions need not always come true

Probability ideas and language can be developed at this level as children make predictions about outcomes. As the children collect data about their experiments, they will need some ways to organize the data. This need connects probability to the experiences that children have had with charts and graphs. (*NCTM 1992, 18*)

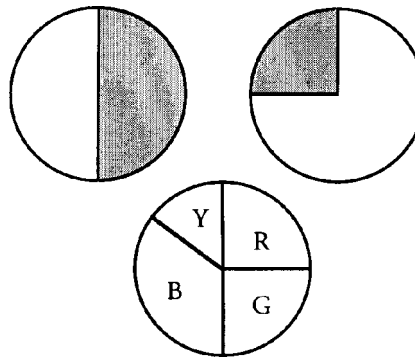
Elaboration–Instructional Strategies/Suggestions

G1 Students should understand that some events are more likely to occur than others, e.g., It is more likely that we will have snow than rain in January; it is more likely that we will get a number greater than two than less than two when we roll a die.

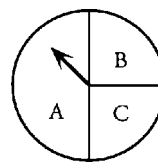
Students should use the terms *always, never, likely, unlikely, and probably* in appropriate situations.

Materials that could be used for probability include number cards, coins, playing cards, dice, and spinners.

- Have students experiment with a variety of spinner faces to determine on which sections the spinners are more or less likely to land, e.g.,



G2 Students should realize that although one outcome may be more likely in theory, it may not happen that way in a given set of tries, e.g., While it is not very likely, it is possible to spin the B section in the spinner face below more often than the A section during a set of 10 tries.



- Have students predict how many heads they will get when they flip a penny 10 times. Have them flip pennies 10 times, keeping tallies of heads and tails. Have them report to class how many heads they got and how this compared to their predictions.

GCO G: Students will represent and solve problems involving uncertainty.

Worthwhile Tasks for Instruction and/or Assessment
Suggested Resources
Performance

G1.1 Ask students to design spinners such that spinning red is more likely than spinning green, but spinning red is less likely than spinning yellow.

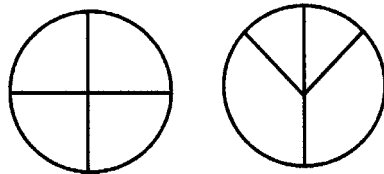
G1.2 Have students roll dice 10 times, recording each outcome. Ask, Do you think that the number which came up most often would again be the most common if you repeated the experiment? Why or why not?

Interview

G1.3 Tell students they win \$1 if the spinner lands on red and lose \$1 if it lands on blue. Ask, How would you like the spinner to be designed?

G1.4 Ask students, Is flipping a coin a fair way to decide whether Anne or Sue should go first in a game? Why or why not?

G1.5 Show the spinners below.



Ask students, If four children are playing a game, why is the spinner on the left more likely to be used to put their names on than the one on the right?

G1.6 Ask students to think of events that are possible, but not very likely, and other events that are very likely, but might not happen.

Grade 3

**Number Concepts/
Number and Relationship
Operations:
Number Sense and
Number Concepts**

GCO A: Students will demonstrate number sense and apply number theory concepts.

Elaboration–Instructional Strategies/Suggestions

KSCO

- (i) By the end of grade 3, students will be expected to construct and communicate number meanings, and to explore and apply estimation strategies, with respect to whole numbers.

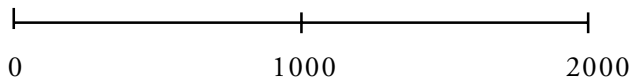
SCO

By the end of grade 3, students will be expected to

- A1** compare and order whole numbers to thousands

A1 Students should be able to identify the greater of two whole numbers and to order a set of numbers. When modelling comparisons, stress the importance of starting with the highest place value. Extend the hundreds chart to show that the system is consistent. Have students continue to model numbers with base-10 materials, both concretely and pictorially. Encourage them to visualize the base-10 block representation of numbers. Put numbers in context, e.g., order populations of provincial towns.

- Prepare a deck of number cards that contain both 2- and 3-digit numbers. Have students deal all the cards face down to the players. Have each player turn the top card over; the one who has the greater (greatest) card wins both or all the cards in play. In the end, the winner is the one who has collected the most cards when all the cards have been turned over.
- Ask students, If $\square 39 > 422$, what can you say about \square ? If $\square 39 > \square 87$, what do you know about the missing digits?
- Shuffle a prepared deck of 40 number cards (4 sets of 0 to 9). Have students select three or four of the cards and arrange them to make the greatest possible number and the least possible number. Ask students to model these numbers with base-10 materials.
- Have students find the approximate location of 850, 1300, and 1890 using a number line like the one below. Have students share their thinking strategies and encourage the class to respond to others' strategies.



- Show the class 1259. Ask them to chant the number that is (a) 300 more, (b) 1000 more, (c) 30 more, and (d) 1 greater.
- Have students put $_ _ _ _$ on a piece of paper. They will try to fill these blanks to make the largest possible 4-digit number. As you roll a die and announce the digit, they will place it in one of their blanks. Once the blank is filled, it cannot be erased. Repeat this process three more times. Students should compare their numbers and share their decision-making strategies.

GCO A: Students will demonstrate number sense and apply number theory concepts.

Worthwhile Tasks for Instruction and/or Assessment
Suggested Resources
Performance

A1.1 Give students some prepared cards containing 3- and 4-digit number. Ask them to order them from least to greatest.

A1.2 Show students the numbers 501 and 398. Ask, Which would take more base-10 flats to represent? How many more? Is this all you need to know to say it is greater? Explain.

A1.3 Ask students to enter a certain number on a calculator, e.g., 2235. Ask, How can you, without clearing the calculator, change the number to 2435? to 2446? to 1234?

A1.4 Give each of two students a 10-section spinner with 10 numbers that are in the hundreds and/or the thousands. Have them spin at the same time and the one who spins the higher number gets a token. The students play until someone has gathered 10 tokens. Select numbers according to the students' level of understanding. Numbers such as 345, 354, 381, 309, 1008, 1800, 1080, 1335, 1353, and 2000 would work well.

Paper and Pencil

A1.5 Ask students to write numbers that are (a) greater than 348 but less than 360, (b) ten more than 165, (c) a little less than 300, (d) one hundred greater than 655, (e) two hundred less than 206, and (f) between 463 and 474.

A1.6 Tell students that *The Guinness Book of Records* reports each of the following to be the largest ever of its kind:

hamburger	2509 kg	doughnut	1700 kg
salami	678 kg	popcorn ball	1080 kg
ice cream sundae	377 kg	lollipop	1369 kg
Easter egg	4765 kg		

Ask students to order these items from least to greatest mass.

Interview

A1.7 Ask students to explain why a 3-digit number is always greater than a 2-digit number.

A1.8 Provide number lines for students. Ask them to estimate the location of numbers that you give them. Have them explain their reasoning.

GCO A: Students will demonstrate number sense and apply number theory concepts.

KSCO

- (i) By the end of grade 3, students will be expected to construct and communicate number meanings, and to explore and apply estimation strategies, with respect to whole numbers.

SCO

By the end of grade 3, students will be expected to

- A2** estimate the size of numbers to the nearest ten or hundred

Elaboration–Instructional Strategies/Suggestions

A2 Discuss rounding with students using base-10 materials and extended hundreds charts as models.

Rather than rounding numbers in isolation, emphasis should be on rounding to estimate in problem-solving and other contexts. Instead of simply applying the “5” rule (5 or greater rounds up, less than 5 rounds down), students need to learn to do what makes sense in given situations. Sometimes it makes sense to underestimate (rounding down), e.g., when determining the number of items you can buy with a fixed amount of money, or when determining the distance you can travel on one-quarter of a tank of gas. Sometimes it makes sense to overestimate (rounding up), e.g., when determining how much food to prepare for a party to make sure there is enough, or when determining the amount of string required to wrap a parcel.

Encourage students to use appropriate compensation techniques when estimating in calculation situations, e.g., in addition, they might round one number up and the other down ($32 + 45$ becomes $30 + 50$, while $27 + 45$ becomes $30 + 40$); in subtraction, they might round both numbers in the same direction to preserve the difference best ($45 - 32$ becomes $40 - 30$, while $45 - 27$ becomes $50 - 30$). Again, it is important that these types of rounding situations be presented in real contexts.

- Have students round the following to the nearest hundred then add:
 (a) $792 + 608$ (b) $582 + 487$ (c) $115 + 378$

GCO A: Students will demonstrate number sense and apply number theory concepts.

Worthwhile Tasks for Instruction and/or Assessment
Suggested Resources
Paper and Pencil

A2.1 Ask students to explain how they would round two or three 2-digit numbers for an estimated sum. To assess the students' understanding of compensation strategies, the following problem might be used: $38 + 48 + 35$.

A2.2 Ask students to name some numbers that could be rounded to 120.

A2.3 Point out that when you go grocery shopping, you sometimes round each price to the nearest 50¢. Ask, Using this system, how much should I have estimated as the total for products priced at 79¢, \$1.38, and \$2.59?

Interview

A2.4 Tell students that a number has been rounded to 40. Ask, What might it have been?

A2.5 Ask students to describe situations in which they would use the number 500 as an estimate for 475.

A2.6 Tell students that Jane estimated $82 - 47$ by using $80 - 50$. Without actually finding the answer, ask how they know that Jane's estimate is low.

A2.7 Tell students that Marla said, "To estimate $46 + 25$, I would add $50 + 20$." Mark said, "It should be 50 plus 30." Ask them to explain whose estimate they think is closer.

A2.8 Tell students that Patrick went to the store with a loonie. He estimated the cost as he picked up a 12¢ pencil, a 25¢ eraser, a 29¢ notepad, and a 19¢ pen. Ask, How might Patrick have estimated? Without finding the actual sum, do you think he had enough money to buy all these items?

GCO A: Students will demonstrate number sense and apply number theory concepts.

Elaboration–Instructional Strategies/Suggestions

KSCO

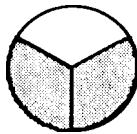
(ii) By the end of grade 3, students will be expected to concretely explore common fractions and decimals in meaningful situations.

SCO

By the end of grade 3, students will be expected to

A3 use simple fractions to describe situations

A3 Students should continue to use simple fractions such as $\frac{1}{10}$, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, and any other fractions that come up in context. These fractions should describe contexts that are *part of a whole* (area models) and *part of a set* (set models), e.g.,



part of a whole

$\frac{2}{3}$ (part of the pizza that is left)



part of a set

$\frac{3}{4}$ (part of the pencils that are long)

Initial experiences should be with part-of-a-whole situations and models.

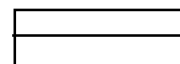
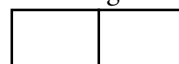
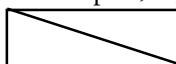
Presenting fractions in contexts makes them much more meaningful to students. It is also helpful, when examining a situation involving a fraction such as $\frac{2}{3}$, to show the related fraction $\frac{1}{3}$. It is also advisable to use horizontal lines when writing fractions.

- Explore fraction relationships among pattern blocks, e.g. if the hexagon is one unit, then the triangle is $\frac{1}{6}$, the rhombus is $\frac{1}{3}$, and the trapezoid is $\frac{1}{2}$. Use other blocks as one unit.
- Use coloured tiles to make a rectangle that is $\frac{3}{4}$ yellow and $\frac{1}{4}$ green. Make another rectangle that is $\frac{7}{10}$ red and $\frac{3}{10}$ blue.
- Students might examine fractions of particular geometric shapes, e.g., $\frac{1}{4}$ of a square and $\frac{1}{6}$ of a hexagon:



Students find fractions as parts of sets more difficult because they have to view the total number as one unit; therefore, initial experiences should be with sets that are contained, e.g., a box of 10 pencils, a package of 8 candies, a six-pack of soft drinks.

Students should see that there are many ways to make the same fractional part, e.g., $\frac{1}{2}$ of a rectangle could be shown as



Models must be used at all grade levels to develop fraction concepts adequately. Further, ... students should have experiences with a wide assortment of models. (Van de Walle 1994, 222–23)

GCO A: Students will demonstrate number sense and apply number theory concepts.


Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

Performance

A3.1 Ask students to use multi-link cubes to model, $\frac{5}{8}$ of the group is girls.

Paper and Pencil

A3.2 Ask students to colour $\frac{3}{4}$ of the circles. 

A3.3 Tell students that Jack has a total of 10 pets that are cats and dogs. Ask, What is the greatest fraction of his pets that could be cats? What is the smallest fraction of his pets that could be cats?

A3.4 Tell students that Lee and Teddy shared a bag of 12 candies. Ask, If Lee had 5 candies, what fraction of the bag did Teddy have?

A3.5 Pair each student with a partner to solve this problem: Eight-year-old Samantha, whose birthday is January 25th, said, “I can’t wait until I’m 8 and $\frac{11}{12}$.” Ask, Why was she excited?

Interview

A3.6 Ask students to tell why, whenever they see a representation of $\frac{3}{8}$, there is always a $\frac{5}{8}$ associated with it.

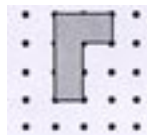
A3.7 Ask students to describe how to find $\frac{1}{3}$ of a strip of 9 squares.

Portfolio

A3.8 Tell students that you have 8 coins. Half of them are pennies. More than $\frac{1}{8}$ of them are quarters. The others are nickels. Have students use coins to represent the situation. Ask, How much money might you have?

Presentation

A3.9 Ask pairs of students to explore the following problem and to present their findings to the class: The shape to the right is $\frac{1}{2}$ of a larger one. What could the larger one look like? How many different possibilities can you find?



Invite students to create similar problems to challenge other students.

GCO A: Students will demonstrate number sense and apply number theory concepts.

KSCO

(iii) By the end of grade 3, students will be expected to read and write whole numbers and to demonstrate an understanding of place value (to four places).

SCO

By the end of grade 3, students will be expected to

A4 demonstrate an understanding of base-10 groupings (units, tens, hundreds, thousands)

Number sense develops over many years, and there may be a wide disparity between the concepts and skills of your third-grade students. The understanding they possess will be fostered if they model numbers in many ways and use numbers to describe real-world situations. (*NCTM* 1992b, 9)

Elaboration–Instructional Strategies/Suggestions

A4 Although some students will have a clear understanding of the base-10 pattern of the place-value system, many will still be in the early stages of its development. It is important that students be provided with regular opportunities to strengthen their knowledge and it is suggested that you use proportional materials so students can see the 10-for-1 relationships, e.g., base-10 blocks, bean sticks, beans and cups, bundles of Popsicle sticks, bundles of paper. While students might be exposed to chip trading, Abacus, money, and other non-proportional models, you should understand that they are not as conceptually based as, and not substitutes for, proportional models. It takes time for the students to thoroughly understand our base-10 system and to recognize and use the place-value notation.

- Say, Show me 489 with your materials. If you add 7, how much do you have? What if you were to add another 4? Explain your trading.
- Provide a shuffled set of 50 cards (5 sets numbered 0 to 9). Have a pair of students draw two cards and decide the greater number they can make. Ask them to add this number using base-10 materials to the value currently on their place-value mats, trading when necessary, e.g., If a pair were to draw a 5 and a 7, they would add 75 to their mat. Three cards may be drawn when students are capable of trading hundreds.
- Have students make bean models by placing 10 beans in small cups and 100 beans in margarine/yogurt tubs. Use these to model a variety of numbers and as substitutes for other base-10 block place-value activities.
- Ask students to enter 4567 on their calculators. Have them change the 5 to an 8 in one step, then the 4 to a 9, and then the 6 to a 3.
- Have students play *Race for a Loonie*. Ask each student to repeatedly toss a die and count out pennies on a mat. Ten pennies should be exchanged for a dime and 10 dimes for a loonie.

GCO A: Students will demonstrate number sense and apply number theory concepts.**Worthwhile Tasks for Instruction and/or Assessment****Suggested Resources***Performance*

A4.1 Ask pairs of students to show twenty-nine hundred seventy-six with base-10 materials. Have them roll a die and add to their models until they reach three thousand, explaining to one another each trade that they make.

A4.2 Have students represent different numbers using base-10 blocks. Have them put the base-10 blocks on blank pieces of paper on their desks. Everyone should circulate and record everyone else's number.

A4.3 Tell students that Mary knows that she needs to collect 300 bottle caps for the contest and she already has 287. Show, using base-10 materials, how many more she must collect to reach her goal.

Paper and Pencil

A4.4 Have students draw pictures of base-10 blocks that would show each of the following numbers: (a) 302 (b) 1027 (c) 1207

A4.5 Have students write each of the following using digits:
(a) 25 tens (b) 22 hundreds (c) 120 tens (d) 59 hundreds

Interview

A4.6 Have students pretend that they have won \$3000. Ask them to determine how many \$100 bills that would be and have them explain their methods of determination.

A4.7 Tell students that Martin said the car cost thirty-four hundred dollars, while Sam said he thought it cost over three thousand dollars. Ask, Are they disagreeing? Explain.

A4.8 Show students the number 8317. Ask, What does the 3 represent in this number? What does the 1 represent? How many thousands are there?

GCO A: Students will demonstrate number sense and apply number theory concepts.

KSCO

(iii) By the end of grade 3, students will be expected to read and write whole numbers and to demonstrate an understanding of place value (to four places).

SCO

By the end of grade 3, students will be expected to

A5 record, model, and interpret numbers up to and including the thousands

The key instructional tool for developing the conceptual knowledge of place value and also for connecting these concepts to symbolism is the use of base ten models. (Van de Walle 1994, 157)

Elaboration–Instructional Strategies/Suggestions

A5 Building numbers with concrete materials helps students develop a better sense of those numbers, e.g., when introducing 1000, it is beneficial to present it first as 10 stacked flats. This helps students visualize 1000 and to understand the 10-for-1 relationship. Students who have had extended experiences with base-10 materials will be able to visualize the models of numbers easily and will be able to make and interpret pictures of base-10 block representations.

It is important to spend time developing the use of zeros in numbers. For some students, the number 3002 looks like three hundred two. Students need many experiences using base-10 materials to make connections with the symbols for numbers with zeros as digits.

After extensive work with the base-10 materials, give students opportunities to record numbers. Ask them to write: three-hundred forty, nine-hundred eight, seven-thousand sixteen, sixty-four hundred thirty-two, and a number that has twenty-two tens and three ones. Experiences involving representing numbers in different ways will help students be able to write numbers described like the last one above. Invite students to take turns giving numbers for the class to write, encouraging them to use a variety of ways to describe them.

Introduce the *read-model-record* triad: read a number to have them record and model it, model a number to have them record and read it, and show the number to have them read and model it.

Provide opportunities for students to interpret numbers in different ways, e.g., 2936 may be interpreted as 2 thousands, 9 hundreds, 3 tens, 6 ones; or as 29 hundreds, 3 tens, 6 ones.

Repeated exercises such as $432 = __h, __t, __o$ should be avoided because they do not necessarily reflect place-value understanding.

- Have students create numbers greater than 500 with 3 in the tens place.
- Provide a set of number cards, such as

1601 **7436** **1462** **814** **5100**

Have one student silently choose one of the numbers and give clues to another student, based on different ways to read the number, so the correct number can be identified. He/she should try to use clues that could apply to more than one number, e.g., *There are more than 15 hundreds* eliminates only 814 and 1482.

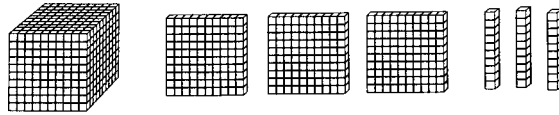
GCO A: Students will demonstrate number sense and apply number theory concepts.

Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

Performance

A5.1 Provide students with base-10 block representations of numbers. Ask them to write and read the number that is modelled, e.g.,



A5.2 Ask students to model 2230 in more than one way using combinations of base-10 blocks.

A5.3 Show students 2006 written symbolically and ask them to model it. Then have them read the number and express it in other ways.

A5.4 Give students 562, 803, and 97. Ask them to write numbers which are (a) 10 greater, and (b) 100 greater.

Paper and Pencil

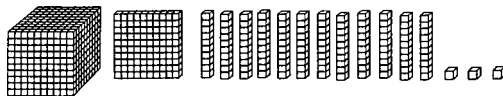
A5.5 Read numbers, or show models of numbers. Have students record them and locate them on number lines.

A5.6 Ask students to describe different items that might cost about \$1250.

A5.7 Ask students to solve this riddle: I am between 1100 and 1200. My last digit is an odd number. What is the smallest number I can be? the greatest number?

Interview

A5.8 Ask students to explain what number is modelled below:



A5.9 Ask students to describe 4000 in more than one way.

A5.10 Ask students, How many tens make 1000?

A5.11 Tell students that Mary won \$5000 in a contest. If she wants all her prize money in \$10 bills, how many bills would she receive?

GCO A: Students will demonstrate number sense and apply number theory concepts.

Elaboration–Instructional Strategies/Suggestions

KSCO

- (iv) By the end of grade 3, students will be expected to order whole numbers and to represent them in multiple ways.

SCO

By the end of grade 3, students will be expected to

A6 read numbers in several ways

A6 Students should understand that numbers can be read in more than one way. Experiences involving interpreting numbers differently help students develop this idea, e.g., 1236 may be read one thousand two hundred thirty-six or as twelve hundred thirty-six. They should understand that both responses are correct, but sometimes one may be more appropriate or used more frequently, e.g., She was born in nineteen hundred; he will graduate in two thousand four; the paint job on the car cost twenty-four hundred dollars.

Although students are unlikely to be dealing with numbers in the tens of thousands, they should be aware that numbers greater than 9999 are written with a space. Since the adoption of the SI Metric System, there are no commas used in numbers. Also, they need to understand that when reading a number, the word *and* is reserved for the decimal and should be avoided in the middle of the number, e.g., 10 453 is read as ten thousand four hundred fifty-three and 2.3 is read as two and three tenths.

GCO A: Students will demonstrate number sense and apply number theory concepts.

Worthwhile Tasks for Instruction and/or Assessment
Suggested Resources
Performance

A6.1 Provide students with the following table:

Dog Breeds in Canada

Breed	No. Registered 1995
Labrador Retriever	9471
Golden Retriever	8699
German Shepherd	8453
Shetland Sheepdog	4775
Rottweiler	4242
Poodle	4050
Shih Tzu	2895
Yorkshire Terrier	2458
Miniature Schnauzer	2381
Siberian Husky	2220

Ask them to read the number of registered dogs both ways, i.e., 2441 as two thousand four hundred forty-one **or** as twenty-four hundred forty-one. Then ask them to round the numbers to the nearest hundred and read their results both ways.

Paper and Pencil

A6.2 Ask students to record and read the number made up of 15 tens and 15 ones.

A6.3 Tell students that a number is made up of 42 tens and fewer than 5 ones. Ask them to write what numbers it could be.

Interview

A6.4 Ask students to read 3241 without using the word thousand(s).

A6.5 Ask students to explain why 320 is the same as 32 tens.

A6.6 Tell students that to subtract 132 from 500 Anne started by saying, “500 is 50 tens, so I’ll regroup one ten and that leaves 49 tens.” Ask why this was a good strategy.

Presentation

A6.7 Ask pairs of students to discuss the following problem: Jane said that 421 has more tens than 139. Peter said that it has fewer. How can they both be correct? Share ideas in a class discussion.

GCO A: Students will demonstrate number sense and apply number theory concepts.

KSCO

- (v) By the end of grade 3, students will be expected to apply number theory concepts, e.g., place-value pattern, in meaningful contexts with respect to whole numbers and commonly used fractions and decimals.

SCO

By the end of grade 3, students will be expected to

- A7 extend the place-value system to model and record numbers involving tenths

Elaboration–Instructional Strategies/Suggestions

A7 This will be the students’ introduction to writing fractions with a denominator of 10 as a decimal. The concept of one-tenth should be developed through several models before the notation is introduced. To help students develop the concept of one-tenth, it is important to clearly establish the whole that they are, or will be, dividing into ten equal parts.

- Have the rod in the base-10 blocks represent a package of candy with the small cubes being the pieces of candy that are tenths of the package. Initially show different numbers of packages (rods) and ask, How many packages do you see? Then show some packages (rods) and two loose candies (cubes), establishing a need to express part of a package.
- Use the rods as a model for one decimetre. Have students use these rods to measure objects in decimetres and thereby establish the need to show part of a decimetre using the small cubes as tenths. Initially they could read and record their measurements in this manner: 2 and 3-tenths decimetres.
- Use a metre stick as the unit of measure, creating a need to show part of a metre. Then 10 rods in the base-10 blocks placed end-to-end will make one metre; thus, the students can show tenths of a metre. Using a number of metre sticks and rods, have the students measure distances and objects, giving the measurements in metres and tenths of metres.
- Have students measure the lengths of various objects in centimetres using rulers, thus creating a need to express part of a centimetre.
- Use chocolate bars with 10 sections to model wholes and tenths. Have students draw pictures to represent various numbers of bars.

After many experiences using words, you could introduce the symbols with the decimal point clearly serving as the *and*. Students should understand that 0.1 and $\frac{1}{10}$ are two ways to write the same number. Then, the decimal notation should be seen as an extension of the 10-for-1 relationship of the place-value system, where 10 of anything to the right makes 1 of the thing to the left. In the same way we know that the 4 in 456 is 4 hundred because of its position, we know that the 3 in 4.3 is 3 tenths.

GCO A: Students will demonstrate number sense and apply number theory concepts.

Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

Performance

A7.1 Ask students to model 0.2 and 1.2 using base-10 blocks.

A7.2 Show students a handful of beans. Ask them to estimate how many would make up about 0.1 of the amount. Have students explain the thinking they used.

Paper and Pencil

A7.3 Ask students to estimate and colour 0.4 of rectangles.

A7.4 Ask students to continue the pattern below:

0.7, 0.8, 0.9, __, __,

A7.5 Provide 2 x 5 grids and ask students to colour 0.3 of them.

A7.6 Ask students to record the number that is 0.2 less than 1.

A7.7 Ask students to record these numbers: four tenths, sixteen and seven tenths, four thousand ninety-nine and nine tenths, and twenty-four hundred six and five tenths.

A7.8 Have students draw segments that are 1.2cm, 5.7cm, 8.5cm, and 10.1cm.

Interview

A7.9 Ask students, Does this model below show 0.4? Explain.



A7.10 Tell students that someone forgot to put the decimal in this number: 427. Ask them where the decimal could be if they were told that the number is less than 100.

A7.11 Ask students, How do you know 1.1 is greater than 0.4?

A7.12 Put 10 beans in each of five cups. Ask students to show you these numbers using cups and beans:

- (a) 3 (b) 4.1 (c) 1.9 (d) 2.5 (e) 0.7

GCO A: Students will demonstrate number sense and apply number theory concepts.

KSCO

- (v) By the end of grade 3, students will be expected to apply number theory concepts, e.g., place-value pattern, in meaningful contexts with respect to whole numbers and commonly used fractions and decimals.

SCO

By the end of grade 3, students will be expected to

- A8** order and compare decimals to tenths

Elaboration–Instructional Strategies/Suggestions

A8 When comparing or ordering decimals, as with performing these tasks with whole numbers, students must realize that there are several points one must keep in mind: (a) the whole number part of a number is a critical part for comparison, e.g., $2.39 < 4.2$ because $2 < 4$; (b) when numbers have the same whole part, the decimal part becomes the critical part for comparison, e.g., $4.3 < 4.7$ because 3-tenths is less than 7-tenths; and (c) it is important to examine the placement of digits, not just the number and size of digits, e.g., $6.2 < 40$ even though both numbers have two digits and those of 6.2 are greater than 40.

Initially, when comparing two decimal numbers, it is useful to use manipulative materials. By matching corresponding parts, students are able to see which number has more parts, e.g., If the rod in the base-10 blocks is one unit, 4 small cubes would show 0.4 and 6 small cubes would show 0.6; therefore, it is obvious that $0.4 < 0.6$.

It is appropriate to use the symbols $<$ and $>$ to represent comparisons, e.g., $3.4 < 5.6$ and $4.2 > 3.9$.

Students should have opportunities to order a list of decimal numbers.

- Have students examine skating scores from a competition and decide which was in first, second, and third place. As a variation, invite three students to perform some stunt and have other students give a decimal score between 0 and 1 for each performance. Then ask students to decide who earned the first, second, and third places.
- Prepare a deck of cards with numbers such as 0.1, 0.2, ... 0.9, 1.0, 1.1, ... 1.9, 2.0, 2.1, ... 2.9 for pairs of students. Each partner gets half the deck. They both turn over one card at a time. The student with the card with the greater number keeps both cards. Play continues until someone has all the cards.
- Have students order the following from greatest to smallest and draw segments using these numbers as measurements in centimetres: 7.2, 5.9, 8.3, 4.5, 9.7, and 6.0.

GCO A: Students will demonstrate number sense and apply number theory concepts.

Worthwhile Tasks for Instruction and/or Assessment
Suggested Resources
Performance

A8.1 Provide students with models of two decimal numbers, using base-10 blocks. Ask them to model another decimal number between these two and then to tell you the three decimal numbers.

A8.2 Using beans and cups of 10 beans, have students represent and order the following numbers from smallest to greatest: 2.7, 1.2, 3.1, 0.8, 2.6, 3.0, 0.2, and 1.5.

A8.3 Show students the decimal 0.7. Ask them to draw a rectangle and shade it to represent this decimal. Draw another rectangle the same size, shade a greater amount, and write the decimal represented by this rectangle.

Paper and Pencil

A8.4 Have students complete a given place-value pattern, e.g., 2.9, ____, 3.1, 3.2, ____, 3.4.

A8.5 Ask students to fill in the blanks with a decimal number that is between the two given numbers:

(a) 2.9, ____, 3.2 (b) 5.9, ____, 8.1 (c) 4.1, ____, 3.7

A8.6 Ask students to create four different number sentences involving decimals in which the $<$ sign is used. Ask them to rewrite each sentence so that the $>$ sign can be used.

A8.7 Ask students to draw two line segments, 4.3cm and 4.7cm, and then compare the two lengths.

Interview

A8.8 Tell students that you are thinking of a number that is greater than 1.5. Ask them to ask relevant questions that would help to determine the number.

Portfolio

A8.9 Ask students to describe or draw pictures of as many items as they can find that have a length between 0.8m and 1.8m.

**Number Concepts/
Number and Relationship
Operations:
Operation Sense and
Number Operations**

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Elaboration–Instructional Strategies/Suggestions

KSCO

- (i) By the end of grade 3, students will be expected to demonstrate an understanding of the connection between relevant, concrete experiences and the mathematical language and symbolism of four basic operations.

SCO

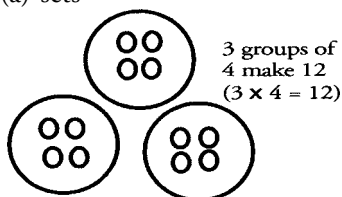
By the end of grade 3, students will be expected to

- B1** recognize several meanings for multiplication

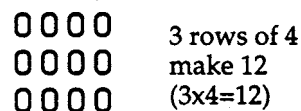
When students have number sense, they understand not only the relationships between numbers but also the effects of arithmetical operations on numbers. They exhibit confidence in their answers and willingness to investigate new situations. (NCTM 1992b, 9)

B1 Students’ understanding of the concept of multiplication should be extended through a variety of application, concrete, pictorial, symbolic, and verbal experiences. These would include contexts that are naturally represented by

(a) sets



(b) arrays



Jeri put 4 cookies on 3 plates. How many cookies were put out?

Mr. Smith set up 3 rows of 4 chairs for the parents. How many chairs did he use?

(c) number line



Michael walked 4km/h for 3 hours. How far did he walk?

In all three types of contexts, students should understand that they could be represented symbolically as a repeated addition sentence, i.e., $4 + 4 + 4 = 12$, or as a multiplication sentence, i.e., $3 \times 4 = 12$. Eventually students should understand that the three types of concrete/pictorial representations could be used interchangeably.

- Provide students with a number of story problems like those above. Ask them to represent each story concretely. Have them record their solutions by drawing pictures and writing addition and multiplication sentences.
- Provide students with a number of set, array, and number line pictures. Ask them to write the multiplication sentence that would represent each picture.

The convention for displaying arrays is for the rows to go left-right as in seats in a theatre rather than up-down as in a classroom, e.g., 2 rows of 5 or 2×5 would be represented by the array

```
X X X X X
X X X X X
```

If this array were rotated a quarter-turn, it would represent 5×2 .

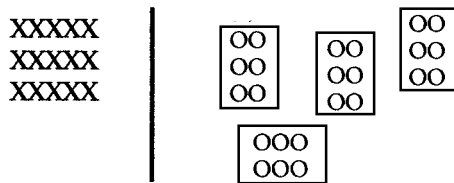
GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

Performance

B1.1 Ask students to record multiplication sentences and to create situations that would be represented by these drawings.



B1.2 Provide students with toothpicks. Ask them to use these toothpicks to make five squares and then state a multiplication sentence that describes how many toothpicks have been used.

B1.3 Ask students to use counters to show why the result is even whether you multiply two even numbers.

Paper and Pencil

B1.4 Have students draw pictures to show what 4×6 means.

B1.5 Give students the multiplication sentence $3 \times 7 = 21$. Ask them to draw array, set, and number line pictures that would represent it.

Interview

B1.6 Ask students what 6×3 might tell about tricycles.

B1.7 Ask students to use counters to make array and set models to represent 4×8 .

Portfolio

B1.8 Ask students to create situations and to draw pictures where multiplication might be used.

Presentation

B1.9 Invite a group of students to create a skit modelling a multiplication situation. Ask other students to write the number sentence being dramatized.

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

KSCO

- (i) By the end of grade 3, students will be expected to demonstrate an understanding of the connection between relevant, concrete experiences and the mathematical language and symbolism of the four basic operations.

SCO

By the end of grade 3, students will be expected to

- B2** recognize several meanings for division

Elaboration–Instructional Strategies/Suggestions

B2 It is important that students understand two meanings of division: *sharing* when the number of groups is known but the number in each group is unknown, and *how many groups* when the number of objects in each group is known but the number of groups is unknown. If 12 cookies are to be shared fairly among 4 people, sharing division would determine how many each person would get. If 12 cookies are to be put in packages of 4, how-many-groups division would determine how many packages would be made.

In both cases, students need to understand that $a \div b$ is equivalent to repeatedly subtracting b from a and counting the number of subtractions to reach zero, e.g., $12 \div 4 = 12 - 4 - 4 - 4$.

Modelling with concrete materials will help students understand the relationship between the two meanings of division. Demonstrate that in sharing 12 items among 3 people, e.g., the actual giving of one item to each person is the same as creating a group of 3. In other words, sharing among 3 people is equivalent to finding how many groups of 3 can be formed.

Give students situations in which there are remainders and ask them to determine how to deal with them. They should begin to understand that the treatment of a remainder depends upon the situation where it occurs. If 17 cookies are to be shared among 3 people, each person would get 5 cookies and the 2 remaining cookies could be shared by cutting them into fractional parts; however, if 17 cookies are to be put in packages of 3, there would be 5 packages and 2 left over cookies. If 17 marbles are to be shared among 3 people, each person would get 5 marbles; some decision would need to be made about what to do with the remaining 2 marbles. If 17 students are to go in 3 vans, 5 students would go in each van and a decision would have to be made about the remaining 2 (6 students in 2 vans, 5 in 1 van; 7 students in 1 van and 5 in 2 vans).

- Brainstorm with students activities suitable for various sized groups of people such as one person reading or drawing, two people playing tennis or chess, and three people skipping or playing marbles. Have them use these groupings to create division story problems involving their class.

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

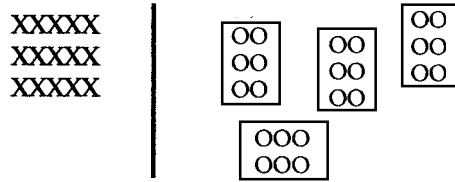
Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

Performance

B2.1 Have students enter 35 on a calculator and repeatedly subtract 7. Ask them to record these subtractions on paper until they reach 0. Have them write the division sentence that expresses what happened.

B2.2 Ask students to record two division sentences for each of the drawings below:



B2.3 Provide students with a variety of division story problems that would have remainders. Ask them to explain what they would do with the remainder in each story.

Paper and Pencil

B2.4 Ask students to create a division story about $30 \div 5$.

B2.5 Ask students to describe situations for which they might have to find the answer to $16 \div 2$.

B2.6 Ask students to draw array and set pictures to represent $13 \div 3$.

Portfolio

B2.7 Ask students to create situations and to draw pictures where division might be used.

Presentation

B2.8 Invite a group of students to act out a skit modelling a division situation. Ask other students to write the number sentence being dramatized.

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Elaboration–Instructional Strategies/Suggestions

KSCO

- (ii) By the end of grade 3, students will be expected to recognize and explain the relationships among the four basic operations.

SCO

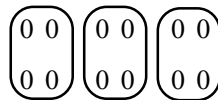
By the end of grade 3, students will be expected to

- B3** recognize the relationship between multiplication and division

B3 Students should understand that for every multiplication situation there is a related division situation and vice versa. The array below could represent three possible situations, each with a corresponding number sentence.

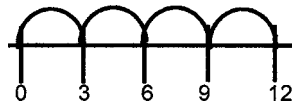
- | | |
|---------------------|--|
| $x \times x \times$ | (a) 2 groups of 4 written as $2 \times 4 = 8$ |
| $x \times x \times$ | (b) 8 shared between 2 written as $8 \div 2 = 4$ |
| | (c) 8 put in groups of 4 written as $8 \div 4 = 2$ |

Similarly, this set picture could have 3 interpretations.



- (a) $3 \times 4 = 12$ for 3 groups of 4
- (b) $12 \div 3 = 4$ for 12 shared among 3
- (c) $12 \div 4 = 3$ for 12 put in groups of 4

Using a number line also helps students see the relationship between multiplication and division.



- (a) $4 \times 3 = 12$ for 4 jumps of 3
- (b) $12 \div 3 = 4$ for 12 in jumps of 3
- (c) $12 \div 4 = 3$ for 12 made in 4 equal jumps

Students should realize that knowing this relationship between multiplication and division is very helpful, particularly when having to divide, e.g., If they have to calculate $36 \div 4$, they can think, 4 times *what* equals 36?

- Provide students with lots of practice with materials to help them visualize the operations. Ask questions as they form sets, e.g., Show me 4 groups of 5. How many do you have altogether? How did you divide up the counters? Encourage the students to respond using the language for describing multiplication and division, e.g., I have 4 groups of 5, which is 20. I started with 20 and divided them into 4 groups: twenty divided by 4 is 5. Extend this learning experience by asking students to start with the same number of counters, dividing them in another way.

The models that we can use to illustrate division concepts are exactly the same as those for multiplication. In fact, when a division is modelled, the result always looks like a multiplication model. (*Van de Walle 1994, 124*)

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

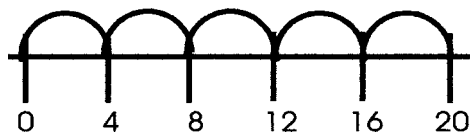
Performance

B3.1 Provide students with toothpicks and ask them to use 12 to make four identical shapes. Ask them what division and multiplication sentences could describe the creation of the shapes.

B3.2 Have students put 28 counters in equal amounts in four cups. Ask them to write the number sentence for this task and to write other number sentences related to this model.

Paper and Pencil

B3.3 Show the following number line. Ask students to record what multiplication and division sentences it shows.



B3.4 Show students the multiplication sentence $5 \times 8 = 40$. Ask them to write related division sentences.

Interview

B3.5 Tell students that there were 3 rows of 4 soldiers marching in the parade. Ask them to show these soldiers using counters, and to give the multiplication and two division sentences that describe it.

B3.6 Explain to students that $26 \div 4$ tells you something about the way the students are grouped in a classroom. Ask them to talk about the size of the group and the number of groups.

Portfolio

B3.7 Ask students to explain how the relationship between multiplying and dividing is like the relationship between adding and subtracting.

B3.8 Ask students to write story problems in which they have to multiply or divide to find the answer. Have them illustrate the solutions and describe the multiplication/division relationship.

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

KSCO

- (iii) By the end of grade 3, students will be expected to create and model problem situations involving whole numbers, using one or more of the four basic operations.

SCO

By the end of grade 3, students will be expected to

- B4** solve and create problems involving addition and/or subtraction
- B5** solve and create problems involving multiplication and division with small numbers

Elaboration–Instructional Strategies/Suggestions

Solving and creating routine problems with the four operations should be an integral part of the development of these concepts. Students should realize that they are learning to add, subtract, multiply, and divide in order to solve problems in the real world. Asking them to create their own story problems requires them to focus on the meaning of the operations and allows them to choose contexts that are meaningful and of interest to them. Such student-generated problems are also a valuable diagnostic tool.

Students should be encouraged to use calculators and estimation strategies when solving story problems involving large and/or decimal numbers and those involving many steps. It is important that they learn to make appropriate use of technology.

You should provide opportunities for students to solve and create multi-step story problems that require two or more operations. Having students working together as partners or in small groups would be very appropriate for such activities.

Students should also solve and create non-routine problems involving the operations, e.g., They might be asked to use the digits 2, 3, 4, and 5 to create the largest difference of two 2-digit numbers; or to arrange 3, 4, and 5 to make $\square \times \square + \square = 19$ true.

B4 Provide students with addition and subtraction story problems of various structures: (a) join and separate problems with the results, the changes, or the starts unknown; (b) part-part-whole problems; and (c) comparison problems. (See *Van de Wale 1994/1998* for a comprehensive discussion of these types of story problems.) You should model and discuss some of the less common structured story problems and invite students to create ones that are like them.

Be sure to include story problems involving money and time as special cases of addition and subtraction. With money, there is a great opportunity to see the advantage of subtracting by counting up in order to make change. With time, attention has to be given to regrouping with 60 minutes to the hour.

B5 Students should solve and create story problems involving various meanings of multiplication and division. (See SCOs, B1 and B2.)

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

Performance

B4.1 Ask students to determine the change from \$5.00, if the bill totalled \$3.59.

Paper and Pencil

B4.2 Have students use only 1s, 2s, and 3s, along with + signs to get a total of 45. Ask, Can this be done using only one +?

B4.3/5.1 Ask students to create story problems that are represented by particular computations, e.g., $212 + 35$ or 8×9 .

B4.4/5.2 Provide students with lists, e.g., food prices. Ask them to use these lists to create and solve problems.

B4.5 Tell students that Martha arrived at 7:45 p.m. and left 20 minutes later. Ask, What time did she leave?

B4.6 Ask students to create addition problems that have the same structure as the following: Elaine had money saved for her vacation. After she won \$250 in the lottery, she had a total of \$815. How much had she saved before winning the lottery?

Interview

B4.7 Observe how students find the time that has passed between 9:42 a.m. and 10:15 a.m.

B4.8 Display two 3-digit numbers using base-10 blocks. Ask students to create situations that would require finding the sum of these two numbers. Have them use the blocks to find the solution.

Portfolio

B4.9/5.3 Have students choose an even number between 1 and 100. Ask them to create story problems in which this number is either the answer or involved somehow in the solution.

B4.10/5.4 Provide students with enrolments in a number of local schools. Ask them to create and solve problems using these data.

Presentation

B5.5 Have pairs of students choose mystery objects and make up a riddles about them, using a multiplication or division sentence as the clue, e.g., I'm thinking of something in this room that shows 4×4 might be a clue for the legs on a group of 4 desks.

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Elaboration–Instructional Strategies/Suggestions

KSCO

- (v) By the end of grade 3, students will be expected to apply computational facts and strategies with respect to the four basic operations and to model addition and subtraction in situations involving whole numbers.

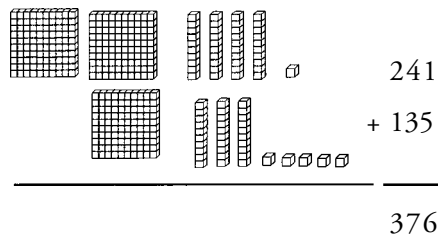
SCO

By the end of grade 3, students will be expected to

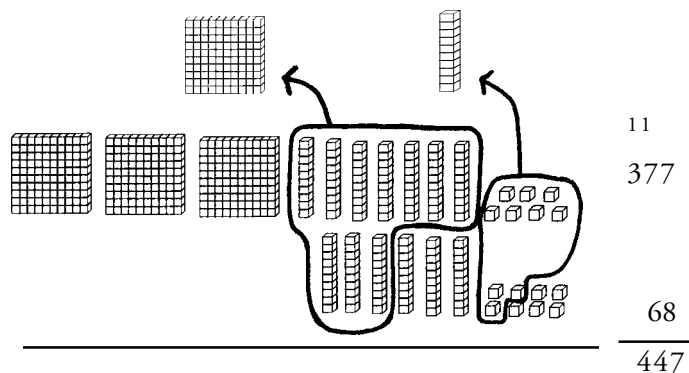
- B6** add and subtract with and without regrouping (up to and including three-digit numbers)

B6 Students need to develop and use alternative paper-and-pencil and mental algorithms for computations, appreciating that they are as valid as traditional algorithms. Mixing addition or subtraction problems requiring, and not requiring, regrouping forces students to examine each question before selecting a strategy.

To develop addition and subtraction with 3-digit numbers, students should use base-10 materials to model the operations, e.g.,



The front-end algorithm could be used for the above example; however, a regrouping strategy might be more useful for $377 + 68$.



For computations like $81 - 37$, students might use a *constant-difference* strategy, i.e., 3 is added to both numbers to maintain the constant difference between them and to make the subtraction easier using $84 - 40$. When students are adding numbers that are close to a multiple of 100, using a *round-and-adjust* strategy, rather than dealing with all of the high digits, is very helpful, e.g., $298 + 399 + 198$ can be solved by adding $300 + 400 + 200$ and subtracting the 5 that was added to round, yielding the answer of 895.

Use terms regrouping and trading rather than borrowing and carrying to convey more meaning about the process.

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

Performance

B6.1 Observe students as they add 125 and 134 and as they subtract 134 from 217 using base-10 materials.

B6.2 Ask students to complete the following addition or subtraction calculations and to explain their strategies:

(a) $38 + 97$ (b) $98 - 44$ (c) $400 - 255$

B6.3 Show students a number of addition and subtraction exercises, some that require regrouping and some that do not. Ask them to loop the exercises that require regrouping.

Paper and Pencil

B6.4 Have students explain in writing why someone might first subtract 30 from 67 in order to calculate $67 - 26$. Ask them to also explain what would be done next.

Interview

B6.5 Display the numbers 124 and 175 with base-10 blocks. Ask students to describe the addition process as they manipulate the models and record the process.

B6.6 Tell students that when Sue was to add $36 + 59$, she said, “36, 96, 95.” Ask them to explain Sue’s thinking.

B6.7 Ask students why someone might find it easier to subtract $123 - 99$ than $123 - 87$.

B6.8 Ask students to explain easy ways to find the sum of \$199, \$298, and \$499 without using calculators.

Portfolio

B6.9 Ask students to prepare displays showing a variety of ways to calculate $57 - 18$, indicating their preferred ways and the reasons for them.

B6.10 Ask students to use sales flyers to create problems for their classmates. Have them include both the problems and their solutions in their portfolios.

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Elaboration–Instructional Strategies/Suggestions

KSCO

- (v) By the end of grade 3, students will be expected to apply computational facts and strategies with respect to the four basic operations and to model addition and subtraction in situations involving whole numbers.

SCO

By the end of grade 3, students will be expected to

- B7 recognize principles of multiplication and division
- B8 relate multiplication and division facts

Students are likely to use the corresponding multiplication fact to think of a division fact. ... It is an interesting question to ask, "When students are working on a page of division facts, are they practising division or multiplication?" (*Van de Walle 1994, 150*)

B7/8 Through exploring multiplication and division situations, students should recognize the following principles:

- (a) sets can be broken down into subsets, e.g.,

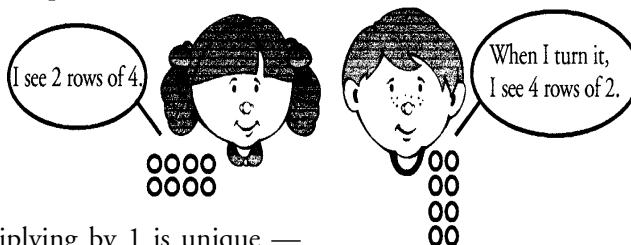
5 sets of 3  can be

4 sets of 3 + 1 set of 3  , or

3 sets of 3 + 2 sets of 3  , or

5 sets of 2 + 5 sets of 1 

- (b) order doesn't matter when you multiply, e.g., 2×4 and 4×2 both have a product of 8



- (c) multiplying by 1 is unique — $1 \times \square$ simply means one group of \square .
- (d) multiplying by 0 is unique — $\square \times 0 = 0$ since many zeros still equal zero
- (e) dividing by 1 is unique — $\square \div 1$ simply means how many 1s in \square

Understanding these principles will help students when they have to master the multiplication and division facts, e.g., 6×8 can be thought of as $5 \times 8 + 1 \times 8$; or $36 \div 6$ as $30 \div 6 + 6 \div 6$.

Students learn division facts by thinking about corresponding multiplication facts. They can reduce the number of separate multiplication facts to be learned by drawing on a relationship previously explored, e.g., any multiple of 4 is twice the same multiple of 2. To help students learn to find one fact based on what they know about another, include, on a regular basis, questions such as, How does knowing $5 \times 4 = 20$ help you to know 6×4 ? What other division fact could help you solve $48 \div 6$?

After students have had extensive experiences modelling the operations, they should begin developing quick recall of some facts using visualization and a variety of other strategies.

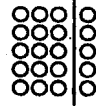
GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

Performance

B7.1 Display a 5 x 4 array of objects and place a ruler on the line shown.

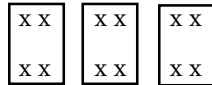


Ask students how this shows that $5 \times 4 = 5 \times 3 + 5 \times 1$. Then ask them to move the ruler to show another way to find 5×4 and to explain their thinking.

B8.1 Have students colour in all the places where 12 appears on a multiplication table. Ask them to describe what they observe.

Paper and Pencil

B7.2 Ask students how the diagram below, which shows 3×4 , also shows 6×2 .



B7.3 Ask students to explain how knowing $4 \times 3 = 12$ helps them solve the following questions:

$4 \times 6 = \square$ $8 \times 3 = \square$ $4 \times 4 = \square$

Interview

B7.4 Ask students to explain why it is easy to multiply by 1 or 0.

B7.5 Have students use counters and stories about packages of candy to compare 3×5 and 5×3 .

B8.2 Ask students to explain how knowing that $5 \times 6 = 30$ could help them to know other multiplication or division facts.

B7.6 Tell students that to solve $42 \div 7$, Alan said, " $21 \div 7 = 3$." Ask them to explain what Alan would do next.

B7.7 Provide students with copies of a multiplication grid. Point out the row beginning with 6, 12, ... Ask them to explain why the numbers increase by 6 each time.

Presentation

B8.3 Ask students to consider the relationships between multiplication and division facts in order to decide the minimum number of facts they think it might be necessary to learn in order to know all of them. Their presentations should include an explanation of how they arrived at the minimum number.

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

KSCO

(vi) By the end of grade 3, students will be expected to apply estimation techniques to predict and justify the reasonableness of results in relevant problem situations involving whole numbers.

SCO

By the end of grade 3, students will be expected to

B9 continue to estimate in addition and subtraction situations

Each mental algorithm can be presented to your class, discussed, and practiced briefly on different days. As new methods are introduced, some students will select different approaches for the same task. These should certainly be discussed and accepted. Once a method has been taught, be sure to encourage continued practice on a daily basis. (*Van de Walle 1994, 203*)

Elaboration–Instructional Strategies/Suggestions

B9 Estimation experiences should include operations involving 2- and 3-digit numbers. One method for estimating is *rounding*, sometimes rounding to the nearest hundred and other times to the nearest ten. It is important they understand that the situation should dictate how closely to estimate, e.g., If you have \$255 and have to estimate the total of two items costing \$79 and \$172, you might round to the nearest \$10; however, if the items cost \$96 and \$88, estimating to the nearest \$100 would be sufficient.

- Have students use number lines to model estimation, imagining gas stations at each multiple of 100 (or 10 if rounding to the nearest 10). To estimate $392 + 475$, they would place their vehicle on 392 and see that they are almost at 400. Therefore, $392 + 475$ is about 875. This also illustrates that it is not always necessary to round both numbers.
- Have students model estimation using base-10 blocks.
- Have students use the *front-end* method to estimate, e.g., $138 + 245$ would be done by thinking $100 + 200$ is 300 and $30 + 40$ is 70, for an estimate of 370; $476 - 348$ would be done by thinking $400 - 300$ is 100, $70 - 40$ is 30 for an estimate of 130.

Sometimes a combination of *front-end* and *clustering* produces a better estimate, e.g., For the computation to the right, add the hundreds digits first ($300 + 300 + 200$, or 800), then cluster the 29, 35, and 42 together to make an additional 100, for an estimate of 900.

$$\begin{array}{r} 329 \\ 335 \\ + 242 \\ \hline \end{array}$$

Students should carefully examine a computation first in order to select a strategy that will give them a good estimate, is efficient, and makes sense to them.

The ultimate goal is for students to estimate on their own, not just when you or the textbook require it. They have to value the skill and they must know that you value it. Estimation should precede all pencil-paper and calculator computations; therefore, it is important that students have efficient estimation strategies. Estimating is a mental activity, one that improves with regular practice. Encouraging students to share their thinking after estimating will contribute to the range of strategies for students to use.

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

Performance

B9.1 Have students toss three dice and list all the possible 3-digit numbers that can be formed from the digits rolled. Ask students to give estimates of how much should be added to each number to get a sum of about 1000.

Paper and Pencil

B9.2 Tell students that the sum of $3 \square 4$ and $5 \square 3$ is about 900. Ask them to suggest what might go in the boxes.

B9.3 Tell students that you were given a subtraction question for which you estimated the answer to be 100. Ask them to list some numbers you might have been subtracting.

B9.4 Tell students that you subtracted a number in the three hundreds from a number in the five hundreds. Ask them to suggest what the numbers might have been if you have correctly estimated the answer to be 100.

B9.5 Show students the number of sports cards in James' collection.

Baseball	247
Football	124
Hockey	138

Ask students to estimate the total number of cards in the collection and to describe the strategies they used.

Interview

B9.6 Ask students which of the following have solutions close to 500: $329 + 189$ $329 + 217$ $329 + 287$

Ask students which of the following has its solution closest to 50:
 $125 - 30$ $168 - 115$ $103 - 82$

In both cases, ask students to explain their thinking.

B9.7 Ask students to identify situations in which exact answers would be required and situations in which estimates would be sufficient.

B9.8 Explain to students that a teeter-totter will hold up to 300 kg. Ask them to estimate the number of students their age who could safely play on it and to explain how they arrived at their estimates.

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

KSCO

- (vi) By the end of grade 3, students will be expected to apply estimation techniques to predict and justify the reasonableness of results in relevant problem situations involving whole numbers.

SCO

By the end of grade 3, students will be expected to

- B10** begin to estimate in multiplication and division situations

Elaboration–Instructional Strategies/Suggestions

B10 In multiplication and division situations, your students are probably still working with products less than 100; therefore, many estimation activities might focus on whether the product is closest to 20, 40, 60, or 80 or the quotient is closest to 1, 5, or 10.

- Help students develop strategies for estimating in a variety of multiplication and division contexts such as the following:
 - $7 \times \$8.99$ will be about \$63 because \$8.99 is almost \$9.00
 - $56 \div 6$ is less than 10 because one would need 60 to make 10 groups of 6
 - $52 \div 8$ would not be close to 10 because 10 groups of 8 is 80, but it will be closer to 6 because 6 groups of 8 is 48
 - 3×17 is closer to 3×20 , or 60, than 3×10 , or 30, and a good estimate would be 50.
- Provide many multiplication and division estimation situations that deal with money.
 - The popsicles in the cafeteria cost 15¢ each. I have a loonie. Can I buy one for myself and each of my five friends? (Student thinking might be: At 10¢ each, I could buy 10 popsicles. At 20¢ each I could buy only 5 popsicles. So, at 15¢ each I can buy somewhere in the middle, between 5 and 10 popsicles.)
 - If scribblers are on sale for \$0.98, and you need to buy 9 of them, about how much change will you get from \$10?
- Ask students to interview parents, relatives, and neighbours to find out when they use estimation in their daily lives. Students could prepare reports and share their findings with the class. Such an activity could be part of students' portfolios. It would reinforce the importance of estimation in real life.

Estimation is a mental activity, one that becomes more precise with practice. Regular attention to estimation activities and strategy sharing is necessary, promoting the use of mental math.

Many researchers advise that teachers promote estimation strategies before students know paper–pencil algorithms. This would mean asking your students for estimates of products of 2-digits by 1-digit and quotients of 2-digits by 1-digit before they know the algorithms for such computations.

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

Paper and Pencil

B10.1 Have students estimate whether the products of the following questions are closest to 20, 40, 60, or 80, recording how they know, without actually calculating the exact answer:

- (a) 4×8.9 (b) 7×11 (c) $5 \times \$3.99$

B10.2 Show students pictures of hexagons with side measurements of 19 cm. Ask them to estimate about how many centimetres it would be around the outside of these hexagons.

B10.3 Tell students that Martha has a loonie and wants to buy three party favours that cost 29¢ each. Ask them to decide whether Martha has enough money and to explain their thinking.

B10.4 Tell students that you have a loonie. Ask, How do you know that I am not able to buy 5 packages of stickers that cost 21¢ each?

Interview

B10.5 Ask students to tell how they know whether the answers to the following questions are closest to 1, 5, or 10, without actually performing the calculations:

- (a) $75 \div 8$ (b) $25 \div 6$ (c) $23 \div 4$

B10.6 Tell students that you bought two dozen hot dogs for a party for seven people. Ask them to estimate to decide whether these will be enough hot dogs.

B10.7 Show students the number of library books signed out at each grade level.

Grade Primary - 21	Grade 3 - 22
Grade 1 - 20	Grade 4 - 18
Grade 2 - 19	

Explain to them that the librarian said that about 100 books were signed out by the five classes. Ask them to explain the librarian's thinking.

Presentation

B10.8 Have students ask grown-ups to describe some situations in which they might use estimates involving multiplication and division at food stores. Ask them to share their findings with their classmates.

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

KSCO

- (vii) By the end of grade 3, students will be expected to select and use appropriate computational techniques (including mental, paper-and-pencil, and technological) in given situations.

SCO

By the end of grade 3, students will be expected to

- B11** mentally add and subtract two-digit and one-digit numbers
B12 mentally add and subtract rounded numbers
B13 use technology to solve problems involving larger numbers

Mental algorithms develop and improve in both quality and quantity over years of practice. ... Mental arithmetic is not a three-week unit but a long-term goal. (*Van de Walle 1994, 203*)

Elaboration–Instructional Strategies/Suggestions

B11 Continue to provide oral practice of addition and subtraction facts that are the foundation for mental math. Reviewing and reinforcing strategies for fact learning should also be part of this practice. When an exact answer is required, students should learn to choose the appropriate method—a mental math strategy, a paper-pencil algorithm, or a calculator.

- In order for students to become more efficient in performing mental calculations, expose them to a variety of strategies.
 - *Make 10*, e.g., For $26 + 7$, you might think 26 and 4 make 30, and 3 more is 33; for $37 + 26$, 37 plus 3 is 40 and 23 more is 63.
 - *Front end*, e.g., For $47 + 8$, think 40 plus 15 ($7 + 8$), or 55; for $24 + 12 + 31$, you might say, “20, 30, and 30 makes 60, 64, 66, 67; the answer is 67.”
 - *Counting on*, e.g., For $50 - 19$, think, “29, 39, 49 is 30 plus one more is 31, so the difference is 31.”
 - *Subtract 10 and compensate*, e.g., For $31 - 8$, you might subtract 10 and add 2 back on.
 - *Constant difference*, e.g., For $54 - 29$, if 1 is added to both numbers, making $55 - 30$, the difference will be the same and the subtraction will be easier, making 25.
 - *Use nearest multiple of ten and compensate*, e.g., For $31 - 8$, you might think, “30 minus 8 is 22 and 1 more is 23.”
 - *Compatible numbers*, e.g., For $25 + 32 + 75$, you might notice that 25 and 75 make 100, then add the 32. Students need to become familiar with number combinations that go together to give sums of 10 and 100.

It is useful to use hundreds charts to help students visualize mental calculations, e.g., When adding 11, go down one (add 10) and over one (add 1).

B12 Students should continue to mentally add and subtract rounded numbers, e.g., $400 + 500$, $100 - 80$, $300 - 30$.

B13 Students should be encouraged to use calculators as tools in problems requiring tedious calculations or those beyond their capabilities. Calculators should not be a substitute for mental and paper-pencil strategies involving 2- and 3-digit numbers.

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

Performance

B11.1 Ask students to add mentally as you draw numbers from a bag and announce them, and to stop you when the sum has passed 40.

B13.1 Ask students to show how they would use calculators to find $4234 + 187$.

B11.2 Show pictures of several items with prices less than one dollar. Have students start with \$2.00 and buy as many items as possible. Ask them to tell you how much is left after each purchase.

Paper and Pencil

B11.3 Have students make lists of calculations involving 2- and 3-digit numbers which would be more quickly done mentally than on paper or with a calculator.

B12.1 Present calculations, orally (or on an overhead), and ask students to record only the answers. Allow only a few seconds for each question, e.g., $300 + 600$, $200 - 40$, $200 + 80 + 30$, and $220 - 40$.

B11.4 Ask students to give the compatible number for each of the following:

- (a) 82 (b) 49 (c) 65 (d) 75 (e) 60

Interview

B11.5 Ask students to describe strategies for calculating $48 - 9$ and $76 + 11$ mentally.

B11.6 Tell students that to find $37 - 8$, Rita said, “37, 27, 29.” Have them explain what might be Rita’s thinking.

B11.7 Tell students that you had three quarters and spent 48¢. Ask them to explain how you might mentally figure out the change you should have.

Portfolio

B11.8 Have students explain why, when adding a single-digit number to a number in the fifties, the answer has to be either in the fifties or in the sixties.

Patterns and Relations

GCO C: Students will explore, recognize, represent, and apply patterns and relationships, both informally and formally.

Elaboration–Instructional Strategies/Suggestions

KSCO

- (i) By the end of grade 3, students will be expected to recognize, describe, extend, and create patterns and sequences in a variety of mathematical and real-world contexts, e.g., geometric, numeric, and measurement.

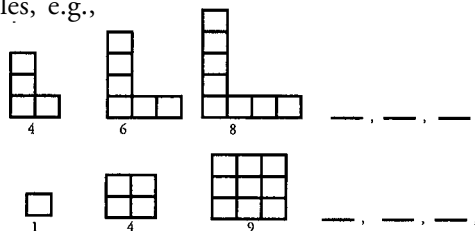
SCO

By the end of grade 3, students will be expected to

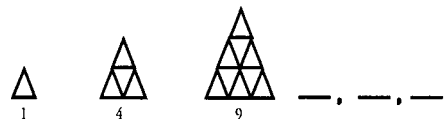
- C1 recognize the pattern implicit in the place-value system
- C2 recognize and create geometric patterns

C1/C2 Students have learned that the digit representing hundreds is to the left of the one representing tens in the place-value system and that it takes ten of one unit to be equivalent to one of the units to its left. Students can use this pattern to understand how we extend the place-value system to decimals, i.e., If 10 ones make 1 ten, 10 tens make 1 hundred, and 10 tenths make 1, the place to the right of the ones should be tenths.

- Provide students with opportunities to connect patterns to number ideas as in the following:
 - Patterns found in the hundreds charts should continue to be explored and expanded.
 - Ask students to respond orally to give a number that is 100 more (100 less, 10 more, 10 less) than 2- or 3-digit numbers that you provide.
 - Using logic blocks, have students create one-difference trains. The difference between each shape could be color, shape, size, or thickness.
 - Have students create L-shaped numbers and square numbers using tiles, e.g.,



- Ask students to create triangular numbers with Δ pattern blocks.



- Have students observe and create patterns with shapes based on transformations (slides, flips, and turns), e.g.,



Almost all patterning activities should involve some form of physical materials to make up the pattern. This is especially true of repeating patterns in grades K to 4. (*Van de Walle 1994, 373*)

GCO C: Students will explore, recognize, represent, and apply patterns and relationships, both informally and formally.

Worthwhile Tasks for Instruction and/or Assessment

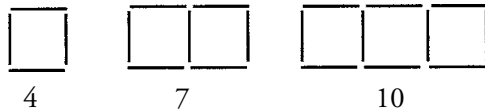
Suggested Resources

Performance

C2.1 Provide students with pattern blocks, asking them to create and describe patterns using flips.

C2.2 Ask students to find, in the classroom, examples of patterns created by slides, flips, or a combination of the two.

C2.3 Ask students to use toothpicks to continue the pattern shown below:



C2.4 Show a picture of the following pattern:



Ask students to use the triangles in the pattern blocks to recreate this pattern and to describe the pattern in terms of slides and flips.

Paper and Pencil

C1.1 Provide parts of number patterns, asking students to complete them to make patterns, e.g.,

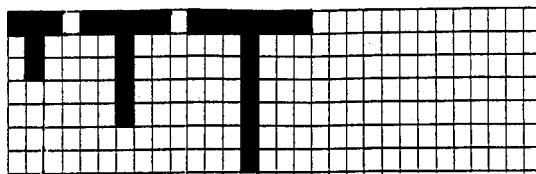
- 475, 575, _____, _____
- 233, _____, 253, _____
- 420, 440, _____, _____
- 556, _____, _____, 889

Interview

C1.2 Ask students, If 2.3 is changed to 2.31, what do you think the 1 represents? If only the tens in 321 are multiplied by 10, what is the new number?

Portfolio

C2.5 Provide cm² paper for students. Have them use coloured pencils to copy and continue the pattern below. Ask students to create their own growing patterns.



GCO C: Students will explore, recognize, represent, and apply patterns and relationships, both informally and formally.

KSCO

- (ii) By the end of grade 3, students will be expected to use patterns to solve problems.

SCO

By the end of grade 3, students will be expected to

- C3 use and recognize the patterns in a multiplication table


Elaboration–Instructional Strategies/Suggestions

C3 Students should find and explain patterns that occur in the multiplication grid. It is important that students understand they can use these patterns to determine other products or quotients.

Multiplication Grid

x	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81

- Have students explore the following patterns in the multiplication table and explain why they work:
 - the numbers in each row and column increase by the same amount
 - the numbers in each row increase by an amount one greater than the increase in the previous row
 - the square numbers are found on the left-right diagonal and the numbers on the left-right diagonal increase by 1, 3, 5, ...
 - the row for products of 4 is double the row for products of 2, the row for products of 6 is double the row for products of 3
 - when you add the corresponding products for 2 and 3, you get the products for 5, e.g., 2×4 (8) plus 3×4 (12) is the same as 5×4 (20)
 - when you “cross multiply” any 4 numbers that form a square on the grid, the product is the same, e.g.,

$2 \times 6 = 3 \times 4$

 - when you “cross add” these numbers and subtract, you always get 1
 - the grid is symmetrical, i.e., numbers under the left-right diagonal are reflections of the numbers over this diagonal

GCO C: Students will explore, recognize, represent, and apply patterns and relationships, both informally and formally.

Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

Paper and Pencil

C3.1 Ask students to fill in the missing numbers to create patterns, explaining the reasons for their choices.

- 4, 8, ___ , 16, 20
- 5, ___ , 15, ___ , 25
- 3, ___ , ___ , 12, 15

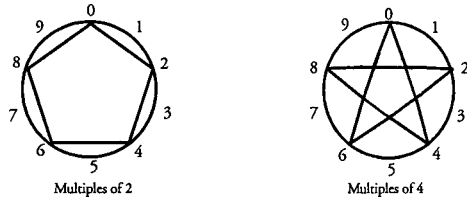
C3.2 Tell the students, Sue says that whenever she needs a multiple of 6, she uses a multiple of 5 and then adds. Ask students to explain what Sue is doing, using 6×8 as an example. Ask them why it works.

Interview

- C3.3 Ask students to look for patterns among the multiplication facts involving nine, explaining any pattern(s) they find.
- C3.4 Ask students to show how they could use the multiplication grid to practise skip counting.
- C3.5 Ask students to explain why some columns and rows have both even and odd numbers.
- C3.6 Ask students to use multiplication grids to explain why 4×5 plus 2×5 is the same as 6×5 .

Portfolio

C3.7 Have students create visual patterns using the final digits of the products for each of the multiples in the multiplication table. In each case, the final digit of one multiple is connected by a line to the final digit of the next, e.g.,



After all the visual patterns are created, ask students to record any observations they make.

GCO C: Students will explore, recognize, represent, and apply patterns and relationships, both informally and formally.

KSCO

- (iii) By the end of grade 3, students will be expected to represent mathematical patterns and relationships in informal ways, including via open sentences, i.e., statements with missing addends.

SCO

By the end of grade 3, students will be expected to

- C4 record a repeated addition pattern using multiplicative notation
- C5 recognize the meaning of open sentences of the forms:
 - $a \times b = \square$
 - $a \times \square = c$
 - $\square \times b = c$

Elaboration–Instructional Strategies/Suggestions

C4/C5 When faced with situations involving repeated addition, students should recognize that these additions can also be written as multiplications. The first factors tells how many times the addend is repeated and the second factors represent the addends, e.g., $5+5+5+5+5+5+5$ means there are 7 sets of 5 and can be written as 7×5 .

- Ask students story problems such as, There were 3 muffins in each package and I bought 6 packages. How many muffins did I buy? Ask students to model the problem, to skip count to determine the answer, and to write the repeated addition pattern as well as the corresponding multiplication notation, 6×3 . Point out that although 3×6 is also 18, the story would involve 3 packages of 6 muffins.

- Provide students with opportunities to look for patterns such as

4 packages	$3 + 3 + 3 + 3$	4×3	12
5 packages	$3 + 3 + 3 + 3 + 3$	----	----
6 packages	-----	----	----
7 packages	-----	----	----

Although open multiplication sentences with missing factors—particularly with a missing first factor—are more challenging for students to solve than open sentences with the product missing, students need experience with all types to develop the facility needed for division.

- Provide students with opportunities to practice stating the meaning for open sentences, e.g.,
 - $4 \times \square = 24$ — Four sets of how many make 24?
 - $\square \times 5 = 15$ — How many sets of 5 equal 15?
 - $3 \times 6 = \square$ — 3 sets of 6 is how many?

To draw attention to the fact that some products occur more frequently and that some open sentences can have more than one solution, have students find all possible solutions to questions such as the following:

$\square \times \square = 12$ $\square \times \square = 5$ $\square \times \square = 24$

GCO C: Students will explore, recognize, represent, and apply patterns and relationships, both informally and formally.

Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

Paper and Pencil

C5.1 Ask students to solve the following open sentences:

$$4 \times 3 = \square$$

$$7 \times \square = 21$$

$$\square \times 4 = 20$$

$$\square \times \square = 18$$

Have them show the meanings in sketches or diagrams.

C4.1 Have students create story problems that show the difference between 8×4 and 4×8 . Ask them to explain why the answers (products) are the same.

C4.2 Ask students to fill in the missing sections of the following chart on baseball, remembering that there are 9 members on a baseball team.

	$9 + 9 + 9 + 9$	4×9	
3 teams	$9 + 9 + 9$		
6 teams		6×9	
			45

Interview

C4.3 Ask students to explain how to calculate $6 + 6 + 6 + 6 + 6 + 6 + 6$ without doing the addition.

C4.4 Ask students to use calculators to find 6×8 without using the multiplication keys.

C5.2 Ask students to create three different open multiplication sentences for which the solutions are all 12.

C5.3 Show students the following equations and ask them to give the meaning for each:

$$6 \times \square = 18$$

$$\square \times 5 = 20$$

$$4 \times 4 = \square$$

Shape and Space: Measurement

GCO D: Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

KSCO

- (ii) By the end of grade 3, students will be expected to identify and use non-standard and standard units of measurement and to appreciate their role in communication.

SCO

By the end of grade 3, students will be expected to

- D1** estimate and measure length in metres, decimetres, and centimetres
- D2** estimate and measure capacity in millilitres and litres

Besides its value outside the classroom, ... estimation in measurement activities helps students focus on the attribute being measured, adds intrinsic motivation, and helps develop familiarity with standard units. (*Van de Walle 1994, 310*)

Elaboration–Instructional Strategies/Suggestions

D1 Students need help to develop estimation strategies for measurement. Initially, you could ask for a comparison estimate rather than a measure, e.g., Ask if the height of the door is more than/less than/equal to 2m, or ask if the length of the textbook is closer to 1dm or 2dm or 3dm. Students need to develop visual referents for units of length in order to have known measures upon which to base their estimates, e.g., A student with a hand span of 15cm uses that measure as a referent when asked to estimate the lengths of objects in centimetres; a student uses the known measure of the height of a doorknob above the floor, usually 1m, when asked to estimate the lengths of things in metres. All measurements made with instruments should be preceded by estimates.

The decimetre unit will be new to most students and, although not yet commonly used in everyday life, it is a convenient unit for classroom use. It is easily introduced because it is the length of the rod in the base-10 blocks; this rod will likely be the visual referent that most students would use for estimating in decimetres. Using the rod length as 1dm, it is easy to line up 10 centimetre cubes to establish that 1dm = 10cm, and to line up 10 rods against a metre stick to establish that 1dm = 0.1m. This context is also an ideal one to help develop the concept of tenths. (See SCO, A7.)

- Set up a mini-Olympics in which students compete in events such as a kleenex kick, a penny thumb toss, and a cotton ball puffing. Have students measure their results to the nearest decimetre, record them, and make comparisons.

D2 Students should continue to estimate and measure capacities using the litre unit and be introduced to the millilitre unit. Graduated measuring cups can be used to provide students with a sense of 25mL, 100mL, 250mL, and 500mL. Graduated medicine cups which typically accompany students' medication or medicine droppers provide good examples of things measured using small numbers of millilitres. Also, a cube the size of the small base-10 unit cube will hold 1mL of liquid; one the size of the large cube will hold 1 litre, or 1000mL—these make good referents for L and mL in classroom estimation activities.

- Have students discuss which unit (mL or L) is more likely to be used in measuring: (a) a dose of medicine, (b) punch in a punch bowl, (c) pop in a can, and (d) water in a bathtub.

GCO D: Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

Performance

D1.1 Ask students to estimate the heights and widths of some objects in the classroom using a metre reference, e.g., doorknob height. Have them check the estimates by doing the measurement.

D2.1 Ask students to find the capacity of a cereal bowl.

Interview

D2.2 Tell students that it has been suggested that we need to drink at least six glasses of water a day. Ask, About how many litres would that be? What would a container look like that would hold enough water for you for a week?

D1.2 Ask students to find objects in the classroom that are about 1 decimetre in length and 2 decimetres in length. Ask them to compare a decimetre in relation to rulers and metre sticks.

D1.3 Tell students that a bald eagle was found to be 109cm from beak to tail. Ask them to estimate and show how long that would be by cutting lengths of string. Have them check their estimates by measuring the strings.

D1.4 Have students put their two index fingers together. Ask them to show you, by moving apart these fingers, estimates for 6 cm, 24 cm, 32 cm, and 58 cm.

D1.5 Tell students that you know a student who is 6 dm tall. Ask them for an approximate age of this student.

Presentation

D2.3 Ask students to devise and carry out plans to determine how much water people waste when taps are not turned off while brushing their teeth. Have them use their findings to determine how much water would be wasted if all the students in the class neglected to turn off the tap for one brushing.

Portfolio

D2.4 Have students check containers at home, at stores, and in flyers to make charts of the various sizes for different products.

D2.6 Have students keep personal records (in mL) of the beverages they consume in one week. Have them translate the total consumption for the week into litres and choose a container (or combination of containers) which would hold that amount.

GCO D: Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

KSCO

- (ii) By the end of grade 3, students will be expected to identify and use non-standard and standard units of measurement and to appreciate their role in communication.

SCO

By the end of grade 3, students will be expected to

- D3** estimate and measure mass in grams and kilograms
D4 estimate and measure area in non-standard units and square centimetres

Let students measure to check estimates. However, it is only necessary that one or two students or one team do the measurement if the focus of the activity is on estimation. If all students are required to follow their estimates with a measure, they may correctly wonder why they bothered estimating. (*Van de Walle 1994, 311*)

Elaboration–Instructional Strategies/Suggestions

D3 Students should continue to estimate and measure masses using the kilogram and be introduced to the gram. Students need to understand that grams are used to measure very light objects. As with all measurement units, it is important that the students have a personal referent for grams and kilograms that they develop with direct experiences to use in estimation activities.

- Plan a variety of mass activities such as the following:
 - Ask, Could you eat a 1kg cantaloupe? 1kg of popcorn?
 - Ask, If sliced meat sells for \$3.50 per g, is it expensive?
 - Measure 20g of unpopped popcorn. Ask students, Will the mass be greater than, the same as, or less than this after it has been popped? Pop the corn and have them also compare the capacities of the popped and unpopped popcorn.
 - Have students discuss which unit (g or kg) is more likely to be used in measuring (a) a bag of potatoes, (b) a box of paper clips, and (c) a baby.
 - Have students estimate the masses of different objects in the classroom. Get two students to measure each object after its mass has been estimated in order to check the estimates and help students refine their estimating skills.

D4 Have students use non-standard units and square centimetres to measure the areas of common items, e.g., They might use index cards or transparent grids to find the area of their desk tops, estimating before they measure.

Students should see that when they use different grids or objects as units, they get different numbers for the areas. After work with non-standard units, introduce a centimetre grid and explain that a square centimetre is one standard unit for finding area. Students should have numerous opportunities to find the area of different surfaces using square centimetre grids (either paper or transparent).

- Provide a variety of interesting area activities.
 - Ask students to devise a way of measuring the surface area of apples. (They could place the peels on a transparent grid and cover with tape.)
 - Have students find the areas of animals drawn on a grid.
 - Have students compare the areas of different sizes of paper, pattern blocks, or envelopes.
 - Have students find the area of typical school photographs.

GCO D: Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

Performance

D4.1 Have students arrange loops of string into three shapes. Ask them to estimate and then determine which of the shapes has the greatest area.

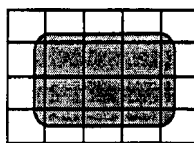
D4.2 Have students find the areas of envelopes using postage stamps as units. Request estimates before they measure.

D3.1 Ask students to sort a variety of objects according to whether they estimate them to be greater than, or less than, the mass of a given object. Compare the objects using a pan balance.

Paper and Pencil

D3.2 Ask students which unit, g or kg, would be the most appropriate to use to measure the mass of (a) a box of paper clips, (b) a carton of books, and (c) a dog.

D4.3 Ask students to explain how they knows that the area of the shape on the right has to be between 6 and 20 units. Ask them for their estimates of the area..

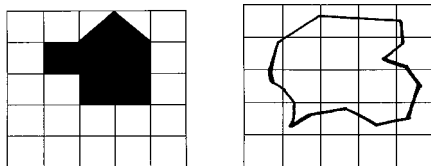


Interview

D4.4 Show three shapes and ask students which they think has the greatest area and to explain why, e.g.,



D4.5 Ask students why it is easier to find the area of the shape on the left than the one on the right.



D4.6 Ask students if the picture below shows 6 units of area and to explain why or why not.



GCO D: Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

KSCO

- (iii) By the end of grade 3, students will be expected to estimate and determine measurements in everyday problem situations and to develop a sense of the relative size of units.

SCO

By the end of grade 3, students will be expected to

- D5** solve problems involving kilometres
D6 use appropriate units for capacity and mass

The purpose of an interview is to uncover how students think about mathematics, so provide opportunities for contradictions in students' beliefs about mathematical concepts to emerge. (*NCTM 1991c, 29*)

Elaboration–Instructional Strategies/Suggestions

D5 Students should know that 1 kilometre is 1000 metres; to fully appreciate how far 1000 metres is, it is important that they have a referent for the distance, e.g., from their school to another landmark. Experiences measuring large distances with a trundle wheel are worthwhile.

Students should solve problems involving kilometres that are meaningful and of interest to them. It is important to include situations in which they can estimate and check these estimates.

- Plan a variety of questions and activities for students.
 - Ask students if they could run 1km without stopping.
 - Ask students how many trips around the playground would they need to walk to cover 1km. Check using a trundle wheel.
 - Ask students whether they think there are enough students in the school to reach 1km if they stood side by side, arms extended. Have them devise a way of calculating this to check.
 - Ask students to determine how many classrooms placed end-to-end would make a kilometre. Check by calculating, using the length of a typical classroom.
 - Ask students to predict how long it will take them to walk 1 kilometre. Then take the students on a 1km walk to check.
 - Have students design a table of other distances familiar to them, e.g., the distance from home to school, and predict how long it would take to walk or bike the distances.

D6 Students should recognize which capacity unit (mL or L) is appropriate in a given situation. You should include situations where each unit is used, e.g., a container of juice for a baby measured in millilitres and one for a family measured in litres.

- Arrange a display of food containers, some with labels indicating millilitres and others with labels indicating litres. Have students examine and arrange them from largest to smallest.
- Have students check food containers to make lists of things measured in millilitres and of things measured in litres.

Students should recognize which mass unit (gram or kilogram) is appropriate for measuring the mass of a specific item. It is helpful for students to investigate how everyday items are measured, e.g., food items. You should include items which are small and dense as well as those which are large and porous.

GCO D: Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

Performance

D6.1 Ask students to estimate, then verify, how much water will pass through a given funnel in 10 seconds.

D6.2 Show students a variety of large and small containers. Ask them what unit would be used for the capacity of each. Ask them to estimate the capacities in these units. Fill one of the containers with the most popular estimate to check the degree of accuracy. Allow students to readjust their other estimates. Check them.

D6.3 Ask students to estimate the masses of a variety of items, using their personal referent. Have them verify their estimates.

Paper and Pencil

D6.4 Have students list five items that would be measured in grams and five items that would be measured in kilograms.

D6.5 Have students draw pictures of objects that they believe to have a mass of about 5 kg.

Interview

D5.1 Have students name landmarks that are approximately 2km away, using their referents for kilometres.

D6.6 Ask students to name three items normally measured in millilitres and three items normally measured in litres.

D5.2 Tell students that the tiger shark can travel about 50km in an hour. Ask, About how far is 50km from the school? About how long would it take you to walk that far?

Portfolio

D5.3 Provide lists of distances (in kilometres) to various places. Ask students to plan trips that do not exceed 500 km.

D5.4 Have students research the heights of the world's largest buildings and structures. Have them determine how many of each, sitting on top of one another, it would take to be more than one kilometre.

GCO D: Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

KSCO

(iii) By the end of grade 3, students will be expected to estimate and determine measurements in everyday problem situations and to develop a sense of the relative size of units.

SCO

By the end of grade 3, students will be expected to

D7 read digital and analog clocks to the nearest five minutes

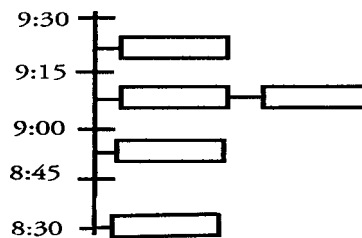
D8 continue to solve a wide variety of measurement problems

Elaboration–Instructional Strategies/Suggestions

D7 Students should read times on both digital and analog clocks to provide information about relevant situations, e.g., comparing start and finish times to determine how much time has passed, and to focus on times when special events are going to happen.

To help them read time to the nearest five minutes, it is important that students are comfortable skip counting by 5s. This also provides the opportunity for students to relate the numbers on a clock to the five-times table. It is helpful to use clocks that mark not only the numbers from 1 to 12, but also the minute amounts.

- Have students track events throughout a specific day by means of a time line divided into 15 minute segments. Students should record the time of the activity or event and note it at the appropriate spot on a time line, e.g.,



D8 Students should continue to explore interesting, real life measurement situations. It is important for students to recognize the approximate nature of measurement because we are limited by the precision of the measuring instrument. To encourage this, use approximate language, e.g., about 12 cm long, a bit more than 1.5 litres, not quite 1kg.

- Have students
 - plan a class party and determine the number of 1 litre cans of juice they need to buy
 - make a cover for a chosen book from an oversized sheet of paper
 - use a roll of dental floss to measure, approximately, the perimeter of the gym
 - determine the size of the containers required to hold 1 kg of potatoes, macaroni, or pattern blocks

GCO D: Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

Worthwhile Tasks for Instruction and/or Assessment

Performance

D7.1 Ask students to move the hands of an analog clock to match the time shown on a digital clock.

D7.2 Ask students to record the temperature on a chart at five different times during the morning.



Paper and Pencil

D7.3 Have students make lists of the times when the two hands of the clock are just about on numbers that are one apart.

D7.4 Have students work in pairs to set up a schedule in which every student will get 10 minutes on the computer starting at 8:30 a.m. Ask them if all students can have time on the computer before noon, and, if not, how long it will take to finish after lunch. At what time will the last one finish? (Remind them to leave time for recess.)

D7.5 Provide students with blank pictures of analog and digital clocks. Give them times to show on these blank clocks.

Interview

D7.6 Ask the student what time it might be if the minute hand and hour hand are opposite one another.

Presentation

D8.1 Have students work in pairs to figure approximately how many students would have to lie down in a line to make a distance of 1 km. Ask them to record how they found their answers and to share it with the class. (Different line formations are likely to be presented, e.g., head-to-toe or arms outstretched over their heads.)

Portfolio

D8.2 Tell the students that you have a map that takes only one page the size of a sheet of loose-leaf. On the map, 1cm represents 1km. Ask them if all of their community can fit on the map and have them provide a written rationale.

Suggested Resources

Shape and Space: Geometry

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

KSCO

- (i) By the end of grade 3, students will be expected to explore and experiment with geometric shapes and relationships.

SCO

By the end of grade 3, students will be expected to

- E1 continue their development of spatial sense with emphasis on perceptual constancy

Spatial understandings are necessary for interpreting, understanding, and appreciating our inherently geometric world. Insights and intuitions about two- and three-dimensional shapes and their characteristics, the interrelationships of shapes, and the effects of changes to shapes are important aspects of spatial sense. Students who develop a strong sense of spatial relationships and who master the concepts and language of geometry are better prepared to learn number and measurement ideas, as well as other advanced mathematical topics. (NCTM 1989, 48)

Elaboration–Instructional Strategies/Suggestions

E1 Perceptual constancy is the ability to recognize figures or objects in space regardless of size, position, or orientation. It involves the recognition of shape or size as stable even if it appears to be different to the observer, e.g., a table top can appear to be a parallelogram or a trapezoid when viewed in certain ways but our experience has taught us it is a rectangle. Similarly, if a shape is reduced or enlarged (as in photographs), the shape is maintained.

- Give students different shapes made from five or six centicubes. Have them make the same shapes using larger multi-link cubes.
- Have students make complex figures on geoboards and make copies on much smaller geopaper. Discuss what has changed and what has stayed the same.
- Show a pattern block design that uses 10 to 12 blocks on an overhead projector. Have students replicate the design at their desks using pattern blocks. Discuss what aspects are constant between the overhead version and those created by students.
- Give students three isometric drawings of shapes, two of which are different views of the same shape (see E1.4). Have them predict which are the two views of the same shape and then build the shape with cubes to check their predictions.
- Have students draw what they see when they look at familiar objects from the top. Have the other students see if they can determine what the objects are, given only these top views. Some objects will be difficult to determine with only a top view and could be more than one thing.
- Put three different shapes made from five multi-link cubes on display. In a bag, place a replica of one of these shapes made from centicubes. Have the students determine by feeling which one of the three it is.

Continue to provide experiences that focus on other spatial abilities, e.g., Using pattern blocks, a visual discrimination activity is to find the difference(s) between two designs made with 8 blocks; a visual memory activity is to have students replicate, from memory, a design that they have been shown only briefly; a spatial relations activity is to solve a puzzle. All of these spatial abilities are inherent in the development of geometric concepts.

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

Performance

E1.1 Make designs on the overhead projector using four or five tangram pieces. Have students replicate these designs using their own pieces. Extend this activity by showing the designs for shorter periods of time before students try to build them from memory.

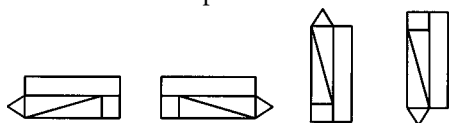
E1.2 Place a variety of 3-D shapes in a bag. Have students feel the shapes in the bag and describe them for other students to try to name them.

Paper and Pencil

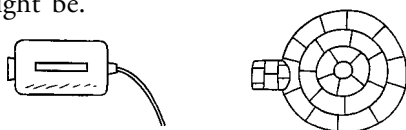
E1.3 Ask students, What 3-D shape has these faces?



E1.4 Ask students, Which of the following are pictures of the same figure? Have students predict and use tracing paper to check.



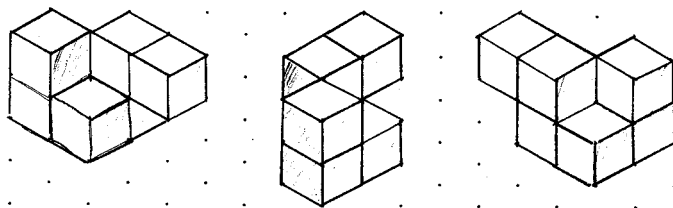
E1.5 Tell students that these are top views of objects. Ask what objects they might be.



(While most students would see the second picture as the top view of an igloo, there could be a variety of suggestions for the first picture including a gas pump, a toaster, and a camcorder.)

Interview

E1.6 Ask students to predict which of the following are two views of the same shape made with six cubes. Have students build them to check.



GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

Elaboration–Instructional Strategies/Suggestions

KSCO

- (i) By the end of grade 3, students will be expected to explore and experiment with geometric shapes and relationships.

SCO

By the end of grade 3, students will be expected to

- E2 recognize and represent angles that are less than/more than right angles
- E3 recognize, name, describe, and represent congruent angles and congruent polygons

E2 Students are not measuring angles in degrees at this stage; they are comparing angles by sight. They have been introduced to the concept of right angle previously; now other angles in shapes should be described as less or more than a right angle. You should prepare a variety of convex polygons including familiar triangles and quadrilaterals for students to use to examine angles.

- Provide students with small rectangular piece of paper or cards. Discuss right angles in relation to the corners of these. Have pairs of students gather a set of six pattern blocks and, using the sheet of paper or card, find shapes that have right angles, angles less than right angles, and angles more than right angles.
- Examine the various members of the quadrilateral family of shapes (trapezoid, parallelogram, rectangle, rhombus, square, and kite) for these angles.
- Have students arrange toothpicks to make right angles and a variety of angles less than/more than right angles.
- Find examples of angles in the classroom/school that would be right, less than right, and more than right. They could use an index card or sheet of paper as a referent for the right angle to do comparisons and make decisions.

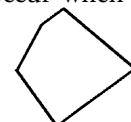
E3 Through a variety of experiences, students should establish that congruent polygons are a perfect match, i.e., they would fit on top of one another exactly, regardless of their relative positions or orientations. Congruent angles in shapes occur when the corners match.



Polygon



Congruent but different orientation



Congruent but different position

- Have students match the corners (angles) of the pattern blocks to find ones that are congruent. Have them compare the angles in squares and rectangles and compare the angles in the three different-sized triangles in the tangram set.
- Ask students to find sets of congruent picture frames on a sheet with pictures of a variety of polygonal picture frames in different positions and orientations. They should try by sight, then check by using tracing paper or by cutting them out.

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

Performance

E2.1 Have students make shapes on geoboards that meet different criteria, e.g., Make shapes that have two right angles; make shapes that have one angle that is more than a right angle; make shapes that have all their angles less than right angles.

E2.2 Provide students with pictures of 12 different angles in different positions on the paper. Have them sort the angles into three sets—right angles, more than right angles, and less than right angles.

E2.3 Have students classify the angles on the faces of 3-D shapes.

E3.1 Ask a group of 10 students each to make one rectangle anywhere on their geoboards. Have students sort these rectangles into congruent groups.

E3.2 Prepare a set of cards with drawings of different polygons—some of them congruent but in different positions/orientations and some similar (reductions, enlargements). Have students match the congruent polygons and discuss why they are/are not congruent.

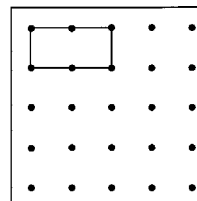
E3.3 Show students a polygon on a geoboard. Have them make slide, flipped, or rotated images of it on their own geoboards. Ask, Is yours congruent to mine? Have them compare the polygons by copying them on geopaper and cutting them out.

Paper and Pencil

E3.4 Give students pictures of a shape, e.g., a parallelogram, and some tracing paper. Ask them to draw three parallelograms congruent to this one but in different positions and/or orientations. Have them draw ones that are not congruent and describe how they are different from the others.

Presentation

E3.5 Have students investigate how many rectangles congruent to this one can be made on a geoboard.



GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

KSCO

- (ii) By the end of grade 3, students will be expected to describe, model, draw, and classify 2-D figures and 3-D shapes.

SCO

By the end of grade 3, students will be expected to

- E4 recognize, name, describe, and represent kite, and some concave, convex, and regular polygons

Elaboration–Instructional Strategies/Suggestions

E4 Show students a real traditional flying kite and read a story about them to help the students make strong associations with the shape called *kite*. Show them a variety of examples of kites, and some that are not, asking the students why they think they are not kites.

Traditional kite



Other examples of kites:



These are not kites:



- Prepare two ropes of 1m and two ropes of 1.5m. Have four students choose ropes and stand holding them to make a kite. Ask, Which ropes are the same length? Have them make a rectangle using the same ropes. Ask them how the positions of the ropes were changed to change a kite into a rectangle.

The students will have worked with some of the *regular polygons*, e.g., the triangle, square, and hexagon in the pattern blocks; however, the name *regular* to describe them will be a new understanding. This could be developed by displaying shapes that are labelled regular and those labelled not regular, asking students to formulate the meaning of regular. Many students will notice that all its sides are equal and all its angles are congruent if a polygon is regular. Make associations to things in the real world, e.g., a stop sign is a regular octagon.

- Have students make all of the different polygons that can be formed with the four triangles made by cutting a square along both diagonals. These can be sorted into two groups, *concave* and *convex* polygons that you name. Ask students to try to describe concave and convex by looking for the patterns in the sort. At this stage, students will probably define them by their overall appearance, e.g., concave polygons are “caved in.”

For your information: A kite is a quadrilateral with two pairs of congruent adjacent sides. While rhombuses are kites too, at this level we treat them as a separate class. Students should not be given formal definitions at this stage of their development.

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

Performance

E4.1 Give each student two long straws and two shorter straws. Have students arrange them on their desks to make a kite. Have them examine each other's to see how many different kites they can find. Ask, How are they the same and how are they different?

E4.2 Have students use pattern blocks to make polygons that are concave and polygons that are convex.

E4.3 Give students 15 toothpicks, all the same length. Have them arrange five of them to make convex pentagons that are not regular, five to make concave pentagons, and five to make regular pentagons.

E4.4 Ask students to examine a collection of 3-D shapes to find ones that have faces that are regular polygons. Have them record the ones they find by tracing those faces on paper.

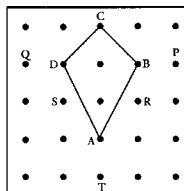
Interview

E4.5 Ask students to compare kites and parallelograms, explaining why they look different.

Presentation

E4.6 Have students use five-pin by five-pin geoboards to make as many different kites as they can. (Remind them that kites are not different if they are congruent to one another.) Record each kite on geopaper.

E4.7 Have students make kite $ABCD$ on geoboards as shown. Ask, If the elastic is moved from peg A to peg T , will the new shape be a kite? If B is moved to P , will the new shape be a kite? Move another peg to make it a kite. If the elastic is then moved from peg P to peg R , would the shape be a kite? Move the elastic from peg Q to make the shape a kite.



Portfolio

E4.8 Ask students to cut a square as shown on the right. Have them use the three pieces to make two examples each of concave and convex polygons and trace them.



GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

KSCO

- (ii) By the end of grade 3, students will be expected to describe, model, draw, and classify 2-D figures and 3-D shapes.

SCO

By the end of grade 3, students will be expected to

- E5 recognize, name, describe, and represent different prisms and pyramids
- E6 cut and assemble net patterns for pentagonal and hexagonal prisms and pyramids
- E7 build skeletons of various prisms and pyramids to focus on edges and vertices

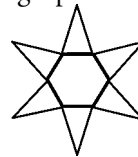
Virtually every activity that is appropriate for K-8 geometry should involve some form of hands-on materials, models, or at least paper such as graph paper or dot paper that lends itself to easy spatial explorations. (*Van de Walle* 1994, 324)

Elaboration–Instructional Strategies/Suggestions

E5 This will build on students’ previous experiences with prisms and pyramids. Prisms and pyramids should be presented together to have the students form impressions of these two families of shapes both of which have “first names” based on the shape of their bases. These could include trapezoidal, pentagonal, hexagonal, and octagonal as well as triangular, rectangular, and square which might already be known.

Commercial 3-D shapes usually have a variety of prisms and pyramids that could be used as models. While the pattern block pieces are prisms, they have been treated as 2-D shapes; however, stacking a number of same-shaped blocks would provide examples of different prisms. Stacking the various polygons from the logic blocks would also provide examples of prisms. This stacking would help students visualize the uniform nature of prisms.

E6 Students should be given copies of nets of these 3-D shapes to cut out and fold up. They should be encouraged to unfold them and examine the 2-D shapes that are connected to make each net. Have them visualize the folding up and unfolding, e.g., a net for a hexagonal pyramid might be



E7 Miniature marshmallows or baking gums and different-sized toothpicks are sources of materials for students to use to build skeletal models. Marshmallows would serve as vertices and toothpicks as edges.

- Have students make skeletons of a variety of prisms and the corresponding pyramids. Discuss how they started off the same but then proceeded differently. Compare the number and shapes of faces, the number of vertices, and edges. See if they can detect any patterns while they are building them, e.g., They always used one more marshmallow as a vertex after the base of a pyramid was made.
- Give students face, edge, and vertex clues to have them name the mystery 3-D shapes.
- Have students feel shapes in bags, describing them by their faces, edges, and vertices so others can try to name them.

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

Worthwhile Tasks for Instruction and/or Assessment

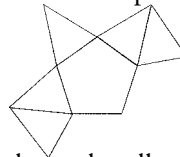
Suggested Resources

Performance

E5.1 Put a variety of prisms and pyramids in a bag. Have students, using only their sense of touch, describe each shape and name it before bringing it out of the bag to check.

E5.2 Place a hexagonal prism and a hexagonal pyramid beside one another. Ask students to name them. Ask them to tell you some things that are the same about them and some things that are different.

E6.1 Present students with nets of prisms and pyramids that have the faces joined in different ways from the ones they have cut out before. Ask them to predict what shapes they would fold up to make. Have them cut them out and fold to check their predictions, e.g., this is not the typical net for a pentagonal pyramid.



E7.1 Ask students each to use toothpicks and marshmallows to build a prism and a pyramid that will have the same number of marshmallow vertices.

E7.2 Play *Who am I?* by giving clues based on faces, edges, and vertices, e.g., I have five faces. I have six vertices. What prism or pyramid am I?

Paper and Pencil

E7.3 Give students two 3-D shapes, asking them to write comparisons between them on the basis of edges, faces, and vertices.

Interview

E7.4 Show students a toothpick and marshmallow pentagon. Ask them to describe what additional material would be needed to make a pentagonal pyramid.

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

KSCO

- (iii) By the end of grade 3, students will be expected to investigate and predict the results of combining, subdividing, and transforming.

SCO

By the end of grade 3, students will be expected to

- E8** predict the results of combining triangles and/or quadrilaterals

Elaboration–Instructional Strategies/Suggestions

E8 Ultimately, students should try to visualize what 2-D and 3-D shapes would be formed by combining triangles and quadrilaterals. Initially, they might be able to do this for some combinations; however, they will probably use a trial-and-error approach for many others. Be sure to encourage students to try to predict before they physically combine the shapes. Afterwards, the students should, in their minds, take the shapes apart and put them back together. It can't be emphasized enough how important of these predicting and reconstructing steps are in the development of spatial visualization. Describing the changing from one shape to another in terms of slides, flips, and/or turns can be helpful in the visualization process.

- Using pattern blocks two at a time, e.g., a trapezoid and a triangle, have students predict what polygons could be made if the blocks are joined on equal sides. Have them check their predictions and trace the blocks to record the new polygons. This should be extended to using three or four blocks; however, you should not be surprised that many students will have difficulty predicting when more than two blocks are used.
- Using the tangram pieces, have students predict, then check, the result(s) of combining
 - two small triangles
 - a small triangle and the square
 - a small triangle and the parallelogram
 - the parallelogram and the square
 - the square and two small triangles
- Make a set of cards, each card containing a face of a prism or pyramid—two each of squares, regular pentagons, and regular hexagons; four equilateral triangles; six rectangles; and six isosceles triangles. Put out sets of face cards for the prisms or pyramids. Have students predict the shapes. Have them check by finding the actual shapes and compare their faces to the cards.

Cardboard 2-D shapes, like those mentioned for the cards in the activity above, could be used by students to predict and experiment with in both 2- and 3-dimensions. There are commercial products that consist of a variety of 2-D shapes that can be manipulated to form 3-D shapes.

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

Performance

E8.1 Provide students with the two triangles formed by cutting rectangles along one diagonal. Have them make and name all the shapes that can be made by combining these two triangles with equal sides matching.

E8.2 Show students two congruent isosceles triangles, one in each hand. Ask them to predict what polygons could be made by combining them. Sketch their predicted shapes. (Remind them to visualize them coming together.) Ask for some predictions, having students explain how they would join the triangles. Ask for a volunteer to make the polygons using your two triangles.

E8.3 Provide students with two congruent squares, one of them cut along a diagonal. Have them investigate the different shapes that can be made using the three pieces with equal sides matched. Ask students to record all the shapes found by tracing around them.

E8.4 Have students use six cardboard congruent squares to build cubes, taping some, but not all, of the edges. Each face should be attached to another along one edge. Have them unwrap the cube and lay it out flat on the desk to make a 2-D shape with six squares. Ask, Is your net for the cube the same or different from others in your class? Ask them to investigate different ways that the net for the cube could be made and draw to record each net.

Paper and Pencil

E8.5 Ask students to predict, check the predictions, and trace to record the shapes that can be made by combining (a) two congruent squares, (b) two congruent rectangles, (c) two congruent parallelograms, and (d) two congruent rhombuses.

Portfolio

E8.6 Provide students with two of each of the six pattern blocks. Have them investigate how many new polygons can be made using two of the same block with equal sides matched. Have them trace to record their findings. Ask, Which of the blocks could make only one polygon? Which could make the most different polygons? As they work, observe whether they recognize the same shapes in different positions or orientations.

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

KSCO

(iii) By the end of grade 3, students will be expected to investigate and predict the results of combining, subdividing, and transforming.

SCO

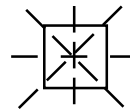
By the end of grade 3, students will be expected to

E9 find the lines of reflective symmetry of polygons

A very useful device for studying symmetry and transformations is the Mira, a piece of red Plexiglass, that stands perpendicular to the table surface. (*Van de Walle 1994, 350*)

Elaboration–Instructional Strategies/Suggestions

E9 Students should begin to appreciate that *reflective symmetry* is a characteristic of some polygons and not others. This can be described by stating how many lines of reflective symmetry a polygon has, e.g., After investigations with miras and paper folding, students should find that a square has four lines of reflective symmetry.



Experiences should provide students with the understanding that a line of reflective symmetry is the fold line where a polygon is folded onto itself so that each half matches exactly, or is the line where a mirror, or mira, can be placed so that the reflection on one side matches the shape on the other.

- Provide students with a variety of polygons, some of which have reflective symmetry. Have the students cut them out and try to fold them onto themselves. When a match is made, have the students crease on the fold line. Have them discuss the polygons and their number of lines of reflective symmetry.

A mira is a good instrument to have students find lines of reflective symmetry in polygons.

Students should also examine shapes that do not have lines of reflective symmetry, e.g., The diagonal of a parallelogram is often mistaken for a line of symmetry because the two triangles are congruent but are not reflective images.

Connect reflective symmetry to regular polygons (see SCO, E4). Try to have students discover the pattern that the number of lines of reflective symmetry for regular polygons is the same as the number of sides it has. These lines join vertices that are opposite each other, midpoints of sides that are opposite each other, or a vertex and the midpoint of the opposite side.

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

Performance

E9.1 Have students fold pieces of rectangular paper to show all of its lines of reflective symmetry. Ask, Why does folding it along a diagonal not make a line of symmetry?

E9.2 Have students make three polygons on a geoboard that have only two lines of reflective symmetry. Ask them to record their polygons on geopaper. Suggest that not all three be the same type of shape.

E9.3 Have students fold pieces of paper in half vertically. Ask them to cut out (along the fold) a shape that, when unfolded, makes a polygon with only one line of symmetry.

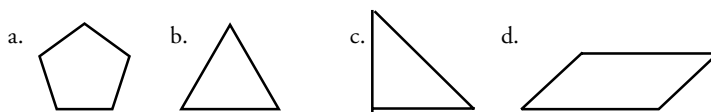
E9.4 Ask students to examine the six shapes in the pattern blocks for lines of symmetry.

E9.5 Have students combine various pieces of tangram puzzles to make polygons with one line of symmetry.

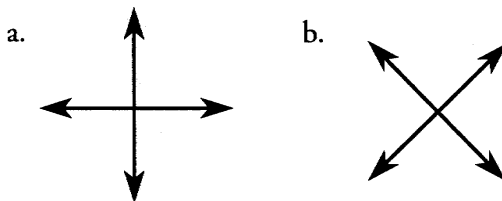
E9.6 Ask students to sort by symmetry the shapes formed by the four triangles made by cutting a square along both diagonals (see SCO, E4).

Paper and Pencil

E9.7 Ask students to use a mira to find any lines of reflective symmetry in the figures below:



E9.8 Ask students to draw shapes, given their two lines of reflective symmetry, as shown below:



GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

Elaboration–Instructional Strategies/Suggestions

KSCO

(iii) By the end of grade 3, students will be expected to investigate and predict the results of combining, subdividing, and transforming.

SCO

By the end of grade 3, students will be expected to

E10 recognize, name, describe, and represent half and quarter turns of 2-D figures

E10 Initial experiences with these turns should be with students' own bodies, turning on the spot, one quarter at a time until a complete revolution is made. You could connect quarter and half turns to the rotations of the minute hand of a clock from 12 to 3 and 12 to 6. They should connect a quarter turn to a right angle as well.

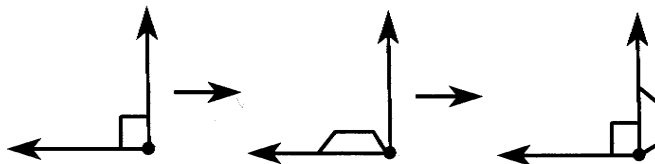
Rotations are the most perceptually challenging of the transformations. Students will need many experiences actually rotating (turning) a shape through quarter and half turns before they are able to recognize and describe such turns. It is suggested that experiences start with cutout 2-D shapes with right angles, e.g., squares and rectangles, and that turning points be one of the vertices of the shape. These turns can be clockwise or counterclockwise. Using squared paper makes quarter and half turns easier for students to do, e.g.,



Experiences could then involve rotating other shapes and pictures of shapes, using tracing paper to find the rotated images, still keeping the vertices as the centres of rotation.

Students should make slide and reflected images of the same shapes that they rotated and compare all these images.

- Have students make quarter turns of a pattern blocks on plain paper by drawing right angles, placing one side of the blocks along one arm of the angle, and rotating the blocks until the same side of the blocks lie along the other arm of the angles. The blocks should be traced in both positions.



GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

Performance

E10.1 Have students draw the half-turn clockwise and counter-clockwise images of the trapezoid in the pattern blocks. Ask, What do you notice?

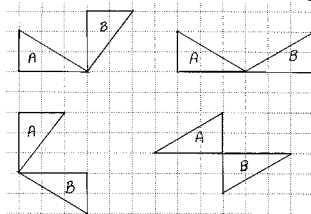
E10.2 Have students make an acute triangle on a geoboard and draw it on geopaper. Ask them to follow these instructions: Rotate the geoboard a quarter turn clockwise and draw the image on geopaper. Rotate the geoboard another quarter turn and draw this image on geopaper. Choose one angle in the original triangle on geopaper and colour it red; find its corresponding angles in the two image triangles and colour them red. Compare the locations of the red angles.

Paper and Pencil

E10.3 Have students draw pictures of squares, rectangles, and parallelograms on squared paper. Have them use tracing papers to make half-turn images of these shapes using one of the vertices as a turning point. Ask them to hold their pencil points firmly on the chosen vertices as they turn the tracing papers the half turns. Using sharp pencils, they should press through the tracing papers to locate the vertices of the turned shapes. Ask, Could these images have been the result of a slide? a reflection? Explain.

E10.4 Provide students with rectangles from the logic blocks. Have them trace these rectangles on paper. Ask them to use one of their vertices as turn centres, to rotate the rectangles a quarter turn counterclockwise, and to trace them. Have them use the same vertex, move the rectangles another quarter turn, and trace them; repeat a third and fourth time. Ask, What happened the fourth time? What would happen if you were to continue?

E10.5 Ask students to describe, for each pair of triangles below, what motion (slide, reflection, rotation) of triangle A would result in triangle B.



GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

KSCO

- (iv) By the end of grade 3, students will be expected to relate geometric ideas to number and measurement ideas and to recognize and apply geometric principles in real-world situations.

SCO

By the end of grade 3, students will be expected to

- E11 recognize and identify various polygons, prisms, and pyramids in real-world contexts
- E12 make the connection for rectangles between the arrays of squares forming them and the describing of their dimensions


Geometry is an important component of the K-4 mathematics curriculum because geometric knowledge, relationships, and insights are useful in everyday situations and are connected to other mathematical topics and school subjects. (*NCTM* 1989, 48)

Elaboration–Instructional Strategies/Suggestions

E11 In the development of geometric concepts, initial experiences should include investigating their real-world uses to help students make associations and form strong visual images, e.g., Since regular polygons are used in many logos and designs, these make natural sources for real-world connections when students are learning about these special polygons. When they are studying any geometric concept, students should be encouraged to find examples of it in magazines, newspapers, cars, stores, school, home, fabric, artworks, wallpaper, and other sources. Mi'kmaq quill work, tole painting, quilts, stencilling, and other handicrafts are good sources of geometric shapes in use. A class collection of boxes and containers can provide examples of various prisms and pyramids.

Collect an assortment of pictures, e.g., streetscapes, formal gardens, buildings, and oriental carpets. Students could find examples of 2-D and 3-D shapes in the pictures.

Students should be encouraged to use geometry in design and art projects, e.g., They could design a quilt using construction paper shapes in different colors; they could design a logo; they could make 3-D ornaments/mobiles using pyramids/prisms with decorated faces.

E12 One of the concrete/pictorial representations of multiplication is arrays, e.g.,  is one way to show 2×3 . This very geometric way of representing multiplication is also the way we often describe rectangles by giving their dimensions, e.g., 2 by 3 or 2×3 would describe the rectangle above.

- Have students use 12 square tiles to build all possible rectangles, recording them on grid paper, and labelling their dimensions.

The connection of arrays to area of squares and rectangles should also be made.

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

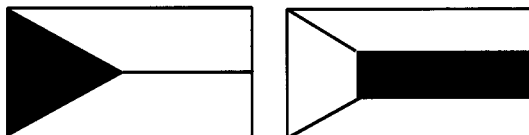
Performance

E11.1 Tell students that many unsharpened pencils are shaped like prisms. Ask them to find such pencils and name the prisms.

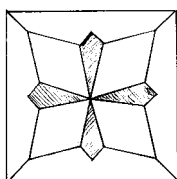
E11.2 Show students a collection of boxes of various shapes. Have them identify and name the shapes of the boxes using geometric vocabulary.

Paper and Pencil

E11.3 Tell students that the following flags of different countries were designed using polygons. Ask them to find, draw, name, and describe the different polygons used in each flag.



E11.4 Tell students that this is one block of a quilt pattern. Ask them to identify, draw, and name some of the different shapes that are used.



Presentation

E11.5 Have students work in groups to explore the different rectangles that can be made using from 1 to 36 squares. Ask them to record their results by giving the possible dimensions in a chart.

Number of Squares	Dimensions of Rectangles
1	1x1
2	1x2, 2x1
3	1x3, 3x1
4	1x4, 4x1, 2x2

Discuss which number of squares made the most different rectangles; which made the smallest number of different rectangles; which had rectangles with 1 as one of its dimensions.

**Data Management
and Probability:
Data Management**

GCO F: Students will solve problems involving the collection, display, and analysis of data.

Elaboration–Instructional Strategies/Suggestions

KSCO

- (i) By the end of grade 3, students will be expected to collect, record, organize, and describe relevant data.

SCO

By the end of grade 3, students will be expected to

F1 select appropriate strategies for collecting, recording, organizing, and describing relevant data

F1 The focus should be on the selection of appropriate strategies for collecting and displaying data, rather than on following a set of instructions.

- Have pairs of students decide on the topic and procedure they will use to collect and display data that will show interesting information about class members.
- Have students conduct a survey to find out what types of things 8- and 9-year olds like to collect. They will need to decide whom to survey and how to organize and present their data.
- Other data collecting and graphing activities might include
 - graphing temperatures over the school week/month
 - collecting and displaying data that represent accomplishments of favourite sport figures or friends, e.g., the number of goals, hits, and points
 - collecting and displaying data that would show the number of pizzas and hot dogs ordered at the cafeteria during one week
 - graphing the distances class members can throw a ball
 - graphing prices of board games
 - collecting and displaying data to represent masses of various fruits/vegetables
 - graphing favourite books of classmates
 - graphing the masses of subject textbooks
 - graphing the masses of different breeds of dogs

Third-grade children are keenly aware of what others think about certain topics. This interest in others broadens and deepens as children gather and analyze opinions from a variety of people. (NCTM 1992b, 20)

GCO F: Students will solve problems involving the collection, display, and analysis of data.

Worthwhile Tasks for Instruction and/or Assessment
Suggested Resources
Performance

F1.1 Ask pairs of students to design surveys that will show favourite school subjects of a class.

F1.2 Ask small groups to brainstorm lists of interesting questions for possible surveys.

Paper and Pencil

F1.3 Ask students to conduct surveys of the types of vehicles in the school parking lot or the number of vehicles passing the school during a half-hour period.

Interview

F1.4 Tell students that Sasha wants to find out whether students in grade 4 like the same television programs she likes. Ask them to suggest ways that Sasha might get this information.

F1.5 Ask students for different strategies they could use to find the high temperatures for each day of a school week.

F1.6 Tell students that Derek noticed the prices of chocolate bars varied from store to store. Ask them to suggest ways Derek could gather information in order to make a graph showing these different prices.

Presentation

F1.7 Ask students to select topics, survey family members and/or neighbours, and present their findings to the class in organized ways.

Portfolio

F1.8 Ask students to keep track of weather conditions over the period of one month and to design ways to present the information in an organized manner.

GCO F: Students will solve problems involving the collection, display, and analysis of data.

KSCO

- (ii) By the end of grade 3, students will be expected to construct concrete and pictorial displays of relevant data,
- (iii) Students will be expected to read and interpret displays of relevant data.

SCO

By the end of grade 3, students will be expected to

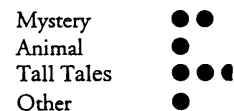
- F2 interpret and create pictographs in which each symbol represents more than one item
- F3 create bar graphs using simple scales

Children sometimes have difficulty with pictographs and other graphs in which each picture or symbol stands for more than one object. But careful development here will help students make sense of units and of multiplication. (*NCTM* 1992b, 20)

Elaboration–Instructional Strategies/Suggestions

F2 Students should make the connection between multiplication and graphing situations in which each pictograph symbol represents more than one item. Simple fractions of symbols, e.g., halves and fourths, may be introduced as well. Students should have experiences interpreting pictographs before creating them after conducting surveys and organizing data.

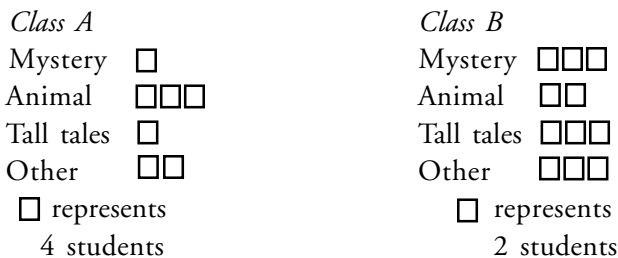
Favourite Types of Stories



● represents 4 children

Students might contrast data on two graphs for which different values are attached to the same symbol, e.g., Ask students, For the graph below, in which class do more students prefer mysteries? Which type of story was preferred by the same number of students?

Favourite Types of Stories



F3 Students have had experiences creating bar graphs for which each section has a value of one. They should learn to interpret and create bar graphs for which each section represents more than 1.

Students should recognize that scales are useful in situations involving larger numbers. For example, data collected about the number of students in grades primary to 2 (48, 64, 56 respectively) would require a minimum of 64 squares to represent this information; therefore, it is preferable to use a scale.

Both horizontal and vertical bar graphs should be considered.

In addition to drawing bar graphs from provided data, students should continue to conduct surveys and organize data to create such graphs, e.g., Have them collect data on the vowels in classmates' first names and create bar graphs to show the total numbers of each vowel collected.

GCO F: Students will solve problems involving the collection, display, and analysis of data.

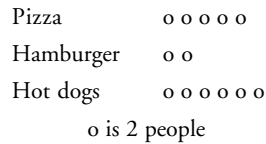
Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

Performance

F2.1 Have students redraw this pictograph so that each symbol represents 4, instead of 2. Ask them to say which graph they prefer and to give reasons for the choice. Ask if there is another way to display the data which might be clearer.

Cafeteria Orders



Paper and Pencil

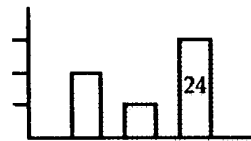
F3.1 Provide the data on the right for students to select scales and create bar graphs.

Favourite Sports

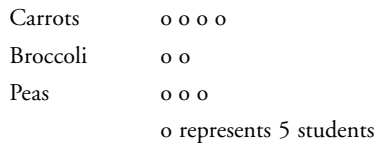
hockey	36
baseball	22
basketball	30
volleyball	16

Interview

F3.2 Show students the graph on the right which represents the favourite basketball teams of a number of grade 3 students. Ask them to tell how many each of the first two bars represents.



F2.2 Ask students to examine the display below and tell how many more students prefer carrots to broccoli. Have them explain what other information is shown by the graph.



Ask why a symbol in a pictograph usually represents more than 1.

Portfolio

F2.4 Ask pairs of students to create pictographs describing their classmates' favourite colours, using one symbol to represent two students.

GCO F: Students will solve problems involving the collection, display, and analysis of data.

Elaboration–Instructional Strategies/Suggestions

KSCO

(iv) By the end of grade 3, students will be expected to generate questions, develop and modify predictions, and implement plans with respect to data analysis.

SCO

By the end of grade 3, students will be expected to

F4 implement plans with respect to the collection of data

F4 When considering how to collect data, students need to consider a variety of issues such as good sources, the number of people to include, the location of the survey, the time to conduct the survey, and the phrasing of the questions.

- Have a small-group or class discussion to explain to students that their assignment is to plan a survey of grades 3 to 5 students. They are to find out whether school uniforms should be introduced. Have the class discuss
 - how many students should be surveyed
 - how many from each grade/class would be surveyed
 - how many girls and how many boys should be surveyed
 - how they will decide which girls and which boys should be surveyed
 - how they should phrase the question they are going to ask the students

Encourage students to discuss the importance of carefully designing the survey plan to ensure that they get a cross-section of the targeted population.

Children need experience in deciding what opinion questions they want answered as well as in collecting and presenting the data. Although students at this level probably will not conduct surveys scientifically, they will begin the process of making decisions on the basis of data. (*NCTM 1992b, 20*)

GCO F: Students will solve problems involving the collection, display, and analysis of data.

Worthwhile Tasks for Instruction and/or Assessment**Suggested Resources***Interview*

F4.1 Ask students what source(s) they might use to find information on the masses of various animals found in animal reserves.

F4.2 Ask students how many people they should ask to determine the number of quarters adults usually carry with them.

Presentation

F4.3 Have students plan how they might conduct a survey to find out the amount of TV people generally watch. Ask them to report on all the issues they think they must consider.

Portfolio

F4.5 Tell students that more books are going to be purchased for the school library. Ask them to design and conduct surveys to help determine what kinds of books should be bought. Tell them that they should present the data they collect in the form of a scaled bar graph.

**Data Management
and Probability:
Probability**

GCO G: Students will represent and solve problems involving uncertainty.

KSCO

- (i) By the end of grade 3, students will be expected to conduct informal investigations of chance and estimate probabilities with respect to games and other simple, everyday situations.

SCO

By the end of grade 3, students will be expected to

- G1** predict and record results in experiments using spinners, coins, dice, coloured cubes, and other simple equipment

From a basic understanding that one event can be more or less likely than another, students can begin to predict specific ratios of outcomes of simple events. Before students have worked with part-to-whole ratios, use language such as “65 out of 100” instead of using fractional probabilities. A discussion of reasons for their predictions is always important. The experiment should then be conducted and results compared with expected outcomes. (*Van de Walle 1994, 385*)

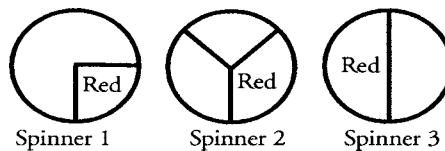
Elaboration–Instructional Strategies/Suggestions

G1 Students should conduct a variety of experiments, predict the results, and record the outcomes. While some students might simply list frequencies of various outcomes, others may be able to use fractions to describe the probability of events.

- Explain to students that they will roll two dice 25 times and count the number of times that the following events occur:
 - the sum of the numbers is 7
 - the difference is 2
 - the product is even

Ask them to predict the results of each event before they carry out the experiment. Some students may be able to record the results for the outcome of each event (sum, difference, product) on a given roll, while others may need to focus on one outcome at a time. Once students have recorded the data, they might make comparative statements, e.g., It is more likely that the product is even than it will have a difference of 2. Have them share their comparative statements with their classmates.

Students could use spinners, predicting how often the arrow will land on particular sections in 12 spins. Then they can spin the spinners, record the results, and compare them with their predictions. Discuss with students how many got what they predicted. Collect the total results of the spins for a group of 10 students (120 spins) and compare the individual predictions to the results of the 120 spins. This will provide the opportunity to discuss the idea that a sample of 12 may not give the predicted results, but that larger samples, such as 120, will come closer to what they expected.



Have students devise ways to record their spins of spinners and their tosses of coins. Ask them to share their ideas.

Provide activities in which students compare their findings of a few experiments (spins/tosses) to those of many.

Students should be expected to use probability language, such as “1 out of 4” or “1 chance in 3.”

GCO G: Students will represent and solve problems involving uncertainty.

Worthwhile Tasks for Instruction and/or Assessment

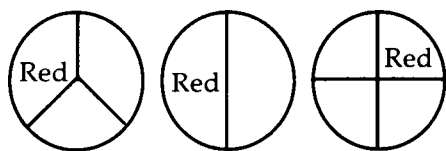
Suggested Resources

Performance

G1.1 Ask students to design spinners on which one is more likely to spin red about 8 times out of ten spins.

G1.2 Have students spill 10 two-colour counters and record the number of each colour which occurs. Ask them to predict whether it is likely that one colour will come up more often than the other with 10 spills. Have them conduct the experiment and report their findings.

G1.3 Show students the spinners below:



For each spinner, have them predict the number of times red will occur in 10 spins. Ask them to conduct the experiment to verify their predictions.

Interview

G1.4 Tell students that you tossed a coin 25 times. Have them predict the approximate number of heads and tails you got. Ask if it is possible that you got 19 heads.

Presentation

G1.5 Show students pictures of a game at the exhibition. Tell them that they will win if the ball lands on an even number. Have them discuss the chances of winning.

3	2	1	4	7
9	7	5	3	1
6	1	9	7	2
1	5	3	8	9
7	9	1	5	3

G1.6 Ask pairs of students to think of something that might happen about half the time when a die is rolled. Students should experiment with dice, record outcomes, and present their results to classmates.

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Mathematics 3



CURRICULUM

Mathematics 3

Mathematics 3

**Implementation Draft
May 2013**

Website References

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Mathematics 3, Implementation Draft

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Introduction

Background and Rationale

Mathematics curriculum is shaped by a vision that fosters the development of mathematically literate students who can extend and apply their learning and who are effective participants in society. It is essential that the mathematics curriculum reflect current research in mathematics instruction. To achieve this goal, Western and Northern Canadian Protocol's (WNCP) *The Common Curriculum Framework for K–9 Mathematics* (2006) has been adopted as the basis for the new mathematics curriculum in Nova Scotia.

The Common Curriculum Framework was developed by the seven ministries of education (Alberta, British Columbia, Manitoba, Northwest Territories, Nunavut, Saskatchewan, and Yukon Territory) in collaboration with teachers, administrators, parents, business representatives, post-secondary educators, and others. The framework identifies beliefs about mathematics, general and specific student outcomes, and performance indicators agreed upon by the seven jurisdictions. The outcomes and performance indicators have been adapted for Nova Scotia. This document is based on both national and international research by the WNCP and the National Council of Teachers of Mathematics (NCTM).

There is an emphasis in the Nova Scotia curriculum on particular key concepts at each grade that will result in greater depth of understanding and, ultimately, stronger student achievement. There is also a greater emphasis on number sense and operations concepts in the early grades to ensure students develop a solid foundation in numeracy.

Purpose

This document provides sets of outcomes and performance indicators to be used as a mandated common base for defining mathematics curriculum expectations. This common base should result in consistent student outcomes in mathematics within the province of Nova Scotia. It should also enable easier transfer for students moving within the province or from any jurisdiction that has adopted the WNCP framework. This document is intended to clearly communicate to all education partners across the province the high expectations for students' mathematical learning.

Program Design and Components

Assessment

Ongoing assessment for learning is essential to effective teaching and learning. Research has shown that assessment for learning (formative assessment) practices produce significant and often substantial learning gains, close achievement gaps, and build students' ability to learn new skills (Black & Wiliam 1998; OECD 2006). Student involvement in assessment promotes learning. Timely and effective teacher feedback and student self-assessment allow students to reflect on and articulate their understanding of mathematical concepts and ideas.

Assessment in the classroom includes

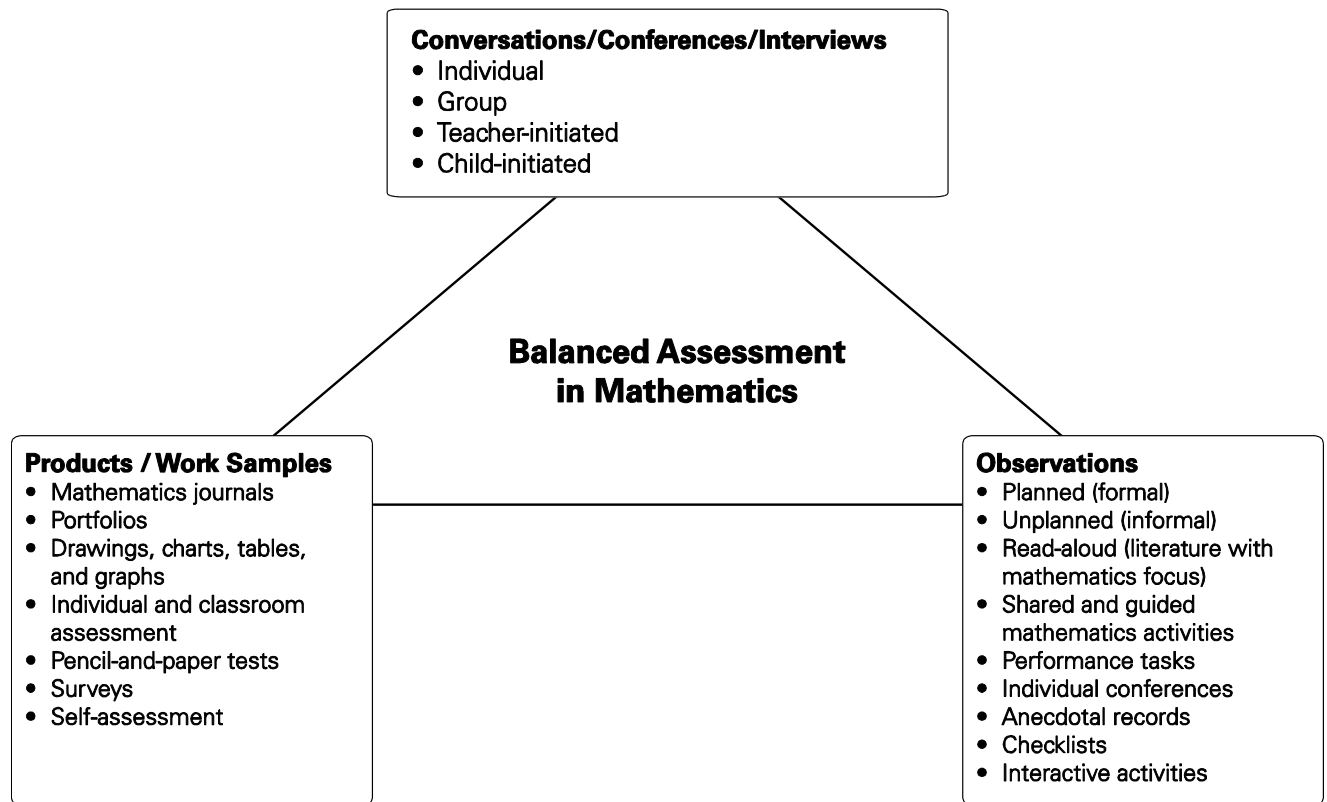
- providing clear goals, targets, and learning outcomes
- using exemplars, rubrics, and models to help clarify outcomes and identify important features of the work
- monitoring progress towards outcomes and providing feedback as necessary
- encouraging self-assessment
- fostering a classroom environment where conversations about learning take place, where students can check their thinking and performance and develop a deeper understanding of their learning

(Davies 2000)

Assessment for learning practices act as the scaffolding for learning, which only then can be measured through assessment of learning (summative assessment). Assessment of learning tracks student progress, informs instructional programming, and aids in decision making. Both forms of assessment are necessary to guide teaching, stimulate learning, and produce achievement gains.

Assessment of student learning should

- align with curriculum outcomes
- clearly define criteria for success
- make explicit the expectations for students' performance
- use a wide variety of assessment strategies and tools
- yield useful information to inform instruction



Time to Learn for Mathematics

The Time to Learn Strategy *Guidelines for Instructional Time: Grades Primary–6* includes time for mathematics instruction in the “Required Each Day” section. In order to support a constructivist approach to teaching through problem solving, it is highly recommended that the 45 minutes required daily in grades primary–2 and the 60 minutes required daily for grades 3–6 mathematics instruction be provided in an uninterrupted block of time.

Time to Learn guidelines can be found at

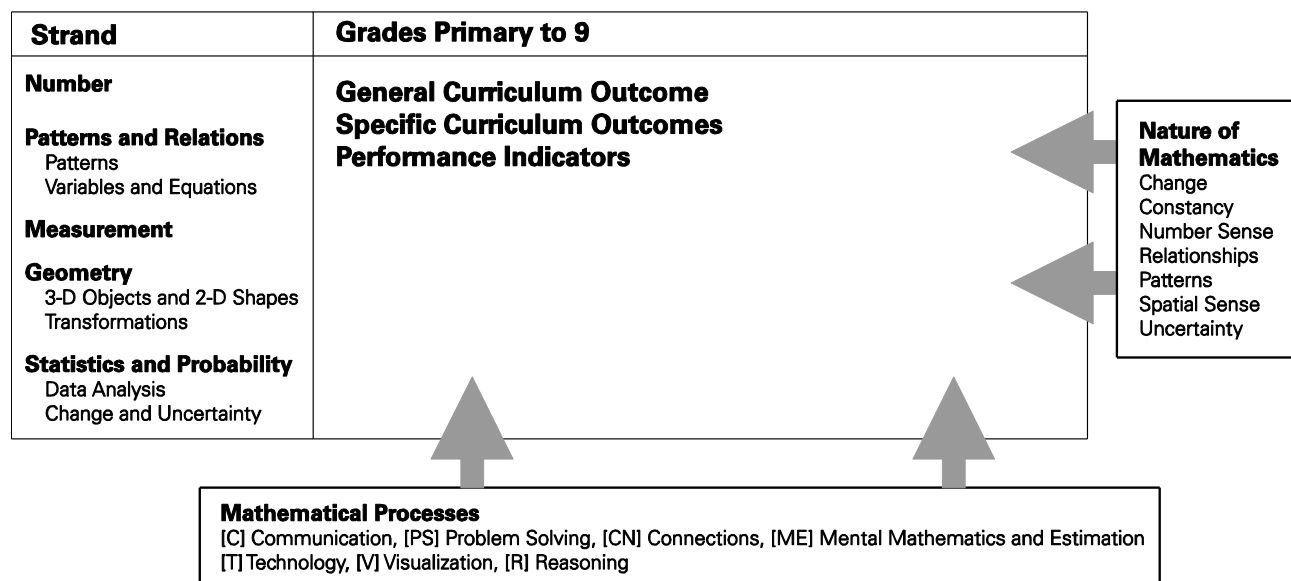
www.ednet.ns.ca/files/ps-policies/semestering.pdf

www.ednet.ns.ca/files/ps-policies/instructional_time_guidelines_p-6.pdf

Outcomes

Conceptual Framework for Mathematics Primary–9

The chart below provides an overview of how mathematical processes and the nature of mathematics influence learning outcomes.



(Adapted with permission from Western and Northern Canadian Protocol, *The Common Curriculum Framework for K–9 Mathematics*, p. 5. All rights reserved.)

Structure of the Mathematics Curriculum

Strands

The learning outcomes in the Nova Scotia Framework are organized into five strands across grades primary to 9.

- Number (N)
- Patterns and Relations (PR)
- Measurement (M)
- Geometry (G)
- Statistics and Probability (SP)

General Curriculum Outcomes (GCO)

Some strands are further subdivided into sub-strands. There is one general outcome (GCO) per sub-strand. GCOs are overarching statements about what students are expected to learn in each strand/sub-strand. The general curriculum outcome for each strand/sub-strand is the same throughout the grades.

NUMBER (N)

GCO: Students will be expected to demonstrate number sense.

PATTERNS AND RELATIONS (PR)

Patterns

GCO: Students will be expected to use patterns to describe the world and solve problems.

Variables and Equations

GCO: Students will be expected to represent algebraic expressions in multiple ways.

MEASUREMENT (M)

GCO: Students will be expected to use direct and indirect measure to solve problems.

GEOMETRY (G)

3-D Objects and 2-D Shapes

GCO: Students will be expected to describe the characteristics of 3-D objects and 2-D shapes and analyze the relationships among them.

Transformations

GCO: Students will be expected to describe and analyze position and motion of objects and shapes.

STATISTICS AND PROBABILITY (SP)

Data Analysis

GCO: Students will be expected to collect, display, and analyze data to solve problems.

Chance and Uncertainty

GCO: Students will be expected to use experimental or theoretical probabilities to represent and solve problems involving uncertainty.

Specific Curriculum Outcomes (SCOs) and Performance Indicators

Specific curriculum outcomes (SCOs) are statements that identify the specific conceptual understanding, related skills, and knowledge students are expected to attain by the end of a given grade.

Performance indicators are statements that identify specific expectations of the depth, breadth, and expectations for the outcome. Teachers use these statements to determine whether students have achieved the corresponding specific curriculum outcome.

Process Standards Key

[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

NUMBER (N)

N01 Students will be expected to say the number sequence forward and backward by

- 1s through transitions to 1000
- 2s, 5s, 10s, or 100s, using any starting point to 1000
- 3s, using starting points that are multiples of 3 up to 100
- 4s, using starting points that are multiples of 4 up to 100
- 25s, using starting points that are multiples of 25 up to 200. [C, CN, ME]

Performance Indicators

- N01.01 Extend the number sequence by 1s, particularly through transition from decade to decade and century to century.
- N01.02 Extend a given skip counting sequence by 2s, 5s, 10s, or 100s, forward and backward, using a given starting point.
- N01.03 Extend a given skip counting sequence by 3s, forward and backward, starting at a given multiple of 3 up to 100.
- N01.04 Extend a given skip counting sequence by 4s, forward and backward, starting at a given multiple of 4 up to 100.
- N01.05 Extend a given skip counting sequence by 25s, forward and backward, starting at a given multiple of 25 up to 200.
- N01.06 Identify and correct errors and omissions in a given skip counting sequence.
- N01.07 Determine the value of a given set of coins (nickels, dimes, quarters, and loonies) by using skip counting.
- N01.08 Identify and explain the skip counting pattern for a given number sequence.

N02 Students will be expected to represent and partition numbers to 1000. [C, CN, V]

Performance Indicators

- N02.01 Read a given three-digit numeral without using the word **and**.
- N02.02 Read a given number word (0 to 1000).
- N02.03 Represent a given number as an expression.
- N02.04 Represent a given number concretely and pictorially in a variety of ways.
- N02.05 Write number words for given multiples of ten to 90.
- N02.06 Write number words for given multiples of a hundred to 900.
- N02.07 Record numerals for numbers expressed orally, concretely, or pictorially.

N03 Students will be expected to compare and order numbers up to 1000. [CN, R, V]

Performance Indicators

- N03.01 Place a given set of numbers in ascending or descending order and verify the result using a number chart or other models.
- N03.02 Create as many different 3-digit numerals as possible, given three different digits. Place the numbers in ascending or descending order.
- N03.03 Identify errors in a given ordered sequence.
- N03.04 Identify missing numbers in parts of a given number chart and on a number line.
- N03.05 Identify errors in a given number chart and on a number line.
- N03.06 Place numbers on a number line containing benchmark numbers for the purpose of comparison.
- N03.07 Compare numbers based on a variety of methods, and record the comparison using words and symbols ($=$, $>$ and $<$).

N04 Students will be expected to estimate quantities less than 1000 using referents. [ME, PS, R, V]

Performance Indicators

- N04.01 Estimate the number of groups of ten in a given quantity using 10 as a referent (known quantity).
- N04.02 Estimate the number of groups of a hundred in a given quantity using 100 as a referent.
- N04.03 Estimate a given quantity by comparing it to a referent.
- N04.04 Select an estimate for a given quantity by choosing among three possible choices.
- N04.05 Select and justify a referent for determining an estimate for a given quantity.

N05 Students will be expected to illustrate, concretely and pictorially, the meaning of place value for numerals to 1000. [C, CN, R, V]

Performance Indicators

- N05.01 Record, in more than one way, the number represented by given proportional and non-proportional concrete materials in traditional and non-conventional formats.
- N05.02 Represent a given number in different ways using proportional and non-proportional concrete materials and explain how they are equivalent; e.g., 351 can be represented as three 100s, five 10s, and one 1s; or two 100s, fifteen 10s and one 1s; or three 100s, four 10s, and eleven 1s.
- N05.03 Record a given number in additive expanded form.
- N05.04 Record a number represented by base-ten blocks arranged in a non-conventional format.

N06 Students will be expected to describe and apply mental mathematics strategies for adding two 2-digit numerals. [C, ME, PS, R, V]

Performance Indicators

- N06.01 Explain mental mathematics strategies that could be used to determine a sum.
 - Ten and some more
 - Tens and some more
 - Quick addition
 - Addition facts to 10 applied to multiples of 10
 - Addition on the hundred chart
 - Adding on

- Make ten
- Compensation
- Compatible numbers

N06.02 Use and describe a personal strategy for determining a sum.

N06.03 Determine a sum of two 2-digit numerals efficiently, using mental mathematics strategies.

N07 Students will be expected to describe and apply mental mathematics strategies for subtracting two 2-digit numerals. [C, ME, PS, R, V]

Performance Indicators

N07.01 Explain mental mathematics strategies that could be used to determine a difference.

- Facts with minuends of 10 or less applied to multiples of 10
- Quick subtraction
- Subtraction on the hundred chart
- Compensation
- Back through ten

N07.02 Use and describe a personal strategy for determining a difference.

N07.03 Determine a difference of two 2-digit numerals efficiently, using mental mathematics strategies.

N08 Students will be expected to apply estimation strategies to predict sums and differences of 1-, 2-, and 3-digit numerals in a problem-solving context. [C, ME, PS, R]

Performance Indicators

N08.01 Explain estimation strategies that could be used to determine an approximate sum or difference.

N08.02 Use and describe a strategy for determining an estimate.

N08.03 Estimate the solution for a given story problem involving the sum or difference of up to two 3-digit numerals.

N09 Students will be expected to demonstrate an understanding of addition and subtraction of numbers (limited to 1-, 2-, and 3-digit numerals) with answers to 1000 by

- using personal strategies for adding and subtracting with and without the support of manipulatives
- creating and solving problems in context that involve addition and subtraction of numbers concretely, pictorially, and symbolically [C, CN, ME, PS, R]

Performance Indicators

N09.01 Model the addition of two or more given numbers using concrete or visual representations and record the process symbolically.

N09.02 Model the subtraction of two given numbers using concrete or visual representations and record the process symbolically.

N09.03 Create an addition or subtraction story problem for a given solution.

N09.04 Determine the sum of two given numbers using a personal strategy, e.g., for $326 + 48$, record $300 + 60 + 14$.

N09.05 Determine the difference of two given numbers using a personal strategy, e.g., for $127 - 38$, record $2 + 80 +$ or $127 - 20 - 10 - 8$.

N09.06 Solve a given problem involving the sum or difference of two given numbers.

N10 Students will be expected to apply mental mathematics strategies and number properties to develop quick recall of basic addition facts to 18 and related basic subtraction facts.

[C, CN, ME, R, V]

Performance Indicators

- N10.01 Describe a mental mathematics strategy that could be used to determine a given basic addition fact up to $9 + 9$.
- N10.02 Explain how the commutative (order-doesn't-matter) property and the identity (no-change-with-zero) property can assist in addition fact learning.
- N10.03 Describe a mental mathematics strategy that could be used to determine a given basic subtraction fact with minuends up to 18 and subtrahends up to 9.
- N10.04 Recognize which facts could be determined by a given strategy.
- N10.05 Quickly recall basic addition facts to 18 and related subtraction facts in a variety of contexts.

N11 Students will be expected to demonstrate an understanding of multiplication to 5×5 by

- representing and explaining multiplication using equal grouping and arrays
- creating and solving problems in context that involves multiplication
- modelling multiplication using concrete and visual representations and recording the process symbolically
- relating multiplication to repeated addition
- relating multiplication to division [C, CN, PS, R]

Performance Indicators

- N11.01 Identify events from experience that can be described as multiplication.
- N11.02 Represent a given story problem (orally, shared reading, written) using manipulatives or diagrams and record in a number sentence.
- N11.03 Represent a given multiplication expression as repeated addition.
- N11.04 Represent a given repeated addition as multiplication.
- N11.05 Create and illustrate a story problem for a given number sentence and/or expression.
- N11.06 Represent, concretely or pictorially, equal groups for a given number sentence.
- N11.07 Represent a given multiplication expression using an array.
- N11.08 Create an array to model the commutative property of multiplication.
- N11.09 Relate multiplication to division by using arrays and writing related number sentences.
- N11.10 Solve a given problem in context involving multiplication.

N12 Students will be expected to demonstrate an understanding of division by

- representing and explaining division using equal sharing and equal grouping
- creating and solving problems in context that involve equal sharing and equal grouping
- modelling equal sharing and equal grouping using concrete and visual representations, and recording the process symbolically
- relating division to repeated subtraction
- relating division to multiplication

(Limited to division related to multiplication facts up to 5×5 .) [C, CN, PS, R]

Performance Indicators

- N12.01 Identify events from experience that can be described as equal sharing.
- N12.02 Identify events from experience that can be described as equal grouping.
- N12.03 Illustrate, with counters or a diagram, a given story problem involving equal sharing, presented orally or through shared reading, and solve the problem.

- N12.04 Illustrate, with counters or a diagram, a given story problem involving equal grouping, presented orally or through shared reading, and solve the problem.
- N12.05 Listen to a story problem, represent the numbers using manipulatives or a diagram and record the problem with a number sentence and/or expression.
- N12.06 Create and illustrate with counters, a story problem for a given number sentence and/or expression.
- N12.07 Represent a given division sentence and/or expression as repeated subtraction.
- N12.08 Represent a given repeated subtraction as a division sentence.
- N12.09 Relate division to multiplication by using arrays and writing related number sentences.
- N12.10 Solve a given problem involving division.

- N13** Students will be expected to demonstrate an understanding of fractions by
- explaining that a fraction represents a part of a whole
 - describing situations in which fractions are used
 - comparing fractions of the same whole with like denominators [C, CN, ME, R, V]

Performance Indicators

- N13.01 Describe everyday situations where fractions are used.
- N13.02 Represent a given fraction concretely or pictorially.
- N13.03 Identify, model, and explain the meaning of numerator and denominator.
- N13.04 Sort a given set of diagrams of regions into those that represent equal parts and those that do not, and explain the sorting.
- N13.05 Name and record the fraction represented by the shaded and non-shaded parts of a given region.
- N13.06 Compare given fractions with the same denominator using models.

PATTERNS AND RELATIONS (PR)

- PR01** Students will be expected to demonstrate an understanding of increasing patterns by describing, extending, comparing, and creating numerical (numbers to 1000) patterns and non-numerical patterns using manipulatives, diagrams, sounds, and actions. [C, CN, PS, R, V]

Performance Indicators

- PR01.01 Identify and describe increasing patterns.
- PR01.02 Describe a given increasing pattern by stating a pattern rule that includes the starting point and a description of how the pattern continues.
- PR01.03 Extend a pattern, using the pattern rule, for the next three terms.
- PR01.04 Compare numeric patterns.
- PR01.05 Identify and explain errors in a given increasing pattern.
- PR01.06 Create a concrete, pictorial, or symbolic representation of an increasing pattern for a given pattern rule.
- PR01.07 Create a concrete, pictorial, or symbolic increasing pattern and describe the pattern rule.
- PR01.08 Solve a given problem using increasing patterns.
- PR01.09 Identify and describe the strategy used to determine a missing term in a given increasing pattern.
- PR01.10 Use ordinal numbers (to 100th) to refer to or to predict terms within an increasing pattern.

PR02 Students will be expected to demonstrate an understanding of decreasing patterns by describing, extending, comparing, and creating numerical (numbers to 1000) patterns and non-numerical patterns using manipulatives, diagrams, sounds, and actions. [C, CN, PS, R, V]

Performance Indicators

- PR02.01 Identify and describe decreasing patterns.
- PR02.02 Describe a given decreasing pattern by stating a pattern rule that includes the starting point and a description of how the pattern continues.
- PR02.03 Extend a pattern using the pattern rule for the next three terms.
- PR02.04 Compare numeric patterns.
- PR02.05 Identify and explain errors in a given decreasing pattern.
- PR02.06 Create a concrete, pictorial, or symbolic representation of a decreasing pattern for a given pattern rule.
- PR02.07 Create a concrete, pictorial, or symbolic decreasing pattern and describe the pattern rule.
- PR02.08 Solve a given problem using decreasing patterns.
- PR02.09 Identify and describe the strategy used to determine a missing term in a given decreasing pattern.
- PR02.10 Use ordinal numbers (to 100th) to refer to or to predict terms within a decreasing pattern.

PR03 Students will be expected to solve one-step addition and subtraction equations involving symbols representing an unknown number. [C, CN, PS, R, V]

Performance Indicators

- PR03.01 Explain the purpose of the symbol in a given addition and in a given subtraction equation with one unknown.
- PR03.02 Create an addition or subtraction equation with one unknown to represent a given combination or separate action.
- PR03.03 Provide an alternative symbol for the unknown in a given addition or subtraction equation.
- PR03.04 Solve a given addition or subtraction equation that represents combining or separating actions with one unknown using manipulatives.
- PR03.05 Solve a given addition or subtraction equation with one unknown using a variety of strategies including guess and check.
- PR03.06 Explain why the unknown in a given addition or subtraction equation has only one value.

MEASUREMENT (M)

M01 Students will be expected to relate the passage of time to common activities using non-standard and standard units (minutes, hours, days, weeks, months, years). [CN, ME, R]

Performance Indicators

- M01.01 Select and use a non-standard unit of measure, such as television shows or pendulum swings, to measure the passage of time and explain the choice.
- M01.02 Identify activities that can or cannot be accomplished in minutes, hours, days, weeks, months, and years.
- M01.03 Provide personal referents for minutes and hours.
- M01.04 Select and use a standard unit of measure, such as minutes, hours, days, weeks, and months to measure the passage of time and explain the choice.

M02 Students will be expected to relate the number of seconds to a minute, the numbers of minutes to an hour, the numbers of hours to a day, and the number of days to a month in a problem-solving context. [C, CN, PS, R, V]

Performance Indicators

- M02.01 Determine the number of days in any given month using a calendar.
- M02.02 Solve a given problem involving the number of seconds in a minute, the number of minutes in an hour, the number of hours in a day, or the number of days in a given month.
- M02.03 Create a calendar that includes days of the week, dates, and personal events.

M03 Students will be expected to demonstrate an understanding of measuring length (cm, m) by

- selecting and justifying referents for the units centimetre or metre (cm, m)
- modelling and describing the relationship between the units centimetre or metre (cm, m)
- estimating length using referents
- measuring and recording length, width, and height [C, CN, ME, PS, R, V]

Performance Indicators

- M03.01 Provide a personal referent for one centimetre and explain the choice.
- M03.02 Provide a personal referent for one metre and explain the choice.
- M03.03 Match a given standard unit to a given referent.
- M03.04 Show that 100 centimetres is equivalent to 1 metre by using concrete materials.
- M03.05 Estimate the length of an object using personal referents.
- M03.06 Determine and record the length and width of a given 2-D shape.
- M03.07 Determine and record the length, width or height of a given 3-D object.
- M03.08 Draw a line segment of a given length using a ruler.
- M03.09 Sketch a line segment of a given length without using a ruler.

M04 Students will be expected to demonstrate an understanding of measuring mass (g, kg) by

- selecting and justifying referents for the units gram and kilogram (g, kg)
- modelling and describing the relationship between the units gram and kilogram (g, kg)
- estimating mass using referents
- measuring and recording mass [C, CN, ME, PS, R, V]

Performance Indicators

- M04.01 Provide a personal referent for one gram and explain the choice.
- M04.02 Provide a personal referent for one kilogram and explain the choice.
- M04.03 Match a given standard unit to a given referent.
- M04.04 Explain the relationship between 1000 grams and 1 kilogram using a model.
- M04.05 Estimate the mass of a given object using personal referents.
- M04.06 Measure, using a balance scale, and record the mass of given everyday objects using the units gram (g) and kilogram (kg).
- M04.07 Provide examples of 3-D objects that have a mass of approximately 1 g, 100 g, and 1 kg.
- M04.08 Determine the mass of two given similar objects with different masses and explain the results.
- M04.09 Determine the mass of an object, change its shape, re-measure its mass and explain the results.

- M05** Students will be expected to demonstrate an understanding of perimeter of regular, irregular, and composite shapes by
- estimating perimeter using referents for centimetre or metre (cm, m)
 - measuring and recording perimeter (cm, m)
 - create different shapes for a given perimeter (cm, m) to demonstrate that many shapes are possible for a perimeter [C, ME, PS, R, V]

Performance Indicators

- M05.01 Measure and record the perimeter of a given regular shape and explain the strategy used.
 M05.02 Measure and record the perimeter of a given irregular or composite shape and explain the strategy used.
 M05.03 Construct a shape for a given perimeter (cm, m).
 M05.04 Construct or draw more than one shape for the same given perimeter.
 M05.05 Estimate the perimeter of a given shape (cm, m) using personal referents.

GEOMETRY (G)

- G01** Students will be expected to describe 3-D objects according to the shape of the faces and the number of edges and vertices. [C, CN, PS, R, V]

Performance Indicators

- G01.01 Identify the faces, edges, and vertices of given 3-D objects, including spheres, cones, cylinders, pyramids, cubes and other prisms.
 G01.02 Identify the shape of the faces of a given 3-D object.
 G01.03 Determine the number of faces, edges, and vertices of a given 3-D object.
 G01.04 Sort a given set of 3-D objects according to the number of faces, edges, or vertices.

- G02** Students will be expected to name, describe, compare, create, and sort regular and irregular polygons, including triangles, quadrilaterals, pentagons, hexagons, and octagons according to the number of sides. [C, CN, R, V]

Performance Indicators

- G02.01 Classify a given set of regular and irregular polygons according to the number of sides.
 G02.02 Identify given regular and irregular polygons having different dimensions.
 G02.03 Identify given regular and irregular polygons having different positions.

STATISTICS AND PROBABILITY (SP)

- SP01** Students will be expected to collect first-hand data and organize it using tally marks, line plots, charts, and lists to answer questions. [C, CN, V]

Performance Indicators

- SP01.01 Record the number of objects in a given set using tally marks.
 SP01.02 Determine the common attributes of line plots by comparing line plots in a given set.
 SP01.03 Organize a given set of data using tally marks, line plots, charts, or lists.
 SP01.04 Collect and organize data using tally marks, line plots, charts, and lists.
 SP01.05 Answer questions arising from a given line plot, chart, or list.
 SP01.06 Answer questions using collected data.

SP02 Students will be expected to construct, label, and interpret bar graphs to solve problems.
[PS, R, V]

Performance Indicators

- SP02.01 Determine the common attributes, title, and axes of bar graphs by comparing bar graphs in a given set.
- SP02.02 Create bar graphs from a given set of data including labelling the title and axes.
- SP02.03 Draw conclusions from a given bar graph to solve problems.
- SP02.04 Solve problems by constructing and interpreting a bar graph.

Mathematical Processes

There are critical components that students must encounter in a mathematics program in order to achieve the goals of mathematics education and encourage lifelong learning in mathematics.

Students are expected to

- communicate in order to learn and express their understanding of mathematics (Communication [C])
- develop and apply new mathematical knowledge through problem solving (Problem Solving [PS])
- connect mathematical ideas to other concepts in mathematics, to everyday experiences, and to other disciplines (Connections [CN])
- demonstrate fluency with mental mathematics and estimation (Mental Mathematics and Estimation [ME])
- select and use technologies as tools for learning and solving problems (Technology [T])
- develop visualization skills to assist in processing information, making connections, and solving problems (Visualization [V])
- develop mathematical reasoning (Reasoning [R])

The Nova Scotia curriculum incorporates these seven interrelated mathematical processes that are intended to permeate teaching and learning. The key to these process standards is presented in a box, as shown below, with each specific curriculum outcome within the strands.

Process Standards Key

[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

Communication [C]

Students need opportunities to read about, represent, view, write about, listen to, and discuss mathematical ideas. These opportunities allow students to create links between their own language and ideas, and the formal language and symbols of mathematics. Communication is important in clarifying, reinforcing, and modifying ideas, knowledge, attitudes, and beliefs about mathematics. Students should be encouraged to use a variety of forms of communication while learning mathematics.

Students also need to communicate their learning using mathematical terminology. Communication can help students make connections between and among the different representational modes—contextual, concrete, pictorial, linguistic/verbal, and symbolic—of mathematical ideas. Students must communicate *daily* about their mathematics learning. This enables them to reflect, to validate, and to

clarify their thinking and provides teachers with insight into students' interpretations of mathematical meanings and ideas.

Problem Solving [PS]

Learning through problem solving should be the focus of mathematics at all grade levels. When students encounter new situations and respond to questions of the type, How would you ...? or How could you ...? the problem-solving approach is being modelled. Students develop their own problem-solving strategies by being open to listening, discussing, and trying different strategies.

In order for an activity to be problem-solving based, it must ask students to determine a way to get from what is known to what is sought. If students have already been given ways to solve the problem, it is not a problem, but practice. A true problem requires students to use prior learning in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement, perseverance, and collaboration.

Problem solving is also a powerful teaching tool that fosters multiple, creative, and innovative solutions. Creating an environment where students openly look for and engage in finding a variety of strategies for solving problems empowers students to explore alternatives and develops confident, cognitive, mathematical risk takers.

When students are exposed to a wide variety of problems in all areas of mathematics, they explore various methods for solving and verifying problems. In addition, they are challenged to find multiple solutions for problems and to create their own problem.

Connections [CN]

Contextualization and making connections to the experiences of learners are powerful processes in developing mathematical understanding. When mathematical ideas are connected to one another or to real-world phenomena, students can begin to view mathematics as useful, relevant, and integrated. Learning mathematics within contexts and making connections relevant to learners can validate past experiences and increase student willingness to participate and be actively engaged. The brain is constantly looking for and making connections.

“Because the learner is constantly searching for connections on many levels, educators need to orchestrate the experiences from which learners extract understanding. ... Brain research establishes and confirms that multiple complex and concrete experiences are essential for meaningful learning and teaching.” (Caine and Caine 1991, 5).

Mathematics should be viewed as an integrated whole rather than as the study of separate strands or units. Connections must also be made between and among the different representational modes—contextual, concrete, pictorial, linguistic/verbal, and symbolic. The process of making connections, in turn, facilitates learning. Concepts and skills should also be connected to everyday situations and other curricular areas. For example, when developing literacy skills students learn to make text-to-world, text-to-text, and text-to-self connections. Students can also make connections to make mathematics come alive through math-to-world, math-to-math, and math-to-self connections.

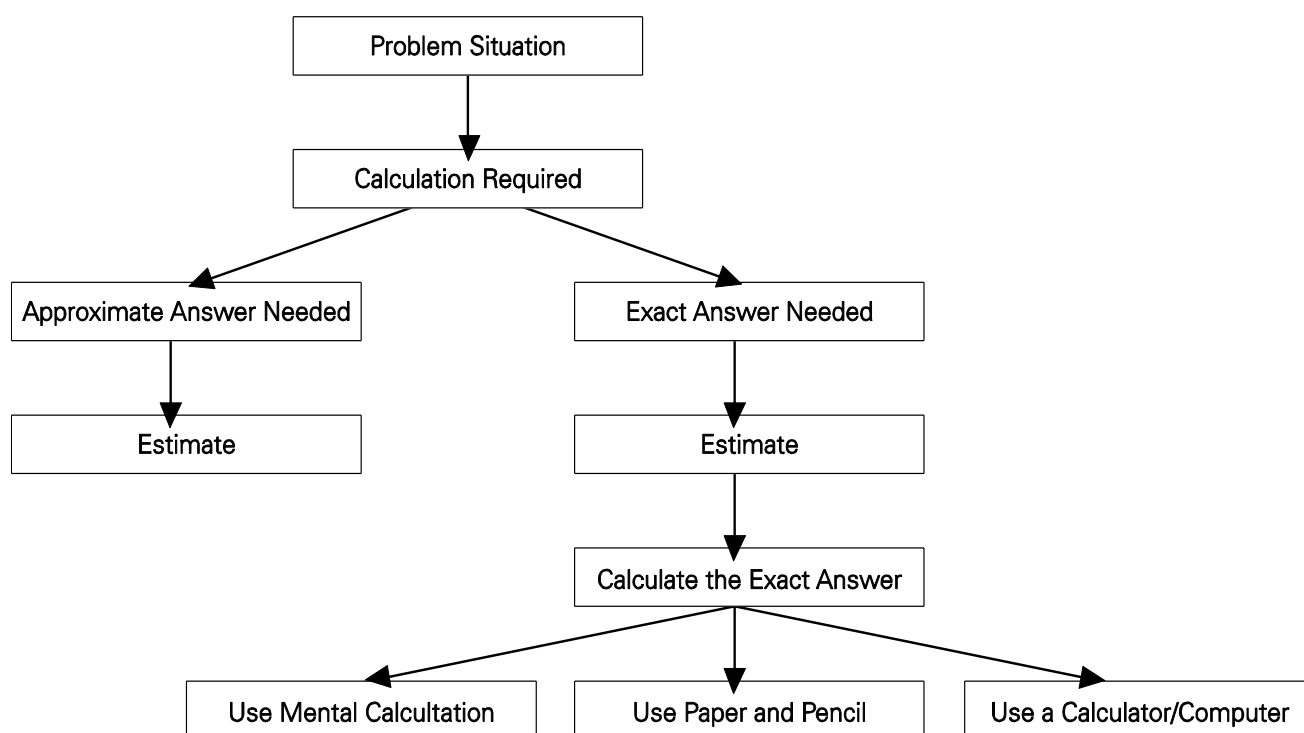
Mental Mathematics and Estimation [ME]

Mental mathematics is a combination of cognitive strategies that enhance flexible thinking and number sense. It is calculating mentally without the use of external aids. Mental mathematics enables students to determine answers without paper and pencil. It improves computational fluency by developing efficiency, accuracy, and flexibility. “Even more important than performing computational procedures or using calculators is the greater facility that students need—more than ever before—with estimation and mental math.” (National Council of Teachers of Mathematics, May 2005).

Students proficient with mental mathematics “become liberated from calculator dependence, build confidence in doing mathematics, become more flexible thinkers, and are more able to use multiple approaches to problem solving.” (Rubenstein 2001) Mental mathematics “provides a cornerstone for all estimation processes, offering a variety of alternative algorithms and nonstandard techniques for finding answers.” (Hope 1988, v)

Estimation is a strategy for determining approximate values or quantities, usually by referring to benchmarks or using referents, or for determining the reasonableness of calculated values. Students need to know how, when, and what strategy to use when estimating. Estimation is used to make mathematical judgments and develop useful, efficient strategies for dealing with situations in daily life.

Students need to develop both mental mathematics and estimation skills through context and not in isolation so they are able to apply them to solve problems. Whenever a problem requires a calculation, students should follow the decision-making process as illustrated below.



The skill of estimation requires a sound knowledge of mental mathematics. Both are necessary to many everyday experiences, and students should be provided with frequent opportunities to practise these skills.

Technology [T]

Technology can be effectively used to contribute to and support the learning of a wide range of mathematical outcomes and enables students to explore and create patterns, examine relationships, test conjectures, and solve problems.

Technology can be used to

- explore and demonstrate mathematical relationships and patterns
- organize and display data
- extrapolate and interpolate
- assist with calculation procedures as part of solving problems
- decrease the time spent on computations when other mathematical learning is the focus
- reinforce the learning of basic facts and test properties
- develop personal procedures for mathematical operations
- create geometric displays
- simulate situations
- develop number sense

The use of calculators is recommended to enhance problem solving, to encourage discovery of number patterns, and to reinforce conceptual development and numerical relationships. They do not, however, replace the development of number concepts and skills. Carefully chosen computer software can provide interesting problem-solving situations and applications.

Technology contributes to a learning environment in which the growing curiosity of students can lead to rich mathematical discoveries at all grade levels. While technology can be used in grades primary to 3 to enrich learning, it is expected that students will achieve all outcomes without the use of technology.

Visualization [V]

Visualization “involves thinking in pictures and images, and the ability to perceive, transform and recreate different aspects of the visual-spatial world.” (Armstrong 1999). The use of visualization in the study of mathematics provides students with opportunities to understand mathematical concepts and make connections among them. Visual images and visual reasoning are important components of number, spatial, and measurement sense. Number visualization occurs when students create mental representations of numbers. These mental images are needed to develop concepts and understand procedures. Images and explanations help students clarify their understanding of mathematical ideas in all strands.

Being able to create, interpret, and describe a visual representation is part of spatial sense and spatial reasoning. Spatial visualization and reasoning enable students to describe the relationships among and between 3-D objects and 2-D shapes.

Measurement visualization goes beyond the acquisition of specific measurement skills. Measurement sense includes the ability to determine when to measure, when to estimate and to know several estimation strategies. (Shaw and Cliatt 1989)

Visualization is fostered through the use of concrete materials, technology, and a variety of visual representations.

Reasoning [R]

Mathematical reasoning helps students think logically and make sense of mathematics. Students need to develop confidence in their abilities to reason and justify their mathematical thinking. High-order questions challenge students to think and develop a sense of wonder about mathematics. Mathematical experiences in and out of the classroom provide opportunities for inductive and deductive reasoning. Inductive reasoning occurs when students explore and record results, analyze observations, make generalizations from patterns, and test these generalizations. Deductive reasoning occurs when students reach new conclusions based upon what is already known or assumed to be true.

Mathematics reasoning involves informal thinking, conjecturing, and validating—these help students understand that mathematics makes sense. Students are encouraged to justify, in a variety of ways, their solutions, thinking processes, and hypotheses. In fact, good reasoning is as important as finding correct answers.

Nature of Mathematics

Mathematics is one way of trying to understand, interpret, and describe our world. There are a number of components that define the nature of mathematics, and these are woven throughout this document. These components include change, constancy, number sense, relationships, patterns, spatial sense, and uncertainty.

Change

It is important for students to understand that mathematics is dynamic and not static. As a result, recognizing change is a key component in understanding and developing mathematics. Within mathematics, students encounter conditions of change and are required to search for explanations of that change. To make predictions, students need to describe and quantify their observations, look for patterns, and describe those quantities that remain fixed and those that change. For example, the sequence 4, 6, 8, 10, 12, ... can be described as

- skip counting by 2s, starting from 4
- an arithmetic sequence, with first term 4 and a common difference of 2
- a linear function with a discrete domain

(Steen 1990, 184).

Constancy

Different aspects of constancy are described by the terms **stability**, **conservation**, **equilibrium**, **steady state**, and **symmetry** (AAAS–Benchmarks 1993, 270). Many important properties in mathematics and science relate to properties that do not change when outside conditions change. Some problems in mathematics require students to focus on properties that remain constant. The recognition of constancy enables students to solve problems. Examples of constancy include the following:

- The area of a rectangular region is the same regardless of the methods used to determine the solution.
- The sum of the interior angles of any triangle is 180° .
- The theoretical probability of flipping a coin and getting heads is 0.5.

Number Sense

Number sense, which can be thought of as intuition about numbers, is the most important foundation of numeracy (British Columbia Ministry of Education 2000, 146). A true sense of number goes well beyond the skills of simply counting, memorizing facts, and the situational rote use of algorithms. Number sense develops when students connect numbers to real-life experiences and use benchmarks and referents. This results in students who are computationally fluent, flexible with numbers, and have intuition about numbers. The evolving number sense typically comes as a by-product of learning rather than through direct instruction. However, number sense can be developed by providing rich mathematical tasks that allow students to make connections.

Relationships

Mathematics is used to describe and explain relationships. As part of the study of mathematics, students look for relationships among numbers, sets, shapes, objects, and concepts. The search for possible relationships involves the collection and analysis of data, and describing relationships visually, symbolically, orally, or in written form.

Patterns

Mathematics is about recognizing, describing, and working with numerical and non-numerical patterns. Patterns exist in all strands, and it is important that connections are made among strands. Working with patterns enables students to make connections within and beyond mathematics. These skills contribute to students' interaction with an understanding of their environment. Patterns may be represented in concrete, visual, or symbolic form. Students should develop fluency in moving from one representation to another. Students must learn to recognize, extend, create, and use mathematical patterns. Patterns allow students to make predictions and justify their reasoning when solving problems. Learning to work with patterns in the early grades helps develop students' algebraic thinking, which is foundational for working with more abstract mathematics in higher grades.

Spatial Sense

Spatial sense involves visualization, mental imagery, and spatial reasoning. These skills are central to the understanding of mathematics. Spatial sense enables students to reason and interpret among and between 3-D and 2-D representations and identify relationships to mathematical strands. Spatial sense is developed through a variety of experiences and interactions within the environment. The development of spatial sense enables students to solve problems involving 3-D objects and 2-D shapes. Spatial sense offers a way to interpret and reflect on the physical environment and its 3-D or 2-D representations. Some problems involve attaching numerals and appropriate units (measurement) to dimensions of objects. Spatial sense allows students to make predictions about the results of changing these dimensions. For example,

- knowing the dimensions of an object enables students to communicate about the object and create representations
- the volume of a rectangular solid can be calculated from given dimensions
- doubling the length of the side of a square increases the area by a factor of four

Uncertainty

In mathematics, interpretations of data and the predictions made from data may lack certainty. Events and experiments generate statistical data that can be used to make predictions. It is important to recognize that these predictions (interpolations and extrapolations) are based upon patterns that have a degree of uncertainty. The quality of the interpretation is directly related to the quality of the data. An awareness of uncertainty allows students to assess the reliability of data and data interpretation. Chance addresses the predictability of the occurrence of an outcome. As students develop their understanding of probability, the language of mathematics becomes more specific and describes the degree of uncertainty more accurately.

Curriculum Document Format

This guide presents the mathematics curriculum so that a teacher may readily view the scope of the outcomes that students are expected to achieve during that year. Teachers are encouraged, however, to examine what comes before and what follows after, to better understand how students' learning at a particular grade level is part of a bigger picture of concept and skill development.

The order of presentation in no way assumes or prescribes a preferred order of presentation in the classroom, but simply lays out the specific curriculum outcomes in relation to the overarching general curriculum outcomes (GCOs).

The footer of the document shows the name of the course, and the strand name is presented in the header. When a specific curriculum outcome (SCO) is introduced, it is followed by the mathematical processes and performance indicators for that outcome. A scope and sequence is then provided, which relates the SCO to previous and next grade SCOs. Also for each SCO, there is background information, assessment strategies, suggested instructional strategies, suggested models and manipulatives, mathematical language, and a section for resources and notes. For each section, the guiding questions should be used to help with unit and lesson preparation.

SCO		
Mathematical Processes		
[C] Communication	[PS] Problem Solving	[CN] Connections
[ME] Mental Mathematics and Estimation		
[T] Technology	[V] Visualization	[R] Reasoning
Performance Indicators		
Describes observable indicators of whether students have achieved the specific outcome.		
Scope and Sequence		
Previous grade or course SCOs	Current grade SCO	Following grade or course SCOs
Background		
Describes the “big ideas” to be learned and how they relate to work in previous grade and work in subsequent courses.		
Additional Information		
A reference to Appendix A, which contains further elaborations for the performance indicators.		
Assessment, Teaching, and Learning		
Assessment Strategies		
Guiding Questions		
<ul style="list-style-type: none"> ▪ What are the most appropriate methods and activities for assessing student learning? ▪ How will I align my assessment strategies with my teaching strategies? 		
ASSESSING PRIOR KNOWLEDGE		
Sample tasks that can be used to determine students’ prior knowledge.		
WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS		
Some suggestions for specific activities and questions that can be used for both instruction and assessment		

FOLLOW-UP ON ASSESSMENT
Guiding Questions
<ul style="list-style-type: none"> ▪ What conclusions can be made from assessment information? ▪ How effective have instructional approaches been? ▪ What are the next steps in instruction for the class and for individual students?
RESPONDING TO ASSESSMENT
Correlations to related resources.
Planning for Instruction
Guiding Questions
<ul style="list-style-type: none"> ▪ Does the lesson fit into my yearly/unit plan? ▪ How can the processes indicated for this outcome be incorporated into instruction? ▪ What learning opportunities and experiences should be provided to promote learning of the outcome and permit students to demonstrate their learning? ▪ What teaching strategies and resources should be used? ▪ How will the diverse learning needs of students be met?
CHOOSING INSTRUCTIONAL STRATEGIES
Suggested strategies for planning daily lessons.
SUGGESTED LEARNING TASKS
Suggestions for general approaches and strategies suggested for teaching this outcome.
Guiding Questions
<ul style="list-style-type: none"> ▪ How can the scope and sequence be used to determine what prior knowledge needs to be activated prior to beginning new instruction?
SUGGESTED MODELS AND MANIPULATIVES
MATHEMATICAL LANGUAGE
Teacher and student mathematical language associated with the respective outcome.
Resources/Notes

Contexts for Learning and Teaching

Beliefs about Students and Mathematics Learning

“Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge.” (National Council of Teachers of Mathematics 2000, 20).

The Nova Scotia mathematics curriculum is based upon several key assumptions or beliefs about mathematics learning that have grown out of research and practice. These beliefs include the following:

- Mathematics learning is an active and constructive process.
- Learning is most effective when standards of expectation are made clear with ongoing assessment and feedback.
- Learners are individuals who bring a wide range of prior knowledge and experiences and who learn via various styles and at different rates.
- Learning is most likely to occur when placed in meaningful contexts and in an environment that supports exploration, risk taking, and critical thinking and that nurtures positive attitudes and sustained effort.

Students are curious, active learners with individual interests, abilities, and needs. They come to classrooms with varying knowledge, life experiences, and backgrounds. A key component in successfully developing numeracy is making connections to these backgrounds and experiences.

Students develop a variety of mathematical ideas before they enter school. Children make sense of their environment through observations and interactions at home and in the community. Mathematics learning is embedded in everyday activities, such as playing, reading, storytelling, and helping around the home. Such activities can contribute to the development of number and spatial sense in children. Curiosity about mathematics is fostered when children are engaged in activities such as comparing quantities, searching for patterns, sorting objects, ordering objects, creating designs, building with blocks, and talking about these activities. Positive early experiences in mathematics are as critical to child development as are early literacy experiences.

Students learn by attaching meaning to what they do and need to construct their own meaning of mathematics. This meaning is best constructed when learners encounter mathematical experiences that proceed from the simple to the complex and from the concrete to the abstract. The use of models and a variety of pedagogical approaches can address the diversity of learning styles and developmental stages of students and enhance the formation of sound, transferable, mathematical concepts. At all levels, students benefit from working with and translating through a variety of materials, tools, and contexts when constructing meaning about new mathematical ideas. Meaningful discussions can provide essential links among concrete, pictorial, contextual, and symbolic representations of mathematics.

The learning environment should value and respect all students’ experiences and ways of thinking, so that learners are comfortable taking intellectual risks, asking questions, and posing conjectures. Students need to explore problem-solving situations in order to develop personal strategies and become mathematically literate. Learners must realize that it is acceptable to solve problems in different ways and that solutions may vary.

Goals for Mathematics Education

The main goals of mathematics education are to prepare students to

- use mathematics confidently to solve problems
- communicate and reason mathematically
- appreciate and value mathematics
- make connections between mathematics and its applications
- become mathematically literate adults, using mathematics to contribute to society

Students who have met these goals will

- gain understanding and appreciation of the contributions of mathematics as a science, a philosophy, and an art
- exhibit a positive attitude toward mathematics
- engage and persevere in mathematical tasks and projects
- contribute to mathematical discussions
- take risks in performing mathematical tasks
- exhibit curiosity about mathematics and situations involving mathematics

Opportunities for Success

A positive attitude has a profound effect on learning. Environments that create a sense of belonging, encourage risk taking, and provide opportunities for student success help develop and maintain positive attitudes and self-confidence. Students with positive attitudes toward learning mathematics are likely to be motivated and prepared to learn, participate willingly in classroom activities, persist in challenging situations, and engage in reflective practices.

To experience success, students must be taught to set achievable goals or assess their progress as they work toward these goals. Striving toward success and becoming autonomous and responsible learners are ongoing, reflective processes that involve revisiting the setting and assessing of personal goals.

Engaging All Learners

“No matter how engagement is defined or which dimension is considered, research confirms this truism of education: *The more engaged you are, the more you will learn.*” (Hume 2011, 6)

Student engagement is at the core of learning. This is critical for teachers to take into account when planning and implementing instruction. Effective instruction engages, embraces, and supports all learners through a range of learning experiences. This curriculum is designed to provide learning opportunities that reflect culturally proficient instructional and assessment practices and are equitable, accessible, and inclusive of the multiple facets of diversity represented in today’s classrooms.

Engagement in learning occurs when students are provided with opportunities to become more invested in their learning. When teachers know their students as individual learners and as individual people, their students are more likely to be motivated to learn, participate in classroom activities, persist in challenging situations, and engage in reflective practices. Students often become more engaged when teachers demonstrate a genuine belief in each student’s potential to learn.

SUPPORTIVE LEARNING ENVIRONMENTS

A supportive and positive learning environment has a profound effect on students' learning. In classrooms where students feel a sense of belonging, are encouraged to actively participate, are challenged without being frustrated, and feel safe and supported to take risks with their learning, students are more likely to experience success. It is realized that not all students will progress at the same pace or be equally positioned in terms of their prior knowledge of and skill with particular concepts and outcomes. Teachers provide all students with equitable access to learning by integrating a variety of instructional approaches and assessment activities that consider all learners and align with the following key principles:

- Instruction must be flexible and offer multiple means of representation.
- Students must have opportunities to express their knowledge and understanding in multiple ways.
- Teachers must provide options for students to engage in learning through multiple ways.

Teachers who know their students well become aware of individual learning differences and infuse this understanding into planned instructional and assessment decisions. They organize learning experiences to accommodate the many ways in which students learn, create meaning, and demonstrate their knowledge and understanding. Teachers use a variety of effective teaching approaches that may include

- providing all students with equitable access to appropriate learning strategies, resources, and technology
- offering a range of ways students can access their prior knowledge to connect with new concepts
- scaffolding instruction and assignments so that individual or groups of students are supported as needed throughout the process of learning
- verbalizing their thinking to model comprehension strategies and new learning
- balancing individual, small-group, and whole-class approaches to learning activities
- involving students in the co-creation of criteria for assessment and evaluation
- providing students with choice in how they demonstrate their understanding according to learning styles and preferences, building on individual strengths, and including a range of difficulty and challenge
- providing frequent and meaningful feedback to students throughout their learning experiences

LEARNING STYLES AND PREFERENCES

The ways in which students make sense of, receive, and process information, demonstrate learning, and interact with peers and their environment both indicate and shape learning preferences, which may vary widely from student to student. Learning preferences are influenced also by the learning context and purpose and by the type and form of information presented or requested. Most students tend to favour one learning style and may have greater success if instruction is designed to provide for multiple learning styles, thus creating more opportunities for all students to access learning. The three most commonly referenced learning styles are

- auditory (such as listening to teacher-presented lessons or discussing with peers)
- kinesthetic (such as using manipulatives or recording print or graphic/visual text)
- visual (such as interpreting information with text and graphics or viewing videos)

While students can be expected to work using all modalities, it is recognized that one or some of these modalities may be more natural to individual students than the others.

A GENDER-INCLUSIVE CURRICULUM

It is important that the curriculum respects the experiences and values of all students and that learning resources and instructional practices are not gender-biased. Teachers promote gender equity and inclusion in their classrooms when they

- articulate equally high expectations for all students
- provide equal opportunity for input and response from all students
- model gender-fair language and respectful listening in their interactions with students

VALUING DIVERSITY: TEACHING WITH CULTURAL PROFICIENCY

Teachers understand that students represent diverse life and cultural experiences, with individual students bringing different prior knowledge to their learning. Therefore, teachers build upon their knowledge of their students as individuals and respond by using a variety of culturally-proficient instruction and assessment strategies. “Instruction that is embedded in socially meaningful contexts, and tasks that are meaningful and relevant to the lives of students, will engage students in high-level problem-solving and reasoning and enhance students’ engagement (Frankenstein 1995; Gutstein 2003; Ladson-Billings 1997; Tate 1995).” (Herzig 2005)

STUDENTS WITH LANGUAGE, COMMUNICATION, AND LEARNING CHALLENGES

Today’s classrooms include students who have diverse backgrounds, abilities, levels of development, and learning challenges. By observing and interacting with students as they work on assigned activities, teachers can identify areas where students may need additional support to achieve their learning goals. Teachers can then respond with a range of effective instructional strategies. Students who have English as an Additional Language (EAL) may require curriculum outcomes at different levels, or temporary individualized outcomes, particularly in language-based subject areas, while they become more proficient in their English language skills. For students who are experiencing difficulties, it is important that teachers distinguish between students for whom curriculum content is challenging and students for whom language-based issues are at the root of apparent academic difficulties.

STUDENTS WHO DEMONSTRATE GIFTED AND TALENTED BEHAVIOURS

Some students are academically gifted and talented with specific skill sets or in specific subject areas. Most students who are gifted and talented thrive when challenged by problem-centred, inquiry-based learning and open-ended activities. Teachers may challenge students who are gifted and talented by adjusting the breadth, the depth, and/or the pace of instruction. Learning experiences may be enriched by providing greater choice among activities and offering a range of resources that require increased cognitive demand and higher-level thinking at different levels of complexity and abstraction. For additional information, refer to *Gifted Education and Talent Development* (Nova Scotia Department of Education 2010).

Connections across the Curriculum

The teacher should take advantage of the various opportunities available to integrate mathematics and other subjects. This integration not only serves to show students how mathematics is used in daily life, but it helps strengthen the students’ understanding of mathematical concepts and provides them with opportunities to practise mathematical skills. There are many possibilities for integrating mathematics in health education, literacy, music, physical education, science, social studies, and visual arts.

Number (N)

GCO: Students will be expected to demonstrate number sense.

Specific Curriculum Outcomes

Process Standards

[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

- N01** Students will be expected to say the number sequence forward and backward by
- 1s through transitions to 1000
 - 2s, 5s, 10s, or 100s, using any starting point to 1000
 - 3s, using starting points that are multiples of 3 up to 100
 - 4s, using starting points that are multiples of 4 up to 100
 - 25s, using starting points that are multiples of 25 up to 200. [C, CN, ME]
- N02** Students will be expected to represent and partition numbers to 1000. [C, CN, V]
- N03** Students will be expected to compare and order numbers up to 1000. [CN, R, V]
- N04** Students will be expected to estimate quantities less than 1000 using referents. [ME, PS, R, V]
- N05** Students will be expected to illustrate, concretely and pictorially, the meaning of place value for numerals to 1000. [C, CN, R, V]
- N06** Students will be expected to describe and apply mental mathematics strategies for adding two 2-digit numerals. [C, ME, PS, R, V]
- N07** Students will be expected to describe and apply mental mathematics strategies for subtracting two 2-digit numerals. [C, ME, PS, R, V]
- N08** Students will be expected to apply estimation strategies to predict sums and differences of 1-, 2-, and 3-digit numerals in a problem-solving context. [C, ME, PS, R]
- N09** Students will be expected to demonstrate an understanding of addition and subtraction of numbers (limited to 1-, 2-, and 3-digit numerals) with answers to 1000 by
- using personal strategies for adding and subtracting with and without the support of manipulatives
 - creating and solving problems in context that involve addition and subtraction of numbers concretely, pictorially, and symbolically [C, CN, ME, PS, R]
- N10** Students will be expected to apply mental mathematics strategies and number properties to develop quick recall of basic addition facts to 18 and related basic subtraction facts. [C, CN, ME, R, V]
- N11** Students will be expected to demonstrate an understanding of multiplication to 5×5 by
- representing and explaining multiplication using equal grouping and arrays
 - creating and solving problems in context that involves multiplication
 - modelling multiplication using concrete and visual representations, and recording the process symbolically
 - relating multiplication to repeated addition
 - relating multiplication to division [C, CN, PS, R]

-
- N12** Students will be expected to demonstrate an understanding of division by
- representing and explaining division using equal sharing and equal grouping
 - creating and solving problems in context that involve equal sharing and equal grouping
 - modelling equal sharing and equal grouping using concrete and visual representations, and recording the process symbolically
 - relating division to repeated subtraction
 - relating division to multiplication
- (Limited to division related to multiplication facts up to 5×5 .) [C, CN, PS, R]
- N13** Students will be expected to demonstrate an understanding of fractions by
- explaining that a fraction represents a part of a whole
 - describing situations in which fractions are used
 - comparing fractions of the same whole with like denominators [C, CN, ME, R, V]

<p>SCO N01 Students will be expected to say the number sequence forward and backward by</p> <ul style="list-style-type: none"> ▪ 1s through transitions to 1000 ▪ 2s, 5s, 10s, or 100s, using any starting point to 1000 ▪ 3s, using starting points that are multiples of 3 up to 100 ▪ 4s, using starting points that are multiples of 4 up to 100 ▪ 25s, using starting points that are multiples of 25 up to 200 <p>[C, CN, ME]</p>			
[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- N01.01** Extend the number sequence by 1s, particularly through transition from decade to decade and century to century.
- N01.02** Extend a given skip counting sequence by 2s, 5s, 10s, or 100s, forward and backward, using a given starting point.
- N01.03** Extend a given skip counting sequence by 3s, forward and backward, starting at a given multiple of 3 up to 100.
- N01.04** Extend a given skip counting sequence by 4s, forward and backward, starting at a given multiple of 4 up to 100.
- N01.05** Extend a given skip counting sequence by 25s, forward and backward, starting at a given multiple of 25 up to 200.
- N01.06** Identify and correct errors and omissions in a given skip counting sequence.
- N01.07** Determine the value of a given set of coins (nickels, dimes, quarters, and loonies) by using skip counting.
- N01.08** Identify and explain the skip counting pattern for a given number sequence.

Scope and Sequence

Mathematics 2	Mathematics 3	Mathematics 4
<p>N01 Students will be expected to say the number sequence by</p> <ul style="list-style-type: none"> ▪ 1s, forward and backward, starting from any point to 200 ▪ 2s, forward and backward, starting from any point to 100 ▪ 5s and 10s, forward and backward, using starting points that are multiples of 5 and 10 respectively to 100 ▪ 10s, starting from any point, to 100 	<p>N01 Students will be expected to say the number sequence forward and backward by</p> <ul style="list-style-type: none"> ▪ 1s through transitions to 1000 ▪ 5s, 10s, or 100s using any starting point to 1000 ▪ 3s using starting points that are multiples of 3 up to 100 ▪ 4s using starting points that are multiples of 4 up to 100 ▪ 25s, using starting points that are multiples of 25 up to 200 	—

Background

Students continue to develop an understanding of number and counting. Students should count with proficiency forward and backward by 1s, 2s, 5s, 10s, or 100s starting at any number from 0 to 1000. “The focus of skip counting in these early years seems to be on helping students see the patterns in our place value system as well as prepare students for work with money.” (Small 2009, 86). Skip counting by 3s, 4s, 5s, 10s, 25s, and 100s is a cornerstone for later multiplicative understanding.

When exploring numbers over 100, spend considerable time focusing on the numbers between 100 and 200. Students need to think about how the counting pattern sounds when they count from 20 to 29 and 30 to 39 and apply it when counting from 120 to 129, and 130 to 139, and when counting through all the decades that follow. Students need multiple opportunities to count the numbers through transition from decade to decade and century to century. For example, when counting from 98 ... 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111 ... a common misconception for students is to think that the next number after the one that ends in “9” is the next big number name. As a result many believe that 200 comes after 109. Students need to see that this same transition will apply for counting into all of the centuries. For example, on a regular basis, they should be asked questions such as, what would come after 199? or count on from 389.

Being able to skip count forward and backward by 5s, 10s, and 100s from any given number; by 3s and 4s, starting at any multiple of 3 and 4 respectively up to 100; and by 25s starting at any multiple of 25 up to 200, will help students be successful with many upcoming concepts such as patterns, money, and place value. Students in Mathematics 2 were skip-counting by 2s forward and backward to 100. Number charts may be used to explore these patterns.

Students should also investigate these skip counting patterns and whether the similar patterns occur when counting by 100s or when counting on hundreds charts beyond 100. A specific focus on looking at the numbers between 100 and 200 can help students to develop a sense of the repeating patterns in the number system. A hundred chart can easily be extended to a 200 chart by adding rows for the next 100 numbers.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Ask students to start at
 - 92 and count forward by 1s (stop at 121)
 - 42 and count forward by 2s (stop at 60)
 - 13 and count by 2s (stop at 35)
 - 78 and count backward by 2s (stop at 58)
 - 30 and count by 10s (stop at 100)
 - 8 and count by 10s (stop at 58)
 - 100 and count backward by 10s (stop at 40)
 - 15 and count by 5s (stop at 60)
 - 85 and count backward by 5s (stop at 55)

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students to skip count following directions, such as
 - Start at 95 and count forward by 5s to 140
 - Start at 349 and count by 100s without going over 1000, but get as close as you can
 - Start at 635 and count forward by 10s to 725
 - Start at 64 and count backward by 2s to 38
 - Start at 399 and count forward by 1s to 420
 - Start at 708 and count backward by 1s to 690
 - Start at 0 and count forward by 25s from 200
 - Start at 4 and count forward by 4s to 32
 - Start at 30 and count backward by 3s to 12
- Provide students with a number of beans (e.g., 60). Ask them to separate the beans from the pile as they count the beans by 3s and then by 4s.
- Ask, Why do you say fewer numbers when counting to 100 by 10s than when counting by 5s?
- Provide students with a skip counting pattern, such as 40, 36, 32, 28, 24, 20, 16, ... Ask them to identify the pattern and then to continue the pattern until they reach 0.
- Ask students to decide which starting point, 6 or 7, is easier when counting by 3s. Ask students to explain their choice.
- Write and say, 25, 50, 60, 65, 70. Ask, What coins am I counting?
- Have students identify and correct the error in a given skip counting sequence, such as
 - 12, 16, 21, 24, 28, 32
 - 27, 30, 33, 35, 39, 42
- Provide students with a set of nickels and ask them to skip count to find the total value of those coins.
- Provide students with a set of coins containing loonies, quarters, dimes, and nickels. Ask them to count the coins and to tell you the total.
- Have students count by 5s until they reach 60. Ask, What other numbers can you count by and still land on 60?

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets 3 (Bauman 2011)

- Checkpoint 1, Tasks 1, 2, and 3, pp. 20–21

Leaps and Bounds toward Math Understanding 3/4 (Small, Lin, and Kubota-Zarivnij 2011)

- Skip Counting

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Give students frequent opportunities to count materials (large quantities) in a variety of ways.
- Highlight the numbers on a number line or hundred chart that occur when skip counting and have students describe the patterns they see. Ensure that the numbers extend beyond 100.
- Provide students with multiple experiences counting both forward and backward, with various starting points.
- Use the calculator constant feature (described below) to count by 2s, 3s, 4s, 5s, 10s, 25s, and 100s. Ask the students to predict what number will come next, before it appears on the display.
- Be sure to use many examples including numbers greater than 100.
- Use situations, such as school fundraisers, as opportunities to count money by skip counting.

SUGGESTED LEARNING TASKS

- Provide students with a hundreds chart and have them colour in the pattern for a given skip counting sequence.
- Provide students with many number patterns to encourage skip counting; for example
 - 25, 50, __, __, 125, __, __
 - 752, 652, __, 452, __, 252, __
 - 110, 105, __, 95, 90, __, __, 75, __, __, __
 - 12, __, 18, __, 24, 27, 30
 - 39, 36, __, 30, __, 24, __, __
 - 4, __, __, 16, __, 24, 28, 32
- Ask students to count a large collection of objects, such as beans in a jar. Ask them how they grouped the beans (e.g., by 2s, 5s, 10s) for ease of counting.
- Use the *constant* (repeat) function (press 0, +, 5, =, =, =, ...) on the calculator to skip count to a target number. For example, if you start at 0 and want to end at 400, by which number(s) could you skip count? What if you started at a different point? What if you wanted to end at a different point?
- Play “What’s in the Can?” Tell students that you are going to drop nickels (or dimes or quarters) into a can. Have the students listen as the coins drop and count to find the total. As an extension, tell the students that there is, for example, 45 cents in the can. Tell them that you are going to add nickels (or dimes) and ask them to keep track to find the total.
- Provide students with a hundreds chart. Point to one of the squares (e.g., 56) and ask, If you start at 28 and count by 4s, will you say this number? Explain. Repeat using other numbers and starting points.
- Provide coins for the students. Ask, Can you use six coins to make 87 cents? Can you make a total of one dollar and 45 cents with only six coins? What are the coins?
- Provide students with play coins. Tell them that you have, for example, five coins in your hand that total 81 cents. Ask, What coins am I holding? (This is a problem situation and may require time.) Have students create puzzle questions like this.
- Provide students with a skip-counting pattern containing an error or an omission. Ask students to correct the error or omission.

SUGGESTED MODELS AND MANIPULATIVES

- calculator
- coins
- hundred chart
- number line
- open number lines
- various objects for counting (e.g. beans, counters)

MATHEMATICAL LANGUAGE

Teacher	Student
<ul style="list-style-type: none"> ▪ forward, backward ▪ nickels, dimes, quarters, loonies ▪ numbers: zero to one thousand ▪ skip counting patterns 	<ul style="list-style-type: none"> ▪ forward, backward ▪ nickels, dimes, quarters, loonies ▪ numbers: zero to one thousand ▪ skip counting patterns

Resources/Notes

Print

- *Making Math Meaningful to Canadian Students K–8* (Small 2009), pp. 86–87, 144
- *Making Math Meaningful to Canadian Students K–8, Second Edition* (Small 2013), pp. 142–143
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 56–58, 61–62, 138–139, 144, 150–152, 285–286
- *Teaching Student-Centered Mathematics, Grades 3–5* (Van de Walle and Lovin 2006), pp. 51–52

Videos

- *Analyzing Patterns (Skip Counting) on a Hundred Board* (27:16 min.) (ORIGO Education 2010)

Notes

SCO N02 Students will be expected to represent and partition numbers to 1000. [C, CN, V]			
[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- N02.01** Read a given three-digit numeral without using the word **and**.
- N02.02** Read a given number word (0 to 1000).
- N02.03** Represent a given number as an expression.
- N02.04** Represent a given number concretely and pictorially in a variety of ways.
- N02.05** Write number words for given multiples of ten to 90.
- N02.06** Write number words for given multiples of a hundred to 900.
- N02.07** Record numerals for numbers expressed orally, concretely, or pictorially.

Scope and Sequence

Mathematics 2	Mathematics 3	Mathematics 4
N04 Students will be expected to represent and partition numbers to 100.	N02 Students will be expected to represent and partition numbers to 1000.	N01 Students will be expected to represent and partition whole numbers to 10 000 concretely, pictorially, and symbolically.

Background

Students must be able to record numbers heard, read written numbers, write numbers in words, and represent written numbers symbolically. They must be able to translate a number from the written to the oral.

Although some students will have a clear understanding of the base-ten pattern of our place value system, many will still be in the early stages of development. It is important that students be provided with regular opportunities to represent numbers concretely, pictorially, symbolically, and in words to strengthen their knowledge.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

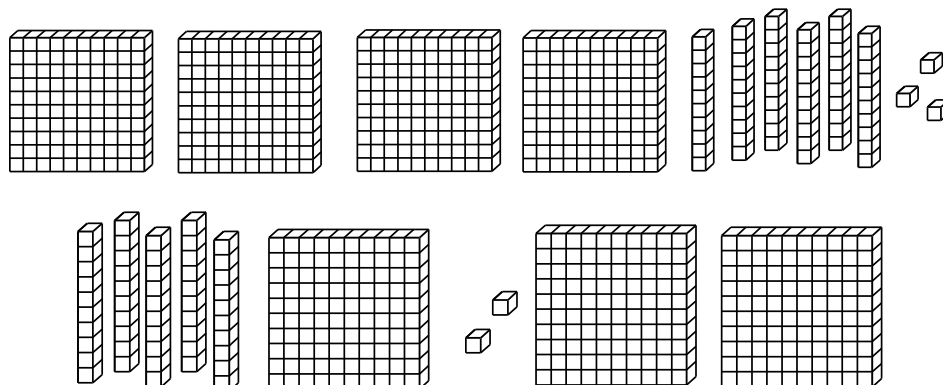
Tasks such as the following could be used to determine students' prior knowledge.

- Ask students to represent 52 (or any 2-digit number) with
 - ten-frames
 - tallies
 - coins
 - base-ten blocks
 - a picture
 - an expression

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students to choose a number less than 1000 and represent it in at least three ways using base-ten blocks. For example, if a student has chosen 527, he or she may represent it with 5 flats, 2 rods, and 7 small cubes; 4 flats, 12 rods, and 7 small cubes; 3 flats, 20 rods, and 27 small cubes or any other combination of blocks that totals 527.
- Present students with a pictorial representation of base-ten blocks illustrating a 3-digit number. Ask students to record the numeral represented by the picture. Ensure that both conventional and non-conventional displays of base-ten blocks are included. For example,



- Ask students to record numbers read orally, both symbolically and with words, making sure to include numbers that have a zero such as 902 or 370.
- Present students with cards on which are written 3-digit numerals. Ask students to read the numbers to you orally and to represent them using base-ten blocks.
- Ask students, Which of the expressions below represents 360? Ask them to explain their thinking.
 - $200 - 160$
 - $380 - 30$
 - $400 - 40$
 - $300 + 60$
 - $100 + 100 + 100 + 50 + 10$
 - $260 + 75 + 25$
 - $357 + 3$
 - $260 + 10$
- Provide students with an open number line and have them place the benchmark numbers, such as 250, 500, 750, and 1000, on it.
- Present students with cards on which are written numbers in words, such as four hundred eighteen or nine hundred seven. Ask students to read the numbers to you and to record the numeral.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets 3 (Bauman 2011)

- Checkpoint 1, Tasks 1, 2, and 3, pp. 20–21

Leaps and Bounds toward Math Understanding 3/4 (Small, Lin, and Kubota-Zarivnij 2011)

- Representing Number

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Ensure students have many opportunities to use a variety of concrete materials.
- Have a mathematics word wall available for students to assist with the correct spelling of number words.
- Provide students with frequent opportunities to represent numbers using words, pictures, and symbols.
- Have students create different expressions for the same numbers, ensuring students understand when describing 3-digit numbers there are more than 99 ones and more than 9 tens.

SUGGESTED LEARNING TASKS

- Have students work in pairs or in small groups. Provide each group with a sheet of chart paper. Ask each group to select a 3-digit number and to represent that number in as many different ways as they can using base-ten blocks. As each base-ten model is created, groups should record it on their chart paper using pictorial representations. Groups should then record the expressions that correspond to each pictorial display. After each group has completed as many different representations for their chosen number as they can, post the chart paper from each group. Have the class examine each chart paper to determine the number represented by the pictorial representations and the expressions. Ask them to explain why all of the pictures and number expressions on a sheet of chart paper are equal.
- Have students create a “thousand” chart by writing the number sequence in 10 blank hundred charts.
- Have students find numbers up to 1000 from different sources, such as newspapers, the Internet, signs, etc., and then ask them to read and model pictorially the numbers they found.
- Ask students to rename a number, less than 1000, as the sum of other numbers.
- Draw a number line labelled 0 and 100 at opposite ends (or 200 and 400, 100 and 600, 100 and 1000, etc.). Mark a few different points on this number line, and ask students what number they think each point might be and why they think that.
- Have students place benchmark numbers on the number line labelled 0 and 1000, for example 250, 500, 750.
- Have students create and solve number riddles, such as, I have written a secret number between 600 and 800. It is an odd number. What might it be?
- Ask students to record a series of numbers that are read to them. Include examples such as “ten less than 652” and numbers that contain a zero.
- Model a number using base-ten materials in an unconventional order and have students say the number.
- Tell students that a number has at least 15 tens and 3 ones. Ask students to decide what number that could be and to explain their thinking.
- Have students work with a partner and record a number with words, exchange with their partner, record that number symbolically, then say that number to their partner.

SUGGESTED MODELS AND MANIPULATIVES

- base-ten blocks
- cards with digits
- counters
- hundred chart
- hundred frame
- linking cubes
- money
- number lines (including walk-on and open number line)
- place-value cards
- place-value chart
- place-value dice

MATHEMATICAL LANGUAGE

Teacher	Student
<ul style="list-style-type: none"> ▪ multiples ▪ number expression ▪ number lines, hundreds charts ▪ number words, symbols, digits ▪ ones, tens, hundreds ▪ represent, partition numbers 	<ul style="list-style-type: none"> ▪ number expression ▪ number lines, hundreds charts ▪ number words, symbols, digits ▪ ones, tens, hundreds ▪ represent, partition numbers

Resources/Notes**Print**

- *Making Math Meaningful to Canadian Students K–8* (Small 2009), pp. 137–144, 146–147
- *Making Math Meaningful to Canadian Students K–8, Second Edition* (Small 2013), pp. 193
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 132–136, 145–148
- *Teaching Student-Centered Mathematics, Grades 3–5* (Van de Walle and Lovin 2006), pp. 39–45, 51–55

Notes

SCO N03 Students will be expected to compare and order numbers to 1000. [CN, R, V]			
[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- N03.01** Place a given set of numbers in ascending or descending order and verify the result using a number chart or other models.
- N03.02** Create as many different 3-digit numerals as possible, given three different digits. Place the numbers in ascending or descending order.
- N03.03** Identify errors in a given ordered sequence.
- N03.04** Identify missing numbers in parts of a given number chart and on a number line.
- N03.05** Identify errors in a given number chart and on a number line.
- N03.06** Place numbers on a number line containing benchmark numbers for the purpose of comparison.
- N03.07** Compare numbers based on a variety of methods, and record the comparison using words and symbols (=, > and <).

Scope and Sequence

Mathematics 2	Mathematics 3	Mathematics 4
N05 Students will be expected to compare and order numbers up to 100.	N03 Students will be expected to compare and order numbers to 1000.	N02 Students will be expected to compare and order numbers to 10 000.

Background

Students should encounter a variety of numbers in context. These contexts help them develop an understanding of number size. In Mathematics 2, students compared and ordered numbers to 100 using benchmarks, number lines, hundred charts, and ten-frames. In Mathematics 3, students will compare numbers to 1000 and will order a set of numbers in ascending and descending order using a variety of methods including number charts, number lines, and place value materials. Visual models encourage reasoning, as students consider how to compare and order numbers. As with all concepts, begin with concrete models before moving to more pictorial and symbolic representations.

Students should recognize that every 2-digit whole number is greater than every 1-digit whole number. Thus, when they compare 2-digit numbers, they should understand that the tens digit is the more vital element of the number; and when they compare two numbers with the same tens digit, they should compare the ones digit. Students should also apply this logic to 3-digit numbers. Given a set of numbers, students should be able to place the numbers in ascending or descending order, and verify the result using a hundreds chart or by drawing a number line.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

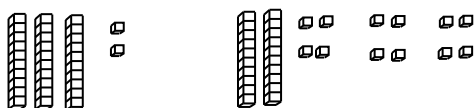
Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Provide students with two representations of the same number. Point to one and ask, Is this number more, less, or are they the same? Have students explain their thinking.



WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask the student to use models to show why 243 is less than 324.
- Ask students to rearrange the digits 1, 4, and 5 to create the number that is closest to 500.
- Ask the student to find a number between 312 and 387 that can be represented using 8 base-ten blocks.
- Ask, What do you do to compare the value of two numbers?
- Show the students two numbers (e.g., 501 and 398) and ask which is greater. Have the students explain their answers. Encourage them to use a variety of models in their explanations.
- Ask, Why are there more numbers greater than 123 than less than 123?
- Ask, If $__39$ is greater than 422, what do you know about $__$ (the missing digit)? If $__39$ is greater than $__87$, what do you know about the missing digits?
- Ask the student to write a number that is
 - greater than 165 but less than 200
 - a little less than 300
 - between 463 and 474
 - greater than 348 but less than 360, etc.
- Ask students to explain why a 3-digit whole number is always greater than a 2-digit whole number. Provide students with specific examples to use in their explanation (e.g., 560 and 56).
- Ask students to select five numbers between 600 and 630, and to write them in increasing order.
- Ask students to make as many numbers as they can using the digits 2, 3, and 4, but using each digit only once. Have them list them in order from least to greatest or greatest to least.
- Provide students with an ordered sequence of numbers that contain an error. Have students identify and correct the error (e.g., 123, 132, 213, 231, 321, 312).

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets 3 (Bauman 2011)

- No Checkpoint for this outcome.

Leaps and Bounds toward Math Understanding 3/4 (Small, Lin, and Kubota-Zarivnij 2011)

- Compare and Order Number

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Have students plot numbers on an open number line (horizontal and vertical) to show their relative positions. Have students share their thinking.
- Give students many opportunities to explore the magnitude of similar digits. For example, How are the digits in 777 similar? How are they different?
- Give students opportunities to build visual representations of numbers. For example, What does 35 look like compared to 353?
- Show students the “greater than” ($>$) and “less than” ($<$) symbols. Tell them that mathematicians use these symbols to replace the words “greater than” and “less than” when comparing numbers. Have students discuss why these symbols may have been chosen.

SUGGESTED LEARNING TASKS

- Ask students to model two different 3-digit numbers using twelve of the base-ten blocks. For example, students might choose to model 642 using 6 flats, 4 rods, and 2 small cubes and 381 using 3 flats, 8 rods, and 1 small cube. After modelling the two numbers, ask students to record the number in symbolic form. Then, ask students to identify which number is greater and to explain their thinking.
- Have students use a prepared deck of 40 numeral cards (4 sets of 0 to 9). Have each student select three of the cards and arrange them to make the greatest possible number and the least possible number. Ask the students to use base-ten blocks to prove they are correct.
- Prepare a deck of cards that contain 2- and 3-digit numerals. Have the students deal all the cards face down to the players. Have each player turn their top card over. The player whose card represents the greater (greatest) numeral wins both or all the cards in play. The winner is the one who has collected the most cards when all the cards have been turned over.
- Provide a set of cards (10 to 15) with each card having a 2- or 3-digit numeral on it. Ask students to order the cards from least to greatest and to explain their thinking using hundreds charts, a number line, or base-ten blocks.
- Have students work in pairs. Provide a set of 10 to 15 cards with each card having a 2- or 3-digit numeral on it. One partner selects 5 cards and secretly decides whether he or she will place them in order from least to greatest or from greatest to least and whether he or she will make an error in the order or not. After arranging the cards according to the decisions made, he or she invites his or her partner to explain how the cards are ordered and whether or not an error has been made. If an error has been made, the partner must correct it and explain his or her thinking.
- As a class activity, repeatedly roll a die and have the students fill in the digits, one at a time, on a place value chart. Alternate by having them try to make the greatest number or the least number. Model the task by placing your digits on an overhead chart or interactive whiteboard. Regularly ask questions such as, What do you need? What don't you want me to roll?
- Play "Guess My Number," in partners, with numbers less than 1000. Use greater than, less than, and is equal to in the response (e.g., Is your number 489? No. My number is greater than that.). Continue the game until the number is guessed, and then change roles and have the other partner guess.
- Give each pair of students two spinners each with 10 numbers that are in the hundreds. Have them spin at the same time. The one who spins the greater number gets a token. The students play until someone has gathered 10 tokens. Select numbers according to the students' level of understanding.
- Provide students with hundreds charts that contain missing numbers. Ask them to fill in the missing numbers and to explain how they know.

SUGGESTED MODELS AND MANIPULATIVES

- base-ten blocks
- calculator
- dice (including place value dice)
- hundred charts
- number cards
- number lines
- open number lines
- place-value charts
- spinners

MATHEMATICAL LANGUAGE

Teacher	Student
<ul style="list-style-type: none"> ▪ benchmark numbers ▪ compare, order, ascending, descending order ▪ hundreds chart, number line ▪ least, greatest ▪ less than, more than, closer to, greater than, ▪ missing numbers, errors 	<ul style="list-style-type: none"> ▪ benchmark numbers ▪ compare, order ▪ hundreds chart, number line ▪ least, greatest ▪ less than, more than, closer to, greater than, ▪ missing numbers, errors

Resources/Notes**Print**

- *Making Math Meaningful to Canadian Students K–8* (Small 2009), pp. 87–89, 143–144, 145
- *Making Math Meaningful to Canadian Students K–8*, Second Edition (Small 2013), pp. 144–145, 196–200
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 142–143
- *Teaching Student-Centered Mathematics, Grades 3–5* (Van de Walle and Lovin 2006), pp. 45–47

Notes

SCO N04 Students will be expected to estimate quantities less than 1000 using referents. [ME, PS, R, V]			
[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- N04.01** Estimate the number of groups of ten in a given quantity using 10 as a referent (known quantity).
- N04.02** Estimate the number of groups of a hundred in a given quantity using 100 as a referent.
- N04.03** Estimate a given quantity by comparing it to a referent.
- N04.04** Select an estimate for a given quantity by choosing among three possible choices.
- N04.05** Select and justify a referent for determining an estimate for a given quantity.

Scope and Sequence

Mathematics 2	Mathematics 3	Mathematics 4
N06 Students will be expected to estimate quantities to 100 using referents.	N04 Students will be expected to estimate quantities less than 1000 using referents.	—

Background

The ability to estimate, a key reasoning skill in mathematics, should develop with regular practise over the course of the year. To develop estimation skills, students should be provided with collections of objects and be asked to estimate the size of the group, using a referent. Estimation helps students develop flexible, intuitive ideas about numbers, further developing number sense.

This outcome should be dealt with in conjunction with outcome N02 as students work toward developing a conceptual understanding of numbers. Making an estimate is often a very difficult task for children. They often do not understand the concept of “about” or “estimate.” Spend a lot of time working with children to help them understand the term **about**. There are many possible estimates for any given estimating situation, and this notion needs to be reinforced with students. Encourage estimation in real-world contexts and emphasize the reasoning underlying the estimate.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

Show students a jar containing about 100 beads. Tell them that one student estimated there were 90 beads in the jar. Another student estimated there were 25 beads in the jar. Ask, Which estimate is closer to the actual number of beads in the jar? Explain your thinking.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Show students a group of items and ask them to choose between three given estimates. Have students explain their reasoning.
- Place a pile of objects on a desk (e.g., paper clips, linking cubes, base-ten units, buttons). Ask students to estimate the number. Observe and interview students to determine if they are using a referent. Guiding questions should include, How did you pick that number? About how many groups of 10 (or 100) are there in the pile?
- Show students a group of objects (e.g., pennies, markers, crayons, stickers, sheets of paper, marbles, etc.) or pictures showing groups (e.g., people in a gym, cars in a parking lot, etc.). Ask students, About how many groups of 10 (or 100) are in the whole group?
- Ask students to describe a strategy used to find an estimate.
- Show students 25 buttons, all buttons touching sides. Say, Susan said, "643 buttons will fit on top of a desk." Do you agree or disagree? Explain.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets 3 (Bauman 2011)

- No Checkpoint for this outcome

Leaps and Bounds toward Math Understanding 3/4 (Small, Lin, and Kubota-Zarivnij 2011)

- No Pathway for this outcome

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Provide students with many opportunities to count groups of 10 and 100 objects in order to develop a sense of these benchmarks. Students should discover that these quantities are not always the same size (e.g., 100 raisins versus 100 oranges).
- Estimate a given quantity by comparing it to a referent (known quantity).
- Estimate the number of groups of 10 and 100 in a given quantity using 10 and 100 as a referent.
- Select between three possible estimates for a given quantity and explain the choice.
- Provide students with opportunities to build referents themselves to determine how many objects are in a specific group.
- Use children’s literature as a context for estimation such as *Counting on Frank* by Rod Clement and *Betcha!* by Stuart Murphy. Discuss how the characters in the stories used estimation.

SUGGESTED LEARNING TASKS

- Show 100 paper clips as a visual referent for the students. Next display a larger group of paper clips. Ask students to estimate how many paper clips there are. Have students explain their thinking.
- Do activity 2.13 of *Teaching Student-Centered Mathematics, Grades 3–5*. Volume Two (Van de Walle and Lovin 2006, 50). Ask students to estimate how many
 - candy bars would cover the floor of your room
 - steps a student would take to walk around the school
 - quarters could be stacked in one stack, floor to ceiling
 - pennies can be laid side by side down an entire room or hallway
 - pieces of notebook paper would cover the gym floor
 - pieces of cereal are in the cereal box

For each scenario, help students identify an appropriate referent and discuss how this referent could be used to determine the total estimate.

- Collect some type of object as a class, with the objective of reaching 1000 (e.g., stickers, pennies, marbles, toy cars, rocks, leaves, buttons).
- Show a quantity of objects such as linking cubes. Ask, If this is 10 linking cubes, what might 143 linking cubes look like?
- Tell students, 100 counters takes up this much space. How much space would 783 counters take up if you placed them flat on the table? If they were in a milk jug? Explain your thinking?
- Have students put some items in a large jar or plastic container. Estimate how many items there are and then count to check. Ask, How far away was your estimate? Change the items in the container each day and repeat the activity. Have students share and discuss their estimation strategies with the class.
- Provide students with different quantities (e.g., 50, 100, 500) of objects of varying sizes and ask them to determine a referent and justify their choice.

SUGGESTED MODELS AND MANIPULATIVES

- variety of containers and objects (e.g., baggies, buckets, beads, marbles, cubes, paper clips, linking cubes, 500-sheet package of photocopy paper)

MATHEMATICAL LANGUAGE

Teacher	Student
<ul style="list-style-type: none"> ▪ about, estimate ▪ estimate quantities ▪ groups of ten, hundred 	<ul style="list-style-type: none"> ▪ about, estimate ▪ groups of ten, hundred

Resources/Notes

Print

- *Making Math Meaningful to Canadian Students K–8*, Second Edition (Small 2013), pp. 144–145, 200
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 58–59, 132
- *Teaching Student-Centered Mathematics, Grades 3–5* (Van de Walle and Lovin 2006), p. 279

Notes

SCO N05 Students will be expected to illustrate, concretely and pictorially, the meaning of place value for numerals to 1000. [C, CN, R, V]			
[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- N05.01** Record, in more than one way, the number represented by given proportional and non-proportional concrete materials in traditional and non-conventional formats.
- N05.02** Represent a given number in different ways using proportional and non-proportional concrete materials and explain how they are equivalent; e.g., 351 can be represented as three 100s, five 10s, and one 1s; or two 100s, fifteen 10s and one 1s; or three 100s, four 10s, and eleven 1s.
- N05.03** Record a given number in additive expanded form.
- N05.04** Record a number represented by base-ten blocks arranged in a non-conventional format.

Scope and Sequence

Mathematics 2	Mathematics 3	Mathematics 4
N07 Students will be expected to illustrate, concretely and pictorially, the meaning of place value for numerals to 100.	N05 Students will be expected to illustrate, concretely and pictorially, the meaning of place value for numerals to 1000.	N01 Students will be expected to represent and partition whole numbers to 10 000 concretely, pictorially, and symbolically.

Background

The development of place-value strategies will help students make sense of larger numbers. Learning to group items in order to make them easier to count is the foundation of place value. When students examine large numbers, they develop a greater sense of the patterning in the place-value system. This exploration will help students to recognize the regularity of the patterns that are inherent in the place-value system. Students should be able to explain that the digits 0–9 are used cyclically to indicate the number of units in any given place. They should also be able to explain the relationship between each place-value position and its neighbour positions, namely a group of ten in one position makes a group of one in the position to the left and a group of one in any position makes a group of ten in the position to the right. Students have used this principle to regroup and trade in previous grades and are now able to state that this pattern continues to work regardless of the size of the number.

Students will need many opportunities to explore the value of the digits in a number using proportional and non-proportional materials. Proper introduction and use of these materials will move student thinking from counting strategies to a deeper understanding of numbers. It is important to understand that students must construct their own understanding of number. This is best accomplished through a variety of materials and through the introduction of those materials as a representation of student thinking.

Students should have had previous experiences with grouping in different ways in preparation for the standard base-ten groupings. Students' initial experience should be with proportional models so that students can see that the piece that represents the 10 is actually 10 times the size of the unit piece.

Initially, students should work with their own personally made proportional materials before moving on to commercially developed materials. Personally made proportional models include toothpicks or small sticks bundled by students into tens and hundreds, beads or buttons strung in tens and hundreds, blocks that can be connected together, and beans glued to wooden stir sticks. Grouping and ungrouping these materials can help prepare students for working with commercially developed materials and to understand the concept of trading 10 little cubes for a rod worth 10. After many experiences with these personally made proportional materials, students should begin to use commercially pre-grouped materials, such as base-ten blocks, ten-frames, and hundreds charts as these models show the magnitude of the number. For example, a base-ten rod is ten times the size of a small cube and 1000 can be represented by ten flats. It is important that this development not be rushed because later problems with number can often be traced back to a poor development of place value.

After extensive work with proportional models, various coloured counters can be used as a non-proportional base-ten model. For example, a red counter can be worth 1, a blue counter can be worth ten times more, and a green counter can be ten times more than the blue. The counters are all the same size but assigned different values.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Ask students to choose a 2-digit number and to record it symbolically. Ask them to represent their 2-digit number using base-ten blocks. Ask them to explain the value of each digit in relation to the base-ten blocks. Then, show the students a 2-digit numeral with both digits the same (e.g., 55). Have students model the value of each digit. Ask students to explain why these digits do not represent the same value.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students to describe 3-digit numbers without using the word **hundred** (e.g., 324 as thirty-two tens, four ones).
- Give students a 3-digit numeral and ask them to represent it with base-ten blocks or other models. Ask students to explain their representation. Ask, Can you represent it another way?
- Ask students to represent numbers with base-ten blocks in different ways and to record each representation using numerals and words (e.g., 132 is 1 hundred, 3 tens, 2 ones; or 13 tens, 2 ones; or 132 ones).
- Ask students to describe 1000 in as many ways as they can. They can use words, materials, pictures, and/or symbols.
- Ask students to explain using words, numbers, and/or pictures how they know that 1000 is the same as 100 tens or that 100 is the same as 10 tens.
- Tell students that pencils can be bought in packages of 1, 10, and 100. Ask students to use stir sticks grouped in 10s, 100s, and 1s to show as many ways as possible to buy 132 pencils.
- Ask students to choose any 3-digit number and tell everything they know about that number.
- Ask how 480 and 680 are the same and how they are different (focus should be on place value).
- Ask how 97 and 907 are the same and how they are different. Ask, Do you think zero (0) is an important number? Why or why not?
- Ask students to select a 3-digit number and to represent that number using base-ten blocks. Then, ask them to represent the number in expanded form.
- Show students a 3-digit number represented in additive expanded form. Ask them to represent that number with base-ten blocks and to record the numeral.
- Ask students to draw pictures of base-ten blocks that would show each of the following numbers: 302, 786, 950, and 878.

FOLLOW-UP ON ASSESSMENT**Guiding Questions**

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets 3 (Bauman 2011)

- No Checkpoint for this outcome

Leaps and Bounds toward Math Understanding 3/4 (Small, Lin, and Kubota-Zarivnij 2011)

- Representing Number

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

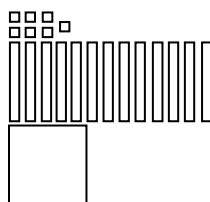
CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Ensure students have opportunities to use both proportional and non-proportional concrete materials. Students should begin working with personally made proportional concrete materials, before working with commercially made proportional concrete materials. Finally, they should work with non-proportional concrete materials.
- Have students represent the same number with different partitions. For example, 254 can be represented using 2 flats and 54 small cubes or 1 flat, 15 rods, and 4 small cubes.
- Provide multiple opportunities for students to show they understand that the position of a digit within a number determines its value.
- Provide students with many experiences modelling numbers with zeros as digits. It is important that students develop a good understanding of the meaning of zero in numbers. For some students, the number 406 looks like 46.

Suggested Learning Tasks

- Provide students with a large collection of objects, such as beans or paper clips. Ask them to group the objects in 10s and 100s and to record the numeral that represents the number of objects in the collection.
- Have students use a number that has all 3-digits the same (e.g., 111). Ask students to use models such as beans, counters, blocks, etc., to explain and show the meaning of each digit.
- Ask students to record the value of the base-ten blocks shown below.



- Ask student to record the number that is made up of 15 tens and 15 ones.
- Ask students to build a model or draw a picture using base-ten blocks. Ask, What is the value of the drawing or model?

- Give each group of students 12 base-ten rods and 16 units. Have them record the numeral the blocks represent.
- Model numbers such as 421 and 139. Discuss which number has more tens and how they know. Students should recognize that 421 has more tens, although it has a smaller digit in the tens place.
- Ask the students to enter a certain number on a calculator (e.g., 235). Ask, How can you, without clearing the calculator, make the number 255? (35? 205? 261?)
- Ask students to record a specific 3-digit number, with all digits different. Ask students to remove the value of one of the digits or the value of one of the places with only one operation. For example, to remove the value of the 3 from the number 734, the student would need to subtract 30.
- Have students create non-proportional concrete models and explain their values.

SUGGESTED MODELS AND MANIPULATIVES

- base-ten blocks
- calculators
- counters
- digit cards
- hundred chart
- hundred frame
- money
- number lines
- place-value chart

MATHEMATICAL LANGUAGE

Teacher	Student
<ul style="list-style-type: none"> ▪ ones, tens, hundreds ▪ place value ▪ small cubes, rods, flats 	<ul style="list-style-type: none"> ▪ ones, tens, hundreds ▪ small cubes, rods, flats

Resources/Notes

Print

- *Making Math Meaningful to Canadian Students K–8* (Small 2009), pp. 138–144, 146–148
- *Making Math Meaningful to Canadian Students, K–8, Second Edition* (Small 2013), pp. 193–202
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 127–134, 136–137, 145–148

Videos

- *Teaching Place Value 20–99* (29:08 min) (ORIGO Education 2010)

Notes

SCO N06 Students will be expected to describe and apply mental mathematics strategies for adding two 2-digit numerals.

[C, ME, PS, R, V]

[C] Communication

[PS] Problem Solving

[CN] Connections

[ME] Mental Mathematics and Estimation

[T] Technology

[V] Visualization

[R] Reasoning

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N06.01 Explain mental mathematics strategies that could be used to determine a sum.

- Ten and some more
- Tens and some more
- Quick addition
- Addition facts to 10 applied to multiples of 10
- Addition on the hundred chart
- Adding on
- Make ten
- Compensation
- Compatible numbers

N06.02 Use and describe a personal strategy for determining a sum.

N06.03 Determine a sum of two 2-digit numerals efficiently, using mental mathematics strategies.

Scope and Sequence

Mathematics 2	Mathematics 3	Mathematics 4
<p>N10 Students will be expected to apply mental mathematics strategies to quickly recall basic addition facts to 18 and determine related subtraction facts.</p>	<p>N06 Students will be expected to describe and apply mental mathematics strategies for adding two 2-digit numerals.</p>	<p>N03 Students will be expected to demonstrate an understanding of addition of numbers with answers to 10000 and subtraction (limited to 3- and 4-digit numerals) by</p> <ul style="list-style-type: none"> ▪ using personal strategies for adding and subtracting ▪ estimating sums and differences ▪ solving problems involving addition and subtraction

Background

When a problem requires an exact answer, students' first consideration should be whether or not they can calculate it mentally. The development of mental mathematics needs to be a major goal of any mathematics program for two major reasons. First, in their day-to-day activities, most people's computational, measurement, and spatial needs can be met by having well-developed mental mathematics strategies. Secondly, because technology has replaced paper-and-pencil as the major tool for complex tasks, people need to have well-developed mental mathematics strategies to be alert to the reasonableness of the results generated by this technology.

Mental calculation refers to getting exact answers by using strategies to do the calculations in one's head. In general, a strategy should be introduced in isolation from other strategies, a variety of different reinforcement activities should be provided until it is mastered, the strategy should be assessed in a variety of ways, and then it should be combined with other previously learned strategies.

Note: The addition facts discussed in SCO N10 should be addressed prior to working on these mental mathematics strategies.

Situations must be regularly provided to ensure that students have sufficient practice with mental mathematics strategies and that they use their skills as required. Using mental mathematics will allow a student to focus on the relationships between numbers and operations rather than relying on completing a traditional algorithm. For example, students may solve $49 + 99$ mentally by adding 100 to 49, then subtracting 1. This method involves using benchmark numbers then compensating by adding or subtracting, whichever operation is necessary.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Ask students to describe in as many ways as possible why $8 + 7 = 15$.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask, How does knowing $8 + 8 = 16$, help you solve $58 + 8$?
- Ask the student to add mentally as you draw numbers from a bag and to stop you when the sum is greater than 40. Have the student explain their thinking process to reach the sum of 40 or more.
- Ask students to explain how many different ways they can add $49 + 34$ in their heads. Ask them to explain which strategy is easier to use.
- Tell students that when adding 9 and 57, Jon said that he would rather add 10 to 57 and subtract 1. Ask them if this works and why.
- Ask students to describe a strategy for solving $76 + 11$ (or other 2-digit addition questions) mentally using models, numbers, words, or pictures.

- Provide students with a sheet of 10 addition practice items, such as
 - $33 + 12$
 - $71 + 24$
 - $98 + 42$
 - $56 + 34$
 - $25 + 65$
 - $87 + 13$
 - $15 + 75$
 - $44 + 52$
 - $76 + 19$
 - $82 + 17$Ask them to circle all the questions they can solve mentally. For the questions they've circled, ask them to explain the strategy they used.
- Ask the student to describe a strategy for solving $68 + 39$ mentally using models, numbers, words, or pictures.
- Ask students how many different ways they can mentally subtract 19 from 43? Which way was easiest?

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets 3 (Bauman 2011)

- Checkpoint 7, pp. 40–41

Leaps and Bounds toward Math Understanding 3/4 (Small, Lin, and Kubota-Zarivnij 2011)

- Mental Math

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Ensure students recognize that mental mathematics is an approach that they should use every time they are required to calculate.
- Require that students recall addition and subtraction facts to 18 with automaticity (see SCO N10).
- As a class, share, discuss, and explore strategies used by individual students. This allows for exposure to a variety of strategies for students to choose ones that make sense to them and are more efficient.
- Review “making a ten” with students. For example, for $28 + 4$, one might think 28 and 2 make 30, and 2 more is 32. This can be extended to the addition of 2-digit numbers. For example, for $38 + 24$, 38 plus 20 is 58, and 2 more is 60, plus 2 is 62.
- Have students add two 2-digit numbers, and explain their thinking. For example, $24 + 31$. Students might say, “20 and 30 makes 50, 4 more makes 54, and 1 more makes 55—the answer is 55.”
- Have students explain their thinking using number lines (e.g., to solve $28 + 37$, think $20 + 30 + 15$).
- Have students use the hundred chart to solve a variety of 2-digit addition problems.
- Have students use two metre sticks and place one under the other so that the numbers are the reverse of the one on the top. This model can be used to explore compatible numbers.

SUGGESTED LEARNING TASKS

- Present calculations such as in the following example, orally (or on an overhead), and ask the student to write only the answer (e.g., $30 + 60$; $20 + 40$; $20 + 80$). They should be able to do this quickly.
- Provide a set of computation practice items and ask students to circle the questions that they could solve mentally and describe the strategy they would use.
- Have students explain how they would use mental mathematics strategies to solve 2-digit addition questions, such as $34 + \square = 69$ or $39 + \square = 64$.
- Have the student make a list of calculations involving 2-digit numbers that would be quicker to do mentally than to do using paper and pencil or a calculator.
- Have students explain how changing the addend will affect the answer (e.g., if $31 + 48$ is changed to $31 + 50$, the sum would be two more than the actual sum).
- Have the student list the doubles facts that might help him or her solve expressions such as $88 + 89$ and $49 + 51$ or $39 + 38$.
- Present students with a variety of mental mathematics questions, such as
 $21 + 43$
 $37 + 59$
 $63 + 41$
 $74 + 46$

Have students share different ways to solve each question.

To solve $21 + 43$ students might

- start with the tens ($20 + 40 = 60$), then add the ones ($1 + 3 = 4$) and then add those sums together ($60 + 4 = 64$)
- start with 43, add on 20 to get 63, and then add on 1 to get 64
- start with 21, add on 40 to get 61, and then add on 3 to get 64

SUGGESTED MODELS AND MANIPULATIVES

- calculator
- hundred chart
- numeral cards
- open number line

MATHEMATICAL LANGUAGE

Teacher	Student
<ul style="list-style-type: none"> ▪ 1-digit, 2-digit ▪ addition facts to 10 applied to multiples of 10: small cubes, rods ▪ addition on the hundred chart ▪ mental math ▪ quick addition: tens ▪ strategy ▪ ten and some more ▪ tens and some more 	<ul style="list-style-type: none"> ▪ 1-digit, 2-digit ▪ small cubes, rods ▪ mental math ▪ tens ▪ strategy ▪ ten and some more

Resources/Notes**Print**

- *Making Math Meaningful to Canadian Students K–8* (Small 2009), pp. 33, 162
- *Making Math Meaningful to Canadian Students K–8, Second Edition* (Small 2013), pp. 29–30, 218
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 51–52, 158, 160–161
- *Teaching Student-Centered Mathematics, Grades 3–5* (Van de Walle and Lovin 2006), p. 44

Videos

- *Comparing Mental Strategies: Addition* (14:42 min.) (ORIGO Education 2010)
- *Powerful Models to Help Struggling Students: Number Lines* (17:37 min.) (ORIGO Education 2010)
- *Powerful Strategies to Help Struggling Students: Bridge to Ten* (13:23 min.) (ORIGO Education 2010)
- *Questions for Developing Mental Computation Strategies* (13:42 min.) (ORIGO Education 2010)
- *Using a Hands-on Approach to Develop Mental Strategies for Addition* (11:04 min.) (ORIGO Education 2010)
- *Using Mental Strategies to Add* (26:15 min.) (ORIGO Education 2010)

Notes

SCO N07 Students will be expected to describe and apply mental mathematics strategies for subtracting two 2-digit numerals.

[C, ME, PS, R, V]

[C] Communication

[PS] Problem Solving

[CN] Connections

[ME] Mental Mathematics and Estimation

[T] Technology

[V] Visualization

[R] Reasoning

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N07.01 Explain mental mathematics strategies that could be used to determine a difference.

- Facts with minuends of 10 or less applied to multiples of 10
- Quick subtraction
- Subtraction on the hundred chart
- Compensation
- Back through ten

N07.02 Use and describe a personal strategy for determining a difference.

N07.03 Determine a difference of two 2-digit numerals efficiently, using mental mathematics strategies.

Scope and Sequence

Mathematics 2	Mathematics 3	Mathematics 4
<p>N10 Students will be expected to apply mental mathematics strategies to quickly recall basic addition facts to 18 and determine related subtraction facts.</p>	<p>N07 Students will be expected to describe and apply mental mathematics strategies for subtracting two 2-digit numerals</p>	<p>N03 Students will be expected to demonstrate an understanding of addition of numbers with answers to 10000 and subtraction (limited to 3- and 4-digit numerals) by</p> <ul style="list-style-type: none"> ▪ using personal strategies for adding and subtracting ▪ estimating sums and differences ▪ solving problems involving addition and subtraction

Background

When a problem requires an exact answer, students' first consideration should be whether or not they can calculate it mentally. Situations must be regularly provided to ensure that students have sufficient practice with mental mathematics strategies and that they use their skills as required. Using mental mathematics will allow a student to focus on the relationships between numbers and operations rather than relying on completing a traditional algorithm. Most mental mathematics strategies provide a more efficient computation method as opposed to pencil and paper. Students should be presented with the horizontal form of an equation to encourage the use of mental mathematics.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

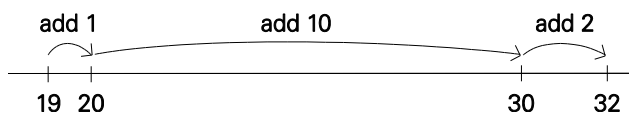
Tasks such as the following could be used to determine students' prior knowledge.

- Ask students to describe in as many ways as possible why $15 - 8 = 7$.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Tell students that when subtracting 7 from 51, Jon said that he would rather subtract 6 from 50. Ask them if this works and why.
- Ask students to describe a strategy for solving $76 - 11$ mentally using models, numbers, words, or pictures.
- Ask students to describe a strategy for solving $68 - 39$ mentally using models, numbers, words, or pictures.
- Have students explain what is wrong with Lisa's method for solving $45 - 26$. Lisa said, " $45 - 25 = 20$ and $20 + 1 = 21$. The answer is 21."
- Ask students to explain different ways that someone could subtract 19 from 43 in their head? Ask them to identify which way was easiest.
- To solve $47 - 29$, Beth said, "47, 27, 20, 18. The answer is 18." Explain the strategy Beth used to solve the problem.
- To solve $32 - 19$, Jonah showed the following jumps on a number line and said the answer was 13.



Ask students to explain how Jonah solved the problem.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets 3 (Bauman 2011)

- Checkpoint 7, pp. 40–41

Leaps and Bounds toward Math Understanding 3/4 (Small, Lin, and Kubota-Zarivnij 2011)

- Mental Math

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Ensure students recognize that mental mathematics is an approach that they should use every time they are required to calculate.
- Require that students recall addition and subtraction facts to 18 with automaticity (see SCO N10).
- As a class, share, discuss, and explore strategies used by individual students. This allows for exposure to a variety of strategies for students to choose ones that make sense to them and are more efficient.
- Have students subtract using quick subtraction and explain their thinking.
- Relate addition to subtraction regularly so students are better able to use this understanding to solve addition and subtraction problems and to check their work. Use missing addends to encourage this.
- Have students model subtraction and explain their thinking using number lines, hundred charts, and base-ten blocks.

SUGGESTED LEARNING TASKS

- Provide a set of computation practice items and ask students to circle the questions that they could solve mentally and describe the strategy they would use.

- Present students with a variety of mental mathematics questions, such as
 - 43 – 21
 - 59 – 37
 - 92 – 73
 - 74 – 46
 Have students share different ways to solve each question.
- Present calculations, such as in the following example, orally (or on an overhead), and ask the student to write only the answer (e.g., $60 - 30$, $40 - 20$, $80 - 30$). They should be able to do this quickly. Ask students to share their strategies.
- Have students make a list of calculations involving 2-digit numbers that would be quicker to do mentally than using paper and pencil or with a calculator.
- Have students explain how changing the subtrahend will affect the answer (e.g., for $100 - 48$ to $100 - 50$, the difference would be two more).
- Have students explain how they would use mental mathematics strategies to solve 2-digit subtraction questions, such as $34 - \square = 19$ or $69 - \square = 54$.

SUGGESTED MODELS AND MANIPULATIVES

- calculator
- number cards
- open number line

MATHEMATICAL LANGUAGE

Teacher	Student
<ul style="list-style-type: none"> ▪ 1-digit, 2-digit ▪ hundred chart ▪ mental mathematics ▪ quick subtraction: tens, minus ▪ small cubes, rods ▪ strategy 	<ul style="list-style-type: none"> ▪ 1-digit, 2-digit ▪ hundred chart ▪ mental mathematics ▪ tens, minus ▪ small cubes, rods ▪ strategy

Resources/Notes

Print

- *Making Math Meaningful to Canadian Students K–8* (Small 2009), pp. 33, 162
- *Making Math Meaningful to Canadian Students K–8, Second Edition* (Small 2013), pp. 29–30, 218
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 51–52, 158, 160–161
- *Teaching Student-Centered Mathematics, Grades 3–5* (Van de Walle and Lovin 2006), p. 44

Videos

- *Powerful Models to Help Struggling Students: Number Lines* (17:37 min.) (ORIGO Education 2010)
- *Questions for Developing Mental Computation Strategies* (13:42 min.) (ORIGO Education 2010)
- *Using a Hands-On Approach to Develop Mental Strategies for Subtraction* (6:45 min.) (ORIGO Education 2010)

Notes

SCO N08 Students will be expected to apply estimation strategies to predict sums and differences of 1-, 2-, or 3-digit numerals in a problem-solving context.

[C, ME, PS, R]

[C] Communication

[PS] Problem Solving

[CN] Connections

[ME] Mental Mathematics and Estimation

[T] Technology

[V] Visualization

[R] Reasoning

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N08.01 Explain estimation strategies that could be used to determine an approximate sum or difference.

N08.02 Use and describe a strategy for determining an estimate.

N08.03 Estimate the solution for a given story problem involving the sum or difference of up to two 3-digit numerals.

Scope and Sequence

Mathematics 2	Mathematics 3	Mathematics 4
—	N08 Students will be expected to apply estimation strategies to predict sums and differences of 1-, 2-, or 3-digit numerals in a problem-solving context.	N03 Students will be expected to demonstrate an understanding of addition of numbers with answers to 10000 and subtraction (limited to 3- and 4-digit numerals) by <ul style="list-style-type: none"> ▪ using personal strategies for adding and subtracting ▪ estimating sums and differences ▪ solving problems involving addition and subtraction

Background

Estimating is a critical skill in today's world. For most people in their daily lives, an estimate is all that is needed to make decisions, and to think about the reasonableness of numerical claims and answers generated by others. In the technology-rich society, it is essential to encourage student thinking and reasoning skills based on estimation, rather than the acceptance of answers generated by computers or calculators.

Before attempting pencil-and-paper or calculator computations, students must find estimates, so they think about the reasonableness of those pencil-and-paper or calculator answers. Always model estimating before personally doing any calculations in front of the class, and constantly remind students to estimate before calculating. After the calculation, students can use their estimate to check their solutions, asking themselves, Does my answer make sense? Help students develop this skill by frequently asking questions such as, Is your answer reasonable? How do you know?

While teaching estimation strategies, it is important to use the language of estimation. Some of the common words and phrases are **about**, **approximately**, **between**, **a little more than**, **a little less than**, **close**, **close to**, and **near**.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Ask students to tell whether 44 is closer to 40 or 50 and to explain how they know.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students to identify situations in which an exact answer would be required and some in which an estimate is sufficient.
- Ask students to explain two different ways to estimate the difference for $54 - 26$.
- Ask, Do you think that 48 might ever be used as an estimate for a sum or difference? Explain your answer.
- Ask students to explain why a good estimate for a subtraction might be greater than the actual answer sometimes, but less other times. Encourage students to use examples to help them explain.
- Tell students that Jason knew there were 35 members in his Karate Club and about 28 in the club in the neighbouring town. When asked to estimate the number of name tags to make for members of both clubs, Jason said, "I think I should make 65." Ask, How do you think Jason estimated? Was it a good estimate?
- Have students toss two dice and create a 2-digit number. Ask them to estimate how much should be added to the number to get a sum of about 200 or ask them to tell how much could be subtracted to get a difference of about 10?
- Tell the student that $4\square + \square 8$ is about 70. Ask what digits might go in the blanks.
- Ask students to identify which of the following questions would have an answer close to 150 and to explain their thinking.
 - $92 + 37$
 - $69 + 82$
 - $77 + 87$

- Show students the number of sports cards in James' collection. Baseball: 48, Football: 19, Hockey: 84. Ask students to estimate the total number of cards in the collection and to describe the strategy they used.
- Tell students that Marc wants to buy a new bike that costs \$135. He has saved \$48. About how much more will he need to save? Ask students to explain how they solved the problem.
- Tell students that a number between 30 and 40 is added to a number between 40 and 50. Ask, What might be a good estimate for the answer? Why?

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets 3 (Bauman 2011)

- No Checkpoint for this outcome.

Leaps and Bounds toward Math Understanding 3/4 (Small, Lin, and Kubota-Zarivnij 2011)

- No Pathway for this outcome.

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Ensure students recognize that estimation should be used every time they are required to solve a problem, make predictions, or check answers.
- Use left-to-right or front-end method. The following is an example:
 - $138 + 149$ ($130 + 140$ is 270, $8 + 9$ is close to 20, for an estimate of 290)
- Use ten-frames for relatively small numbers. Displaying 23 on ten-frames, for example, clearly shows that 23 is closer to 20 than to 30.

- Use base-ten blocks or a hundred chart to help students as they begin estimating with larger numbers. For example, using base-ten blocks will help a student see that 37 (3 rods and 7 ones) is closer to 4 rods than to 3 rods. Eventually, students should realize that estimating can easily be performed without the base-ten blocks.
- Use quarters (25¢) as a model to think of multiples of 25. Have students consider multiples of 25 when estimating numbers.

SUGGESTED LEARNING TASKS

- Have students use estimation in story problem situations, such as
 - Tali baked 49 whole wheat rolls and Miranda baked 158. Do they have enough to feed two hundred parents coming to Mathematics Night?
 - Play “A Fast Ten” with students. Students turn over two playing cards (a deck of cards numbered 1–9 only) to build a 2-digit number. The student who determines to which multiple of ten that number is closest gets the cards. This game could be extended to add or subtract estimates of two pairs of cards.
 - Tell the student that the sum of two numbers has been estimated to be about 120. Ask the student to list four possible pairs of numbers that might have been added.
 - Have students explore open-ended questions, such as, The difference between two numbers was estimated to be 50. What could the numbers have been? The sum of two numbers was estimated to be 700. What could the two numbers have been?
 - Have students explain whether an estimate is reasonable. For example, tell students that Juan estimated the difference for $689 - 276$ to be 500. Ask whether that estimate is reasonable or not.

SUGGESTED MODELS AND MANIPULATIVES

- base-ten blocks
- metre sticks
- money
- number lines
- ten-frames

MATHEMATICAL LANGUAGE

Teacher	Student
<ul style="list-style-type: none"> ▪ 1-digit, 2-digit, 3-digit ▪ about, between, a little more than, a little less than, close, close to, and near. ▪ estimate ▪ front-end estimation ▪ front-end estimation adjusted 	<ul style="list-style-type: none"> ▪ 1-digit, 2-digit, 3-digit ▪ about, between, a little more than, a little less than, close, close to, near ▪ estimate

Resources/Notes

Print

- *Making Math Meaningful to Canadian Students K–8* (Small 2009), pp. 160–161, 173
- *Making Math Meaningful to Canadian Students, K–8, Second Edition* (Small 2013), pp. 29–30, 218
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 144–145

Notes

<p>SCO N09 Students will be expected to demonstrate an understanding of addition and subtraction of numbers (limited to 1-, 2-, and 3-digit numerals) with answers to 1000 by</p> <ul style="list-style-type: none"> using personal strategies for adding and subtracting with and without the support of manipulatives creating and solving problems in context that involve addition and subtraction of numbers concretely, pictorially, and symbolically. <p>[C, CN, ME, PS, R]</p>			
[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- N09.01** Model the addition of two or more given numbers using concrete or visual representations and record the process symbolically.
- N09.02** Model the subtraction of two given numbers using concrete or visual representations and record the process symbolically.
- N09.03** Create an addition or subtraction story problem for a given solution.
- N09.04** Determine the sum of two given numbers using a personal strategy, e.g., for $326 + 48$, record $300 + 60 + 14$.
- N09.05** Determine the difference of two given numbers using a personal strategy, e.g., for $127 - 38$, record $2 + 80 + 7$ or $127 - 20 - 10 - 8$.
- N09.06** Solve a given problem involving the sum or difference of two given numbers.

Scope and Sequence

Mathematics 2	Mathematics 3	Mathematics 4
<p>N09 Students will be expected to demonstrate an understanding of addition (limited to 1- and 2-digit numerals) with answers to 100 and the corresponding subtraction by</p> <ul style="list-style-type: none"> using personal strategies for adding and subtracting with and without the support of manipulatives creating and solving problems that involve addition and subtraction explaining and demonstrating that the order in which numbers are added does not affect the sum explaining and demonstrating that the order in which numbers are subtracted matters when finding a difference. 	<p>N09 Students will be expected to demonstrate an understanding of addition and subtraction of numbers with answers to 1000 (limited to 1-, 2-, and 3-digit numerals) by</p> <ul style="list-style-type: none"> using personal strategies for adding and subtracting with and without the support of manipulatives creating and solving problems in context that involve addition and subtraction of numbers concretely, pictorially, and symbolically. 	<p>N03 Students will be expected to demonstrate an understanding of addition of numbers with answers to 10000 and subtraction (limited to 3- and 4-digit numerals) by</p> <ul style="list-style-type: none"> using personal strategies for adding and subtracting estimating sums and differences solving problems involving addition and subtraction

Background

This outcome involves the development of two critical abilities—the ability to solve the full range of addition and subtraction story problems efficiently, and the ability to add and subtract up to 3-digit numbers efficiently. For the most part, these two abilities should be taught simultaneously; however, there will be times when some lessons should focus on one or the other.

Students should be presented with addition and subtraction story problems of all structures.

- Join (result, change, and start unknown)
- Separate (result, change, and start unknown)
- Part-part-whole (part and whole unknown)
- Compare (difference, smaller, and larger unknown).

Join story problems all have an action that causes an increase, while separate story problems have an action that causes a decrease. Part-part-whole story problems, on the other hand, do not involve any actions, and compare story problems involve relationships between quantities rather than actions.

Examples of these various types of problems appear in the table provided below.

Join			Part-Part-Whole	Compare
Result Unknown	Change Unknown	Start Unknown	Whole Unknown	Difference Unknown
Mike earned \$328 last year selling newspapers. This year he earned \$415. How much money did he earn in all? $328 + 415 = ?$	Last week Katie picked 115 kg of blueberries. She picked some more blueberries this week giving her a total of 236 kg. How many kilograms of blueberries did she pick this week? $115 + ? = 236$ or $236 - 115 = ?$	The grade 4 class is fund raising for a community centre. A donor just gave them \$563 and now they have \$998. How much money did they have before the donation? $? + 563 = 998$ or $998 - 563 = ?$	There are 317 boys and 248 girls in a school. How many students are in the school? $317 + 248 = ?$	Mary sold 278 greeting cards for the school fund raiser. Chantella sold 195. How many more greeting cards did Mary sell than Chantella sold? $195 + ? = 278$ or $278 - 195 = ?$
Separate			Part-Part-Whole	Compare
Result Unknown	Change Unknown	Start Unknown	Part Unknown	Smaller or Larger Unknown
Gavin collected 239 toy cars in his bucket. He gave his brother 103 of those toy cars. How many toy cars does he have left? $239 - 103 = ?$	Kayla had 156 g of sugar. She used some to make cookies and has 83 g left. How much sugar did she use? $156 - ? = 83$ or $156 - 83 = ?$	A company had some books to donate to schools. They gave the first school 256 of them. They still have 517 books to give away. How many books did they have to begin with? $? - 256 = 517$ or $256 + 517 = ?$	There were 735 people at the community dinner. If 352 of them were children, how many were adults? $352 + ? = 735$ or $735 - 352 = ?$	Joe collected 387 bottles for the recycling project. Sue collected 185 more bottles than Joe. How many bottles did Sue collect? $387 + 185 = ?$ If Joe collected 74 more than Dan, how many did Dan collect? $387 - 74 = ?$

In the table, there are number sentences that students may generate depending upon how they think about the problem. For example, consider the Join (change unknown) problem: Last week Katie picked 115 kg of blueberries. She picked some more blueberries this week giving her a total of 236 kg. How

many kilograms of blueberries did she pick this week? If students solved this by starting with 115, adding on until they reached 236, and determining what they added on, they would represent the problem in symbols as $115 + ? = 236$. On the other hand, if students solved it by starting with 236, removing 115, and determining what was left, they would represent the problem as $236 - 115 = ?$ Students should be encouraged to model the story problems with base-ten blocks and write number sentences that reflect their thinking. All these story problems can also be modelled using a variety of pictorial representations including student-generated pictures, those described on page 67 of *Teaching Student-Centered Mathematics, Grades K–3* by John Van de Walle and LouAnn Lovin, or strip diagrams as described in Appendix A. It is important that the pictures students draw represent their thinking and should mirror their work with models.

When students have to compute sums and differences involving numbers with up to 3-digits, they should use strategies that are reliable, accurate, and efficient. Through the sharing of strategies, students will be exposed to a variety of possible addition and subtraction strategies, and each student will adopt ones that he or she understands well and has made his or her own. That is why these strategies are often referred to as “personal strategies.” The most appropriate strategy used may vary depending on the student and the numbers involved in the problem.

While some of these strategies may have emerged directly from students work with base-ten blocks, other strategies should be modelled by students using the base-ten blocks to help understand the logic behind them. Research has shown that children who create personal strategies almost universally start by adding from the left. Many students in Mathematics 3 are not developmentally ready to work with groups of 10 and groups of 100 with deep understanding; therefore, these students are more likely to understand alternative strategies that highlight the actual quantities represented by the digits in the tens and hundreds places.

The paper-and-pencil recording of students’ personal strategies should reflect their thinking and must be reliable, accurate, and efficient. The symbolic recording used need not be the standard algorithm. Regardless of the strategy used, the teacher must monitor each student’s symbolic recording of the strategy to ensure that the recording is mathematically correct, organized, and efficient.

Two examples of strategies and symbolic recordings are shown below. Additional examples are provided in Appendix A.

If students are asked to add 237 and 478, students could determine the sum by

- Start by writing 237 as $200 + 30 + 7$ and 478 as $400 + 70 + 8$.
- Add 200 and 400 to get a sum of 600.
- Add 30 and 70 to get a sum of 100.
- Add 7 and 8 to get a sum of 15.
- Add 600, 100, and 15 to get a sum of 715.

This may be recorded on paper as

$237 + 478 = 200 + 30 + 7 + 400 + 70 + 8$	or	237
$200 + 400 = 600$		$+ 478$
$30 + 70 = 100$		<u>600</u>
$7 + 8 = 15$		100
$600 + 100 + 15 = 715$		<u>+ 15</u>
		715

If we introduce subtraction using word problems, students can begin modelling their solutions. Consider the following problem: On our vacation, we went to visit our aunt in Fredericton. We drove 239 km and stopped for lunch. If the distance to our aunt’s house is 526 km, how much further do we have to drive?

Students could explain and record their solution as shown below:

- We knew we had to subtract 239 from 526. So we started with 5 flats, 2 rods, and 6 small cubes to show 526. We removed 2 flats. Then, we had to remove 3 rods, so we changed one flat to 10 rods. Finally we removed 9 small cubes, after we traded 1 rod for 10 small cubes. This could be recorded on paper as
 $526 - 239 = ?$
 $526 - 200 = 326$
 $326 - 30 = 296$
 $296 - 9 = 287$
We have to travel 287 km more.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students’ prior knowledge.

- Give students an addition or a subtraction number sentence involving 1- and 2-digit numbers. Ask them to create a story problem that would be solved using the number sentence. Ask them to model the story problem using concrete materials or pictures.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Tell a student that someone told you that you do not have to learn to subtract if you know how to add. Ask, Do you agree? Why or why not?

- Observe the student as he or she adds 125 and 134 or subtracts 134 from 217 using base-ten blocks or an open number line.
- Provide the following addition or subtraction calculations for the student to complete. Ask the student to explain and record symbolically his or her strategy.
38 + 97
98 – 44
400 – 255
- Show the student a number of addition and subtraction questions, some of which require regrouping and some of which do not. Ask him or her to circle the questions they could do quickly and explain why they made those choices.
- Have the student explain in writing why someone might first subtract 30 from 74 in order to calculate $674 - 26$. Ask what would be done next.
- Display the numbers 124 and 75 with base-ten blocks. Ask the student to describe the addition process as he or she manipulates the models.
- Tell the student that Sue had to add $36 + 59$ and said, “36, 96, 95.” Have the student explain Sue’s thinking.
- Ask why someone might find it easier to subtract $123 - 99$ than $123 - 87$.
- Ask the student to prepare a display showing a variety of ways to calculate $287 + 162$ indicating his or her preference and the reason for it.
- Ask the students to use a sales flyer to create some problems for his or her classmates. Have them record both problems and solutions.
- Using the numbers 811 and 543 create a subtraction problem that can be solved using addition. Then, ask students to solve the problem.
- Ask, How does knowing $13 - 6 = 7$, help you solve $153 - 6$?
- Ask students to add 125 and 78 and describe the process using an open number line.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets 3 (Bauman 2011)

- No Checkpoint for this outcome.

Leaps and Bounds toward Math Understanding 3/4 (Small, Lin, and Kubota-Zarivnij 2011)

- Addition
- Subtraction

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Provide examples of situations in which students will have to devise some method for regrouping. For example, tell them that one student found that there were 155 M & M'S in one bag and 258 in another. Ask them to determine how many there were in the two bags. Ask them to model the question and to explain how they solved it.
- Create a sheet with 10 ten-frames to represent 100. Provide each student with two sheets and have him or her show each of the two addends (e.g., $67 + 76$). This will help students visualize how the numbers could be combined (e.g., $60 + 70$, $7 + 6$). Students may need to explore moving the amounts to be on the same sheet if they are having trouble finding the sum.
- Examine number patterns to help students understand the connection between addition and subtraction facts and 2-digit plus 2-digit and 3-digit plus 3-digit addition and subtraction. For example, $6 + 7 = 13$, so $60 + 70 = 130$ and $600 + 700 = 1300$ and $13 \text{ tens} \text{ minus } 6 \text{ tens} = 7 \text{ tens}$.
- Have students model their thinking on an open number line.

SUGGESTED LEARNING TASKS

- Tell students that Fran had 187 stickers. She gave 59 of them to her friend. Ask students to explain how they go about solving the problem. Providing a model for the students, such as a number line, may be helpful.
- Set up a “store” within the classroom and have the students take turns being the cashier. Model for them how to “count on” when making change.
- Create sheets or overhead transparencies containing completely and partially filled ten-frames representing one part of a target number. Students apply strategies that make sense to them to determine the missing part. Since this is essentially a subtractive problem, many students will use “think addition” to work their way up to the target number.
- Use the following digits to create two, 2-digit numbers that have the greatest possible sum: 2, 3, 4, 5. Use the same digits to create the greatest difference.
- Have pairs of students roll place value dice to create pairs of 2- and 3-digit numbers and find the sum or difference. As a variation, give students a “target number” (e.g., 100) that they try to reach by adding or subtracting the numbers they create with the dice.

SUGGESTED MODELS AND MANIPULATIVES

- | | |
|-------------------|--------------------|
| ▪ base-ten blocks | ▪ money |
| ▪ calculator | ▪ open number line |
| ▪ dice | ▪ place-value dice |
| ▪ hundred chart | ▪ place-value mats |
| ▪ hundred frames | ▪ ten-frames |

MATHEMATICAL LANGUAGE

Teacher	Student
<ul style="list-style-type: none"> ▪ addition, subtraction ▪ number sentence ▪ small cubes, rods, flats ▪ strip diagram: part, whole ▪ sum, difference ▪ trading, grouping ▪ unknown 	<ul style="list-style-type: none"> ▪ addition, subtraction ▪ number sentence ▪ small cubes, rods, flats ▪ strip diagram: part, whole ▪ sum, difference ▪ trading, grouping ▪ unknown

Resources/Notes**Print**

- *Making Math Meaningful to Canadian Students K–8* (Small 2009), pp. 104–110, 162–172
- *Making Math Meaningful to Canadian Students K–8*, Second Edition (Small 2013), pp. 160–166, 215–217
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 66–70, 70–75, 157–172
- *Teaching Student-Centered Mathematics, Grades 3–5* (Van de Walle and Lovin 2006), pp. 53, 108–113

Videos

- Using Language Stages to Develop Addition Concepts (15:38 min.) (ORIGO Education 2010)
- Using Language Stages to Develop Subtraction Concepts (18:32 min.) (ORIGO Education 2010)
- Using Static Problems to Relate Addition and Subtraction and Introduce Equality (13:25 min.) (ORIGO Education 2010)
- Using Static Problems to Relate Addition and Subtraction and Introduce Functions (18:59 min.) (ORIGO Education 2010)

Notes

SCO N10 Students will be expected to apply mental mathematics strategies and number properties, to develop quick recall of basic addition facts to 18 and related basic subtraction facts.

[C, CN, ME, R, V]

[C] Communication

[PS] Problem Solving

[CN] Connections

[ME] Mental Mathematics and Estimation

[T] Technology

[V] Visualization

[R] Reasoning

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- N10.01** Describe a mental mathematics strategy that could be used to determine a given basic addition fact up to $9 + 9$.
- N10.02** Explain how the commutative (order-doesn't-matter) property and the identity (no-change-with-zero) property can assist in addition fact learning.
- N10.03** Describe a mental mathematics strategy that could be used to determine a given basic subtraction fact with minuends up to 18 and subtrahends up to 9.
- N10.04** Recognize which facts could be determined by a given strategy.
- N10.05** Quickly recall basic addition facts to 18 and related subtraction facts in a variety of contexts.

Scope and Sequence

Mathematics 2	Mathematics 3	Mathematics 4
N10 Students will be expected to apply mental mathematics strategies to quickly recall basic addition facts to 18 and determine related subtraction facts.	N10 Students will be expected to apply mental mathematics strategies and number properties, to develop quick recall of basic addition facts to 18 and related basic subtraction facts.	—

Background

It is expected that, by the end of Mathematics 2, students will have achieved quick recall of addition facts up to $9 + 9$. In Mathematics 3, therefore, the majority of time should be spent on the related subtraction facts. It is important to note, however, that students will struggle with subtraction facts if the related addition facts are not well known. Quick recall for most students should be three seconds, or less, but there may be some students who need one or two additional seconds to mentally determine some of the facts. Knowing that students develop facility at different times of the year, we should not promote competition between students but should instead focus on individual self-improvement. Whatever the time limit, to achieve this outcome students should not be counting on their fingers or using other cumbersome strategies. They should be processing the facts in their minds to get the sums and differences. Through understanding of relationships, repetition, and practise, however, students may achieve automaticity (instant recall) for some or all of the facts without any strategy processing whatsoever.

Why should students learn the basic facts? Having an instant recall of the basic facts

- means students have one of the building blocks for mental computation that is the most common means of computation in society today
- gives students a sense of empowerment
- means that students can concentrate on learning newer concepts and procedures that use these facts, rather than devoting neural energy to finding these sums and differences

It is important to provide opportunities for practice using games and meaningful contexts, rather than just learning the facts for their own sake. Furthermore, students should be expected to use quick recall of facts in their everyday work in mathematics and other subjects, not just during the times allocated for fact learning. Once facts are mastered, there should be no need to use counting strategies when two numbers are combined in addition or subtraction. Many students continue to use counting strategies from force of habit. These students need to be encouraged and challenged to use facts in order to sustain those facts and to help students develop new habits.

“Memorizing basic facts, perhaps with the use of flash cards, is very different from internalizing number combinations. *Memorized* knowledge is knowledge that can be forgotten. *Internalized* knowledge can’t be forgotten because it is a part of the way we see the world. Children who memorize addition and subtraction facts often forget what they have learned. On the other hand, children who have internalized a concept or relationship can’t forget it; they know it *has* to be that way because of a whole network of relationships and interrelationships that they have discovered and constructed in their minds.” (Richardson 1999, 43)

Students should continue to practise quick recall of addition facts that were expected in Mathematics 2, by using appropriate strategies or by instant recall. While there are 100 addition facts up to $9 + 9$, students should understand that the order-doesn’t-matter property (commutative property) of addition means that 90 facts are in 45 pairs, such as $5 + 8$ and $8 + 5$, so there are really only 55 addition facts to be learned (10 double facts and 45 commutative pairs).

Students should review addition fact strategies and the clusters of facts to which these strategies apply, sustaining the three-second or less response time expected in Mathematics 2. These strategies include

- Associations for the Double Facts ($1 + 1, \dots, 9 + 9$)
- Next Number for the Plus-1 Facts ($2 + 1, 1 + 2, 3 + 1, 1 + 3, \dots, 9 + 1, 1 + 9$)
- Double, Next Number for the 1-Apart Facts ($2 + 3, 3 + 2, 3 + 4, 4 + 3, \dots, 8 + 9, 9 + 8$)
- Next Even or Odd for the Plus-2 Facts ($4 + 2, 2 + 4, 5 + 2, 2 + 5, \dots, 9 + 2, 2 + 9$)
- No Change for the Plus-0 Facts ($0 + 0, 1 + 0, 0 + 1, 2 + 0, 0 + 2, \dots, 9 + 0, 0 + 9$)
- Make-10 for the Plus-9 Facts ($9 + 3, 3 + 9, 9 + 4, 4 + 9, \dots, 9 + 7, 7 + 9$)
- Make-10 for the Plus-8 Facts ($8 + 3, 3 + 8, 8 + 4, 4 + 8, 8 + 5, 5 + 8, 8 + 6, 6 + 8$)
- Make-10 for the Plus-7 Facts ($7 + 3, 3 + 7, 7 + 4, 4 + 7, 7 + 5, 5 + 7$)
- Double the Number between for the 2-Apart Facts ($5 + 3, 3 + 5, 6 + 4, 4 + 6, 7 + 5, 5 + 7$)
- Variety of Strategies for the Last Two Facts ($6 + 3, 3 + 6$)

Students may also share other very effective strategies besides the ones listed above. Sharing and discussing alternative strategies should be part of fact learning. It opens eyes and minds to the flexibility of numbers and operations. Having such flexibility is a critical aspect of the development of number sense and operation sense.

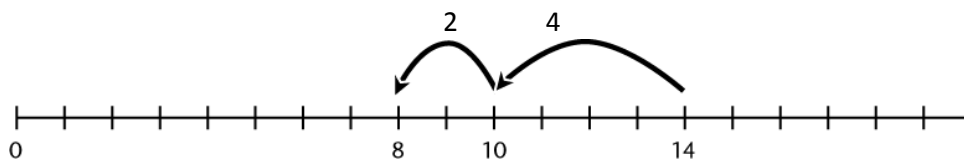
While facts involving zero appear to be the easiest of all, they are often a source of error. Because students have generalized that the addition of one number to another, changes the number, they resist the idea that no change has occurred even though there has been an addition. Furthermore, students have not likely experienced reading a story problem that results in writing a number sentence with zero as an addend. To develop the strategy for the Plus 0 facts, it is suggested that such story problems are created. Students will find them funny, and this will help them internalize the no-change aspect of these facts. Zero is the additive identity because under the operation of addition it does not affect a change.

Students will already have had experiences with subtraction fact strategies in Mathematics 2. Now, the focus will be on using the strategies to efficiently develop quick recall of these facts. Perhaps the most useful strategy for subtraction facts is the think-addition strategy, in which students get answers to

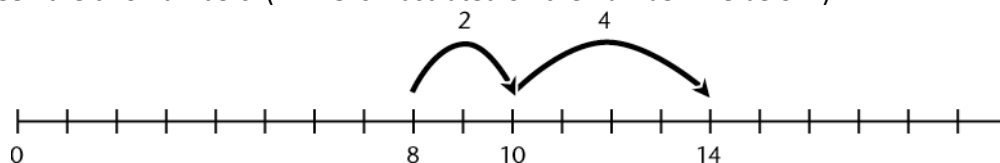
subtraction facts by recalling the related addition facts. For example, for $15 - 7$, students think of what they would add to 7 to get 15, recall that $7 + 8 = 15$, and so know that $15 - 7$ must be 8. This strategy rests on students' understanding the inverse relationship between addition and subtraction, and on knowing addition facts. Also, this strategy focuses on the difference meaning of subtraction (how far apart the two numbers are) rather than a take-away meaning.

Besides the think-addition strategy, there are two other very useful subtraction strategies that not only can be used for quick fact recall, but also can be applied in subtraction situations involving larger numbers. These strategies are as follows.

- Back-through-10:** This strategy involves subtracting in two steps—one part of the subtrahend is subtracted to get to ten and the other part of the subtrahend is then subtracted from 10. This strategy is most effective when only 1 or 2 has to be subtracted from 10 in the second step. Modelling this strategy on a number line as a “take-away” would help students visualize the steps—Mark the minuend, show the leap from the minuend to 10, and then the final leap to the answer. ($14 - 6$ is illustrated on the number line below.)



- Up-through-10:** This strategy involves finding the difference between the two numbers in steps: first, the difference between the subtrahend and 10 is found and then the difference between 10 and the minuend is found, and finally these two differences are added to give the total difference. Modelling this strategy on a number line will also help students visualize the steps: Mark both the minuend and the subtrahend on the number line, show the leap from the subtrahend to 10, show the leap from 10 to the minuend, and see that the two leaps together represent the total difference between the two numbers. ($14 - 8$ is illustrated on the number line below.)



Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Orally and visually present a series of addition facts, one at a time, to students allowing 3–5 seconds for them to recall each fact. Ask them to record each sum before proceeding to the next question.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students to explain, using a model, why he or she knows that $3 + 4$ has to equal $4 + 3$ even before finding the total.
- Ask, Why is it easy to add or subtract 0 to numbers? (interview student or write response in a journal)
- Ask, Why is it easy to add the numbers $5 + 5 + 6 + 4 + 8 + 2$?
- Ask, How can you use addition to solve $16 - 7$?
- Have students write all of the number facts they can for a provided sum or difference (e.g., 6 as a difference: $6 - 0$, $7 - 1$, $8 - 2$, $9 - 3$, $10 - 4$, $11 - 5$, $12 - 6$, $13 - 7$, $14 - 8$, $15 - 9$).
- Show students a mathematics fact. Ask students to record answers on individual white boards (or use plastic plates or plastic sheet protectors). Have students share their personal strategies. This activity could be extended to include open frame questions (e.g., $7 + \underline{\quad} = 13$) as explored in outcome PR03.
- Ask students to describe as many different ways as possible to solve $18 - 9$.

FOLLOW-UP ON ASSESSMENT**Guiding Questions**

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets 3 (Bauman 2011)

- Checkpoint 3, pp. 26–27
- Checkpoint 7, pp. 40–41

Leaps and Bounds toward Math Understanding 3/4 (Small, Lin, and Kubota-Zarivnij 2011)

- Mental Math

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Provide students with a variety of models to practise and help visualize the basic facts. Students can also use other strategies, such as drawing pictures and role-playing to represent various sums and differences in a problem-solving context.
- Use mathematical vocabulary with students, including **sum**, **difference**, and **number sentence**.
- Use the open number line to explore “making ten” or “bridging through 10.”
- Ensure students have the opportunity to discuss their strategies with others. The focus should be on the efficiency of the strategy.
- Provide lots of opportunities for practise (visually/orally) with immediate feedback over an extended period of time.
- Use the addition table to explore patterns and help students identify the facts that they have mastered. The known facts can be coloured in with the goal of having the entire table coloured.
- Have students create realistic word problems related to addition and subtraction.
- Provide many opportunities where the focus is on the relationship between the numbers.

Suggested Learning Tasks

- Have students roll two number cubes (dice). They either add or subtract these values. For example, if a 5 and a 2 are rolled, they will work with either 5, 2, 7, or 5, 2, 3. Ask the students to make up a subtraction story based on these numbers, and write the corresponding number sentence. (**Note:** Ten-sided dice work well, as do prepared numeral cards.) If students are proficient with the addition facts, adapt this activity so that they must focus on subtraction.
- Play “Missing Part” game for two students to practise their fact recall. One student places a number of counters in front of them (e.g., 16) and then the student covers some of the counters with their hand. The other student must determine how many counters are hidden as quickly as possible.
- Use a “Looping Activity” where every student is given a card with a basic fact number sentence in which one of the numbers is missing, written as “Who has ...?” (e.g., Who has $5 + \underline{\quad} = 11$). The card also has the answer from someone else’s card written as “I have ...” Students take turns reading their cards in sequence by responding when their card answers someone else’s question.
- Provide students with cards with a subtraction number sentence (e.g., $13 - 7 = \underline{\quad}$). Have students rewrite the sentence as a missing addend number sentence (e.g., $7 + \square = 13$) and solve it.

SUGGESTED MODELS AND MANIPULATIVES

- 10-sided dice
- addition chart
- counters
- dominoes (double nine)
- double ten-frames
- linking cubes
- number cards
- number cubes
- open number line

MATHEMATICAL LANGUAGE

Teacher	Student
<ul style="list-style-type: none"> ▪ addition, subtraction facts ▪ double, plus-0, plus-1, plus-2, 1-apart, 2-apart, make-ten, back-through-10, up-through-10 ▪ mental mathematics strategy ▪ order, no change, order does not matter 	<ul style="list-style-type: none"> ▪ addition, subtraction facts ▪ double, plus-0, plus-1, plus-2, 1-apart, 2-apart, make-ten, back-through-10, up-through-10 ▪ mental mathematics strategy ▪ order, no change, order does not matter

Resources/Notes**Print**

- *Making Math Meaningful to Canadian Students K–8*, Second Edition (Small 2013), pp. 166–172
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 20, 94–119
- *Teaching Student-Centered Mathematics, Grades 3–5* (Van de Walle and Lovin 2006), pp. 74–99

Videos

- An Introduction to Teaching Addition Number Facts (15:51 min.) (ORIGO Education 2010)
- Teaching the Bridge-to-10 Strategy for Addition Number Facts (17:11 min.) (ORIGO Education 2010)
- Teaching the Count-on Strategy for Addition Number Facts (17:49 min.) (ORIGO Education 2010)
- Teaching the Think-Addition Subtraction Fact Strategy (13:41 min.) (ORIGO Education 2010)
- Teaching the Use- Doubles Strategy for Addition Number Facts (14:20 min.) (ORIGO Education 2010)

Notes

- SCO N11** Students will be expected to demonstrate an understanding of multiplication to 5×5 by
- representing and explaining multiplication using equal grouping and arrays
 - creating and solving problems in context that involves multiplication
 - modelling multiplication using concrete and visual representations and recording the process symbolically
 - relating multiplication to repeated addition
 - relating multiplication to division

[C, CN, PS, R]

[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- N11.01** Identify events from experience that can be described as multiplication.
- N11.02** Represent a given story problem (orally, shared reading, written) using manipulatives or diagrams and record in a number sentence.
- N11.03** Represent a given multiplication expression as repeated addition.
- N11.04** Represent a given repeated addition as multiplication.
- N11.05** Create and illustrate a story problem for a given number sentence and/or expression.
- N11.06** Represent, concretely or pictorially, equal groups for a given number sentence.
- N11.07** Represent a given multiplication expression using an array.
- N11.08** Create an array to model the commutative property of multiplication.
- N11.09** Relate multiplication to division by using arrays and writing related number sentences.
- N11.10** Solve a given problem in context involving multiplication.

Scope and Sequence

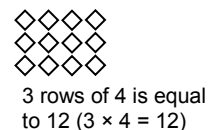
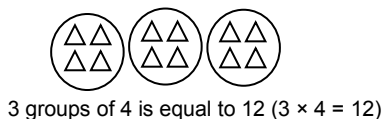
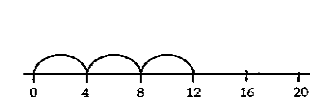
Mathematics 2	Mathematics 3	Mathematics 4
—	<p>N11 Students will be expected to demonstrate an understanding of multiplication to 5×5 by</p> <ul style="list-style-type: none"> ▪ representing and explaining multiplication using equal grouping and arrays ▪ creating and solving problems in context that involve multiplication ▪ modelling multiplication using concrete and visual representations, and recording the process symbolically ▪ relating multiplication to repeated addition ▪ relating multiplication to division 	<p>N05 Students will be expected to describe and apply mental mathematics strategies to recall basic multiplication facts to 9×9 and to determine related division facts.</p> <p>N06 Students will be expected to demonstrate an understanding of multiplication (2- or 3-digit by 1-digit) to solve problems by</p> <ul style="list-style-type: none"> ▪ using personal strategies for multiplication with and without concrete materials ▪ using arrays to represent multiplication ▪ connecting concrete representations to symbolic representations ▪ estimating products ▪ applying the distributive property

Background

Students should be introduced to multiplication through situations (equal-group story problems) that lend themselves to modelling with sets, arrays, and linear or measurement models, such as number lines. For example, students would likely represent three plates of five cookies by making three groups of five counters (set model), represent three rows of five cadets on parade by making three rows of five counters (array model). After students have modelled story problems, they should explain the connection between their models and the story problems using verbal expressions such as “groups of,” “rows of,” and “jumps of.” Their initial number sentences, based on their prior knowledge, would involve repeated addition, such as $5 + 5 + 5 = 15$. It is important not to begin too soon using the word “times” and the multiplication symbol because this may interfere with students’ understanding of multiplication situations. The formal writing of multiplication sentences should be delayed until students understand the meaning of multiplication; that is, they can correctly interpret and create story problems, model them concretely, record them pictorially, and write repeated addition number sentences. Students will need help to translate these repeated addition number sentences to multiplication number sentences. Students should first verbalize repeated addition sentences as “ ___ groups of ___ is ___” and then learn that the symbol \times can be used for “groups of.” For example, $5 + 5 + 5 = 15$, would be described as “3 groups of 5 is 15” and then written as $3 \times 5 = 15$.

In number sentences for multiplication, the numbers being multiplied are called factors and the answers are called products. For example, in $3 \times 5 = 15$, the factors are 3 and 5, and the product is 15. Students should understand and use these terms, **factors** and **product**, when describing multiplication situations. Students should understand the different ways that factors and products can be represented, such as

- by repeated addition (3×4 means $4 + 4 + 4$)
- by making sets of equal groups (3 groups of 4 items)
- by making an array (3 rows of 4 columns)
- by showing jumps on a number line (3 jumps of 4)



Students in Mathematics 3 should determine the products of two factors by creating or visualizing one of these representations, and skip counting to get the total. While students may initially use one-to-one counting to get products, they need to be convinced that skip counting is a more efficient and desirable strategy. Ideally, by the end of Mathematics 3, students would use a double-count strategy to find products. For example, for 4×5 , a student would touch 1 finger and say 5, a second finger and say 10, a third finger and say 15, and a fourth finger and say 20. They used fingers to keep track of one count (the number of groups) while they kept track of the second count (the number in the groups) by orally skip counting.

As students explore the meaning of multiplication with concrete and pictorial models, they will encounter commutative pairs, such as 2×3 and 3×2 , or 4×5 and 5×4 . Through focused questions, students should discover what is alike about these pairs (the same product) and what is different (the number of groups in one expression is the number in each group in the other expression). Array models of these commutative pairs best illustrate this property of multiplication because the arrays are quarter-turn rotated images of each other. Students should begin to realize that this commutative property of multiplication can often make determining products easier. For example, it is easier for most students to skip count by 5s than by 4s; therefore, to find 5×4 , it would be easier to think of 4×5 , saying 5, 10, 15, 20. Even though 5×4 would actually be 4, 8, 12, 16, 20, it has the same product as 4×5 .

Students in Mathematics 3 are initially likely to view multiplication and division (SCO N12) as two very distinct concepts, so they will need focused experiences to help them begin to see the inverse relationship between these two operations. It is through the concrete and pictorial representations of the two operations that this relationship is most vivid. For example, if students are asked to use counters to show 3×4 and to show $12 \div 3$, their final displays may both be 3 groups of 4 counters or 3 by 4 arrays, even though the processes they used to create those displays were different. As operations, this inverse relationship means that multiplication undoes division and division undoes multiplication. For example, if 3 is multiplied by 5 to get 15, then 15 divided by 5 is 3. After both concepts are introduced, it is important to have specific experiences to help students make this inverse relationship between multiplication and division.

Note: There is no expectation in Mathematics 3 for students to learn multiplication facts. All multiplication problems should involve either pictures and/or context.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Ask students to skip count forward by 2s, starting at 2 and ending at 10.
- Ask students to skip count forward by 5s, starting at 5 and ending at 25.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Have students represent story problems using models or diagrams and record the corresponding number sentences.
- Create and illustrate a story problem for 2×4 .
- Have students create a real-life story problem that involves multiplication and solve it.
- Have students represent a given multiplication sentence, such as 5×3 , using an array.
- Have students represent a given repeated addition as multiplication and vice versa.

- Have students represent equal groups for a given number sentence concretely or pictorially.
- Ask students to model as many arrays as possible with 16 counters. Have them write the related multiplication and division facts for each array.
- Solve a contextual problem such as, Jacques has 3 bags of apples. Each bag has 4 apples. How many apples does he have?
- Ask students to put 10 tiles into rows of 5. Ask how many rows there are.
- Show students an array and have them provide the related multiplication and division sentences.
- Use an array to show that 2×3 is the same amount as 3×2 .

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets 3 (Bauman 2011)

- Checkpoint 9, p. 48 (Line Master 9.1)

Leaps and Bounds toward Math Understanding 3/4 (Small, Lin, and Kubota-Zarivnij 2011)

- No Pathway for this outcome.

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Brainstorm objects that come in sets of 2, 3, 4, and 5, such as hands, eyes, ears, mittens, shoes, wheels on a bicycle, for things that come in 2s; wheels on a tricycle, sides on a triangle, juice packs for things that come in 3s; sides on a square, tires on a car, legs on a horse for things that come in 4s, and fingers on a hand, days in a school week for things that come in 5s. These contexts can be used when students are creating story problems for multiplication.

- Play “Broken Calculator” to relate multiplication and addition. Students use the constant feature of the calculator to find various products without using the multiplication key. Challenge the students to model their product using counters.
- Show an array. Ask students to write the fact family illustrated in the array (multiplication and division).
- Ask students to show multiple representations of a given multiplication fact.
- Ask students to draw pictures showing various situations in which multiplication might be used.
- Give students many opportunities to solve missing factor problems. Example: It takes 4 toothpicks to build a square. How many of the same sized squares can be built with 16 toothpicks? ($4 \times \square = 16$)

SUGGESTED LEARNING TASKS

- Using counters, have students build as many arrays as possible for a given number, and write the corresponding equations.
- Provide students with a number of multiplication and division story problems. Ask them to represent each story concretely. Have them record their solutions by drawing pictures and writing repeated addition and multiplication sentences.
- Ask students to draw pictures to show what 3×4 means.
- Create arrays on cards and cut off a corner so that some counters are missing but the intended number of rows and columns remains clear. Show cards to students and ask them how many counters the card had initially if all the rows and columns had the same number of counters.
- Have students investigate what happens when you multiply a number by 0, 1, 2, 3, 4, 5. Are there patterns in the products?
- Invite a group of students to act out a skit modelling either a multiplication or division situation. Ask other students to suggest the number sentence being dramatized.
- Have the students create a realistic story problem to go with a given number sentence (e.g., 4×5) or describe a situation for which you might have to find the answer to 5×3 . Students may exchange these story problems and solve them using pictures and concrete models.
- Students can create multiplication booklets for a given number (limited to 5) using computer-generated or magazine pictures. For example, students could create a booklet for multiplying by 2s. On the front of a page, students could insert a picture and write a story problem that could be solved with multiplication. On the back of that page, students could write the repeated addition and the multiplication equations that would be used to solve their story problem.
- Present students with pictures of equal groups, arrays, or number lines showing jumps. Ask them to record the repeated addition and the multiplication sentence represented by the picture. Then, ask them to explain how they would find the product.
- Ask students to use concrete materials to model the solution to a multiplication story problem, such as, Tomas and his two friends each have 3 pencils. How many pencils do they have altogether?

SUGGESTED MODELS AND MANIPULATIVES

- array examples
- calculator
- counters
- ruler
- square tiles

MATHEMATICAL LANGUAGE

Teacher	Student
<ul style="list-style-type: none">▪ factors, product▪ groups of, rows of, jumps of▪ multiplication, division▪ number line▪ number sentence, number expression▪ repeated addition, equal groups, number of groups▪ sets, arrays	<ul style="list-style-type: none">▪ factors, product▪ groups of, rows of, jumps of▪ multiplication, division▪ number line▪ number sentence, number expression▪ repeated addition, equal groups, number of groups

Resources/Notes

Print

- *Making Math Meaningful to Canadian Students K–8* (Small 2009), pp. 118–124
- *Making Math Meaningful to Canadian Students Grades K–8, Second Edition* (Small 2013) pp. 172–174, 176–187
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 77–84, 86–89

Notes

- SCO N12** Students will be expected to demonstrate an understanding of division by
- representing and explaining division using equal sharing and equal grouping
 - creating and solving problems in context that involves equal sharing and equal grouping
 - modelling equal sharing and equal grouping using concrete and visual representations, and recording the process symbolically
 - relating division to repeated subtraction
 - relating division to multiplication.

(Limited to division related to multiplication facts up to 5×5 .)

[C, CN, PS, R]

[C] Communication

[PS] Problem Solving

[CN] Connections

[ME] Mental Mathematics and Estimation

[T] Technology

[V] Visualization

[R] Reasoning

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- N12.01** Identify events from experience that can be described as equal sharing.
- N12.02** Identify events from experience that can be described as equal grouping.
- N12.03** Illustrate, with counters or a diagram, a given story problem involving equal sharing, presented orally or through shared reading, and solve the problem.
- N12.04** Illustrate, with counters or a diagram, a given story problem involving equal grouping, presented orally or through shared reading, and solve the problem.
- N12.05** Listen to a story problem, represent the numbers using manipulatives or a diagram and record the problem with a number sentence and/or expression.
- N12.06** Create and illustrate with counters, a story problem for a given number sentence and/or expression.
- N12.07** Represent a given division sentence and/or expression as repeated subtraction.
- N12.08** Represent a given repeated subtraction as a division sentence.
- N12.09** Relate division to multiplication by using arrays and writing related number sentences.
- N12.10** Solve a given problem involving division.

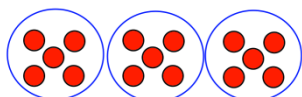
Scope and Sequence

Mathematics 2	Mathematics 3	Mathematics 4
—	<p>N12 Students will be expected to demonstrate an understanding of division by</p> <ul style="list-style-type: none"> ▪ representing and explaining division using equal sharing and equal grouping ▪ creating and solving problems in context that involves equal sharing and equal grouping ▪ modelling equal sharing and equal grouping using concrete and visual representations, and recording the process symbolically ▪ relating division to repeated subtraction ▪ relating division to multiplication <p>(Limited to division related to multiplication facts up to 5×5.)</p>	<p>N07 Students will be expected to demonstrate an understanding of division (1-digit divisor and up to 2-digit dividend) to solve problems by</p> <ul style="list-style-type: none"> ▪ using personal strategies for dividing with and without concrete materials ▪ estimating quotients ▪ relating division to multiplication

Background

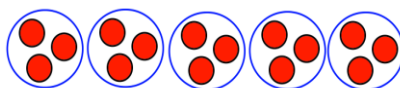
Students should be introduced to division through story problems. For this introduction, there are two types of situations, equal sharing and equal grouping, which need to be considered.

Equal-sharing problems are those in which the number of groups is known and the number in each group needs to be found. For example, Three friends want to share 15 candies. How many candies will each friend get? If students model this problem with counters, they would most likely start with 15 counters and distribute the counters, one at a time, into three groups. After all the counters have been distributed, they see there are 5 counters in each group, indicating that each friend will get 5 candies.



This may be described verbally as, When 15 is divided into 3 groups, there are 5 in each group. (**Note:** Some mathematics educators refer to these problems simply as sharing problems. While students' prior experiences of sharing in the social sense may not have guaranteed the shares were equal, they need to understand that equal sharing is fair sharing and is essential in the mathematics concept of division.)

Equal-grouping problems are those in which the number in each group is known and the number of groups needs to be found. For example, Friends want to share 15 candies by each taking 3 candies. How many friends will get candies? If students model this problem with counters, they would most likely start with 15 counters, take 3 of these counters to give to 1 friend, take another 3 to give to a second friend, and continue doing this until they run out of counters; thus, 5 friends will get candies.



This may be described verbally as, When 15 is divided into groups of 3, there are 5 groups. (**Note:** Some mathematics educators refer to these problems as measurement problems.)

Students should solve several examples of both of these types of division problems by modelling them concretely, recording them pictorially, and describing the division in words before they are introduced to division sentences. Students can be introduced to the symbol for division as a symbolic translation of the verbal descriptions. For example, “When 15 is divided into 3 groups, there are 5 in each group” is translated $15 \div 3 = 5$; “When 15 is divided into groups of 3, there are 5 groups” is translated $15 \div 3 = 5$. Students are surprised that these two very different situations result in the same division sentence. These two interpretations of a division sentence need to get equal attention.

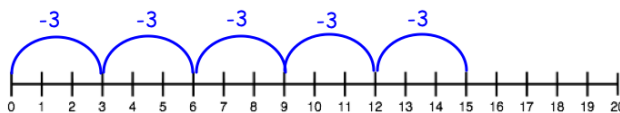
In division sentences, there are three numbers—one is called the dividend, one is called the divisor, and one is called the quotient. The dividend is the quantity being divided, the divisor is the number of groups (equal-sharing situations) or the number in each group (equal-grouping situations), and the quotient is the number in each group (equal-sharing situations) or the number of groups (equal-grouping situations). For example, for $15 \div 3 = 5$, 15 is the dividend, 3 is the divisor, and 5 is the quotient. The interpretations of the divisor and the quotient depend upon whether the situation represented in symbols is an equal-sharing situation or an equal-grouping situation.

Students should also relate arrays to the two types of division situations. For example, for $15 \div 3 = 5$, the divisor 3 may represent the number of rows (equal-sharing situations) as in the array on the left, or may represent the number in each row (equal-grouping situations) as in the array on the right.



It is important that students model story problems that do not involve remainders.

Just as multiplication is a short-hand for repeated addition, division is a short-hand for repeated subtraction. This is most apparent in equal-grouping situations. For example, when 15 candies are divided into groups of 3, 3 candies at a time are removed from the set of 15 until there are no candies left. Students are easily convinced that $15 - 3 - 3 - 3 - 3 - 3 = 0$ represents the action in this situation, especially if it is also modelled on a number line.



Most students will need help, however, to see equal-sharing situations as repeated subtraction because they usually share the quantities into the groups one at a time. Students need to understand that if they keep track of how many are shared in each round of distribution, they can also write a repeated subtraction sentence for these situations. For example, when 15 candies are divided into 3 groups, sharing them 1 at a time, there are a total of 3 distributed in each round (each time each group gets 1). Thinking about this equal-sharing situation in this way, students can appreciate why the number line picture (above) can also represent it.

Students in Mathematics 3 are initially likely to view multiplication (SCO N11) and division as two very distinct concepts, so they will need focussed experiences to help them begin to see the inverse relationship between these two operations. It is through the concrete and pictorial representations of the two operations that this relationship is most vivid. For example, if students are asked to use counters to show 3×4 and to show $12 \div 3$, their final displays may both be 3 groups of 4 counters or both be 3 by 4 arrays, even though the processes they used to create those displays were different. As operations, this inverse relationship means that multiplication undoes division and division undoes multiplication. For example, if you start with 3 and multiply it by 5, you get 15; then, if you divide 15 by 5, you get back to 3. After both multiplication and division are introduced, it is important to have specific experiences to help students make this inverse relationship between these two operations.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

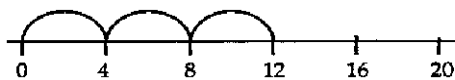
Tasks such as the following could be used to determine students' prior knowledge.

- Ask students to skip count backward by 2s, starting at 12 and ending at 0.
- Ask students to skip count backward by 5s, starting at 25 and ending at 0.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students to write a division story about $25 \div 5$.
- Have students draw a picture or use counters to show what $12 \div 3$ means.
- Ask students to describe a situation for which you might have to find the answer to $15 \div 3$.
- Ask students to draw pictures showing various situations in which either multiplication or division might be used.
- Show the number line below. Ask students to record what multiplication and division sentences it might be showing.



- Show students the multiplication sentence $5 \times 3 = 15$. Ask them to write related division sentences.
- Tell students that amusement park rides are priced as follows:
 - \$1 for the ferris wheel
 - \$2 for the bullet
 - \$3 for the twister
 Ask, How many rides, and of which kind, can you have for \$12? Are there other possibilities?
- Show students an array of up to 25 counters. Ask students which multiplication and division family is shown by the array.
- Have students listen to a story problem and represent the numbers using models or an illustration and record the problem with a number sentence. For example, Emma has 16 stickers to share with 4 friends. How many stickers will each friend get? ($16 \div 4 = 4$)

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets 3 (Bauman 2011)

- Checkpoint 8, pp. 44–45

Leaps and Bounds toward Math Understanding 3/4 (Small, Lin, and Kubota-Zarivnij 2011)

- No Pathway for this outcome.

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Explore various types of division problems: equal share, equal groups, and repeated subtraction. Expect students to use concrete materials and pictures to model these situations.
- Use concrete materials to help students understand the relationship between the meanings of division. Demonstrate that, in sharing 12 items equally among 3 people, for example, the actual giving of 1 item to each person is the same as creating a group of 3. In other words, sharing among 3 people is equivalent to finding how many groups of 3 can be formed.
- Play “Broken Calculator.” Students work in groups to find ways to use the calculator to solve division exercises without using the divide key.
- Provide problem solving situations in which solutions can be found using either multiplication or division.

SUGGESTED LEARNING TASKS

- Provide the student with some toothpicks and ask him or her to use 12 to make 4 identical shapes. Ask the student what division and multiplication sentences could describe the creation of the shapes.
- Set up a 3×4 array and ask the student to give two multiplication and two division sentences that describe it by looking at the array from different perspectives.
- Invite a group of students to act out a skit modelling either a multiplication or division situation. Ask other students to identify the number sentence being dramatized.
- Ask the student to write problems in which one has to multiply or divide to find the answer. Have him or her illustrate the solutions and describe the multiplication/division relationship.
- Ask students to solve a division problem in as many ways as possible (including multiplication).
- Ask students to solve division problems using the same numbers; one where the result is “equal shares” and the other where the result is “equal groups.” Have students represent these problems with counters or with a diagram.

SUGGESTED MODELS AND MANIPULATIVES

- arrays
- counters
- number lines

MATHEMATICAL LANGUAGE

Teacher	Student
<ul style="list-style-type: none"> ▪ arrays, repeated subtraction ▪ divided into ▪ equal sharing/fair sharing, equal grouping ▪ multiplication, division ▪ number in each group, number of groups ▪ number sentence, number expression 	<ul style="list-style-type: none"> ▪ repeated subtraction ▪ divided into ▪ multiplication, division ▪ number in each group, number of groups ▪ number sentence, number expression

Resources/Notes

Print

- *Making Math Meaningful to Canadian Students K–8* (Small 2009), pp. 118–124
- *Making Math Meaningful to Canadian Students K–8, Second Edition* (Small 2013), pp. 174–187
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 77–84, 86–89

Notes

SCO N13 Students will be expected to demonstrate an understanding of fractions by

- explaining that a fraction represents a part of a whole
- describing situations in which fractions are used
- comparing fractions of the same whole with like denominators

[C, CN, ME, R, V]

[C] Communication

[PS] Problem Solving

[CN] Connections

[ME] Mental Mathematics and Estimation

[T] Technology

[V] Visualization

[R] Reasoning

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N13.01 Describe everyday situations where fractions are used.

N13.02 Represent a given fraction concretely or pictorially.

N13.03 Identify, model, and explain the meaning of numerator and denominator.

N13.04 Sort a given set of diagrams of regions into those that represent equal parts and those that do not, and explain the sorting.

N13.05 Name and record the fraction represented by the shaded and non-shaded parts of a given region.

N13.06 Compare given fractions with the same denominator using models.

Scope and Sequence

Mathematics 2	Mathematics 3	Mathematics 4
—	<p>N13 Students will be expected to demonstrate an understanding of fractions by</p> <ul style="list-style-type: none"> ▪ explaining that a fraction represents a part of a whole ▪ describing situations in which fractions are used ▪ comparing fractions of the same whole with like denominators. 	<p>N08 Students will be expected to demonstrate understanding of fractions less than or equal to one by using concrete, pictorial, and symbolic representations to</p> <ul style="list-style-type: none"> ▪ name and record fractions for the parts of a whole or a set ▪ compare and order fractions ▪ model and explain that for different wholes, two identical fractions may not represent the same quantity ▪ provide examples of where fractions are used.

Background

Fractions are very complex, and it will take several years for students to develop a full understanding of them. In Mathematics 3, the focus is only on the development of students' understanding of fractions less than one whole in situations that can be represented by regional (area) models. Students should investigate the more common fraction families such as halves, thirds, fourths, fifths, sixths, eighths, tenths, and twelfths. Students should be introduced to these fractions through contexts that are real to them before investigating other contexts. Students should model these fractions using created and commercial concrete materials, draw and interpret pictures of these fractions, and describe them using oral language before they are carefully introduced to the symbolic representations. Often it is the symbols that are the source of difficulty with fractions, especially if those symbols are introduced too early in the development of the concept.

While this is the first grade in which students are formally introduced to fractions, they would have encountered them and heard them used in their daily lives. One-half, or half, is likely the most familiar fraction for most students, and you may hear them using the word, maybe not always correctly. Because of their familiarity, halves are the family of fractions that you should start with to establish some general understandings about fractions before moving on to other families of fractions. These understandings include the following:

- Fractional parts are equal shares or equal-sized portions of one whole. (For example, if you hear students referring to “the bigger half,” it is an indication that they do not yet understand that one-half is one of two equal portions.)
- A fraction is a number that describes a relationship between a part and a whole. (For example, oral language should include “1 out of 2 equal parts” and “one-half.”)
- The equal parts of the whole do not have to be the same shape. (For example, join the diagonals of one of two congruent squares and the mid-points of two opposite sides of the other square: the two triangles and the two rectangles formed are all one-half of the square and are all equal to one another.)
- When creating fractional parts of a whole, the whole amount must be used. (For example, if a student is asked to cut a given length of string into halves and makes the cut, only to find that one piece is longer than the other, he or she cannot just cut off the extra, throw it away, and claim the two pieces are halves of the original piece of string.)
- Two fractions with the same name are not equal amounts unless they represent the part(s) of the same whole. (For example, one-half of the white board is not equal to one-half of a credit card.)

The concrete materials that students use to model fractions should be a balance between commercially produced models, such as The Fraction Factory, fraction circles, pattern blocks, and tangrams, and models created by the students themselves by cutting and folding paper and creating shapes on geoboards. When students have to create their own fraction models, they must concentrate on making all the parts equal. If they only use prepared materials, they may concentrate on the number of pieces and/or the colours rather than on the equality of the parts. Also, when using materials, what is designated the whole should vary. For example, if pattern blocks are used, sometimes the hexagon should be designated the whole and other times one of the other blocks should be the whole. This varying of the whole helps students develop flexibility in thinking about fractions, and forces them to focus on the relationship aspect of a part to a whole.

Students’ initial pictures of fractions may involve them tracing the concrete models; however, they should move on to creating their own parts of a whole, such as a square, rectangle, or circle, being careful to make the parts equal in area. When they are asked to identify fractions in pictures with parts of shapes shaded, those pictures should be varied. For example, include some pictures in which the parts are not all the same shape but equal in area, some in which the parts are not equal, and some that will require the students to partition to make the parts.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Give students two pieces of paper. Ask them to fold one piece of paper in half. Ask them to fold the second piece of paper in half, but in a different way.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students, If you are really hungry and want a large piece of vegetarian pizza, would you cut the pizza into thirds, fourths, or tenths? Have them explain their thinking.
- Provide students with a square piece of paper and ask them to show fourths by folding the paper. Have the students compare their fourths. Are they the same shape? Are they all really fourths?
- Show students a region with a shaded part. Ask students to name and record the fraction represented by the shaded part. Ask them to name and record the fraction represented by the unshaded part.
- Ask students to sort various shapes that show equal and unequal parts shaded. Ask students to explain in writing how they sorted the shapes.
- Ask students to use the hexagon from the pattern block as one whole. Ask them to model and name fractions represented by the triangle, rhombus and/or trapezoid.
- Ask students, Is half a lot or a little? Have them explain their thinking.
- Ask students to identify the numerator and denominator of a given fraction.
- Provide students with fractions with the same denominator and have them identify the larger (or smaller) fraction and explain their reasoning using models.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets 3 (Bauman 2011)

- Checkpoint 4, pp. 29–30 (Line Master 4.1)

Leaps and Bounds toward Math Understanding 3/4 (Small, Lin, and Kubota-Zarivnij 2011)

- Fractions

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Have students explore various models for fractions—part of a region and part of a length.
- Ensure students develop an understanding that “A fraction is a number that describes a relationship between a part (represented by the numerator) and a whole (represented by the denominator). Although students see two numbers, they have to think of one idea, the relationship.” (Small 2009, 196). Students should be able to describe what is the whole and what are the parts.
- Provide students with rectangles and number lines that are the same length. Ask students to colour half of a rectangle, and indicate where half is on the number line. Once students understand the concept of half, this activity could be extended to fourths (quarters) and thirds, etc.

SUGGESTED LEARNING TASKS

- Ask students to fold a strip of paper into equal parts (e.g., halves, quarters, thirds).
- Give students pieces of scrap paper that are different sizes. Have students tear off a piece and describe what part of the whole it represents. Compare the pieces with classmates, and discuss why some students may have the same fraction, but the sizes of their pieces of paper are different.
- Give students some pattern blocks or Cuisinaire rods. Have them model one-half, one-fourth, and one-third (or other fractions) using various blocks or rods.
- Have students work in groups of four. Provide each student in the group with a piece of string but ensure that each piece of string has a different length. Ask each student to cut his or her piece of string in half. Then, ask students to compare their half with that of other students in their group. Ask, If everyone in your group has half a piece of string, why aren't the strings all the same length? (Students need to understand that the “whole” was different for each person in the group.)

- Show students three pictures of varying sizes of the same item, all items cut into the same number of pieces. Ask students which of the pieces they would like to have. Explain why they made that choice.
- Ask students to model a specific fraction using five pattern blocks. Draw their model on isometric grid paper and colour the fractional part they have represented with their model.
- Have students model on a number line (0 to 1) where one-half, one-third, one-fourth (or another fraction) would be. Ask them to explain their thinking.
- Show students a 2-D shape. Tell students that the shape is a part of a whole. What could the whole be? Discuss the various possible answers and reasons why there is more than one correct answer.
- Represent a 2-D shape on a geo-board. Tell students that the shape is a one-half (or another fraction) of a whole. Ask them to use geo-boards to show what the whole might look like.
- Give students a sheet on which multiple copies of a particular regular polygon are printed. Have students explore the various fractions that can be represented on their given polygon.

SUGGESTED MODELS AND MANIPULATIVES

- colour tiles
- Cuisinaire rods
- fraction pieces
- geo-boards
- isometric grid paper
- number lines
- pattern blocks

MATHEMATICAL LANGUAGE

Teacher	Student
<ul style="list-style-type: none"> ▪ fact families: halves, thirds, fourths, fifths, sixths, eighths, tenths, twelfths ▪ fractions ▪ greater than, less than ▪ numerator/top number, denominator/bottom number ▪ one-half, half, one-fourth ▪ part of a whole, equal parts, fair shares ▪ whole, one whole, one 	<ul style="list-style-type: none"> ▪ halves, thirds, fourths, fifths, sixths, eighths, tenths, twelfths ▪ fractions ▪ greater than, less than ▪ top number, bottom number ▪ one-half, one-fourth ▪ equal parts ▪ whole, one whole, one

Resources/Notes

Print

- *Making Math Meaningful to Canadian Students K–8* (Small 2009), pp. 195–200, 203–204, 206–210
- *Making Math Meaningful to Canadian Students, K–8, Second Edition* (Small 2013), pp. 249–254, 257–258
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 251–255, 256–258, 267
- *Teaching Student-Centered Mathematics, Grades 3–5* (Van de Walle and Lovin 2006), pp. 131–135, 136–140

Notes

Patterns and Relations (PR)

GCO: Students will be expected to use patterns to describe the world and solve problems.

GCO: Students will be expected to represent algebraic expressions in multiple ways.

Specific Curriculum Outcomes

Process Standards

[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

- PR01** Students will be expected to demonstrate an understanding of increasing patterns by describing, extending, comparing, and creating numerical (numbers to 1000) patterns and non-numerical patterns using manipulatives, diagrams, sounds, and actions. [C, CN, PS, R, V]
- PR02** Students will be expected to demonstrate an understanding of decreasing patterns by describing, extending, comparing, and creating numerical (numbers to 1000) patterns and non-numerical patterns using manipulatives, diagrams, sounds, and actions. [C, CN, PS, R, V]
- PR03** Students will be expected to solve one-step addition and subtraction equations involving symbols representing an unknown number. [C, CN, PS, R, V]

SCO PR01 Students will be expected to demonstrate an understanding of increasing patterns by describing, extending, comparing, and creating numerical (numbers to 1000) patterns and non-numerical patterns using manipulatives, diagrams, sounds, and actions.

[C, CN, PS, R, V]

[C] Communication

[PS] Problem Solving

[CN] Connections

[ME] Mental Mathematics and Estimation

[T] Technology

[V] Visualization

[R] Reasoning

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- PR01.01** Identify and describe increasing patterns.
- PR01.02** Describe a given increasing pattern by stating a pattern rule that includes the starting point and a description of how the pattern continues.
- PR01.03** Extend a pattern, using the pattern rule, for the next three terms.
- PR01.04** Compare numeric patterns.
- PR01.05** Identify and explain errors in a given increasing pattern.
- PR01.06** Create a concrete, pictorial, or symbolic representation of an increasing pattern for a given pattern rule.
- PR01.07** Create a concrete, pictorial, or symbolic increasing pattern and describe the pattern rule.
- PR01.08** Solve a given problem using increasing patterns.
- PR01.09** Identify and describe the strategy used to determine a missing term in a given increasing pattern.
- PR01.10** Use ordinal numbers (to 100th) to refer to or to predict terms within an increasing pattern.

Scope and Sequence

Mathematics 2	Mathematics 3	Mathematics 4
<p>PR02 Students will be expected to demonstrate an understanding of increasing patterns by describing, reproducing, extending, and creating numerical patterns (numbers to 100) and non-numerical patterns using manipulatives, diagrams, sounds, and actions.</p>	<p>PR01 Students will be expected to demonstrate an understanding of increasing patterns by describing, extending, comparing, and creating numerical (numbers to 1000) patterns and non-numerical patterns using manipulatives, diagrams, sounds, and actions.</p>	<p>PR01 Students will be expected to identify and describe patterns found in tables and charts, including a multiplication chart.</p> <p>PR02 Students will be expected to translate among different representations of a pattern, such as a table, a chart, or concrete materials.</p> <p>PR03 Students will be expected to represent, describe, and extend patterns and relationships, using charts and tables to solve problems.</p>

Background

Patterns are the foundation for many mathematical concepts. They should be taught throughout the year in situations that are meaningful to students, as patterns are embedded in all areas of mathematics. Providing students with the opportunity to discover and create patterns then describe and extend those patterns will result in more flexible thinking across strands.

In Mathematics 2, students had experiences describing, reproducing, extending, and creating repeating and increasing patterns. They used ordinal numbers (to tenth) to describe elements of repeating patterns. Students in Mathematics 3 will continue to explore increasing patterns, both numerical patterns with numbers to 1000 and non-numerical patterns with concrete materials, pictures, sounds, and actions. They will use ordinal numbers (to 100th) to refer to or to predict terms within an increasing pattern.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Ask students to use tiles to create a growing pattern. Ask them to explain their pattern rule.
- Ask students to skip count by 10s starting at 23 and ending at 93.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Give students a diagram showing a square table with 4 chairs (one on each side). Tell students that if 2 tables were put together, you would seat 6 people. Ask, How many people can we seat with 6 tables? 8? 10? What if we started with a table of 6? Have students explain their reasoning.
- Ask students to show you different ways these patterns could be extended.
20, 40, __, __, __, ...
1, 4, __, __, ...
- Tell students, "I am thinking of a pattern. I have landed on 50. What could I be counting by?" Accept any reasonable answer that includes an explanation.
- Ask students to begin skip counting by 100 from a given 2- or 3-digit number. Ask them to record the numbers that they say.
- Give students an increasing pattern modelled with tiles and ask them to describe, recreate, and extend the pattern in another way.

- Have students identify the pattern rule of the following increasing patterns and extend the pattern 3 more terms.
4, 7, 10, 13, 16, ...
13, 18, 23, 28, 33, ...
- Have students identify the errors in the following increasing patterns and correct them
3, 6, 9, 12, 15, 19, 21, 24, 28, 30, ...
40, 45, 50, 60, 65, 75, ...
- Provide students with an increasing pattern such as 5, 10, 15, 20, ... Ask them to predict the 11th term in the pattern.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets 3 (Bauman 2011)

- Checkpoint 10, Task 1, pp. 50–51

Leaps and Bounds toward Math Understanding 3/4 (Small, Lin, and Kubota-Zarivnij 2011)

- Patterns

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Provide students with a variety of concrete and pictorial materials such as linking cubes, colour tiles, or pattern blocks, to create and extend increasing patterns.
- Expect students to discuss and write about how patterns increase and can be extended.
- Encourage students to identify the attributes of different increasing patterns (e.g., increases by the same amount).

- Have students describe errors or missing elements within an increasing pattern.
- Expect students to demonstrate their understanding of patterns by representing the same pattern in many different ways—concretely, pictorially, symbolically, orally, rhythmically, and physically.

SUGGESTED LEARNING TASKS

- Show students the first two elements of an increasing pattern made with tiles. Ask them to copy and continue the pattern.
- Have students work in groups of 6. Ask each group to represent an increasing pattern using actions/movements. After groups have practised their patterns, have them present their increasing patterns to the class. Students should then describe each of the increasing patterns in words.
- Have students explore hundreds charts to 1000 (1–100, 101–200, 201–300, etc.). Look for increasing patterns when counting forward by 2s, 5s, 10s, 25s, and 100.
- Give students one of the elements of an increasing pattern (not necessarily the first element). Ask students to model as many possible ways to extend the pattern as they can. For example, if the third element is 12, possible solutions could be
4, 8, 12, 16, ...
3, 7, 12, 18, ...
2, 6, 12, 20, ...
6, 9, 12, 15, ...
- Take students on a “Pattern Hunt” identifying increasing patterns in their school environment. Have them use numbers, pictures, and words to describe the patterns they discover.
- Have students add 2, 10 and/or 25 to a number. Ask students to describe what they notice.
- Provide students with the first 3 or 4 elements of an increasing pattern. Have them use appropriate materials to extend and explain the pattern.

SUGGESTED MODELS AND MANIPULATIVES

- coloured tiles
- grid paper
- hundred charts (up to 1000)
- linking cubes
- pattern blocks

MATHEMATICAL LANGUAGE

Teacher	Student
<ul style="list-style-type: none"> ▪ describing, extending, comparing, creating ▪ increasing patterns ▪ pattern rule ▪ starting point, increasing by, ... ▪ term 	<ul style="list-style-type: none"> ▪ increasing patterns ▪ pattern rule ▪ starting point, increasing by, ... ▪ term

Resources/Notes

Print

- *Making Math Meaningful to Canadian Students K–8* (Small 2009), pp. 570–573, 579–581
- *Making Math Meaningful to Canadian Students, K–8, Second Edition* (Small 2013), pp. 608–615
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 57–58, 138, 281–282, 285–288
- *Teaching Student-Centered Mathematics, Grades 3–5* (Van de Walle and Lovin 2006), pp. 293–295

Notes

SCO PR02 Students will be expected to demonstrate an understanding of decreasing patterns by describing, extending, comparing, and creating numerical (numbers to 1000) patterns and non-numerical patterns using manipulatives, diagrams, sounds, and actions.

[C, CN, PS, R, V]

[C] Communication

[PS] Problem Solving

[CN] Connections

[ME] Mental Mathematics and Estimation

[T] Technology

[V] Visualization

[R] Reasoning

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- PR02.01** Identify and describe decreasing patterns.
- PR02.02** Describe a given decreasing pattern by stating a pattern rule that includes the starting point and a description of how the pattern continues.
- PR02.03** Extend a pattern using the pattern rule for the next three terms.
- PR02.04** Compare numeric patterns.
- PR02.05** Identify and explain errors in a given decreasing pattern.
- PR02.06** Create a concrete, pictorial, or symbolic representation of a decreasing pattern for a given pattern rule.
- PR02.07** Create a concrete, pictorial, or symbolic decreasing pattern and describe the pattern rule.
- PR02.08** Solve a given problem using decreasing patterns.
- PR02.09** Identify and describe the strategy used to determine a missing term in a given decreasing pattern.
- PR02.10** Use ordinal numbers (to 100th) to refer to or to predict terms within a decreasing pattern.

Scope and Sequence

Mathematics 2	Mathematics 3	Mathematics 4
—	<p>PR02 Students will be expected to demonstrate an understanding of decreasing patterns by describing, extending, comparing, and creating numerical (numbers to 1000) patterns and non-numerical patterns using manipulatives, diagrams, sounds, and actions.</p>	<p>PR01 Students will be expected to identify and describe patterns found in tables and charts, including a multiplication chart.</p> <p>PR02 Students will be expected to translate among different representations of a pattern, such as a table, a chart, or concrete materials.</p> <p>PR03 Students will be expected to represent, and describe, and extend patterns and relationships, using charts and tables to solve problems.</p>

Background

Patterns are the foundation for many mathematical concepts. They should be taught throughout the year in situations that are meaningful to students, as patterns are embedded in all areas of mathematics. Providing students with the opportunity to discover and create patterns then describe and extend those patterns will result in more flexible thinking across strands. Students should initially describe non-numerical patterns, such as shape, action, sound, and then incorporate numerical patterns by connecting them to the non-numerical patterns.

A large focus in Mathematics 3 is the introduction and development of decreasing patterns. Students use their knowledge of increasing patterns to make connections to the concept of decreasing patterns, since similar understandings are developed. Several of the same tasks that were suggested with work on increasing patterns can be used with modifications to represent decreasing patterns.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Ask students to skip count backward by 10s starting at 123. Ask them to symbolically record the numbers that they say and to describe how the numbers change.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students to show you different ways these decreasing patterns could be extended.
80, 40, ...
925, 825, ...
1000, 500, ...
- Tell students, I am thinking of a decreasing pattern. I have landed on 50. What could I be counting by? Accept any reasonable answer that includes an explanation.
- Ask the student to say a number that is 100 less (10 less) than a 2- or 3-digit number that is provided.
- Give students a decreasing pattern modelled with tiles and ask them to describe, recreate, and extend the pattern in another way.
- Have students identify the pattern rule of the following decreasing patterns and extend the pattern three more terms.
25, 22, 19, 16, ...
24, 20, 16, 14, 10, 6
83, 78, 73, 68, 63

- Have students identify the errors in the following decreasing patterns and correct them.
138, 128, 118, 108, 88, 78
30, 28, 24, 21, 19, 15, 12, 9, 6, 3
40, 35, 29, 25, 20, 15, 10, 5
576, 566, 556, 546, 536, 516, 506, 486

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets 3 (Bauman 2011)

- No Checkpoint for this outcome.

Leaps and Bounds toward Math Understanding 3/4 (Small, Lin, and Kubota-Zarivnij 2011)

- Patterns

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

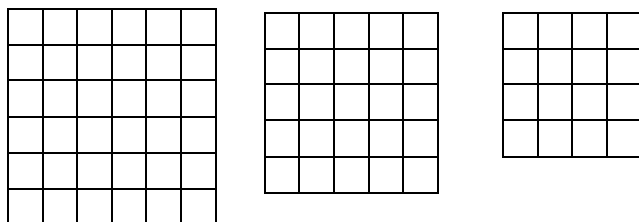
CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Provide students with a variety of concrete and pictorial materials such as linking cubes, colour tiles, or pattern blocks to create and extend decreasing patterns.
- Expect students to discuss and write about how patterns decrease and can be extended.
- Encourage students to identify the attributes of different decreasing patterns (e.g., increases by the same amount). Ask, What is changing in the pattern? What remains the same?
- Have students describe errors or missing elements within a decreasing pattern.
- Expect students to demonstrate their understanding of patterns by representing the same pattern in many different ways—concretely, pictorially, symbolically, orally, rhythmically, and physically.

SUGGESTED LEARNING OPPORTUNITIES

- Provide students with the first three elements of a decreasing pattern, such as the one below,



Ask students to continue the pattern using tiles and then to record the pattern numerically.

- Have students explore hundreds charts to 1000 (1–100, 101–200, 201–300, etc.). Look for patterns when skip counting backward by 2s, 5s, 10s, 25s, and 100.
- Give students one of the elements of a decreasing pattern (not necessarily the first element). Ask students to model as many possible ways to extend the pattern as they can (e.g., if the third element is 12, possible solutions could be 32, 22, 12, 2, ... or 18, 15, 12, 9, ... or 20, 16, 12, 8, ...).
- Take students on a “Pattern Hunt” identifying decreasing patterns in their school environment. Have them use numbers, pictures, and words to describe the patterns they discover.
- Have students repeatedly subtract 2, 5, or 10. Ask students to describe what they notice.
- Provide students with the first three or four elements of a decreasing pattern. Have them use appropriate materials to extend and explain the pattern.

SUGGESTED MODELS AND MANIPULATIVES

- coloured tiles
- linking cubes
- grid paper
- pattern blocks
- hundreds charts (up to 1000)

MATHEMATICAL LANGUAGE

Teacher	Student
<ul style="list-style-type: none"> decreasing patterns describing, extending, comparing, creating pattern rule starting point, decreasing by ... term 	<ul style="list-style-type: none"> decreasing patterns pattern rule starting point, decreasing by ... term

Resources/Notes**Print**

- Making Math Meaningful to Canadian Students K–8* (Small 2009), pp. 570–573, 579–581
- Making Math Meaningful to Canadian Students K–8*, Second Edition (Small 2013), pp. 608–615
- Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 57–58, 138, 281–282, 285–288
- Teaching Student-Centered Mathematics, Grades 3–5* (Van de Walle and Lovin 2006), pp. 293–295

Notes

SCO PR03 Students will be expected to solve one-step addition and subtraction equations involving symbols representing an unknown number.

[C, CN, PS, R, V]

[C] Communication

[PS] Problem Solving

[CN] Connections

[ME] Mental Mathematics and Estimation

[T] Technology

[V] Visualization

[R] Reasoning

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- PR03.01** Explain the purpose of the symbol in a given addition and in a given subtraction equation with one unknown.
- PR03.02** Create an addition or subtraction equation with one unknown to represent a given combination or separate action.
- PR03.03** Provide an alternative symbol for the unknown in a given addition or subtraction equation.
- PR03.04** Solve a given addition or subtraction equation that represents combining or separating actions with one unknown using manipulatives.
- PR03.05** Solve a given addition or subtraction equation with one unknown using a variety of strategies including guess and check.
- PR03.06** Explain why the unknown in a given addition or subtraction equation has only one value.

Scope and Sequence

Mathematics 2	Mathematics 3	Mathematics 4
<p>PR03 Students will be expected to demonstrate and explain the meaning of equality and inequality by using manipulatives and diagrams (0 to 100).</p> <p>PR04 Students will be expected to record equalities and inequalities, symbolically, using the equal symbol or the not equal symbol.</p>	<p>PR03 Students will be expected to solve one-step addition and subtraction equations involving symbols representing an unknown number.</p>	<p>PR05 Students will be expected to express a given problem as an equation in which a symbol is used to represent an unknown number.</p> <p>PR06 Students will be expected to solve one-step equations involving a symbol to represent an unknown number.</p>

Background

In Mathematics 2, students learned the concepts of equality and inequality and the meaning of the symbols = and \neq . This knowledge is extended in Mathematics 3 to solving equations that include symbols that represent unknowns. An equation is a mathematical statement that includes an equal sign and may have been called a number sentence in the earlier grades. The equal sign tells us that the quantity on the left is the same as the quantity on the right. The equal sign is a symbol of equivalence and balance.

The focus of this outcome is to ask students to develop strategies to help them solve equations when there is a symbol representing an unknown number. Students should solve equations of the following forms with the equal sign and/or the unknown symbol in different locations:

$$6 + 3 = \bigcirc$$

$$5 + \diamond = 8$$

$$\Delta + 4 = 24$$

$$8 - 5 = ?$$

$$8 - ? = 3$$

$$\Delta = 12$$

$$\diamond - 15 = 5$$

$$6 = 3 + \Delta$$

$$6 = ? + 5$$

$$\Delta = 16 - 12$$

$$4 + ? = 5 + 7$$

Students may find some of the equations above difficult to solve and need to have many opportunities to explore all of the different forms. It is also very important to read and interpret equations in a meaningful way. In reading $9 + \Delta = 16$ you may say, What do I need to add to 9 to get 16? or If 16 is made up of two parts, and one part is 9, how many are in the other part?

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Ask students to write number sentences using the equal sign (=) and/or the not equal sign (\neq) and then explain their reasoning.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students to find the number that makes each equation true.
 - $5 + \square = 13$
 - $38 = \square + 16$
 - $16 - \square = 7$
 - $\square = 24 - 18$
 - $\square = 6 + 4$
 - $\square - 44 = 25$

Have students explain the strategies they used. Ask, Can there be more than one answer for each? Why or why not?

- Ask, How might you use counters (or another model) to find the number to make this equation true? $\square + 18 = 25$ Have students write a story problem for this equation.
- Have students write the corresponding equation for a word problem and solve it. For example, Gabrielle had some stickers and gave her friend 9. Now she has 8 left. How many did she have at the start? ($\square - 9 = 8$)
- Present students with two numbers and ask them to create equations where one of the numbers is unknown. For example, for 15 and 8 some possible equations are $15 - 8 = \square$, $8 + \square = 15$, $15 = \square + 8$, $\square = 15 - 8$. Ask students to explain what a symbol represents in an equation (e.g., it represents an unknown).
- Show students a mathematics fact that includes a symbol for the missing number. Ask students to record their answers on individual whiteboards or paper. Have students share their strategies for solving the question.
- Tell students that when Amy solved the equation $13 = 7 + \square$, she said that the answer was 20. Is she correct? Explain using models, pictures, numbers, and/or words.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets 3 (Bauman 2011)

- Checkpoint 5, pp. 33–34 (Line Master 5.1 and 5.2)
- Checkpoint 6, pp. 36–37 (Line Master 6.1)

Leaps and Bounds toward Math Understanding 3/4 (Small, Lin, and Kubota-Zarivnij 2011)

- Equality

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Ensure students see and use a variety of symbols representing the unknown.
- Ensure that students use oral and written language to read and interpret equations in a meaningful way.
- Re-emphasize the part-part-whole relationship of addition and subtraction. This will help students solve a variety of equations by thinking of them in a different way (e.g., $12 - \square = 8$ can be thought of as $8 + \square = 12$).
- Provide story structures that involve more than basic fact knowledge in the equations (e.g., $\square + 15 = 36$). Have students explore how to solve for the unknown. They could use the relationship between addition and subtraction to solve equations.
- Have students use models to help solve equations.
- Have students solve equations that originate from word problems. Ensure that students are able to explain how to find the unknown in a variety of equations. Use a variety of forms of equations (e.g., start unknown, change unknown, result unknown).

SUGGESTED LEARNING TASKS

- Ask students to match equations with word problems where the unknown is in different locations. In the following examples, an addition **or** a subtraction equation could be used to represent each problem.
 - Mia has 15 cherries and eats some. Now she has 6. How many did she eat?
 $15 - 6 = \square$ or $\square + 6 = 15$
 - Edmond has 6 hockey cards, but he would like to have 15. How many more does he need?
 $15 - \square = 6$, $6 + \dots = 15$
 - Zane has 15 markers, but 6 of them no longer work. How many does he have that work?
 $6 + \square = 15$, $15 - 6 = \square$
 - Some cookies are on a plate. Six cookies are in a jar making 15 cookies altogether. How many cookies are on the plate?
 $\square + 6 = 15$, $15 - 6 = \square$
 - Have students create problems to represent equations such as the following:
 $4 + 7 = \square$ $\square - 8 = 8$ $\square + 4 = 13$
- Show the students a balance scale using linking cubes to represent an equation and a piece of paper with a question mark on it to represent the unknown. Ask students to record the equation and solve it. Students can replace the paper with linking cubes to help solve the equation or to check their answer.

SUGGESTED MODELS AND MANIPULATIVES

- balance scales
- base-ten blocks
- counters
- linking cubes

MATHEMATICAL LANGUAGE

Teacher	Student
<ul style="list-style-type: none"> ▪ addition and subtraction equation ▪ guess and check ▪ solve ▪ symbol, unknown 	<ul style="list-style-type: none"> ▪ addition and subtraction equation ▪ guess and check ▪ solve ▪ symbol, unknown

Resources/Notes

Print

- *Making Math Meaningful to Canadian Students K–8* (Small 2009), pp. 115, 582, 642–643
- *Making Math Meaningful to Canadian Students K–8*, Second Edition (Small 2013), pp. 170, 620–621, 624–629
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 297–302
- *Teaching Student-Centered Mathematics, Grades 3–5* (Van de Walle and Lovin 2006), pp. 306–312

Notes

Measurement (M)

GCO: Students will be expected to use direct and indirect measure to solve problems.

Specific Curriculum Outcomes

Process Standards

[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

- M01** Students will be expected to relate the passage of time to common activities using non-standard and standard units (minutes, hours, days, weeks, months, years). [CN, ME, R]
- M02** Students will be expected to relate the number of seconds to a minute, the numbers of minutes to an hour, the numbers of hours to a day, and the number of days to a month in a problem-solving context. [C, CN, PS, R, V]
- M03** Students will be expected to demonstrate an understanding of measuring length (cm, m) by
- selecting and justifying referents for the units centimetre or metre (cm, m)
 - modelling and describing the relationship between the units centimetre or metre (cm, m)
 - estimating length using referents
 - measuring and recording length, width, and height [C, CN, ME, PS, R, V]
- M04** Students will be expected to demonstrate an understanding of measuring mass (g, kg) by
- selecting and justifying referents for the units gram and kilogram (g, kg)
 - modelling and describing the relationship between the units gram and kilogram (g, kg)
 - estimating mass using referents
 - measuring and recording mass [C, CN, ME, PS, R, V]
- M05** Students will be expected to demonstrate an understanding of perimeter of regular, irregular, and composite shapes by
- estimating perimeter using referents for centimetre or metre (cm, m)
 - measuring and recording perimeter (cm, m)
 - create different shapes for a given perimeter (cm, m) to demonstrate that many shapes are possible for a perimeter [C, ME, PS, R, V]

SCO M01 Students will be expected to relate the passage of time to common activities using non-standard and standard units (minutes, hours, days, weeks, months, years).

[CN, ME, R]

[C] Communication

[PS] Problem Solving

[CN] Connections

[ME] Mental Mathematics and Estimation

[T] Technology

[V] Visualization

[R] Reasoning

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- M01.01** Select and use a non-standard unit of measure, such as television shows or pendulum swings, to measure the passage of time and explain the choice.
- M01.02** Identify activities that can or cannot be accomplished in minutes, hours, days, weeks, months, and years.
- M01.03** Provide personal referents for minutes and hours.
- M01.04** Select and use a standard unit of measure, such as minutes, hours, days, weeks, and months to measure the passage of time and explain the choice.

Scope and Sequence

Mathematics 2	Mathematics 3	Mathematics 4
M01 Students will be expected to demonstrate an understanding of the calendar and the relationships among days, weeks, months, and years.	M01 Students will be expected to relate the passage of time to common activities using non-standard and standard units (minutes, hours, days, weeks, months, years).	M01 Students will be expected to read and record time using digital and analog clocks, including 24-hour clocks.

Background

Time, as a unit of measurement, presents a unique challenge to students because it cannot be seen. Students need the opportunity to explore and discuss daily activities that involve the passage of time and to make connections to their real-world experiences. Through the use of non-standard or standard units, students will understand that time, as a measurement, is about the duration of an event from beginning to end. Personal referents will allow students to better estimate time. The goal is for students to be able to determine the appropriate unit of time to describe an event.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Ask students to name the days of the week in order beginning with Sunday. Ask them to name the months of the year beginning with January.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Have students describe the duration of something (e.g., physical education class) using their personal referent.
- Ask students to name an activity that takes minutes (hours, weeks, months, or years) to complete.
- Ask students,
 - What is something you can do in a second? In a minute?
 - What is something you can do about 10 times in a minute? In an hour?
- Give students a set of time cards (minutes, hours, days, months, years) and have students hold up the appropriate card to describe the duration of an event said by the teacher or by another student (e.g., recess—student holds up the “minutes” card).

FOLLOW-UP ON ASSESSMENT**Guiding Questions**

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets 3 (Bauman 2011)

- Checkpoint 11, pp. 54–55 (Line Master 11.1)

Leaps and Bounds toward Math Understanding 3/4 (Small, Lin, and Kubota-Zarivnij 2011)

- Time

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Engage students in daily conversations whereby they need to select an appropriate unit of time to describe activities (e.g., Does it take minutes or hours to eat your lunch?).
- Ask students to identify events that take exactly one minute. More than one minute? Less than one minute? This should be extended to other durations of time.
- Have students create their own non-standard unit timers to compare durations, such as plastic water bottles to create a water timer (see *Teaching Student-Centred Mathematics, Grades K–3, Volume 1*, Van de Walle and Lovin, 2006, 242, Fig. 8.14) or a pendulum by using a tennis ball suspended on a long string.
- Discuss the duration of various school events occurring throughout the school day and year.
- Use children’s literature that focuses on durations of time to provide connections for students with this outcome.

SUGGESTED LEARNING TASKS

- Ask students to estimate how many times one can count to ten, while walking heel-to-toe across the classroom. Have the student verify his or her estimate. Ask why another student might get a different result.
- Have students work in pairs to predict which of two specified activities will take longer. One student times the other who is performing the two activities, then the roles are reversed. Activities could include
 - printing your name five times
 - walking the length of the classroom heel to toe
 - making a chain of 25 “links,” paper clips, or linking cubes
 - completing 10 jumping jacks
- Ask students to build a timeline reflecting the time and duration of a sequence of events.
- Ask students to identify and correct errors in your statements about time. For example, it will take me about fifteen days to eat my lunch. My favourite TV show is about one second long.

SUGGESTED MODELS AND MANIPULATIVES

- calendar
- links
- linking cubes
- timeline

MATHEMATICAL LANGUAGE

Teacher	Student
<ul style="list-style-type: none"> ▪ estimate, measure ▪ minutes, hours, days, weeks, months, years ▪ time 	<ul style="list-style-type: none"> ▪ estimate, measure ▪ minutes, hours, days, weeks, months, years ▪ time

Resources/Notes

Print

- *Making Math Meaningful to Canadian Students K–8* (Small 2009), pp. 441–448
- *Making Math Meaningful to Canadian Students K–8*, Second Edition (Small 2013), pp. 487–490
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 242–243, 244
- *Teaching Student-Centered Mathematics, Grades 3–5* (Van de Walle and Lovin 2006), pp. 269–270

Notes

SCO M02 Students will be expected to relate the number of seconds to a minute, the number of minutes to an hour, the number of hours to a day, and the number of days to a month in a problem-solving context.

[C, CN, PS, R, V]

[C] Communication

[PS] Problem Solving

[CN] Connections

[ME] Mental Mathematics and Estimation

[T] Technology

[V] Visualization

[R] Reasoning

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

M02.01 Determine the number of days in any given month using a calendar.

M02.02 Solve a given problem involving the number of seconds in a minute, the number of minutes in an hour, the number of hours in a day, or the number of days in a given month.

M02.03 Create a calendar that includes days of the week, dates, and personal events.

Scope and Sequence

Mathematics 2	Mathematics 3	Mathematics 4
M01 Students will be expected to demonstrate an understanding of the calendar and the relationships among days, weeks, months, and years.	M02 Students will be expected to relate the number of seconds to a minute, the number of minutes to an hour, the number of hours to a day, and the number of days to a month in a problem-solving context.	M01 Students will be expected to read and record time using digital and analog clocks, including 24-hour clocks.

Background

Students will solve problems that relate the number of seconds to a minute, minutes to an hour, and days to a month using calendars and personal events. Although it is not an expectation that students will be assessed on their ability to use a clock to tell time, clocks may be referred to during the learning of this outcome. Students have had previous experiences relating the number of days to a week and months to a year.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Point to a date on the calendar. Ask students to read the date. They should tell you the month, the day of the week, and the date (e.g., Tuesday, April 6).

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students to tell how many
 - seconds in a minute
 - minutes in an hour
 - days in a given month
- Ask students,
 - What is something you can do in a second? In a minute?
 - What is something you can do about 10 times in a minute? In an hour?
- Tell students that
 - Ashram took 90 seconds to run a race and Logan took 3 minutes and ask, Who was faster?
 - it took Julie 125 minutes to drive to her grandparent's house; and ask, How many hours did it take?
- Give students a set of time cards (minute, hour, day, month, year) and have students hold up the appropriate card to describe the duration of an event said by the teacher (e.g., 60 seconds—students hold up the "minute" card; 52 weeks—students hold up the "year" card).
- Show students a calendar for the year and ask them to
 - identify ways in which months are the same and ways in which they differ
 - point out today's date and to find out what the date will be in six weeks
- Have students create a calendar and include their birth date and three other important dates for them.
- Provide students with a calendar for the year. Ask them to find a date that would become a new month in six days. How do they know?

FOLLOW-UP ON ASSESSMENT**Guiding Questions**

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets 3 (Bauman 2011)

- No Checkpoint for this outcome.

Leaps and Bounds toward Math Understanding 3/4 (Small, Lin, and Kubota-Zarivnij 2011)

- Time

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Use a calendar throughout the school year. Engage students in discussions about the number of days in any given month, the cycle of days, and the number patterns within the calendar.
- Ask students to solve problems using the calendar.
- Direct students' attention to the analog clock. Count the seconds aloud with the students to verify it takes 60 seconds for the minute hand to move from one tick mark to the next on an analog clock. Ask students to point to the minute hand, tell them that when the minute hand moves from one tick mark to the next, one minute or sixty seconds has passed.
- Use the calendar to plan, keep track of appointments, and measure time. Focus on the structure of the month and numerical patterns.
- Have students build and create their own monthly calendar. They will need to write the months and the days of the week in order, number the days, and fill in any special dates for that month, such as class trips and physical education days.
- Use children's literature containing references to time to provide connections for students with this outcome.

SUGGESTED LEARNING TASKS

- As a class, watch the second hand on an analog clock count off seconds. Extend this to include watching the second hand count 60 seconds. After these experiences, ask students to estimate the passage of specific amounts of time. For example, students could be asked to raise their hands when they think 10 seconds (30 seconds, 60 seconds) have passed.
- Provide a calendar for the year, and have the students figure out how many school days each month will have. How many Friday 13ths are there in the year? On what days of the week do the birthdays of friends and family fall? Ask the students to write about their findings.
- Ask pairs of students to predict how many weeks there are in a year. Have them use a calendar for the year to check their prediction.

- Show students a calendar for the year. Ask them to point out the day's date and to find out what date it will be in six weeks. Seven weeks?
- Ask students to build a timeline reflecting the time and duration of a sequence of events.
- Create time circles for days of the week and months of the year to demonstrate the cyclical nature of the passage of time.

SUGGESTED MODELS AND MANIPULATIVES

- calendar
- clocks
- linking cubes
- links
- sand timer
- timeline

MATHEMATICAL LANGUAGE

Teacher	Student
<ul style="list-style-type: none">▪ clock, calendar▪ seconds, minutes, hours, days, weeks, months	<ul style="list-style-type: none">▪ clock, calendar▪ seconds, minutes, hours, days, weeks, months

Resources/Notes

Print

- *Making Math Meaningful to Canadian Students K–8* (Small 2009), pp. 441–448
- *Making Math Meaningful to Canadian Students K–8, Second Edition* (Small 2013), pp. 487–490
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 242–243, 244
- *Teaching Student-Centered Mathematics, Grades 3–5* (Van de Walle and Lovin 2006), pp. 269–270

Notes

SCO M03 Students will be expected to demonstrate an understanding of measuring length (cm, m) by

- selecting and justifying referents for the units centimetre and metre (cm, m)
- modelling and describing the relationship between the units centimetre and metre (cm, m)
- estimating length using referents
- measuring and recording length, width, and height

[C, CN, ME, PS, R, V]

[C] Communication

[PS] Problem Solving

[CN] Connections

[ME] Mental Mathematics and Estimation

[T] Technology

[V] Visualization

[R] Reasoning

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

M03.01 Provide a personal referent for one centimetre and explain the choice.

M03.02 Provide a personal referent for one metre and explain the choice.

M03.03 Match a given standard unit to a given referent.

M03.04 Show that 100 centimetres is equivalent to 1 metre by using concrete materials.

M03.05 Estimate the length of an object using personal referents.

M03.06 Determine and record the length and width of a given 2-D shape.

M03.07 Determine and record the length, width or height of a given 3-D object.

M03.08 Draw a line segment of a given length using a ruler.

M03.09 Sketch a line segment of a given length without using a ruler.

Scope and Sequence

Mathematics 2	Mathematics 3	Mathematics 4
<p>M02 Students will be expected to relate the size of a unit of measure to the number of units (limited to non-standard units) used to measure length and mass.</p> <p>M03 Students will be expected to compare and order objects by length, height, distance around, and mass using non-standard units and make statements of comparison.</p> <p>M04 Students will be expected to measure length to the nearest non-standard unit by using multiple copies of a unit and using a single copy of a unit (iteration process).</p> <p>M05 Students will be expected to demonstrate that changing the position of an object does not alter the measurements of its attributes.</p>	<p>M03 Students will be expected to demonstrate an understanding of measuring length (cm, m) by</p> <ul style="list-style-type: none"> ▪ selecting and justifying referents for the units centimetre and metre (cm, m) ▪ modelling and describing the relationship between the units centimetre and metre (cm, m) ▪ estimating length using referents ▪ measuring and recording length, width, and height. 	<p>M03 Students will be expected to demonstrate an understanding of area of regular and irregular 2-D shapes by</p> <ul style="list-style-type: none"> ▪ recognizing that area is measured in square units ▪ selecting and justifying referents for the units cm^2 or m^2 ▪ estimating area by using referents for cm^2 or m^2 ▪ determining and recording area (cm^2 or m^2) ▪ constructing different rectangles for a given area (cm^2 or m^2) in order to demonstrate that many different rectangles may have the same area.

Background

Working with standard units is integral to students' understanding of measurement. Students may start using standard units to measure length when they realize that non-standard units mean different things to different people. For example, if someone says a book is 15 cm long, everyone knows how long that is, but to say the book is 15 cards long would be more difficult to interpret. They need to develop a familiarity with standard units and explore the relationship between them.

Students in Mathematics 3 will be introduced to two basic standard units of length—centimetre and metre. They have had previous experiences with measuring, using non-standard units, the attributes of length, height, and width, which will enable them to measure 3-D objects as well as 2-D shapes.

Estimation in measurement is an essential part of the measurement process that has applications to real-world situations. Estimates are sometimes all that is needed and at other times reassure us about the reasonableness of our answers. Through estimation, students become more familiar with the standard units. It is also engaging for students to challenge themselves to have their estimates as close as possible to the actual measurement.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Ask students to estimate the number of cubes it would take to measure the length of a big book. Provide students with a collection of linking cubes and ask them to measure the length of the big book (multiple copies of a non-standard unit). Then, ask them to measure the length of the big book again using only one cube (single copy of the same unit many times). Ask them to explain the results.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students to estimate the length of a book using a personal referent for centimetres.
- Ask students to estimate the length of the classroom using a personal referent for metres.
- Ask students to cut a length of about 1 m from a ball of string. Have them verify their estimates.
- Ask students to draw a line segment that is about 7 cm long without using a ruler.
- Show students a line segment that is 95 cm and have students estimate its length and then measure it with a ruler.
- Have students use materials to show that a metre is the same as 100 centimetres.
- Provide students with a shoebox or other box and have them measure its length, width, and height.
- Have students use a ruler to measure the length of a pencil or other object without using zero as the starting point.
- Provide students with a photograph and have the student measure the length and width of the picture.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets 3 (Bauman 2011)

- No Checkpoint for this outcome.

Leaps and Bounds toward Math Understanding 3/4 (Small, Lin, and Kubota-Zarivnij 2011)

- Length

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Provide opportunities for students to discover and share their personal referents for centimetres and metres. They should be able to explain their choices and recognize that there are many appropriate referents for each unit.
- Include measurement situations that are of interest to the students and that provide useful information, such as measuring book heights for a new bookcase or determining if a large piece of furniture can fit through the door.
- Have students create their own rulers. Initially numbers should not be included so students need to count the number of units, rather than looking at the number on the ruler. As they become more familiar with its use, numbers can be added.
- Present situations requiring students to choose the most appropriate unit of measure.

SUGGESTED LEARNING TASKS

- Have students relate lengths to their own bodies. For example, "My legs are about half a metre long, my nose is 4 cm long, and 8 of my footprints would make a metre."
- Compare 100 centimetre cubes (base-ten blocks) linked together to a metre.
- "Challenge students to find different ways to measure the same length with one ruler. Start from either end; start at a point not at the end; measure different parts of the object and add the results." (Van de Walle and Lovin, 2006, 233).
- Read the book, *How Big Is a Foot?* by Rolf Myller, and relate the story to why standard units of measurement are valuable. As a follow-up, discuss why it is not a good idea to tell someone how long a table is by using pieces of paper as a measurement unit.
- Have students develop a book on measurement that they can add to over time. This could include drawings of their personal referents, pictures of objects that they have estimated and measured, and descriptions of length, width, and height.
- Give each student a metre-long piece of twine and ask them to use it to measure objects in their homes. Have them make lists of items that are less than one metre, one metre, or a little more than one metre. Have the students enter their findings in a table such as the one shown below.

Less Than One Metre	One Metre	More Than One Metre

- Set up a mini-Olympics in which students compete in events such as a tissue kick, a penny thumb toss, and cotton ball puffing. Have students measure all results to the nearest centimetre or metre, and then record and compare them.
- Ask students to explain how they could use the twine to identify objects that are about half a metre in length.

SUGGESTED MODELS AND MANIPULATIVES

- centimetre cubes
- Cuisenaire rods
- metre sticks
- rulers
- string

MATHEMATICAL LANGUAGE

Teacher	Student
<ul style="list-style-type: none"> ▪ estimate, measure ▪ length, width, height ▪ line segment ▪ personal referent ▪ ruler, straight edge ▪ standard units: centimetre, metre 	<ul style="list-style-type: none"> ▪ estimate, measure ▪ length, width, height ▪ line segment ▪ referent ▪ ruler, straight edge ▪ centimetre, metre

Resources/Notes**Print**

- *Making Math Meaningful to Canadian Students K–8* (Small 2009), pp. 363–368, 376–378
- *Making Math Meaningful to Canadian Students K–8, Second Edition* (Small 2013), pp. 411–414, 423–425
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 223–233
- *Teaching Student-Centered Mathematics, Grades 3–5* (Van de Walle and Lovin 2006), pp. 252–260

Notes

SCO M04 Students will be expected to demonstrate an understanding of measuring mass (g, kg) by			
<ul style="list-style-type: none"> ▪ selecting and justifying referents for the units grams and kilograms (g, kg) ▪ modelling and describing the relationship between the units grams and kilograms (g, kg) ▪ estimating mass using referents ▪ measuring and recording mass 			
[C, CN, ME, PS, R, V]			
[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- M04.01** Provide a personal referent for one gram and explain the choice.
- M04.02** Provide a personal referent for one kilogram and explain the choice.
- M04.03** Match a given standard unit to a given referent.
- M04.04** Explain the relationship between 1000 grams and 1 kilogram using a model.
- M04.05** Estimate the mass of a given object using personal referents.
- M04.06** Measure, using a balance scale, and record the mass of given everyday objects using the units gram (g) and kilogram (kg).
- M04.07** Provide examples of 3-D objects that have a mass of approximately 1 g, 100 g, and 1 kg.
- M04.08** Determine the mass of two given similar objects with different masses and explain the results.
- M04.09** Determine the mass of an object, change its shape, re-measure its mass and explain the results.

Scope and Sequence

Mathematics 2	Mathematics 3	Mathematics 4
<p>M02 Students will be expected to relate the size of a unit of measure to the number of units (limited to non-standard units) used to measure length and mass.</p> <p>M03 Students will be expected to compare and order objects by length, height, distance around, and mass using non-standard units and make statements of comparison.</p>	<p>M04 Students will be expected to demonstrate an understanding of measuring mass (g, kg) by</p> <ul style="list-style-type: none"> ▪ selecting and justifying referents for the units grams and kilograms (g, kg) ▪ modelling and describing the relationship between the units grams and kilograms (g, kg) ▪ estimating mass using referents ▪ measuring and recording mass. 	—

Background

When addressing this outcome, note that the terms **mass** and **weight** are similar, but they are not the same. Weight measures how heavy an object is and depends upon gravity, so it will vary with height above sea level. On the other hand, mass measures the amount of matter in an object and will be the same at all heights above sea level.

As with all measurement units, it is important that students have a personal referent for a gram and a kilogram. Students should recognize which mass unit (gram or kilogram) is appropriate for measuring the mass of a specific item. It is helpful for students to investigate how everyday items, such as food

items, are measured. Include items that are small and dense, such as a golf ball, as well as those that are large and hollow or porous, such as a beach ball. Students need to understand that grams are used to measure very light objects and kilograms are more appropriate units for heavier objects.

Students have had previous experiences investigating mass using non-standard units. They will now begin to estimate and measure masses, using the gram (g) and kilogram (kg). By lifting and holding a variety of objects that have a mass of 1 kg, such as a bag of sugar, they should also develop a sense of what a kilogram feels like.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Provide a set of objects for students. Ask them to measure the mass of each object using non-standard units and then to order them from heaviest to lightest.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask, Could you eat 1 kg of watermelon? 1 kg of popcorn? Have students explain their thinking.
- Have students discuss which unit (g or kg) is more likely to be used in measuring the mass of:
 - a bag of potatoes
 - a box of paper clips
 - an apple
 - a bicycle
- Ask students to draw a picture of an object that they think would have a mass of about 1 kilogram.
- Display a set of five objects of similar size and a sixth target object. Ask students to sort them into groups with masses less than and greater than the target object.
- Provide students with a golf ball and a Ping-Pong ball. Ask if they can tell which has a larger mass by looking at them (comparing the sizes of the two balls). Have them find the mass of the balls.

- Ask students, Do bigger objects always have greater mass than smaller objects? Explain your thinking.
- Have students measure the mass of a ball of modelling clay. Have them use all of the clay to make a new object. Ask them to predict the mass of the new object and verify their prediction.
- Ask students to say which would be a more reasonable estimate for the mass of an adult cat—50 g or 5 kg—and explain their reasoning?
- Provide students with a collection of objects. Ask students to predict which have a mass of about 1 g, 100 g, and 1 kg. Have students explain their choices.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets 3 (Bauman 2011)

- No Checkpoint for this outcome.

Leaps and Bounds toward Math Understanding 3/4 (Small, Lin, and Kubota-Zarivnij 2011)

- Mass

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Have students compare the mass of objects to an established gram, 100 grams and 1 kg mass.
- Have students create masses of 1 g, 100 g, 1 kg (e.g., ask students to fill containers with various materials until they think a mass of 1 kg is reached).
- Have students find common items that are measured in grams and kilograms. Create a classroom display.
- Have students measure mass on a balance scale or other more accurate scales. Bathroom scales can be harder to interpret.

- Ensure students estimate and measure mass, using grams and kilograms as the units.
- Provide situations in which students make comparisons between the masses of two objects; one in grams, the other in kilograms.
- Provide opportunities for students to explore what happens to the mass of the same object if the shape of the object changes.

SUGGESTED LEARNING TASKS

- Have students estimate and then measure the mass of different objects in the classroom.
- Ask students to predict, from a collection of objects, which one has a mass of about 1 kilogram.
- Ask students to choose a small item. Next have the student estimate and determine how many of the items would be required to make a mass of a kilogram.
- Ask students to find something that has the same mass as two bags of marbles.
- Ask students to find the number of potatoes in 2 kg. Ask, Will the number always be the same? Why or why not?
- Have students predict and measure the number of pennies needed for a mass of 100 grams. Repeat with other coins. Ask students how much 1 kg of that coin would be worth.
- Have students measure 20 g of unpopped popcorn. Have students predict if the mass will be greater than, the same as, or less than 20 g after it has been popped? Have students compare how much space is taken up by the popped versus unpopped popcorn.
- Investigate the number of kilograms students could comfortably carry in their backpack or the total number of kilograms of a group of books on a shelf, etc.
- Use balance scales to have students investigate the mass of different kinds of balls (e.g., Ping-Pong ball vs. golf ball).
- Have students write what they know about the relationship between 1000 grams and a kilogram.
- Have the students select a personal referent for 1 g and 1 kg and explain their choice.

SUGGESTED MODELS AND MANIPULATIVES

- base-ten blocks
- kitchen scale
- pan balance or beam balance
- sets of standard weights
- variety of objects to weigh

MATHEMATICAL LANGUAGE

Teacher	Student
<ul style="list-style-type: none"> ▪ balance scale ▪ estimate, measure ▪ mass ▪ personal referent ▪ standard units: gram, kilogram 	<ul style="list-style-type: none"> ▪ balance scale ▪ estimate, measure ▪ mass ▪ referent ▪ gram, kilogram

Resources/Notes

Print

- *Making Math Meaningful to Canadian Students K–8* (Small 2009), pp. 433–435, 437
- *Making Math Meaningful to Canadian Students K–8*, Second Edition (Small 2013), pp. 480–481
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 241–242, 245–248
- *Teaching Student-Centered Mathematics, Grades 3–5* (Van de Walle and Lovin 2006), pp. 268–268, 274–279

Notes

SCO M05 Students will be expected to demonstrate an understanding of perimeter of regular, irregular, and composite shapes by

- estimating perimeter using referents for centimetre or metre (cm, m)
- measuring and recording perimeter (cm, m)
- create different shapes for a given perimeter (cm, m) to demonstrate that many shapes are possible for a perimeter

[C, ME, PS, R, V]

[C] Communication

[PS] Problem Solving

[CN] Connections

[ME] Mental Mathematics and Estimation

[T] Technology

[V] Visualization

[R] Reasoning

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- M05.01** Measure and record the perimeter of a given regular shape and explain the strategy used.
- M05.02** Measure and record the perimeter of a given irregular or composite shape and explain the strategy used.
- M05.03** Construct a shape for a given perimeter (cm, m).
- M05.04** Construct or draw more than one shape for the same given perimeter.
- M05.05** Estimate the perimeter of a given shape (cm, m) using personal referents.

Scope and Sequence

Mathematics 2	Mathematics 3	Mathematics 4
<p>M02 Students will be expected to relate the size of a unit of measure to the number of units (limited to non-standard units) used to measure length and mass.</p> <p>M03 Students will be expected to compare and order objects by length, height, distance around, and mass using non-standard units and make statements of comparison.</p> <p>M04 Students will be expected to measure length to the nearest non-standard unit by using multiple copies of a unit and using a single copy of a unit (iteration process).</p> <p>M05 Students will be expected to demonstrate that changing the position of an object does not alter the measurements of its attributes.</p>	<p>M05 Students will be expected to demonstrate an understanding of perimeter of regular, irregular and composite shapes by</p> <ul style="list-style-type: none"> ▪ estimating perimeter using referents for centimetre or metre (cm, m) ▪ measuring and recording perimeter centimetre or metre (cm, m) ▪ create different shapes for a given perimeter centimetre or metre (cm, m) to demonstrate that many shapes are possible for a perimeter 	<p>M03 Students will be expected to demonstrate an understanding of area of regular and irregular 2-D shapes by</p> <ul style="list-style-type: none"> ▪ recognizing that area is measured in square units ▪ selecting and justifying referents for the units cm^2 or m^2 ▪ estimating area by using referents for cm^2 or m^2 ▪ determining and recording area (cm^2 or m^2) ▪ constructing different rectangles for a given area (cm^2 or m^2) in order to demonstrate that many different rectangles may have the same area

Background

Students should be able to explain that perimeter is the distance around a shape or the length of the boundary of an enclosed region. An understanding that perimeter is not distinct from linear measurement will be key to students' success when exploring perimeter. Students will need to understand that perimeter is the same as measuring linear distance that is not in a straight line. They

should already be proficient using non-standard and standard tools (SCO M03) to measure length. In Mathematics 3, they will develop proficiency with measuring perimeter in standard units (centimetre and metre) using rulers, measuring tapes, metre sticks, and trundle wheels.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Ask students to estimate and measure the distance around the top of their desk using non-standard units such as linking cubes.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Give students regular and irregular, including composite, shapes and have them find the perimeter and explain their strategy. For example,
 - have students construct a shape with a given perimeter using grid paper
 - ask students to construct two different shapes with the same given perimeter using grid paper
- Provide students with a geo-board. Have them create,
 - a rectangle with a perimeter of 12 units
 - a second rectangle of 12 units but with different dimensions
 - a different shape (not a rectangle or triangle) with a perimeter of 12 units
- Ask students to estimate the perimeter of a given shape. Have them measure and record the actual length.
- Ask students to solve the following problem: Farmer Bill has 24 metres of fencing. How many different rectangular chicken coops can he make?
- Provide students with three shapes and ask whether it is possible that they all have the same perimeter. Explain. Have them find the perimeter of each.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets 3 (Bauman 2011)

- No Checkpoint for this outcome.

Leaps and Bounds toward Math Understanding 3/4 (Small, Lin, and Kubota-Zarivnij 2011)

- No Pathway for this outcome.

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Ask students to predict the perimeter prior to making their measurements.
- Provide students with frequent opportunities to construct, measure, and record perimeter of regular and irregular, including composite, shapes.
- Ask students to construct or draw more than one shape for the same given perimeter.
- Use perimeter problem-solving situations that provide a context for students (e.g., border around rooms or bulletin boards, frames, fences, trim).
- Provide many opportunities for students to measure the perimeter of irregular shapes using indirect measure with materials such as a string and ruler.
- Ask students to make comparisons between the perimeter of various shapes and estimate which shapes have a similar perimeter.

SUGGESTED LEARNING TASKS

- Ask students, How can we find the distance around a shape? (Provide regular and irregular shapes).
- Give each group a metre stick, tape measure and a 30-cm ruler, and string. Ask them to figure out how to find the perimeter of shapes around the classroom. Discuss different results.
- Give students pieces of string (different lengths) and ask, How many different objects can you find with a perimeter that is equal to the length of your string?"
- Ask students, How many shapes can you find with a perimeter of 10 cm? 30 cm? 1 m? 3 m?
- Provide students with geo-boards or grid paper and ask, How many different shapes can you make with a given perimeter?
- Tell students, I have drawn a shape in grid paper with a perimeter of 24 cm. What might my shape look like?
- Have students trace the outline of their bodies on a large surface with sidewalk chalk and then estimate and measure the perimeter of their bodies.
- Provide students with a set of 12 pentominoes and have them find the perimeter of each piece. Do all of the shapes have the same perimeter? Which perimeter is the most common?
- Use a trundle wheel to find the perimeter of the gym or playground.

SUGGESTED MODELS AND MANIPULATIVES

- | | |
|---------------------------|-----------------|
| ▪ 30-cm rulers | ▪ pentominoes |
| ▪ base-ten units and rods | ▪ string |
| ▪ geo-boards | ▪ tape measure |
| ▪ grid paper | ▪ trundle wheel |
| ▪ metre sticks | |

MATHEMATICAL LANGUAGE

Teacher	Student
<ul style="list-style-type: none"> ▪ measure, estimate ▪ perimeter, distance around ▪ personal referent ▪ standard units: metre, centimetre 	<ul style="list-style-type: none"> ▪ measure, estimate ▪ perimeter, distance around ▪ referent ▪ metre, centimetre

Resources/Notes**Print**

- *Making Math Meaningful to Canadian Students K–8* (Small 2009), pp. 380–387
- *Making Math Meaningful to Canadian Students Grades K–8, Second Edition* (Small 2013), pp. 427–428
- *Teaching Student-Centered Mathematics, Grades 3–5* (Van de Walle and Lovin 2006), pp. 264–265

Notes

Geometry (G)

GCO: Students will be expected to describe the characteristics of 3-D objects and 2-D shapes and analyze the relationships among them.

GCO: Students will be expected to describe and analyze position and motion of objects and shapes.

Specific Curriculum Outcomes

Process Standards

[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

G01 Students will be expected to describe 3-D objects according to the shape of the faces and the number of edges and vertices. [C, CN, PS, R, V]

G02 Students will be expected to name, describe, compare, create, and sort regular and irregular polygons including triangles, quadrilaterals, pentagons, hexagons, and octagons according to the number of sides. [C, CN, R, V]

SCO G01 Students will be expected to describe 3-D objects according to the shape of the faces and the number of edges and vertices. [C, CN, PS, R, V]			
[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- G01.01** Identify the faces, edges, and vertices of given 3-D objects, including spheres, cones, cylinders, pyramids, cubes and other prisms.
- G01.02** Identify the shape of the faces of a given 3-D object.
- G01.03** Determine the number of faces, edges, and vertices of a given 3-D object.
- G01.04** Sort a given set of 3-D objects according to the number of faces, edges, or vertices.

Scope and Sequence

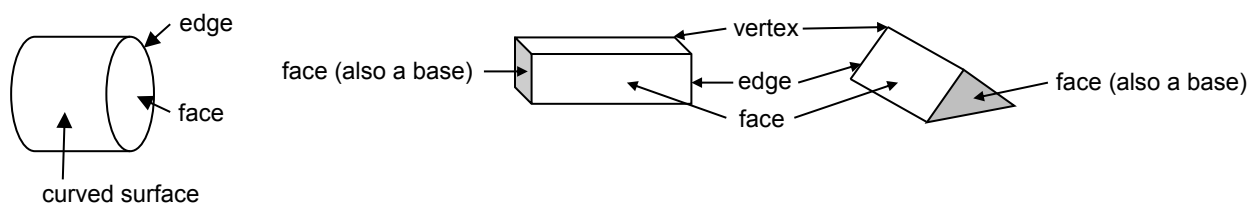
Mathematics 2	Mathematics 3	Mathematics 4
<p>G01 Students will be expected to sort 2-D shapes and 3-D objects using two attributes and explain the sorting rule.</p> <p>G04 Students will be expected to identify 2-D shapes as parts of 3-D objects in the environment.</p>	<p>G01 Students will be expected to describe 3-D objects according to the shape of the faces and the number of edges and vertices.</p>	<p>G01 Students will be expected to describe and construct rectangular and triangular prisms.</p>

Background

Students in Mathematics 2 had experiences identifying, sorting, comparing, describing, and constructing 2-D shapes and 3-D objects. Students in Mathematics 3 will continue to develop their knowledge by describing and sorting 3-D objects according to their geometric attributes. Students will identify the faces, edges, and vertices of 3-D objects including spheres, cones, cylinders, pyramids, and cubes and other prisms.

The geometric attributes of 3-D objects are as follows:

- **Face:** A 2-D shape that forms part of a 3-D object. It is a flat surface that can be traced. Both the shape of the face and the number of faces should be considered as an attribute.
- **Edge:** Occurs where two surfaces of a 3-D object join.
- **Vertex (vertices):** A point where three or more edges meet. (**Note:** On a cone and a pyramid the highest point above the base is called the apex. In a pyramid the apex is also a vertex, but for a cone, it is a mistake to refer to the apex as a vertex as there are no edges that meet.)



Provide students with opportunities to explore these attributes through sorting and constructing activities. As they become more familiar with identifying the attributes, students can determine the number of faces, edges, and vertices. Students will use informal language at this stage rather than precise mathematical language. Students may say “corners” rather than “vertices” and “sides” rather than “faces” for 3-D objects.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students’ prior knowledge.

- Tell students that you have a 3-D object in a bag. One of its faces is round (a circle). Ask what the object could be. Repeat for other 3-D objects and faces of different shapes.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students to describe objects according to their attributes, making sure correct mathematical terms are used for names of objects and faces, edges, and vertices.
- Have 3-D objects sorted by attribute, and ask students to identify the sorting rule.
- Place a triangular prism and triangular pyramid beside one another. Ask students to name them. Ask them to tell you some things that are the same about them and some things that are different.
- Ask students to solve riddles such as, I have 5 faces, 8 edges, and 5 vertices. Who am I? Have students create and solve their own 3-D riddles.
- Have students sort a group of objects according to the number of faces, edges, and vertices.
- Have students identify the shape of the faces of a given 3-D object.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets 3 (Bauman 2011)

- Checkpoint 15, Task 2, pp. 70–71
- Checkpoint 16, Task 2, pp. 73–74

Leaps and Bounds toward Math Understanding 3/4 (Small, Lin, and Kubota-Zarivnij 2011)

- 3-D Objects

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Provide students with concrete models of given 3-D objects (geometric solids or other objects), including cubes and other prisms, spheres, cones, cylinders, and pyramids.
- Identify and sort 3-D objects according to the number of faces, edges, and vertices.
- Use cross-curricular opportunities to explore 3-D objects in art and science classes.
- Ask students to identify particular 3-D objects in their environment and in pictures and to justify their answers.
- Read children’s literature that includes geometry concepts, such as *Sir Cumference and the Sword in the Cone* by Cindy Neuschwander and *The Greedy Triangle* by Marilyn Burns. Discuss.

SUGGESTED LEARNING TASKS

- Say to students, In a bag I have an object that has flat faces, and straight edges. What might this object be? (Other attributes should be used to extend this activity.)
- Tell students, The object behind my back is able to roll. Ask what might it be? (Other attributes should be used to extend this activity.)
- Have students create a mini book about 3-D objects that include a picture of each object and its attributes.
- Ask students to build a wall using 3-D objects. Discuss what 3-D objects could and could not be used.
- Ask students, What can you tell me about a pyramid? A cone? A prism? Have them focus on the attributes.
- Give each group a collection of 3-D objects. Have them sort the objects according to the geometric attributes and provide the sorting rule.
- Have students choose two different 3-D shapes. Have them write and illustrate three ways they are the same, and three ways they are different.
- Ask students to play a game of “Name That 3-D Shape.” Students must determine the object from the clues given.
- Have students make “Wanted” posters for 3-D objects, describing number of faces, edges, and vertices, and shapes of faces.
- Cut out and label pictures from magazines to build a collage and to identify 3-D objects in the environment.

SUGGESTED MODELS AND MANIPULATIVES

- | | |
|--------------------|--------------|
| ▪ geometric solids | ▪ Polydrons |
| ▪ marshmallows | ▪ straws |
| ▪ modelling clay | ▪ toothpicks |

MATHEMATICAL LANGUAGE

Teacher	Student
<ul style="list-style-type: none"> ▪ 3-D objects ▪ apex ▪ attributes ▪ cubes, spheres, cones, cylinders, pyramids, prisms ▪ faces, edges/sides, vertices/corners ▪ flat surface, curved surface 	<ul style="list-style-type: none"> ▪ 3-D objects ▪ cubes, spheres, cones, cylinders, pyramids, prisms ▪ faces, edges/sides, vertices/corners ▪ flat surface, curved surface

Resources/Notes

Print

- *Making Math Meaningful to Canadian Students K–8* (Small 2009), pp. 285–286, 291–292, 303–304
- *Making Math Meaningful to Canadian Students K–8*, Second Edition (Small 2013), pp. 343–344, 347–348, 358–359
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 186–193, 195, 204–207, 217
- *Teaching Student-Centered Mathematics, Grades 3–5* (Van de Walle and Lovin 2006), pp. 220–224, 204–211

Notes

SCO G02 Students will be expected to name, describe, compare, create, and sort regular and irregular polygons, including triangles, quadrilaterals, pentagons, hexagons, and octagons according to the number of sides.

[C, CN, R, V]

[C] Communication

[PS] Problem Solving

[CN] Connections

[ME] Mental Mathematics and Estimation

[T] Technology

[V] Visualization

[R] Reasoning

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

G02.01 Classify a given set of regular and irregular polygons according to the number of sides.

G02.02 Identify given regular and irregular polygons having different dimensions.

G02.03 Identify given regular and irregular polygons having different positions.

Scope and Sequence

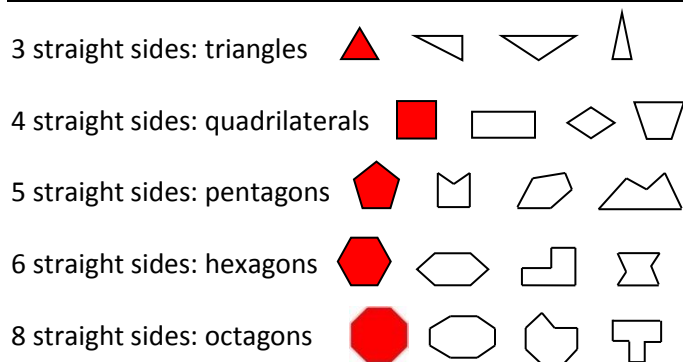
Mathematics 2	Mathematics 3	Mathematics 4
G03 Students will be expected to recognize, name, describe, compare and build 2-D shapes, including triangles, squares, rectangles, and circles.	G02 Students will be expected to name, describe, compare, create, and sort regular and irregular polygons including triangles, quadrilaterals, pentagons, hexagons, and octagons according to the number of sides.	—

Background

Students have had many opportunities to explore 2-D shapes through sorting, patterning, and building activities. Students' previous experiences with describing and comparing polygons included squares, triangles, and rectangles. Students in Mathematics 3 will extend their knowledge to include both regular and irregular polygons.

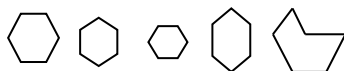
Polygons are closed 2-D shapes with three or more straight sides. They have the same number of sides as angles and are classified by their number of sides. Regular polygons have all equal sides and angles, such as equilateral triangles, squares, and the hexagon pattern blocks. In an irregular polygon, all the sides are not the same length and/or all the angles are not the same size.

Students should focus on comparing the number of sides as the key attribute for classifying polygons. In this outcome, students should be able to name the specific polygons triangle, quadrilateral, pentagon, hexagon, and octagon. In the following diagram, the shaded polygons are regular polygons, and all others are irregular polygons.



Regular polygons are shaded.

Although pattern blocks are frequently used for geometric inquiry, most of these shapes are regular. Students may develop the misconception that only certain familiar polygons meet the criteria for these shapes. For example, students may not initially recognize all of the following shapes as hexagons.



Ask students to find examples of polygons in the world around them, perhaps even collect as many types of a shape as they can find. Sort the shapes according to the number of sides. By sorting polygons according to the number of sides, students can learn the names for the polygons.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Show students a set of triangles in various sizes and positions. Tell them that one student thinks these shapes are all triangles, but another student doesn't agree. Ask them to explain who is correct.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Provide students with a sheet that includes a number of different polygons (regular and irregular) that are different sizes, forms, and/or orientations. Have the students sort and name the polygons. Observe that the students recognize the same shape in different positions or orientations.
- Have students create two different pentagons (or other polygons) on a geo-board.
- Provide students with pattern blocks. Have them create new polygons by using two blocks (equal sides should be matched) and trace the shape of the new polygon. Have them write the type of polygon they created.
- Have students explain how an octagon and a hexagon (or other shapes) are similar and different.
- Show students two groups of sorted polygons. Ask, What might the sorting rule have been? Include different types of polygons that are regular and irregular and different sizes.
- Ask students, If you draw a pentagon (or another shape) and your friend draws a pentagon, will the two shapes look exactly the same? Why or why not? What will be the same every time?

FOLLOW-UP ON ASSESSMENT**Guiding Questions**

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets 3 (Bauman 2011)

- No Checkpoint for this outcome.

Leaps and Bounds toward Math Understanding 3/4 (Small, Lin, and Kubota-Zarivnij 2011)

- 2-D Shapes

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Ask questions that focus on the attributes of polygons. For example, What other shapes look like this one? In what way are the shapes alike? In which ways are they different?
- Provide opportunities for students to develop their own definitions for the different types of polygons. Have students sort shapes such as those included as black line masters in *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006). Have students explain their reasoning to others.
- Have students create different polygons on geo-boards or dot paper. Challenge the students to create different types of triangles (quadrilaterals, pentagons, etc.).
- Have students create a book for a particular polygon. Include a variety of examples (regular and irregular) for each type of polygon included in this outcome. This could be extended to include other types of polygons.
- Use geo-strips or strips of paper of different lengths to create various polygons.
- Use children’s literature, such as *The Greedy Triangle* by Marilyn Burns to further explore the attributes of polygons.
- Integrate art activities using these shapes. For example, create a piece of art using only a single 2-D shape, but change the other attributes (e.g., size, orientation, length of sides, colour).

SUGGESTED LEARNING TASKS

- Ask students to make a triangle on a geo-board that has two pegs inside, then one that has three. Ask, What is the greatest number of pegs that can be inside a triangle on a geo-board? Repeat this activity with other shapes.
- Have students sort a collection of pattern blocks by the type of polygon.
- Provide students with sets of tangrams and pentominoes. Have them sort the shapes into triangles, quadrilaterals, pentagons, hexagons, and octagons. (**Note:** There are no pentagons and there are some shapes that have more than 8 sides.)
- Have groups of students create a “path of polygons” using sidewalk chalk on a large surface and drawing a sequence of different polygons.
- Ask students to create riddles about a chosen polygon. Riddles can be exchanged for other students to solve.
- Ask students to explain why it is possible to have a polygon with more than eight sides but it is not possible to have an octagon with more than eight sides.
- Have students complete a concept map or Frayer model for a given polygon.

SUGGESTED MODELS AND MANIPULATIVES

- dot paper
- geo-board
- geo-strips
- pattern blocks
- pentominoes
- tangrams

MATHEMATICAL LANGUAGE

Teacher	Student
<ul style="list-style-type: none"> ▪ dimension, size ▪ position ▪ regular, irregular polygons ▪ triangles, quadrilaterals, pentagons, hexagons, octagons ▪ turning (rotating), flipping (reflection), sliding (translations) 	<ul style="list-style-type: none"> ▪ sizes ▪ polygons ▪ triangles, quadrilaterals, pentagons, hexagons, octagons ▪ turning, flipping, sliding

Resources/Notes

Print

- *Making Math Meaningful to Canadian Students K–8* (Small 2009), pp. 287–288, 292–296
- *Making Math Meaningful to Canadian Students K–8*, Second Edition (Small 2013), pp. 343–344, 348–352
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 186–193, 193–195, 196–199, 202–204, 206–208
- *Teaching Student-Centered Mathematics, Grades 3–5* (Van de Walle and Lovin 2006), pp. 204–211, 211–214, 220–222, 225–226

Notes

Statistics and Probability (SP)

GCO: Students will be expected to collect, display, and analyze data to solve problems.

GCO: Students will be expected to use experimental or theoretical probabilities to represent and solve problems involving uncertainty.

Specific Curriculum Outcomes

Process Standards

[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

SP01 Students will be expected to collect first-hand data and organize it using tally marks, line plots, charts, and lists to answer questions. [C, CN, V]

SP02 Students will be expected to construct, label, and interpret bar graphs to solve problems. [PS, R, V]

SCO SP01 Students will be expected to collect first-hand data and organize it using tally marks, line plots, charts, and lists to answer questions.

[C, CN, V]

[C] Communication

[PS] Problem Solving

[CN] Connections

[ME] Mental Mathematics and Estimation

[T] Technology

[V] Visualization

[R] Reasoning

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- SP01.01** Record the number of objects in a given set using tally marks.
- SP01.02** Determine the common attributes of line plots by comparing line plots in a given set.
- SP01.03** Organize a given set of data using tally marks, line plots, charts, or lists.
- SP01.04** Collect and organize data using tally marks, line plots, charts, and lists.
- SP01.05** Answer questions arising from a given line plot, chart, or list.
- SP01.06** Answer questions using collected data.

Scope and Sequence

Mathematics 2	Mathematics 3	Mathematics 4
SP01 Students will be expected to gather and record data about self and others to answer questions.	SP01 Students will be expected to collect first-hand data and organize it using tally marks, line plots, charts, and lists to answer questions.	SP01 Students will be expected to demonstrate an understanding of many-to-one correspondence.

Background

Students will be given opportunities to collect, organize, and display data to answer questions. Previously, students have constructed concrete graphs and pictographs to answer questions and solve problems. In Mathematics 3, students will use tally marks, lists, charts, line plots, and bar graphs to organize data relevant to their everyday life. Opportunities for collecting data to answer questions should naturally occur throughout the year. These opportunities may include putting students' names on a birthday chart to display in the classroom, deciding on a lunchtime activity, comparing shoe sizes, or comparing bedtimes. When working with data, students discover not only answers to questions, but meaningful information that can evoke change in their world. The expectation is that students will collect, organize, and display data to answer questions.

At this level, students should be encouraged to become more independent in the selection of appropriate strategies for collecting and organizing data. For example, ask pairs of students to decide on the strategy they will use to collect and organize data that will show interesting information about classmates.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

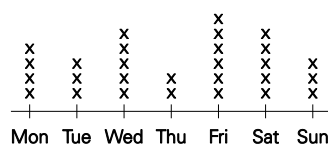
Tasks such as the following could be used to determine students' prior knowledge.

- Provide students with a collection of about 25 linking cubes in three or four different colours. Ask them to organize the cubes and record the date in a chart using tally marks or another method. Ask them to write two questions that the tally marks would answer.

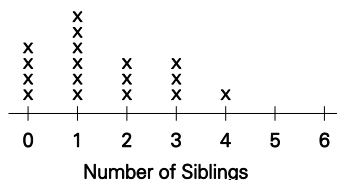
WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students to select a topic, survey family members and/or neighbours, and present their findings to the class in an organized chart, list, or line plot.
- Ask the students to keep track of weather conditions over the period of one month and to design a way to present the information in an organized chart, list, or line plot.
- Show students a line plot such as the one below and ask what it may represent.



- Ask students how they would represent the sports the children in their class play and how many students play each sport?
- Show students the following line plot and ask questions such as, What is the most common number of siblings? How many students have two siblings or less? How many students have four siblings? (Ensure students know that siblings are what their brothers and sisters are called.)



- Show students a set of data presented in chart form. Ask them to represent the data in another way, such as tally marks or a line plot.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets 3 (Bauman 2011)

- Checkpoint 17, p. 77
- Checkpoint 18, pp. 79–80 (Line Master 18.1)

Leaps and Bounds toward Math Understanding 3/4 (Small, Lin, and Kubota-Zarivnij 2011)

- Sorting and Organizing Data

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

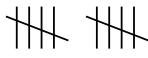
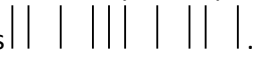
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Have pairs of students decide on the procedure they will use to collect and display data using tally marks, line plots, charts, or lists showing interesting information about class members.
- Have students plan and conduct an in-class survey about a favourite _____ (e.g., toy, television program, hockey player). Ask them to present the results of the survey in an organized chart or list.
- Have students conduct a survey to find out what objects 8- and 9-year-olds like to collect. They will need to decide who to survey and how to organize and present their data.
- Ask small groups of students to brainstorm an interesting question for a possible survey. Then, have each group conduct their survey, and collect and organize their data.

SUGGESTED LEARNING TASKS

- Ask students why it is easier to count the “yes” responses when they are shown like this  rather than like this .
- Have students collect, record, and organize data in a line plot, chart, tally marks, or list to describe the favourite books of their classmates (or other relevant topic).
- Model recording a set of data in a line plot, list, and chart format and discuss the advantages and disadvantages of each type of data display.
- Collect and display data that represent
 - accomplishments of favourite sport figures or friends (e.g., the number of goals, hits, points)
 - the distance class members can throw a ball
 - mass of various fruits or vegetables
 - mass of subject textbooks
 - mass of different breeds of dogs
- Give students a list of questions and have them identify the questions that might be used for a particular graph or set of data.
- Ask students to describe what they would expect to find in a “well-made” line plot.
- Show students an organized list of first-hand data and have them pose relevant questions about the data.

SUGGESTED MODELS AND MANIPULATIVES

- craft sticks
- linking cubes
- paper clips

MATHEMATICAL LANGUAGE

Teacher	Student
<ul style="list-style-type: none"> ▪ collect, organize, display, interpret data ▪ first-hand data ▪ tally marks, line plots, charts, lists ▪ title, label, horizontal axis, dots, crosses 	<ul style="list-style-type: none"> ▪ collect, organize, display ▪ tally marks, line plots, charts, lists ▪ title, label, horizontal axis, dots, crosses

Resources/Notes

Print

- *Making Math Meaningful to Canadian Students K–8* (Small 2009), pp. 471–473, 484, 525–528, 530–532
- *Making Math Meaningful to Canadian Students K–8*, Second Edition (Small 2013), pp. 517–518, 568–574, 530
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 310–312, 317–319, 322, 329–330
- *Teaching Student-Centered Mathematics, Grades 3–5* (Van de Walle and Lovin 2006), pp. 320–322, 329–330, 333

Notes

SCO SP02 Students will be expected to construct, label, and interpret bar graphs to solve problems. [PS, R, V]			
[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- SP02.01** Determine the common attributes, title, and axes of bar graphs by comparing bar graphs in a given set.
- SP02.02** Create bar graphs from a given set of data including labelling the title and axes.
- SP02.03** Draw conclusions from a given bar graph to solve problems.
- SP02.04** Solve problems by constructing and interpreting a bar graph.

Scope and Sequence

Mathematics 2	Mathematics 3	Mathematics 4
SP02 Students will be expected to construct and interpret concrete graphs and pictographs to solve problems.	SP02 Students will be expected to construct, label, and interpret bar graphs to solve problems.	SP02 Students will be expected to construct and interpret pictographs and bar graphs involving many-to-one correspondence to draw conclusions.

Background

A bar graph is a useful tool for organizing data. Students will explore both vertical and horizontal bar graphs, making the connection that the height or length of the bars represents a number. Ensure that all graphing activities are based on one-to-one correspondence. Initially, students will focus on reading and interpreting given bar graphs. Spacing the bars on a graph is necessary because the data is discrete, that is, the data represents separate categories. When reading a bar graph, ask students to use a ruler, index card, or finger to accurately find the number on the appropriate axis that aligns with the top or end of each bar. Once they have explored reading and interpreting given bar graphs, they will then construct their own. Once students have constructed a bar graph, they should make observations about the data and interpret the data to answer questions. They should also be given experiences answering questions about the data displayed on other bar graphs found in newspapers, magazines, and television or on the Internet.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Provide students with a set of data about a topic of interest to them. Ask them to create a pictograph and a concrete graph that represents the data.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Show students a bar graph on a topic of interest to students. Have them answer questions about the graphs and have them make up questions about the graph.
- Provide students with data. Have them construct a bar graph on grid paper. Ensure that students include a title, and labels on both axes.
- Ask students, What would happen if the bars in a graph were rearranged? Would the graph still give you the same information? Explain.
- Have students answer the following: This is a graph of a survey I did with my Mathematics 3 class. What might the survey be about? Label the graph, make up a title, and then, make up three questions that could be answered with this graph.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets 3 (Bauman 2011)

- Checkpoint 18, pp. 79–80 (Line Master 18.2)

Leaps and Bounds toward Math Understanding 3/4 (Small, Lin, and Kubota-Zarivnij 2011)

- Displaying Data

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Emphasize the use of real data when constructing graphs.
- Use grid paper to ensure bar graphs are as accurate as possible.
- Determine common attributes of bar graphs by examining examples from various sources.
- Make use of opportunities to integrate graphing concepts in other areas, such as science, morning message, social studies, etc.

SUGGESTED LEARNING TASKS

- Provide several bar graphs. Have students compare and determine the common attributes, making sure title, axes, and labels are included.
- Provide several bar graphs. Have students draw conclusions and answer questions about the graphs.
- Provide opportunities for students to match created bar graphs with data organized in charts, lists, tally marks, or line plots.
- Ask students to create a bar graph to show the kinds of pets students in the class have at home. Have them write two questions about their graph.
- Create a bar graph for a set of data on a grid on the floor.
- Provide students with a real-life problem to solve such as, What game should we play in physical education? or What special activity should be at the Celebration Assembly? or What book should be read during literacy time? Create a bar graph from collected data, and use it to make decisions or solve problems.
- Give students a bar graph that has been created, but does not contain labels or title. Ask students to explain why the graph is difficult to read and interpret.

SUGGESTED MODELS AND MANIPULATIVES

- grid paper
- pre-made bar graphs

MATHEMATICAL LANGUAGE

Teacher	Student (oral language)
<ul style="list-style-type: none"> ▪ bar graph, bar(s) ▪ data, collect, organize, display, interpret ▪ title, labels, scale, axis, axes ▪ vertical, horizontal 	<ul style="list-style-type: none"> ▪ bar graph, bar(s) ▪ data, collect, organize, display ▪ title, labels, scale, axis, axes ▪ vertical, horizontal

Resources/Notes**Print**

- *Making Math Meaningful to Canadian Students K–8* (Small 2009), pp. 474–478, 480–481
- *Making Math Meaningful to Canadian Students K–8, Second Edition* (Small 2013), pp. 526–529
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 317–318, 319–321
- *Teaching Student-Centered Mathematics, Grades 3–5* (Van de Walle and Lovin 2006), pp. 329–331

Notes

Appendices

Appendix A:

Performance Indicator Background

Number (N)

<p>SCO N01 Students will be expected to say the number sequence forward and backward by</p> <ul style="list-style-type: none"> ▪ 1s through transitions to 1000 ▪ 2s, 5s, 10s, or 100s, using any starting point to 1000 ▪ 3s, using starting points that are multiples of 3 up to 100 ▪ 4s, using starting points that are multiples of 4 up to 100 ▪ 25s, using starting points that are multiples of 25 up to 200 			
[C, CN, ME]			
[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- N01.01** Extend the number sequence by 1s, particularly through transition from decade to decade and century to century.
- N01.02** Extend a given skip counting sequence by 2s, 5s, 10s, or 100s, forward and backward, using a given starting point.
- N01.03** Extend a given skip counting sequence by 3s, forward and backward, starting at a given multiple of 3 up to 100.
- N01.04** Extend a given skip counting sequence by 4s, forward and backward, starting at a given multiple of 4 up to 100.
- N01.05** Extend a given skip counting sequence by 25s, forward and backward, starting at a given multiple of 25 up to 200.
- N01.06** Identify and correct errors and omissions in a given skip counting sequence.
- N01.07** Determine the value of a given set of coins (nickels, dimes, quarters, and loonies) by using skip counting.
- N01.08** Identify and explain the skip counting pattern for a given number sequence.

Performance Indicator Background

N01.01 When exploring numbers over 100, spend considerable time focusing on the numbers between 100 and 200. Students need to think about how the counting pattern sounds when they count from 20 to 29 and 30 to 39 and apply it when counting from 120 to 129 and 130 to 139, and when counting through all the decades that follow. Students need multiple opportunities to count the numbers through transition from decade to decade and century to century. For example, when counting from 98—98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, ... a common misconception for students is to think that the next number after the one that ends in “9” is the next big number name (29 → 30; 49 → 50). As a result many believe that 200 comes after 109. Students need to see that this same transition will apply for counting into all of the centuries. For example, on a regular basis, they should be asked questions such as, what would come after 199? or count on from 389.

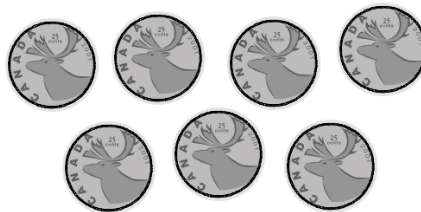
N01.02, N01.03, N01.04, and N01.05 Students in Mathematics 2 skip-counted by 2s forward and backward to 100. In Mathematics 3, students will skip count forward and backward by 5s, 10s, and 100s from any given number; by 3s and 4s, starting at any multiple of 3 and 4 respectively up to 100; and by 25s, starting at any multiple of 25 up to 200. Being able to skip-count will help students be successful with many upcoming concepts such as patterns, money, and place value. Number charts may be used to explore these patterns.

Students should also investigate these skip-counting patterns and whether the similar patterns occur when counting by 100s or when counting on hundreds charts beyond 100. A specific focus on looking at the numbers between 100 and 200 can help students to develop a sense of the repeating patterns in the number system. A hundred chart can easily be extended to a 200 chart by adding rows for the each of the next 100 numbers.

Ask students to use number lines and hundreds charts to skip count forward or backward by 4s starting at different multiples of 4. Ask them to record their jumps on the number line or colour in the number they jump to on the hundred charts.

N01.06 and N01.08 Students should be able to identify skip-counting patterns and any errors and omissions that may occur in these patterns, whether presented forward or backward. Using number lines and hundreds charts are effective visuals to help students identify the patterns and the errors or omissions. For example, display a part of a hundreds chart with numbers missing in a skip-counting sequence. Ask students to identify the skip-counting pattern and to fill in the missing numbers.

N01.07 Students will skip-count to determine the value of a group of one type of coins, such as nickels, dimes, quarters, and loonies. For example, the following set of quarters could be used to count 25, 50, 75, 100, 125, 150, 175. **Note:** The final count is recorded and said as a whole number, 175 cents; not using decimal notation.



SCO N02 Students will be expected to represent and partition numbers to 1000.

[C, CN, V]

[C] Communication

[PS] Problem Solving

[CN] Connections

[ME] Mental Mathematics and Estimation

[T] Technology

[V] Visualization

[R] Reasoning

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- N02.01** Read a given three-digit numeral without using the word **and**.
- N02.02** Read a given number word (0 to 1000).
- N02.03** Represent a given number as an expression.
- N02.04** Represent a given number concretely and pictorially in a variety of ways.
- N02.05** Write number words for given multiples of ten to 90.
- N02.06** Write number words for given multiples of a hundred to 900.
- N02.07** Record numerals for numbers expressed orally, concretely, or pictorially.

Performance Indicator Background

N02.01 and N02.02 The number 205 is read, two hundred five. Students must be able to record numbers heard and read numbers written symbolically. When reading numbers, the word **and** is reserved for the decimal, which will be discussed in Mathematics 4.

N02.03 Students who have a deep understanding of numbers up to 1000 will be able to represent numbers in a variety of ways. It is important to model the correct use of the term **expression** to students. An expression names a number. Sometimes an expression is a number such as 150. Sometimes an expression shows an arithmetic operation, such as $125 + 25$. 150 may also be represented by its partitions, such as $80 + 70$, $100 + 50$, and $50 + 50 + 50$. Numbers can also be represented by a difference expression, such as $175 - 25$.

N02.04 It is important that students see the numbers up to 1000 in different ways in order to realize a number can cover a big area or a small area, depending on the size of the items being counted. Provide opportunities for students to use hundreds charts and collections of materials such as straws, buttons, commercial counters, kidney beans, and paper clips to represent given numbers. Students will decide on various ways to count the objects, perhaps grouping them in tens and/or hundreds, then presenting their numbers in pictures.



325 represented pictorially with buttons

Students should recognize that 1000 is just another expression for ten hundreds or 100 tens.

N02.05, N02.06, and N02.07 Students will also need to be able to write the number words for the multiples of ten to ninety (twenty, thirty, forty, ...) and multiples of a hundred to nine hundred (two hundred, three hundred, ...). It should also be noted that when writing one thousand symbolically, there is no space or comma between the thousands and hundreds place; 1000 not 1 000, nor 1,000. We do not use the comma because in many countries using the metric system, the comma is used as the decimal point.

SCO N03 Students will be expected to compare and order numbers to 1000. [CN, R, V]			
[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

Performance Indicators

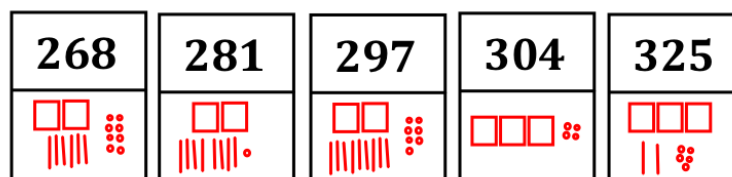
Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- N03.01** Place a given set of numbers in ascending or descending order and verify the result using a number chart or other models.
- N03.02** Create as many different 3-digit numerals as possible, given three different digits. Place the numbers in ascending or descending order.
- N03.03** Identify errors in a given ordered sequence.
- N03.04** Identify missing numbers in parts of a given number chart and on a number line.
- N03.05** Identify errors in a given number chart and on a number line.
- N03.06** Place numbers on a number line containing benchmark numbers for the purpose of comparison.
- N03.07** Compare numbers based on a variety of methods, and record the comparison using words and symbols (=, > and <).

Performance Indicator Background

N03.01 Students should be given opportunities to place a given set of numbers in ascending or descending order. For example, a student may be given six or eight base-ten cards with different amounts on them and be asked to sort them from least to greatest, or vice versa. It is important for teachers to mix up the amounts so that the sets do not always represent consecutive numbers. Students could also be given cards with numbers on them and be asked to model and sort the numbers. This type of task reinforces modelling while providing an opportunity to put numbers in order.

The student could verify the order by looking for the numbers on hundreds charts or by drawing a number line.



N03.02 Given any three digits, students should make as many 3-digit numbers as they can, then put them in order from least to greatest. For example, if given cards containing the digits 3, 5, and 2, students can make 235, 253, 325, 352, 523, and 532. Students should be able to explain how they determined all possible numbers and how they ordered them. Students should also be able to arrange them from greatest to least or should be able to place the numbers on an open number line.

N03.03 Students should be able to identify when a given sequence of numbers is not in the correct order and be able to correct it by rearranging them. They should be encouraged to talk about how they made their corrections.

N03.04 and N03.05 Students should have enough familiarity with the hundreds chart that they are able to identify errors or the values of missing numbers. A student could be given a hundreds chart with numbers missing and be asked to fill in the missing values and to explain how they decided what number went in each empty position.

N03.06 When comparing two numbers, students should be encouraged to make use of benchmarks. Students should say that 48 is less than 95 since both numbers are to the left of 100 on a number line but only 48 is to the left of 50. Similarly, 37 is greater than 27 since 37 is to the right of 30 and 27 is to the left of 30 on a number line. This reasoning process is part of having number sense. Students will often refer to the number of tens in a number in order to compare it to another; for example, 47 is greater than 21 since 47 is more than 4 tens, but 21 is only a bit more than 2 tens. This type of language is preferable to 4 is more than 2 so 47 is greater, particularly since students should focus on the fact that the 4 in 47 represents 40, not 4, and the 2 in 21 represents 20, not 2. This work should be connected to the use of base-ten materials, number lines, and hundreds charts.

N03.07 An understanding of place value (explored in greater depth in SCO N05) is essential for students to compare and order numbers. For example, to compare 667 and 607, students should notice that both numbers have 6 hundreds, but that the 667 is greater than 607 because it has more tens in the tens place. The numbers could also be compared by considering their relative position in the counting sequence: 667 comes after 607, so 667 is greater than 607. Students should be able to compare two or more numbers, each less than 1000, to determine relative sizes. Include situations in which numbers are located on hundreds charts and number lines. When numbers are represented in their standard or symbolic form, students can use the number of digits to get a sense of their size in order to compare them. Three-digit numbers are less than a 1000 but greater than any 2-digit number. SCO N05 should be done before or in conjunction with comparing 3-digit numbers.

SCO N04 Students will be expected to estimate quantities less than 1000 using referents. [ME, PS, R, V]			
[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- N04.01** Estimate the number of groups of ten in a given quantity using 10 as a referent (known quantity).
- N04.02** Estimate the number of groups of a hundred in a given quantity using 100 as a referent.
- N04.03** Estimate a given quantity by comparing it to a referent.
- N04.04** Select an estimate for a given quantity by choosing among three possible choices.
- N04.05** Select and justify a referent for determining an estimate for a given quantity.

Performance Indicator Background

N04.01, N04.02, and N04.03 Students use reasoning skills to estimate a total using a visual referent. Students may create a mental picture (visualization) of an amount and use that picture to estimate a total. This process of making connections between similar visuals will lead to greater proportional understanding and reasoning. It is essential that students develop referents in order to be effective at estimating.

For example, knowing how much 10 stars is, helps students to estimate the quantity in the larger group of stars.



Students must build on strategies developed in earlier grades involving ten to include a sense of one hundred. Using their knowledge of one hundred, students can then estimate larger quantities. For example, a bag of one hundred counters may be used to determine how many counters are in a larger pile, by estimating how many groups of 100 are in the pile.

N04.04 Students should be able to select an estimate for a given quantity by choosing from among three possible choices. For example, show students a collection of a certain object and have them choose the best estimate for the quantity from three possible choices, and record it. Objects such as beans, raisins, toothpicks, or wooden stir sticks could be used. After recording their estimates, students should begin counting the objects. As they count to certain benchmarks, allow students to change their estimates.

N04.05 Through the process of selecting and using referents, students will be able to justify a referent for determining an estimate for a given quantity. For example, when asked to determine the number of jellybeans in a jar, students select a useful visual referent, such as the number of jellybeans on the top layer, to determine a reasonable estimate.

SCO N05 Students will be expected to illustrate, concretely and pictorially, the meaning of place value for numerals to 1000. [C, CN, R, V]			
[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- N05.01** Record, in more than one way, the number represented by given proportional and non-proportional concrete materials in traditional and non-conventional formats.
- N05.02** Represent a given number in different ways using proportional and non-proportional concrete materials and explain how they are equivalent; e.g., 351 can be represented as three 100s, five 10s, and one 1s; or two 100s, fifteen 10s and one 1s; or three 100s, four 10s, and eleven 1s.
- N05.03** Record a given number in additive expanded form.
- N05.04** Record a number represented by base-ten blocks arranged in a non-conventional format.

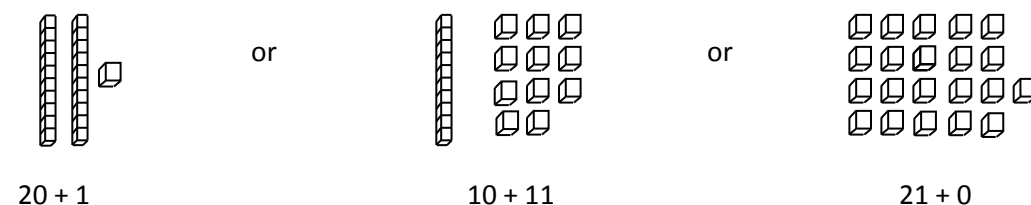
Performance Indicator Background

N05.01 Students should be given many opportunities to record numbers. When provided with a model representation, students need to be able to record numbers in more than one way for the given model. For example, a model or picture of 2 flats, 3 rods, and 4 small cubes can be recorded as, 234; 200, 30, and 4; or 2 hundreds, 3 tens, 4 ones.

It is important to spend time developing a good understanding of the meaning and use of zero in numbers. Students need many experiences using base-ten materials to model numbers with zeros as digits. Teachers should ask students to write the numerals for numbers such as three hundred forty and nine hundred eight. When you write a number in its symbolic form using digits, you call the digit 0 a place holder. If you did not have the digit 0, the number would be recorded as 32, and you would mistakenly think that the 3 represented 30 instead of 300. Students need many experiences using base-ten materials to make connections with the symbols for numbers with zeros as digits.

N05.02 Students must have a deep understanding of numbers up to 1000 and be able to rename numbers in a variety of ways. For example, 842 is the same as 84 tens and 2 ones; 8 hundreds and 42 ones; 8 hundreds, 4 tens, and 2 ones; or 7 hundreds, 14 tens, and 2 ones. Provide opportunities for students to represent each digit in a 3-digit number, using concrete materials, explaining the value of each digit.

Students should come to understand that the position of a digit determines its value. Students should recognize and work with the idea that the value of a digit varies, depending on its position or place, in a numeral. Another area of place value that may cause some confusion for students is that one number, such as 21, can be represented in a variety of ways.



N05.03 Once students have ample opportunities with concrete, pictorial, and verbal representations of base-ten models, they can record the base-ten partitions in an expression such as 256 is $200 + 50 + 6$. This is known as the additive expanded form.

SCO N06 Students will be expected to describe and apply mental mathematics strategies for adding two 2-digit numerals.

[C, ME, PS, R, V]

[C] Communication

[PS] Problem Solving

[CN] Connections

[ME] Mental Mathematics and Estimation

[T] Technology

[V] Visualization

[R] Reasoning

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N06.01 Explain mental mathematics strategies that could be used to determine a sum.

- Ten and some more
- Tens and some more
- Quick addition
- Addition facts to 10 applied to multiples of 10
- Addition on the hundred chart
- Adding on
- Make ten
- Compensation
- Compatible numbers

N06.02 Use and describe a personal strategy for determining a sum.

N06.03 Determine a sum of two 2-digit numerals efficiently, using mental mathematics strategies.

Performance Indicator Background

N06.01 and N06.02 Students will develop, apply, and describe mental mathematics strategies to add two 2-digit numbers.

Ten and Some More

Most students know that teen numbers can be made with a ten and some more even with limited knowledge of place value. They need to experience finding sums with a 10 and a single-digit number to be convinced they do not need to count on: the answer is automatic. This strategy should be reinforced before fact-strategy learning is undertaken. For example, for $10 + 5$, think: 10 and 5 more makes 15.

Tens and Some More

As students develop understanding of place value, they realize that the answers to the addition of single-digit numbers to multiples of 10 (20, 30, 40, ..., 90) are as easy as adding single digits to 10. Answers can be quickly stated without finger counting or counting on. Through place-mat activities and ten-frame activities in regular classroom time, students should become convinced of this easy addition. Afterwards, these addition questions can be reinforced in mental mathematics time. For example, for $30 + 5$, think: 30 and 5 more makes 35.

Quick Addition

Initially you should have students add 1-digit numbers to 2-digit numbers for questions that require no regrouping. Students need to be convinced that questions such as $32 + 7$ and $74 + 5$ are as easy as $2 + 7$ and $4 + 5$. Therefore, these questions should be modelled using ten-frames so students can see that 32 (3 full ten-frames and a 2) plus 7 just requires the addition of the 2 and 7, or that 74 (7 full ten-frames and a 4) plus 5 just requires the addition of 4 and 5. The single digit facts required are those with sums less than 10. For example, for $34 + 3$, think: 4 and 3 is 7, so 30 and 7 more is 37.

Subsequently, they can apply “quick addition” as an addition strategy for solving combinations that require no regrouping and result in answers under 100. For example, for $56 + 23$, think and record: 5 tens and 2 tens is 7 tens, and 6 and 3 is 9, so 70 and 9 more is 79.

Addition Facts to 10 Applied to Multiples of 10

Through modelling with small cubes and rods from the base-ten materials, students should be convinced that adding two sets of rods is no different than adding two sets of small cubes. For example, adding 4 rods and 2 rods results in 6 rods in the same way that adding 4 small cubes and 2 small cubes results in 6 small cubes. Therefore, when asked to find sums such as $20 + 30$, $40 + 10$, and $30 + 50$, students should make the connections to the facts $2 + 3$, $4 + 1$, and $3 + 5$.

You should restrict questions to combinations that result in sums to 100. Students would solve these questions by applying their knowledge of facts to 10. Since this is an extension of the facts (three-second response), a response time goal of five seconds would be reasonable for these questions. For example, for $50 + 20$, think: 5 tens and 2 tens is 7 tens, or 70.

Addition on the Hundred Chart

Display a hundred chart, present students with addition questions involving two 2-digit numbers, ask them to visualize the additions on the chart, and record (or state) their answers. After many experiences, students may be able to visualize the hundred chart and do the addition completely in their heads.

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

For example,

- for $45 + 21$, think: Starting at 45, go down two spaces to 65 and then go 1 space to the right to 66. The answer is 66.
- for $34 + 63$, think: Starting at 34, go down 6 spaces to 94 and then go 3 spaces to the right to get to 97; or Starting at 63, go down 3 spaces to 93 and then go 4 spaces to the right to get to 97. The answer is 97.

N06.03 The objective of this outcome is that students have flexibility with numbers and can efficiently add or subtract two 2-digit numbers mentally. They should have many experiences practising these mental mathematics strategies in a variety of contexts as well as in isolation. Students should become comfortable enough with these strategies that they spontaneously use them whenever appropriate during the time in mathematics, other subjects, and in their daily lives. When students are responding to these questions, it would be reasonable to expect a five- to seven-second response time for most students.

SCO N07 Students will be expected to describe and apply mental mathematics strategies for subtracting two 2-digit numerals.

[C, ME, PS, R, V]

[C] Communication

[PS] Problem Solving

[CN] Connections

[ME] Mental Mathematics and Estimation

[T] Technology

[V] Visualization

[R] Reasoning

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N07.01 Explain mental mathematics strategies that could be used to determine a difference.

- Facts with minuends of 10 or less applied to multiples of 10
- Quick subtraction
- Subtraction on the hundred chart
- Compensation
- Back through ten

N07.02 Use and describe a personal strategy for determining a difference.

N07.03 Determine a difference of two 2-digit numerals efficiently, using mental mathematics strategies.

Performance Indicator Background

N07.01 and N07.02 Students will develop, apply, and describe mental mathematics strategies to subtract two 2-digit numbers.

Facts with Minuends of 10 or Less Applied to Multiples of 10

Through modelling with small cubes and rods from the base-ten materials, students should be convinced that subtracting two sets of rods is no different than subtracting two sets of small cubes. For example, subtracting 3 rods from 9 rods results in 6 rods in the same way that subtracting 3 small cubes from 9 small cubes results in 6 small cubes. Therefore, when asked to find differences such as $50 - 10$, $40 - 20$, and $90 - 50$, students should make the connections to the facts $5 - 1$, $4 - 2$, and $9 - 5$.

Questions should involve minuends of 20, 30, 40, ..., 100 so students can solve the questions by applying their knowledge of subtraction facts to 10. A response time goal of 5 seconds is reasonable for these questions. For example, for $50 - 20$, think: 5 tens minus 2 tens is 3 tens, or 30.

Quick Subtraction

This strategy is used when two 2-digit numbers are to be subtracted and there is no regrouping needed. Starting at the highest place value, simply subtract and record each place value's digits. For example, for $56 - 12$, think about each place value difference: Starting at the front end, 5 tens minus 1 ten is 4 tens, 6 minus 2 is 4, so, 40 and 4 more is 44.

Because this strategy only applies to questions with no regrouping, students must examine each question as a whole to decide whether this strategy can be used. This habit of thinking needs to pervade all mental mathematics lessons. For example, present students with a list of 20 questions, some of which require regrouping, and direct students to apply quick subtraction to the appropriate questions, leaving out the ones for which this strategy cannot be used.

Subtraction on the Hundred Chart

This strategy involves actions on a hundred chart—where going straight up one space represents subtracting 10, and where going left one space represents subtracting 1. Display a hundred chart, present students with subtraction questions involving two 2-digit numbers, ask them to visualize the subtractions on the chart, and record (or state) their answers. After many experiences, students may be able to visualize the hundred chart and do the subtraction completely in their heads.

For example,

- for $45 - 21$, think: Starting at 45, go up two spaces to 25 and then go 1 space to the left to 24. The answer is 24.
- for $74 - 65$, think: Starting at 74, go up 6 spaces to 14 and then go 5 spaces to the left to get to 9. The answer is 9.

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

N07.03 Students need flexibility with numbers to efficiently add or subtract two 2-digit numbers mentally. They should have many experiences practising these mental mathematics strategies in a variety of contexts as well as in isolation. Students should become comfortable enough with these strategies that they spontaneously use them whenever appropriate during the time in mathematics, other subjects, and in their daily lives.

SCO N08 Students will be expected to apply estimation strategies to predict sums and differences of 1-, 2-, or 3-digit numerals in a problem-solving context.

[C, ME, PS, R]

[C] Communication

[PS] Problem Solving

[CN] Connections

[ME] Mental Mathematics and Estimation

[T] Technology

[V] Visualization

[R] Reasoning

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- N08.01** Explain estimation strategies that could be used to determine an approximate sum or difference.
- N08.02** Use and describe a strategy for determining an estimate.
- N08.03** Estimate the solution for a given story problem involving the sum or difference of up to two 3-digit numerals.

Performance Indicator Background

N08.01 and N08.02 The front-end estimation strategy involves adding or subtracting the values in the highest place-value position to get an estimate. These estimates provide answers that are close enough to judge pencil-and-paper and technology-generated answers. Because the other place values are not considered, the front-end estimates for addition questions will always be less than the actual answers. Therefore, you can always use the phrase “more than” in describing your estimate. For example, to estimate $213 + 347$, think: $200 + 300 = 500$, so the estimate is 500 and the answer must be more than 500. In subtraction questions, however, without considering the other place values, you can only use the word “about” in describing your estimate. For example, to estimate $423 - 145$, think: $400 - 100 = 300$, so the estimate is about 300.

The front-end adjusted strategy for addition estimation begins by getting a front-end estimate and then adjusting that estimate to get a better, or closer, estimate; by clustering all the values in the other place values to decide whether there would be enough together to account for an adjustment. For example, to estimate $337 + 545$, think: 300 plus 500 is 800, but this can be adjusted by thinking 37 and 45 make almost another 100; so, the adjusted estimate would be 900.

N08.03 Students should be presented with contexts in which an estimate is all that is required. Story problems should be carefully constructed or selected so that students have opportunities to make decisions based on estimates rather than exact answers.

<p>SCO N09 Students will be expected to demonstrate an understanding of addition and subtraction of numbers (limited to 1-, 2-, and 3-digit numerals) with answers to 1000 by</p> <ul style="list-style-type: none"> using personal strategies for adding and subtracting with and without the support of manipulatives creating and solving problems in context that involve addition and subtraction of numbers concretely, pictorially, and symbolically. <p>[C, CN, ME, PS, R]</p>			
[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- N09.01** Model the addition of two or more given numbers using concrete or visual representations and record the process symbolically.
- N09.02** Model the subtraction of two given numbers using concrete or visual representations and record the process symbolically.
- N09.03** Create an addition or subtraction story problem for a given solution.
- N09.04** Determine the sum of two given numbers using a personal strategy, e.g., for $326 + 48$, record $300 + 60 + 14$.
- N09.05** Determine the difference of two given numbers using a personal strategy, e.g., for $127 - 38$, record $2 + 80 +$ or $127 - 20 - 10 - 8$.
- N09.06** Solve a given problem involving the sum or difference of two given numbers.

Performance Indicator Background

N09.01 and N09.02 Students should be able to model the sum and difference of two given numbers up to 3-digits using base-ten blocks, and use symbols to record the processes that reflect their actions with those blocks. For example, to subtract 137 from 265, students may represent 137 with 1 flat, 3 rods, and 7 small cubes in one set; in another set they may place 1 flat saying 237, 2 rods saying 257, and 8 small cubes saying 265; determine that 128 is the total in the second set; and state the difference between 265 and 137 is 128. The students would record $137 + 100 + 20 + 8 = 265$ and $100 + 20 + 8 = 128$ to reflect the counting-up strategy they did with the blocks.

After students have modelled and solved a number of addition and subtraction situations, they may be introduced to strip diagrams as another way to represent the situations. For example, Bobby was given 63 green stamps. He already had 127 stamps. How many stamps does he have now? The strip diagram for this problem is as follows:

127	63
?	

As another example, Bobby had 98 stamps. He was given more by his friend. Then he had 137 stamps. How many stamps did his friend give him? The strip diagram for this problem is as follows:

98	?
137	

Because students have to decide where to place in the diagram the two given numbers in the story problem, they have to carefully read the problem to determine whether each given quantity is a part or a whole. If the quantity is a part, it would be placed in one section of the top rectangle; if the quantity is a whole it would be placed in the bottom rectangle. They should put a question mark in the bottom rectangle or one of the sections in the top rectangle, depending upon what is missing (what they are asked to find).

The principal use of strip diagrams is as a strategy to help students interpret story problems. Students will solve the problems using their personal strategies; however, through extensive use of strip diagrams, some students may generalize that subtraction is the operation that will always find a missing part and that addition will always find a missing whole.

N09.03 Students should be able to create story problems given an addition or subtraction number sentence. In order for their story problems to go beyond simple result-unknown types, they will need to have very specific experiences in which they create story problems similar to ones that are modelled. For example, students should be presented with four or five join (change unknown) story problems and, after they solve those problems, they should be asked to create a story problem similar to the join problems they were presented, but in a different context.

N09.04 and N09.05 It is expected that students will be able to symbolically add and subtract two 3-digit numbers using reliable and efficient strategies. Students should be able to explain their strategy and whether their solution is reasonable based on their prior estimate.

Examples of strategies and symbolic recordings for addition and subtraction are shown below.

If students are asked to add 237 and 478, students could determine the sum by doing the following:

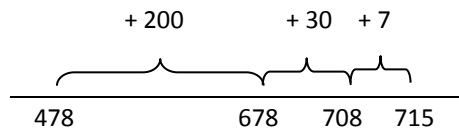
- Start by writing 237 as $200 + 30 + 7$ and 478 as $400 + 70 + 8$.
- Add 200 and 400 to get a sum of 600.
- Add 30 and 70 to get a sum of 100.
- Add 7 and 8 to get a sum of 15.
- Add 600, 100, and 15 to get a sum of 715.

This may be recorded on paper as follows:

$237 + 478 = 200 + 30 + 7 + 400 + 70 + 8$	or	$\begin{array}{r} 237 \\ + 478 \\ \hline 600 \\ 100 \\ + 15 \\ \hline 715 \end{array}$
$200 + 400 = 600$		
$30 + 70 = 100$		
$7 + 8 = 15$		
$600 + 100 + 15 = 715$		

- Start with the larger number 478.
- Add 200 to get a sum of 678.
- Add 30 to 678 to get a sum of 708.
- Add 7 to 708 to get a sum of 715.

This can be represented as jumps on a number line.



This may be recorded on paper as follows:

$$\begin{aligned}
 &237 + 478 \\
 &478 + 200 = 678 \\
 &678 + 30 = 708 \\
 &708 + 7 = 715
 \end{aligned}$$

- Start by placing one addend below the other.
- Add 8 and 7 and place the sum of 15 below the two addends in line 1.
- Add 30 and 70 and place the sum of 100 in line 2.
- Add 200 and 400 and place the sum of 600 in line 3.
- Add the three lines to get a sum of 715.

This may be recorded on paper as follows:

$$\begin{array}{r}
 237 \\
 + 478 \\
 \hline
 15 \text{ (line 1)} \\
 100 \text{ (line 2)} \\
 + 600 \text{ (line 3)} \\
 \hline
 715
 \end{array}$$

This same strategy might also be recorded as follows:

$$\begin{aligned}
 237 + 478 &= 7 + 8 + 30 + 70 + 200 + 400 \\
 7 + 8 &= 15 \\
 30 + 70 &= 100 \\
 200 + 400 &= 600 \\
 15 + 100 + 600 &= 715
 \end{aligned}$$

- Start by decomposing 237 into 22 and 215.
- Combine the 22 with 478 to get a sum of 500.
- Add 500 and 215 to get a sum of 715.

This may be recorded on paper as follows:

$$\begin{aligned}
 237 + 47 &= 22 + 215 + 478 \\
 478 + 22 &= 500 \\
 500 + 215 &= 715
 \end{aligned}$$

- Start by adding 500 to 237 to get a sum of 737.
- Subtract 22 from 737 to get a difference of 715.

This may be recorded on paper as follows:

$$\begin{aligned} 237 + 478 \\ 237 + 500 = 737 \\ 737 - 22 = 715 \\ 237 + 478 = 715 \end{aligned}$$

- Start by placing one addend below the other.
- Add 7 ones and 8 ones to get a sum of 15 ones.
- Regroup the 15 ones into 1 ten and 5 ones.
- Record a 1 in the tens place above the addends.
- Record a 5 in the ones place below the line.
- Add 3 tens, 7 tens, and 1 ten (from regrouping the ones) to get a sum of 11 tens.
- Regroup the 11 tens into 1 hundred (10 of the tens) and 1 ten.
- Record a 1 in the hundreds place above the addends.
- Record 1 in the tens place below the line.
- Add 2 hundreds, 4 hundreds, and 1 hundred (from the regrouping of the tens) to get a sum of 7 hundreds.
- Record a 7 in the hundreds place below the line.

This may be recorded on paper as follows:

$$\begin{array}{r} 11 \\ 237 \\ + 478 \\ \hline 715 \end{array}$$

If we introduce subtraction using word problems, students can begin modelling their solutions. Consider the following problem: On our vacation, we went to visit our aunt in Fredericton. We drove 239 km and stopped for lunch. If the distance to our aunt's house is 526 km, how much further do we have to drive?

Students could use base-ten blocks, number lines, or mental mathematics strategies to solve the problem in different ways. Possible solutions may include the following:

Group 1

We knew we had to subtract 239 from 526. So we started with 5 flats, 2 rods and 6 small cubes to show 526. We removed 2 flats. Then, we had to remove 3 rods, so we changed 1 flat to 10 rods. Finally we removed 9 small cubes, after we traded 1 rod for 10 small cubes.

This could be recorded on paper as follows:

$$\begin{aligned} 526 - 239 = ? \\ 526 - 200 = 326 \\ 326 - 30 = 296 \\ 296 - 9 = 287 \end{aligned}$$

We have to travel 287 km more.

Group 2

We started with 2 flats, 3 rods, and 9 small cubes to show 239. Then, we added 3 flats, but we knew that was too much because we had 539. So, we removed 9 small cubes, and we had 530. We still needed to remove 4 more small cubes to get to 526. So, we traded 1 rod for 10 small cubes, and removed the 4 small cubes.

This could be recorded on paper as follows:

$$239 + ? = 526$$

$$239 + 300 = 539$$

$$539 - 9 = 530$$

$$530 - 4 = 526$$

$$300 - 13 = 287$$

So, we know we have 287 km to drive.

Group 3

We used an empty number line. We put 239 and 526 on the line. We made a jump of 1 from 239 to 240. Next, we made a jump of 60 from 240 to 300. Then we made a jump of 200 from 300 to 500. Then we made a jump of 26, from 500 to 526. So, we combined all of our jumps, $1 + 60 + 200 + 26$, to get 287. We have 287 km more to travel.

This could be recorded on paper as follows:

$$239 + ? = 526$$

$$239 + 1 = 240$$

$$240 + 60 = 300$$

$$300 + 200 = 500$$

$$500 + 26 = 526$$

$$1 + 60 + 200 + 26 = 287$$

Group 4

We started with 2 flats, 3 rods, and 1 small cube to show 239. We added 2 flats and had 439. We added 6 rods to get to 499. We added 1 small cube to get to 500. Then, we added on 2 rods and 6 small cubes to get to 526. So, we looked at everything we had added on (2 flats, 6 rods, 1 small cube, 2 rods, 6 small cubes) and knew that we had added on 287.

This could be recorded on paper as follows:

$$239 + ? = 526$$

$$239 + 200 = 429$$

$$429 + 60 = 499$$

$$499 + 1 = 500$$

$$500 + 26 = 526$$

$$200 + 60 + 1 + 26 = 287$$

Group 5

We knew we had to subtract 239 from 526. We decided to subtract 240 instead because it was easier to work with. So, we started at 526, jumped back 200 to 326. Then we jumped back 20 to 306, and then jumped back another 20 to 286. But we knew we had jumped back 1 too many and so we moved to 287.

This could be recorded on paper as follows:

$$526 - 200 = 326$$

$$326 - 20 = 306$$

$$306 - 20 = 286$$

$$286 + 1 = 287$$

Group 6

We wanted to subtract a friendly number. It would be nice to subtract 300. So, we changed 239 to 300 by adding on 61. Since we added 61 to 239, we had to add 61 to 526 to keep our constant difference. Then, we had a nice question to solve mentally, $587 - 300 = 287$.

This could be recorded on paper as follows:

$$239 + 61 = 300$$

$$526 + 61 = 587$$

$$587 - 300 = 287$$

$$526 - 239 = 587 - 300 = 287$$

Regardless of the strategy used, the teacher must monitor each student's recording of the strategy to ensure that the recording is mathematically correct, organized, and efficient. For example, to solve $237 + 478$, a student could accurately record his or her thinking as follows:

Method A

$$237 + 478 = 200 + 30 + 7 + 400 + 70 + 8$$

$$200 + 400 = 600$$

$$30 + 70 = 100$$

$$7 + 8 = 15$$

$$600 + 100 + 15 = 715$$

Method B

$$237$$

$$+ 478$$

$$\hline 600$$

$$100$$

$$+ 15$$

$$\hline 715$$

Method C

$$237 + 478 = 200 + 400 + 30 + 70 + 7 + 8 = 600 + 100 + 15 = 715$$

However, if a student recorded his or her thinking as

$$237 + 478 = 200 + 30 + 7 + 400 + 70 + 8 = 200 + 400 = 600 + 30 + 70 = 700 + 7 + 8 = 715,$$

it would be necessary to work with the student to correct the recording error. Correction would be necessary, as this is an example of the incorrect use of the equal sign. It may result from a student's misunderstanding of the meaning of the equal sign. One way to address this is to have students verify the accuracy of the recording by asking them to read the equal sign as "is the same as." In the example above, it is correct to say

- $237 + 478$ is the same as $200 + 30 + 7 + 400 + 70 + 8$
- $237 + 478$ is the same as $600 + 100 + 15$
- $237 + 478$ is the same as 715
- $200 + 30 + 7 + 400 + 70 + 8$ is the same as $600 + 100 + 15$
- $200 + 30 + 7 + 400 + 70 + 8$ is the same as 715
- $600 + 100 + 15$ is the same as 715

However, it is incorrect to say

- $237 + 478$ is the same as $200 + 400$
- $237 + 478$ is the same as $600 + 30 + 70$

- $237 + 478$ is the same as $200 + 400$
- $200 + 30 + 7 + 400 + 70 + 8$ is the same as $200 + 400$, etc.

N09.06 Students should be able to solve story problems of different types by writing the most efficient open number sentences and computing the sums or differences to find the solutions. They should be able to do this either directly upon reading the problem, or by drawing or visualizing pictures that represent the problem.

SCO N10 Students will be expected to apply mental mathematics strategies and number properties, to develop quick recall of basic addition facts to 18 and related basic subtraction facts.

[C, CN, ME, R, V]

[C] Communication

[PS] Problem Solving

[CN] Connections

[ME] Mental Mathematics and Estimation

[T] Technology

[V] Visualization

[R] Reasoning

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- N10.01** Describe a mental mathematics strategy that could be used to determine a given basic addition fact up to $9 + 9$.
- N10.02** Explain how the commutative (order-doesn't-matter) property and the identity (no-change-with-zero) property can assist in addition fact learning.
- N10.03** Describe a mental mathematics strategy that could be used to determine a given basic subtraction fact with minuends up to 18 and subtrahends up to 9.
- N10.04** Recognize which facts could be determined by a given strategy.
- N10.05** Quickly recall basic addition facts to 18 and related subtraction facts in a variety of contexts.

Performance Indicator Background

N10.01 Students should be familiar with a mental strategy that can be used to quickly recall a given addition fact and be able to explain the strategy using that fact.

N10.02 Students should understand and use the commutative property of addition without an expectation that they know the word **commutative**. It is sufficient that they know the meaning; that is, the order in which numbers are combined in addition does not affect the answer. This knowledge is applied throughout fact learning. For example, $5 + 8$ and $8 + 5$ are both determined by making 8 a 10 and then adding 3, and $6 + 7$ and $7 + 6$ are both determined by doubling 6 and adding 1. There are 45 such pairings in the 100 addition facts.

Students should observe that the sum of a number and zero will always result in that number. It is the only number that will not result in a change in either addition or subtraction. If asked, What number can you add to 8 or subtract from 8 and get the same answer? students should realize that number is 0.

N10.03 Students should be familiar with a mental strategy that can be used to quickly recall a given subtraction fact and be able to explain the strategy using that fact. In addition, students should be able to state three other facts (one subtraction and two addition) that are related to this fact. If called on, they can apply the back-through-10 strategy to a fact such as $16 - 7$. Similarly, students could apply the up-through-10 strategy to a fact such as $15 - 9$.

N10.04 If students are presented with a set of facts, they should be able to identify the ones that could be found by a stated strategy, or sort the facts into groups that could be determined by the same strategy.

N10.05 The ultimate objective of this outcome is that students have quick recall of both addition and subtraction facts. Either the students use a mental strategy to determine the fact or have instant recall (they just know it). They should have many experiences practising these facts in a variety of contexts as well as in isolation. Students should understand that knowledge of addition and subtraction facts replaces counting strategies, and this fact knowledge should be applied all the time in mathematics, other subjects, and in their daily lives.

- SCO N11** Students will be expected to demonstrate an understanding of multiplication to 5×5 by
- representing and explaining multiplication using equal grouping and arrays
 - creating and solving problems in context that involves multiplication
 - modelling multiplication using concrete and visual representations and recording the process symbolically
 - relating multiplication to repeated addition
 - relating multiplication to division

[C, CN, PS, R]

[C] Communication

[PS] Problem Solving

[CN] Connections

[ME] Mental Mathematics and Estimation

[T] Technology

[V] Visualization

[R] Reasoning

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- N11.01** Identify events from experience that can be described as multiplication.
- N11.02** Represent a given story problem (orally, shared reading, written) using manipulatives or diagrams and record in a number sentence.
- N11.03** Represent a given multiplication expression as repeated addition.
- N11.04** Represent a given repeated addition as multiplication.
- N11.05** Create and illustrate a story problem for a given number sentence and/or expression.
- N11.06** Represent, concretely or pictorially, equal groups for a given number sentence.
- N11.07** Represent a given multiplication expression using an array.
- N11.08** Create an array to model the commutative property of multiplication.
- N11.09** Relate multiplication to division by using arrays and writing related number sentences.
- N11.10** Solve a given problem in context involving multiplication.

Performance Indicator Background

N11.01 Challenge students to think of real-life objects that come in equal groups, such as wheels on bicycles or tricycles, legs on chairs, legs on stools, leaves on three-leaf clovers, legs on animal, eggs in a carton, fingers on hands, and days in a week. Ask them to create situations using these objects that would be described as multiplication situations, such as the number of eyes in a group of 4 people, the number of wheels on 5 bicycles, and the number of shoes in 3 pairs.

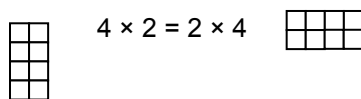
N11.02 The most meaningful way to apply and practice multiplication is in problem-solving contexts. Students should sometimes have story problems read to them, sometimes have them in print, and other times turn situations in story books into multiplication situations. No matter how the story problems are presented, students should model their solutions using concrete materials and/or pictures or diagrams from which they get solutions by skip counting the equal groups. Students will also be able to write the corresponding repeated addition number sentences and, eventually, the appropriate multiplication number sentences.

N11.03 and N11.04 Students should be able to translate any multiplication number expression into a repeated addition number expression. For example, given 5×4 , students write $4 + 4 + 4 + 4 + 4$ because they understand the meaning of the two factors in multiplication expressions. Interpreting the first factor as the number of groups cannot be overemphasized because of students' prior experiences with addition expressions in which both addends were countable quantities of objects. Similarly, students should be able to translate a repeated addition expression into a multiplication expression. For example, given $3 + 3 + 3 + 3 + 3$, students can write this as 5×3 .

N11.05 Students should be able to create appropriate story problems that correspond to given multiplication number expressions. Having created the problems, they can draw pictures or diagrams that provide the solutions. For example, given 2×5 , a student’s problem may be, Sam and Mary each had 5 pencils. How many pencils did they have altogether? The student may draw two stick people and show five pencils in each person’s right hand, writing 10 next to the picture.

N11.06 and N11.07 Students should be able to model concretely or pictorially a given multiplication number expression or sentence as equal groups and as an array. For example, given $5 \times 2 = 10$ or $2 \times 5 = 10$, students can show this as 5 groups with 2 counters in each group and as 2 rows with 5 counters in each row.

N11.08 The array is a powerful model to illustrate the order or commutative property in multiplication. For example, the first array below has 4 rows of 2 columns and therefore is a model for 4×2 . The second is a 2×4 array. Both have an answer of 8.



N11.09 After students have been introduced to multiplication and division, they should understand that one array represents a multiplication sentence and two corresponding division sentences. For example, the array below is a representation for $4 \times 2 = 8$ and also a representation for eight divided into groups of two ($8 \div 2 = 4$) and for eight divided into four groups ($8 \div 4 = 2$).



N11.10 Students should have experiences solving equal-group story problems in contexts that are new to them (and perhaps novel as well) using a strategy of their own choosing. Some students may choose to model it with some concrete materials, other students may draw a picture or diagram, while still others may immediately write a number sentence. Seeing the same problem solved in a variety of ways is enriching for all students when the solution strategies are shared. If students solve the problem concretely or pictorially, encourage them to write a multiplication sentence to represent it.

- SCO N12** Students will be expected to demonstrate an understanding of division by
- representing and explaining division using equal sharing and equal grouping
 - creating and solving problems in context that involves equal sharing and equal grouping
 - modelling equal sharing and equal grouping using concrete and visual representations, and recording the process symbolically
 - relating division to repeated subtraction
 - relating division to multiplication.

(Limited to division related to multiplication facts up to 5×5 .)

[C, CN, PS, R]

[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- N12.01** Identify events from experience that can be described as equal sharing.
- N12.02** Identify events from experience that can be described as equal grouping.
- N12.03** Illustrate, with counters or a diagram, a given story problem involving equal sharing, presented orally or through shared reading, and solve the problem.
- N12.04** Illustrate, with counters or a diagram, a given story problem involving equal grouping, presented orally or through shared reading, and solve the problem.
- N12.05** Listen to a story problem, represent the numbers using manipulatives or a diagram and record the problem with a number sentence and/or expression.
- N12.06** Create and illustrate with counters, a story problem for a given number sentence and/or expression.
- N12.07** Represent a given division sentence and/or expression as repeated subtraction.
- N12.08** Represent a given repeated subtraction as a division sentence.
- N12.09** Relate division to multiplication by using arrays and writing related number sentences.
- N12.10** Solve a given problem involving division.

Performance Indicator Background

N12.01 and N12.02 Students should be encouraged to identify everyday experiences that would be equal-sharing situations and equal-grouping situations. For example, students should see that dividing the class into two groups, sharing 12 pieces of paper with 4 students, and sharing a large bag of candy into 3 small bags are all examples of equal-sharing situations. They should also see that dividing the class into groups of 5, giving each student 4 pencils, and placing books into stacks of 4 are all examples of equal-grouping situations.

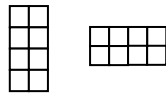
N12.03 and N12.04 The most meaningful way to apply and practise division is in problem-solving contexts. Students should sometimes have story problems read to them, sometimes have them in print, and other times turn situations in story books into division situations. No matter how the story problems are presented, students should model their solutions using concrete materials and/or pictures or diagrams, from which they get solutions. Story problems should be of the two types—equal sharing and equal grouping.

N12.05 Students should be able to interpret a division story problem (equal sharing or equal grouping) that is read three times, model that problem with concrete materials or diagrams, and write the division sentence that represents the problem and its solution.

N12.06 Students should be able to create appropriate story problems that correspond to given division number expressions. Having created the problems, they can draw pictures or diagrams that provide the solutions. For example, given $12 \div 4$, a student's problem may be, Sam had 12 pencils and shared them with 4 friends. How many pencils did each friend get? The student may draw 4 stick people and draw pencils, one at a time, in each person's right hand, until the 12 are all shared and each person has 3 pencils. The student would write $12 \div 4 = 3$ to represent this story and its solution.

N12.07 and N12.08 Students should be able to translate repeated subtraction sentences into a division sentence, and to translate a division sentence into a repeated subtraction sentence. For example, given $8 - 2 - 2 - 2 - 2 = 0$, students should write $8 \div 2 = 4$; given $12 \div 3 = 4$, students should write $12 - 3 - 3 - 3 - 3 = 0$. For $12 \div 3 = 4$, students should think "How many 3s can be subtracted from 12 to get to 0."

N12.09 After students have been introduced to multiplication and division, they should understand that one array represents a multiplication sentence and two corresponding division sentences. For example, the first array below has 4 rows of 2 columns and therefore is a model for $4 \times 2 = 8$. The second is a 2×4 array. Both have an answer of 8.



N12.10 Students should have experiences solving division story problems in contexts that are new to them (and perhaps novel as well) using a strategy of their own choosing. Some students may choose to model it with some concrete material, other students may draw a picture or diagram, while still others may immediately write a number sentence. Seeing the same problem solved in a variety of ways is enriching for all students when the solution strategies are shared. If students solve the problem concretely or pictorially, they should be encouraged to write a division sentence that would represent the problem and its solution.

SCO N13 Students will be expected to demonstrate an understanding of fractions by

- explaining that a fraction represents a part of a whole
- describing situations in which fractions are used
- comparing fractions of the same whole with like denominators

[C, CN, ME, R, V]

[C] Communication

[PS] Problem Solving

[CN] Connections

[ME] Mental Mathematics and Estimation

[T] Technology

[V] Visualization

[R] Reasoning

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- N13.01** Describe everyday situations where fractions are used.
- N13.02** Represent a given fraction concretely or pictorially.
- N13.03** Identify, model, and explain the meaning of numerator and denominator.
- N13.04** Sort a given set of diagrams of regions into those that represent equal parts and those that do not, and explain the sorting.
- N13.05** Name and record the fraction represented by the shaded and non-shaded parts of a given region.
- N13.06** Compare given fractions with the same denominator using models.

Performance Indicator Background

N13.01 It is likely that students will have heard fractional language in their everyday life, such as half an apple, half moon, one-third cup of flour, etc. Students will benefit from learning about fractions concretely, and in the context of real life.

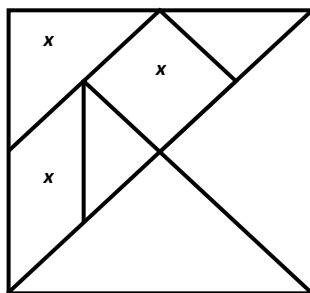
As students are exposed to fractional language in meaningful contexts, they will begin to develop an understanding of fractions. Students in Mathematics 3 should be given the opportunity to discuss simple unit fractions in the context of division stories of regional models, for example, when a pizza is shared by three people, each person has one-third of the original amount of the pizza.

It is also helpful to recognize, when examining a situation involving a regional model, if one-third of a pizza is eaten, then two-thirds of that pizza remains. Informal experiences will help students see that when wholes are divided into a greater number of fair shares, the shares are smaller. This will help later when comparing fractions.

N13.02 and 13.05 Part of a whole—This is when one unit is partitioned into equal parts. The sharing of an apple, or a piece of paper is commonplace to students. The more opportunities they have to partition fairly, the better their visual concept will be for fractions. The emphasis should be on equal parts or fair shares. Students should understand that while the parts are equal in area, they do not need to be identical in shape; this can be a misconception. A tangram set demonstrates this idea clearly where the square, the medium-sized triangle, and the parallelogram all have equivalent area but are not identical in shape. It is important that the representation of the whole, one whole or one, is clear so students understand which region they are taking apart; this concept is essential for comparing fractions.



$\frac{3}{4}$ of the bar is shaded.



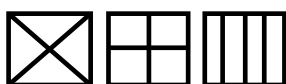
The parallelogram, the square, and the medium-sized triangle each represent $\frac{1}{8}$ of the whole region.

In Mathematics 3, students will be introduced to, and explore, the parts of a whole that results when the whole has been divided into equal-sized portions or “fair shares.” It is important to use the terms **whole**, **one whole**, or **one** to ensure that students have a common language to use regardless of the model used.

Teachers should model the use of language such as “1 of 3 equal parts” and help students connect the language with its symbol $\frac{1}{3}$. Point out to students that $\frac{1}{4}$ should be read “one-fourth” rather than “one-quarter.”

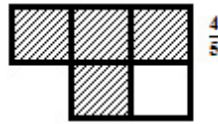
The use of models and concrete representations is essential to their understanding. It is important to use a wide variety of models so that fractions do not simply become apple pieces, granola bars, or pizza slices. Fractional concepts can be strengthened by linking other areas of mathematics such as geometry, money, number, and patterning. It is important that students develop visual images for fractions and be able to tell “about how much” a particular fraction represents and have exposure to common benchmarks, such as one-half. These areas play a key role in consolidating a student’s conceptual understanding of fractions.

Students should see that there are many ways to make the same fractional part. Using pattern blocks where the hexagon is designated as the whole, students could find how many different ways they can make $\frac{1}{2}$, $\frac{1}{3}$, etc. Or using a square, find how many different ways to make $\frac{1}{4}$. This can help with the understanding of equivalence.



Students need to see and explore a variety of models of fractions with a key focus on halves, thirds, fourths, fifths, sixths, eighths, and tenths. Initially, students explore unit fractions such as $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ before moving onto other proper fractions such as $\frac{2}{3}$, $\frac{5}{8}$, and so on. Beginning with fractional terms such as halves, thirds, fourths, etc., and their pictorial representations, provides a bridge to the more challenging concept of the symbolic representations.

N13.03 The meaning of the numerator (top number) and the denominator (bottom number) needs to be emphasized as these can be misleading to students. This is best accomplished by introducing and focusing on the numerator or denominator separately and using visual models linked to the symbols. For example, ask students to use grid paper to represent a fraction as in the picture below. Ask them to name the fraction ($\frac{4}{5}$), and identify and explain the meaning of numerator and denominator. The 4 is the numerator because it tells how many parts of the shape are shaded. The 5 is the denominator because it tells how many equal parts the whole shape is divided into.



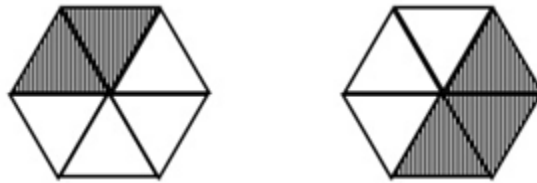
N13.04 It is important that students see and represent non-examples of the area model for fractions as with the rectangle or other shapes.

N13.06 Discuss with students the fact that if two fractions have the same denominator, the fraction with the greater numerator represents the larger piece of the whole. If the denominators of two fractions from the same whole are the same, then the parts are the same.

Students will compare fractions with the same denominator. Students will use language and/or pictures to indicate which fraction is greater than or less than the other. Pattern blocks can be used to demonstrate this concept. For example, the hexagon can be used to represent a pizza and the triangles to represent the slices.

Ask the students, If John ate $\frac{2}{6}$ of the pizza and Gina ate $\frac{3}{6}$ of the pizza, who ate the most pizza?

Ask students to model their answers using the pattern blocks and record the fraction symbols showing which is greater and which is less. Discuss with students how they know.



The shaded part of the first picture ($\frac{2}{6}$) is less than the shaded part of the second ($\frac{3}{6}$).

Patterns and Relations (PR)

SCO PR01 Students will be expected to demonstrate an understanding of increasing patterns by describing, extending, comparing, and creating numerical (numbers to 1000) patterns and non-numerical patterns using manipulatives, diagrams, sounds, and actions.

[C, CN, PS, R, V]

[C] Communication

[PS] Problem Solving

[CN] Connections

[ME] Mental Mathematics and Estimation

[T] Technology

[V] Visualization

[R] Reasoning

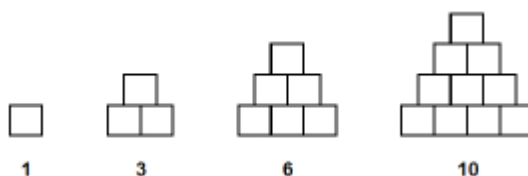
Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- PR01.01** Identify and describe increasing patterns.
- PR01.02** Describe a given increasing pattern by stating a pattern rule that includes the starting point and a description of how the pattern continues.
- PR01.03** Extend a pattern, using the pattern rule, for the next three terms.
- PR01.04** Compare numeric patterns.
- PR01.05** Identify and explain errors in a given increasing pattern.
- PR01.06** Create a concrete, pictorial, or symbolic representation of an increasing pattern for a given pattern rule.
- PR01.07** Create a concrete, pictorial, or symbolic increasing pattern and describe the pattern rule.
- PR01.08** Solve a given problem using increasing patterns.
- PR01.09** Identify and describe the strategy used to determine a missing term in a given increasing pattern.
- PR01.10** Use ordinal numbers (to 100th) to refer to or to predict terms within an increasing pattern.

Performance Indicator Background

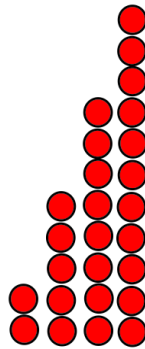
PR01.01 and **PR01.10** Students should be able to describe an increasing pattern. An increasing pattern is a growing pattern where the size of the term increases in a predictable way. The terms in an increasing pattern grow by either a constant amount or by an increasing amount each time. Students need sufficient time to explore increasing patterns through various manipulatives, such as Cube-A-Links, tiles, toothpicks, counters, pattern blocks, base-ten blocks, ten-frames, bread tags, stickers, or buttons, to realize they increase in a predictable way. As students describe increasing shape patterns, help them recognize that each term has a numeric value.



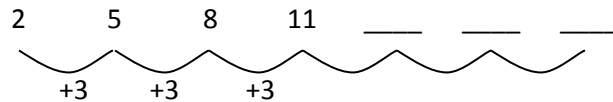
A hundred chart is a valuable tool to use with students when exploring increasing patterns. Students should be able to locate and describe various increasing patterns found on a hundred chart, such as horizontal, vertical, and diagonal patterns. For example, when skip counting by 3, use only starting points that are multiples of 3 (3, 6, 9, 12, ...) and this will result in a diagonal representation on a hundred chart. Skip counting with 5, starting at 0, the pattern is two vertical columns with numbers ending in the digits 5 and 0. Students should also explore hundred charts to 1000 (1–100, 101–200, 201–300, ...) and look for patterns when counting by 2s, 5s, 10s, 25s, and 100s.

A counting sequence is an increasing pattern where each number represents a term in the pattern. For example, in the counting sequence 1, 2, 3, 4, ..., 1 represents the first term, 2 the second term, 3 the third term ... This counting sequence can be connected to ordinal numbers where students should be able to recognize that the 34th term is 34 and that 57 is the 57th term. These ordinal number patterns should be investigated for numbers up to 100.

PR01.02 Students should be able to describe a given increasing pattern by stating the pattern rule. A pattern rule tells how to make the pattern and can be used to extend an increasing pattern. Give students the first three or four terms of an increasing pattern. Ask them to state the pattern rule by identifying the term that represents the starting point and describing how the pattern continues. For example, in the pattern below, the pattern rule is, start with 2 counters and add 3 counters each time.



As students describe concrete or pictorial patterns, help them recognize that each term has a numeric value. For example, the above pattern can be expressed as, 2, 5, 8, 11, ... by counting the number of counters in each term. Students may also find it useful to record the change from one number to the next as shown below:



PR01.03 Students should be able to extend a pattern by identifying the rule, and use the rule to build and draw the next three terms. Initially, students should replicate the first three terms with concrete materials and then extend the pattern. The use of the concrete materials allows them to make changes if necessary and to build onto one term to make the next term. Students should be able to explain why their extension follows the pattern. It is important to note that for some patterns, there may be more than one way to extend the pattern. For example, if only one term is given, such as the third term 12, some possible solutions could be

- 4, 8, 12, 16, ...
- 3, 7, 12, 18, ...
- 2, 6, 12, 20, ...
- 6, 9, 12, 15, ...

PR01.04 Students need opportunities to compare numeric patterns, discussing how they are the same and how they are different. When comparing increasing patterns, compare the starting points and how each term increases. For example, one way students may address this is by using a page with four small hundred charts. Ask them to skip count and shade one chart by 2s, one chart by 5s, one chart by 10s, and one chart by 25s. Then discuss the pattern rule in each chart comparing the starting points and the amount of increases.

PR01.05 Students should be provided with a variety of increasing patterns that contain errors, and be able to identify and explain the errors. For example, given the pattern 3, 7, 11, 15, 19, 23, 26, 31, 35, 39, students should state the pattern rule: Start at 3 and add 4 each time. Therefore, 26 is an error since it is only adding on 3 not 4. A second error is 31 since it is adding 5 and not 4. To help students visualize this pattern, they can shade the numbers on a hundred chart and look for the mistakes. Students may see that 26 and 31 do not fit the number pattern; therefore, they are errors.

PR01.06 Students should be able to create various representations of an increasing pattern that follow a given pattern rule. Initially students will create patterns with concrete materials, then pictures, and then numbers. Some students may create a simple pattern, while other students may create a more complex pattern.

PR01.07 Students should be able to create increasing patterns, concretely, pictorially, and symbolically, and be able to describe the pattern rule they used to create their representations. When creating increasing patterns, initially students need to choose a starting point and then decide on the amount of increase. The amount of increase may be either a constant amount or an increasing amount. Students should be able to describe their pattern by clearly explaining how it changes from one term to the next. Students may share their patterns and the strategies they used to create their pattern.

PR01.08 Students should have frequent experiences with solving real-world problems that interest and challenge them using increasing patterns. They should use concrete materials or pictures to model the problem before determining the pattern rule. A variety of strategies may be used to solve the problem such as using a number line, a hundred chart, a picture, concrete materials, or skip counting.

PR01.09 Students should be able to identify and describe the strategy used to determine a missing term in a given increasing pattern. Since patterns increase in a predictable way, to determine a missing term, students should first look at the term that comes before and after. One strategy may be to identify and use the pattern rule. Other possible strategies may include using a number line, a hundred chart, a picture, concrete materials, or skip counting.

SCO PR02 Students will be expected to demonstrate an understanding of decreasing patterns by describing, extending, comparing, and creating numerical (numbers to 1000) patterns and non-numerical patterns using manipulatives, diagrams, sounds, and actions.

[C, CN, PS, R, V]

[C] Communication

[PS] Problem Solving

[CN] Connections

[ME] Mental Mathematics and Estimation

[T] Technology

[V] Visualization

[R] Reasoning

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

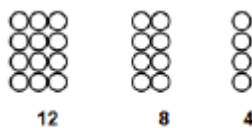
- PR02.01** Identify and describe decreasing patterns.
- PR02.02** Describe a given decreasing pattern by stating a pattern rule that includes the starting point and a description of how the pattern continues.
- PR02.03** Extend a pattern using the pattern rule for the next three terms.
- PR02.04** Compare numeric patterns.
- PR02.05** Identify and explain errors in a given decreasing pattern.
- PR02.06** Create a concrete, pictorial, or symbolic representation of a decreasing pattern for a given pattern rule.
- PR02.07** Create a concrete, pictorial, or symbolic decreasing pattern and describe the pattern rule.
- PR02.08** Solve a given problem using decreasing patterns.
- PR02.09** Identify and describe the strategy used to determine a missing term in a given decreasing pattern.
- PR02.10** Use ordinal numbers (to 100th) to refer to or to predict terms within a decreasing pattern.

Performance Indicator Background

PR02.01 and **PR02.10** Students should be able to describe a decreasing pattern. A decreasing pattern is a shrinking pattern where the size of the term decreases in a predictable way. The terms in a decreasing pattern shrink by either a constant amount or by an increasing amount each time. Students need sufficient time to explore decreasing patterns through various manipulatives, such as Cube-A-Links, tiles, toothpicks, counters, pattern blocks, base-ten blocks, ten-frames, bread tags, stickers, and buttons. Sometimes students are more comfortable during the exploration stage if they can experiment first, using manipulatives, then pictures, and eventually numbers.

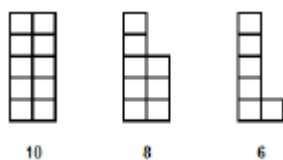
As students begin to investigate patterns, they sometimes confuse repeating patterns with decreasing patterns. Remind them to look for a core first. If they cannot find a core, then the pattern is not a repeating pattern.

Earlier, students became familiar with assigning a numeric value to each element in an increasing pattern. This expectation also applies to decreasing patterns.

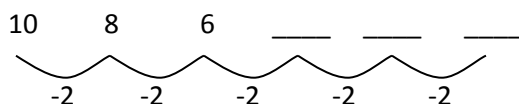


Students should be able to identify and describe various decreasing patterns found on a hundred chart, such as horizontal, vertical, and diagonal patterns. This can be connected to skip counting in outcome N01. Provide copies of hundred charts. Ask students to begin with 100 and skip count backward, shading in the number for each count all the way to 1. Then they write a description of the pattern. For example, if they chose 5, the pattern is two vertical columns, with numbers ending in the digits 5 or 0.

PR02.02 Students should be able to describe a given decreasing pattern by stating the pattern rule. A pattern rule includes a term representing a starting point and a description of how the pattern continues. A pattern rule tells how to make the pattern and can be used to extend a pattern. For example, in the pattern below, the pattern rule is to start with 10 squares and decrease by 2 squares each time.



As students describe decreasing shape patterns, help them recognize that each term has a numeric value. The above pattern can be expressed as 10, 8, 6, ... by counting the number of squares in each term. Students may also find it useful to record the change from one term to the next as shown below:



Remind students that a pattern rule must have a starting point or the pattern rule is incomplete. For example, if a student describes the pattern 10, 8, 6, ... as a decrease by 2 pattern without indicating that it starts at 10, the pattern rule is incomplete.

PR02.03 Students should be able to extend a pattern by identifying the rule, and use the rule to build and draw the next three terms. Students should replicate the first three terms with concrete materials and then extend the pattern. The use of the concrete materials allows them to make changes if necessary and to build onto one term to make the next term. Students should be able to explain why their extension follows the pattern. It is important to note that for some patterns, there may be more than one way to extend the pattern and this is quite acceptable.

PR02.04 Students need opportunities to compare numeric patterns, discussing how they are the same and how they are different. When comparing decreasing patterns, compare the starting points and how each term decreases using a variety of representations such as shape patterns, hundred charts, and number patterns. For example, give students a page with four small hundred charts. Ask them to skip count backward and shade one chart by 2s, one chart by 5s, one chart by 10s, and one chart by 25s. Then discuss the pattern rule in each chart indicating the starting point and the amount of decrease.

PR02.05 Students should be provided with a variety of decreasing patterns that contain errors and be able to identify and explain the errors. For example, given the pattern 89, 86, 83, 80, 77, 75, 71, ..., they would state the pattern rule: Start at 89 and subtract 3 each time. Therefore, 75 is an error since it is only subtracting 2 not 3. A second error is 71 since it is subtracting 4 and not 3. To help students visualize this pattern they can shade the numbers on a hundred chart and look for the mistakes. Students may see that there are fewer than three numbers between 77 and 75 and more than three numbers between 75 and 71, therefore, it is an error.

PR02.06 Students should be able to create various representations of a decreasing pattern that follow a given pattern rule. Initially students will create patterns with concrete materials, then pictures, and then numbers. Some students may create a simple pattern, while other students may create a more complex pattern.

PR02.07 Students should be able to create decreasing patterns, concretely, pictorially, and symbolically, and be able to describe the pattern rule they used to create their representations. When creating decreasing patterns, initially students need to choose a starting point and then decide on the amount of the decrease. The amount of decrease may be either a constant amount or an increasing amount. Students should be able to describe their pattern by clearly explaining how it changes from one term to the next. Students may share their patterns and the strategies they used to create their pattern.

PR02.08 Students should have frequent experiences with solving real-world problems that interest and challenge them using decreasing patterns. They should use concrete materials or pictures to model the problem before determining the pattern rule. A variety of strategies may be used to solve the problem such as using a number line, a hundred chart, a picture, concrete materials, or skip counting.

PR02.09 Students should be able to identify and describe the strategy used to determine a missing term in a given decreasing pattern. Since patterns decrease in a predictable way, to determine a missing term the students should first look at the term that comes before and after. One strategy may be to identify and use the pattern rule. Other possible strategies may include using a number line, a hundred chart, a picture, concrete materials, or skip counting.

SCO PR03 Students will be expected to solve one-step addition and subtraction equations involving symbols representing an unknown number.

[C, CN, PS, R, V]

[C] Communication

[PS] Problem Solving

[CN] Connections

[ME] Mental Mathematics and Estimation

[T] Technology

[V] Visualization

[R] Reasoning

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- PR03.01** Explain the purpose of the symbol in a given addition and in a given subtraction equation with one unknown.
- PR03.02** Create an addition or subtraction equation with one unknown to represent a given combination or separate action.
- PR03.03** Provide an alternative symbol for the unknown in a given addition or subtraction equation.
- PR03.04** Solve a given addition or subtraction equation that represents combining or separating actions with one unknown using manipulatives.
- PR03.05** Solve a given addition or subtraction equation with one unknown using a variety of strategies including guess and check.
- PR03.06** Explain why the unknown in a given addition or subtraction equation has only one value.

Performance Indicator Background

PR03.01 The unknown value in an equation can be shown using a variety of symbols, such as a circle, a triangle, and an open box. It is important that a variety of symbols are used so students do not develop the misconception that an unknown can only be represented by one of these symbols. Students should be able to explain that in the equation $15 + \Delta = 18$, the triangle symbol represents the missing part, 3.

PR03.02 Students are expected to create an addition or subtraction equation with one unknown to represent a given combination or separation action. For example, when students are presented with the following story problem they are expected to create an addition or subtraction equation. Mary has 24 stickers on a roll and gives 10 stickers to her friend Betty. How many stickers are left on the roll? One student wrote, $24 - 10 = ?$ as his number sentence to represent the problem. Another student wrote, $24 - ? = 10$ as his number sentence. Have several students explain the reasoning for each number sentence.

PR03.03 When students create equations, they should know that they can use different symbols to represent the unknown to help solve story problems. For example, Josh has some marbles and he bought 12 more. Now he has 33 marbles. How many marbles did he have at the start? This can be represented with the equation: $\diamond + 12 = 33$ or $? + 12 = 33$.

PR03.04 and PR03.05 To solve given addition and subtraction problems, students should initially use manipulatives. For example, Ms. Best needs 18 pieces of construction paper for art class. She has 7 pieces, how many more pieces of construction paper does she need? Therefore, the equation for this problem would be $18 - 7 = ?$ or $7 + ? = 18$. Students may use counters to model the problem. Observe to see if students start with 18 counters and separate 7 from the group to find the unknown or if they start with 7 counters and add up to 18.

To solve addition or subtraction equations with one unknown, students need to explore and explain different strategies. Some examples of strategies may include, but are not limited to, the following:

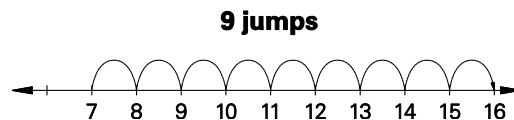
Guess and Check Strategy: This strategy is based on trying different numbers. The key is to think after each try and change or revise the guess when necessary. For example, $7 + \Delta = 16$.

- Think $7 + 7 = 14$, that is too low.
- Think $7 + 8 = 15$, that is too low but closer to 16.
- Think $7 + 9 = 16$. So the missing number is 9.

Mental Mathematics Strategy: For example, $7 + \Delta = 16$.

- Think doubles. I know $7 + 7 = 14$, and 14 is only 2 away from 16, so the missing number must be 9.

Number Line Strategy: Create a number line with the start point being 7. Then count up to 16, keeping track of the jumps on the number line.



It is important that students read and solve equations when the unknown number is on either the left side or the right side of the equal sign.

PR03.06 Students should be able to explain why the unknown in a given addition or subtraction equation has only one value. Present students with an equation such as $8 + \Delta = 17$. Starting with 8 counters on the table, secretly place 9 counters under a cup. Ask students to tell you how many you put under the cup by viewing how many more are needed to make 17. Once the students have determined that there are 9 counters under the cup, ask if there could be any other answer. Could the number be anything else? How do you know? Listen carefully to students' reasoning and explanations. This will provide insight as to how the students are thinking.

Measurement (M)

SCO M01 Students will be expected to relate the passage of time to common activities using non-standard and standard units (minutes, hours, days, weeks, months, years). [CN, ME, R]			
[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- M01.01** Select and use a non-standard unit of measure, such as television shows or pendulum swings, to measure the passage of time and explain the choice.
- M01.02** Identify activities that can or cannot be accomplished in minutes, hours, days, weeks, months, and years.
- M01.03** Provide personal referents for minutes and hours.
- M01.04** Select and use a standard unit of measure, such as minutes, hours, days, weeks, and months to measure the passage of time and explain the choice.

Performance Indicator Background

M01.01 Initially, it is essential that students are able to choose non-standard units, such as pendulum swings, television shows, sand-timers, or recesses that measure the passage of time in a uniform and appropriate manner. Hand claps do not always ensure uniformity of time from person to person. Ask students to select and justify an appropriate non-standard unit to estimate how long it would take them to do activities such as tying their laces, writing their names, walking down the hall, or going to a movie.

M01.02 Students will explore the concept of the passage of time and use time vocabulary such as minutes, hours, days, weeks, months, and years. It is useful to discuss throughout the day the duration of long and short events to develop a sense of the various standard units of time. Students will identify activities that can or cannot be completed in a given amount of time. For example, ask students to describe something they do that takes a minute, an hour, a week, a month, or a year.

Engage students in daily conversations whereby they need to select an appropriate unit of measurement for activities such as brushing their teeth, walking to school, reading a story, extracurricular activities, sleeping, summer vacation, or building a highway. Ask questions such as, Would it take hours or minutes to tie your shoes? Can a house be built in days, weeks, or months? Do we measure the growth of trees by days or years?

M01.03 Personal referents, such as the length of a favourite television show or how long it takes to properly wash their hands, allow students to better estimate time. Students can also use activities with known durations as referents for estimating the duration of other activities. For example, I know that it takes one hour to watch my favourite TV show, which is about the same time it takes to read a chapter in my book.

SCO M02 Students will be expected to relate the number of seconds to a minute, the number of minutes to an hour, the number of hours to a day, and the number of days to a month in a problem-solving context.

[C, CN, PS, R, V]

[C] Communication

[PS] Problem Solving

[CN] Connections

[ME] Mental Mathematics and Estimation

[T] Technology

[V] Visualization

[R] Reasoning

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

M02.01 Determine the number of days in any given month using a calendar.

M02.02 Solve a given problem involving the number of seconds in a minute, the number of minutes in an hour, the number of hours in a day, or the number of days in a given month.

M02.03 Create a calendar that includes days of the week, dates, and personal events.

Performance Indicator Background

M02.01 Using a calendar throughout the school year strengthens the students' sense of time. Each month brings a new calendar page to explore. As students are examining the calendar to determine the number of days in any given month, some students may find it easy to remember, using the jingle, Thirty days hath September, April, June, and November. All the rest have 31, ... Others may enjoy the knuckle method for remembering the number of days in each month. Make a fist showing four knuckles, start by pointing to the first knuckle and saying, January. The space between knuckles is February, the second knuckle is March, and so on. After saying, July, go back to the beginning making August land on the first knuckle and continue until year end. The months that land on the knuckles each has 31 days.

M02.02 Before engaging in problem-solving activities, students need to consolidate their understanding of number of seconds in a minute and minutes in an hour. Some examples follow.

- Direct students' attention to the analog clock. How many big numbers are on the clock? Ask students to point to the hour hand, tell them that when the hour hand moves from one number to the next, one hour has passed, or sixty minutes. Ask students to point to the minute hand, tell them that when the minute hand moves from one tick mark to the next, one minute has passed, or sixty seconds.
- Count the seconds aloud with the students to verify it takes 60 seconds for the minute hand to move from one tick mark to the next on an analog clock.
- Challenge students to estimate how long one minute is by having them place their heads on their desks. When they think one minute is up, they should raise their hand without looking up. At the end of one minute, tell the students that one minute is up.

M02.03 In the real world, the calendar is used to plan, keep track of appointments, and measure time. This is how it should be used in the classroom. In order to focus on the structure of the month and numerical patterns, have students build and create their own monthly calendar. They will need to write the months and the days of the week in order, number the days, and fill in any special dates for that month, such as birthdays, class trips, and physical education days. Having a one page, year-long calendar nearby will help students see and understand where the current month fits into a year's progression. Let students take their calendar home for scheduling personal activities.

SCO M03 Students will be expected to demonstrate an understanding of measuring length (cm, m) by			
<ul style="list-style-type: none"> ▪ selecting and justifying referents for the units centimetre and metre (cm, m) ▪ modelling and describing the relationship between the units centimetre and metre (cm, m) ▪ estimating length using referents ▪ measuring and recording length, width, and height 			
[C, CN, ME, PS, R, V]			
[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- M03.01** Provide a personal referent for one centimetre and explain the choice.
- M03.02** Provide a personal referent for one metre and explain the choice.
- M03.03** Match a given standard unit to a given referent.
- M03.04** Show that 100 centimetres is equivalent to 1 metre by using concrete materials.
- M03.05** Estimate the length of an object using personal referents.
- M03.06** Determine and record the length and width of a given 2-D shape.
- M03.07** Determine and record the length, width or height of a given 3-D object.
- M03.08** Draw a line segment of a given length using a ruler.
- M03.09** Sketch a line segment of a given length without using a ruler.

Performance Indicator Background

M03.01 and M03.02 When introducing centimetres and metres, it is important for students to have the opportunity to discover personal referents for these standard units of length. Students should think about how they could tell if something is about 1 cm or 1 m long if they did not have a ruler or a metre stick. They should identify and explain why the width of their finger is a personal referent for 1 cm and why the height of a doorknob from the floor is a personal referent for 1 m. Having these personal referents helps students visualize measurements and estimate more accurately. Personal referents also make the units easier for students to remember.

M03.03 Students should identify objects from around the classroom, that would be an appropriate referent for a centimetre or a metre; for example, a pencil, a garbage can, a teacher desk, or a glue stick.

M03.04 Students should recognize that a metre is 100 centimetres long. Although many metre sticks are marked up to 100, it is often still not clear to students. Working in groups with base-ten materials, students need to explore how many small cubes would line up along a metre stick to consolidate their understanding of the equivalence of 100 cm to 1 m.

M03.05 Students need to estimate the length of an object using personal referents. Students should find items in the classroom that are close to given centimetre or metre lengths using their personal referents as a measurement.

M03.06 This is the first year where students will begin to use a standard tool to measure length. It is valuable to initially use simpler rulers that are created by the students. Then move on to tools that are easy for students to read. Students should use rulers (or the side of the ruler) that show only numbered centimetres and not millimetres.

It is important for students to line up the 0 mark with one end of the shape being measured. Emphasis should be placed on counting the intervals between the numbers, rather than looking at the number on the ruler that is aligned with the end of the object. Lining up small cubes from base-ten materials along the ruler will demonstrate that the numbers on the ruler correspond to the number of small cubes, starting at 0.

It is also important to observe how students use a ruler to measure a shape that is longer than the ruler. Show students how to measure something that is longer than a ruler by marking, recording, and starting again.

M03.07 Using a centimetre ruler, students should measure the length, width, or height of a given 3-D object in the classroom, such as a lunch box, their desk, or a cereal box. Students can record their measurements using both the number and the measurement unit; for example, 3 cm or 3 centimetres. Ensure students are clear about the distance they should be measuring.

M03.08 Before creating a specific length, students should practise drawing a variety of straight lines, such as drawing a triangle, rectangle, or house. Once they establish comfort with the ruler, students should draw lines of given lengths. Students need to attend to the starting point when drawing a line segment of a given length.

M03.09 Once students have ample opportunities to measure and draw line segments of a particular length, they should be able to draw a line segment that is about a certain measure using a straight edge but not a ruler. Students should develop personal referents to aid them in this skill. For example, my thumb is 1 cm wide, and my hand-span is 15 cm wide. Students should sketch line segments of 6 cm, 10 cm, and 20 cm, and discuss how the use of their personal referent could help them with this activity.

SCO M04 Students will be expected to demonstrate an understanding of measuring mass (g, kg) by			
<ul style="list-style-type: none"> ▪ selecting and justifying referents for the units grams and kilograms (g, kg) ▪ modelling and describing the relationship between the units grams and kilograms (g, kg) ▪ estimating mass using referents ▪ measuring and recording mass 			
[C, CN, ME, PS, R, V]			
[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- M04.01** Provide a personal referent for one gram and explain the choice.
- M04.02** Provide a personal referent for one kilogram and explain the choice.
- M04.03** Match a given standard unit to a given referent.
- M04.04** Explain the relationship between 1000 grams and 1 kilogram using a model.
- M04.05** Estimate the mass of a given object using personal referents.
- M04.06** Measure, using a balance scale, and record the mass of given everyday objects using the units gram (g) and kilogram (kg).
- M04.07** Provide examples of 3-D objects that have a mass of approximately 1 g, 100 g, and 1 kg.
- M04.08** Determine the mass of two given similar objects with different masses and explain the results.
- M04.09** Determine the mass of an object, change its shape, re-measure its mass and explain the results.

Performance Indicator Background

M04.01 Using their understanding of a kilogram, ask students to brainstorm items that may have a mass of 1 gram. They may also use a small base-ten cube as a personal referent of a gram. You may wish to provide students with items such as a raisin, bean seed, jelly bean, or a paper clip, to conceptualize the sense of how a gram feels.

M04.02 It would be beneficial for students to have an opportunity to make a kilogram mass of their own. Provide students with materials such as sand, flour, sugar, and small cubes from base-ten materials to fill a container until it exactly balances with a 1 kg mass on a balance scale. Using this kilogram container they can now compare its mass to items in the classroom to help them find a personal referent for 1 kg.

M04.03 Using objects from the classroom, for example a counter, a raisin, a paper clip, a textbook, a sneaker, or a lunch box, ask students to identify whether the object is an appropriate referent for grams or kilograms.

M04.04 It is important for students to know that 1000 grams is equal to a kilogram. Using food items of various benchmark masses, such as 2 bags of 500 g, 4 boxes of 250 g, or you may wish to have a pre-counted bag of 1000 jellybeans, model how 1000 g is equal to 1 kg using a balance scale.

M04.05 Estimating mass is more difficult than estimating other measures, as the object’s size and shape is not directly related to its mass. Once students have established a personal referent for 1 g and 1 kg, they can now use their referents to estimate the mass of common objects such as an eraser, an apple, a juice box, or a textbook, or to estimate whether an object is heavier or lighter than 1 kg.

M04.06 Model how a balance scale can be used to determine the mass of everyday objects. Provide a variety of objects for students to use as they explore measuring mass. Students can record their measurements using both the number and the measurement unit, for example 3 kg or 3 kilograms.

M04.07 Students need many opportunities to find examples of 3-D objects that have a mass of approximately 1 g, 100 g, and 1 kg. With the mass of a gram being so small, it is important to provide students with opportunities to work with masses of varying benchmark sizes in an effort to develop a conceptual understanding for working with grams and kilograms. For example, a shoelace has a mass of 1 g, a nickel has a mass of about 5 g, a rod has a mass of about 10 g, 10 rods could be used to show 100 g, or 10 flats could be used to show 1 kg.

M04.08 Determine and record the mass of two similar items such as a Ping-Pong ball and a golf ball. Ask students to explain why two objects that appear to be so similar can have different masses. For example, a Ping-Pong ball is made of lighter material, is hollow, and intended to move short distances, whereas a golf ball is made of heavier material, is solid, and intended to travel long distances. Measuring and comparing similar items with different masses will help students understand the necessity for using the same unit of measurement when comparing the amount of matter those objects contain.

M04.09 Using manipulatives such as multilink cubes, ask students to create a shape and measure its mass. Then ask them to change the shape of their creation using the exact same material and measure its mass again. Having opportunities to compare objects that have been rearranged will strengthen the understanding that the same object rearranged will maintain its original mass.

SCO M05 Students will be expected to demonstrate an understanding of perimeter of regular, irregular, and composite shapes by

- estimating perimeter using referents for centimetre or metre (cm, m)
- measuring and recording perimeter (cm, m)
- create different shapes for a given perimeter (cm, m) to demonstrate that many shapes are possible for a perimeter

[C, ME, PS, R, V]

[C] Communication

[PS] Problem Solving

[CN] Connections

[ME] Mental Mathematics and Estimation

[T] Technology

[V] Visualization

[R] Reasoning

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- M05.01** Measure and record the perimeter of a given regular shape and explain the strategy used.
- M05.02** Measure and record the perimeter of a given irregular or composite shape and explain the strategy used.
- M05.03** Construct a shape for a given perimeter (cm, m).
- M05.04** Construct or draw more than one shape for the same given perimeter.
- M05.05** Estimate the perimeter of a given shape (cm, m) using personal referents.

Performance Indicator Background

M05.01 Students should find the perimeter of many different regular-, irregular-, and composite-shaped objects, before being introduced to pictorial forms. Students should explore their own methods for determining the perimeter of a shape and should not develop or follow a formula for calculating perimeter. For example, provide students with various regular and irregular polygons (e.g., squares, rectangles, and triangles), some string, and a ruler. Small groups of students should be asked to find the perimeter in a variety of ways. Some may use the string, while others go directly to measuring the sides with the ruler.

Metres are used when measuring the perimeter of large shapes such as a window, a door, or a room. Discuss with students possible strategies for determining the perimeter of the classroom. Students should explore and record the measurements as they go. Students need to be able to explain the strategies they used for finding perimeter as they proceed. Ask students what number sentence could be used to find the perimeter.

M05.02 Pentominoes may be used to illustrate measuring and recording the perimeter of a given composite shape. Pentominoes are shapes each made up of five squares, all of which must have at least one side matching up with the side of another. In addition to composite-shaped objects with straight sides, it is important to expose students to other shapes such as their handprint. Working with a partner, ask students to trace around their closed hand. Using string they can outline their handprint and then cut the string to determine the perimeter of their handprint by measuring the length of the string with their ruler. Again, students need to be able to explain the strategies they used for finding perimeter as they proceed.

M05.03 Students should be given opportunities to construct shapes of a given perimeter. Discuss with students that when constructing shapes for a given perimeter, they must remember that their shapes should be completely enclosed. It would be easier for students to begin their constructions drawing rectangles using centimetre grid paper and horizontal and vertical lines only.

M05.04 Students need to draw more than one shape for the same given perimeter. Students may use a geo-board or centimetre grid paper to explore various shapes with the same perimeter. They may explore various rectangles before exploring other more complicated shapes.

M05.05 Students should use personal referents when estimating perimeter. Through estimation, students can verify whether their measurements are reasonable. For example, provide students with a playing card and ask them how they could find the card's perimeter using the width of their finger. Invite students to estimate the card's perimeter using this personal referent. Then, using a ruler, ask students to find the actual perimeter and compare it to their estimate.

Brainstorm a list of possible referents for a metre, such as a wrapping paper roll, their arm span, or the height of the door knob from the floor. Ask students to select a referent to determine the perimeter of a given shape, such as a bulletin board, a bookshelf, or a table. Estimation in some circumstances may be the only measurement necessary.

SCO G01 Students will be expected to describe 3-D objects according to the shape of the faces and the number of edges and vertices.

[C, CN, PS, R, V]

[C] Communication

[PS] Problem Solving

[CN] Connections

[ME] Mental Mathematics and Estimation

[T] Technology

[V] Visualization

[R] Reasoning

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

G01.01 Identify the faces, edges, and vertices of given 3-D objects, including spheres, cones, cylinders, pyramids, and cubes and other prisms.

G01.02 Identify the shape of the faces of a given 3-D object.

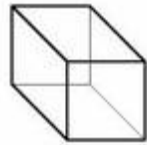
G01.03 Determine the number of faces, edges, and vertices of a given 3-D object.

G01.04 Sort a given set of 3-D objects according to the number of faces, edges, or vertices.

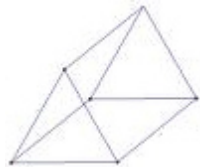
Performance Indicator Background

G01.01, G01.02, and G01.03 Students should be able to identify the faces, edges, and vertices as well as the shape of the faces of a given 3-D object.

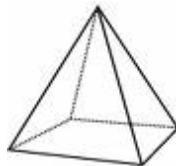
This prism has 6 rectangular faces, 8 edges, and 8 vertices.



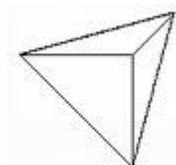
This prism has 3 rectangular faces, 2 triangular faces, 9 edges, and 6 vertices.



This pyramid has 1 rectangular face, 4 triangular faces, 8 edges, and 5 vertices.



This pyramid has 4 triangular faces, 6 edges, and 4 vertices.



Show students models and real-life objects of cylinders, cones, and spheres. Ask students what the difference is between these solids and the prisms and pyramids already studied. Show students the faces, edges, and vertices of each solid. Brainstorm, with the students, what each term means.

Students should be able to determine the number of faces, edges, and vertices of a given 3-D object.

- A cylinder is a 3-D object with 2 faces, 1 curved surface, 2 edges, and 0 vertices.
- A cone is a 3-D object with 1 face, 1 curved surface, 1 edge, and 1 apex.
- A sphere is a 3-D object with 1 curved surface, 0 faces, 0 edges, and 0 vertices.

G01.04 Students should compare and sort 3-D objects by observing the number of faces, edges, and vertices. A student may sort objects in various ways, such as those that have all square faces, those that have circular faces, those that have 8 vertices, or those that have straight edges. Students should play games with their peers where they sort objects and ask their peers to guess the sorting rule according to the number of faces, edges, and vertices.

SCO G02 Students will be expected to name, describe, compare, create, and sort regular and irregular polygons, including triangles, quadrilaterals, pentagons, hexagons, and octagons according to the number of sides.

[C, CN, R, V]

[C] Communication

[PS] Problem Solving

[CN] Connections

[ME] Mental Mathematics and Estimation

[T] Technology

[V] Visualization

[R] Reasoning

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

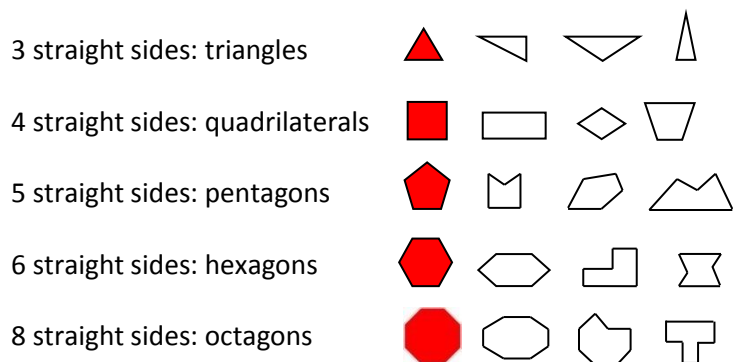
G02.01 Classify a given set of regular and irregular polygons according to the number of sides.

G02.02 Identify given regular and irregular polygons having different dimensions.

G02.03 Identify given regular and irregular polygons having different positions.

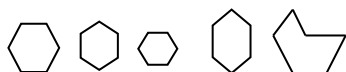
Performance Indicator Background

G02.01 Students should focus on comparing the number of sides as the key attribute for classifying polygons. In this outcome, students should be able to name the specific polygons—triangle, quadrilateral, pentagon, hexagon, and octagon. In the diagram below, the shaded polygons are regular polygons, and all others are irregular polygons.



Regular polygons are shaded.

Although pattern blocks are frequently used for geometric inquiry, most of these shapes are regular. Students may develop the misconception that only certain familiar polygons meet the criteria for these shapes. For example, students may not initially recognize all of the shapes below as hexagons.



Ask students to find examples of polygons in the world around them, perhaps even collect as many types of a shape as they can find. Sort the shapes according to the number of sides. By sorting polygons according to the number of sides, students can learn the names for the polygons.

G02.02 and G02.03 Students should be given opportunities to explore both regular and irregular polygons varying the positions and dimensions of the shapes. Provide students with a polygon to trace as they experiment with different positions by turning (rotating), flipping (reflection), and sliding (translations). Through many experiences with identifying polygons in a variety of positions, students should begin to realize that a polygon, regardless of its position, remains the same shape.

Provide students with various sizes of a particular polygon. Have students count the number of sides and identify the polygon. Having a variety of these experiences with different polygons, students should begin to realize that a polygon, regardless of its dimensions, remains the same shape.

Statistics and Probability (SP)

SCO SP01 Students will be expected to collect first-hand data and organize it using tally marks, line plots, charts, and lists to answer questions. [C, CN, V]			
[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- SP01.01** Record the number of objects in a given set using tally marks.
- SP01.02** Determine the common attributes of line plots by comparing line plots in a given set.
- SP01.03** Organize a given set of data using tally marks, line plots, charts, or lists.
- SP01.04** Collect and organize data using tally marks, line plots, charts, and lists.
- SP01.05** Answer questions arising from a given line plot, chart, or list.
- SP01.06** Answer questions using collected data.

Performance Indicator Background

SP01.01, SP01.03, and SP01.04 Students should be encouraged to collect, organize and record their data using a tally system, line plots, charts and lists to answer questions. Using tally marks is a simple way for students to keep track of information as they collect it. Lists are a way for students to record the objects collected. A list can be made into a chart on which, like last year, students would record their tally marks. Grouping the tally marks in fives makes it easier for students to total the numbers in each category by skip counting. When making a chart, students should always give it a title or heading to inform the reader about the meaning of the data. Students could then organize the data on line plots.

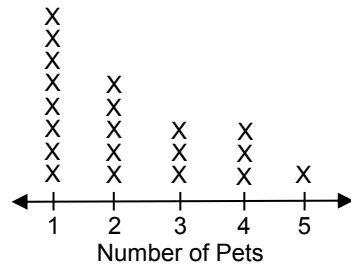
A line plot is a graph that uses a number line as a horizontal axis. Instead of a number line, the horizontal axis could just be a list of the collected data. A line plot provides a bridge from tally charts to bar graphs. At first students should create their line plots using grid paper, with one dot or cross per grid paper square. The dots or crosses are placed one above the other for each tally mark for each item in the list or chart. A line plot offers students a visual comparison of the different quantities of every piece of data.

SP01.02 Students should place different line plots they have made together, and have a discussion about what they might consider their common attributes. For example, they should notice that the attributes that are common include the title, the labels, the horizontal axis, and the use of dots or crosses. They should also notice that the common attributes can differ; for example there could be different titles, different use of the horizontal axis, and different labels. They might also notice that when a line plot does not have a title as in the picture below, it is hard to make sense of the graph.

Tally system

Number of Pets	Number of Students
1	TTTT III
2	TTTT
3	III
4	III
5	I

Line plot



Chart/List

Number of Pets	Number of Students
1	8
2	5
3	3
4	3
5	1

SP01.05 After a display of the data is constructed, discussing the information that can be obtained from the display is a valuable exercise. Students should work together to formulate questions that can be answered by other students using the data in the line plot, chart, or list. For example, from a line plot that displays the number of letters in your last name, students might formulate questions such as, What is the most common number of letters in a name? How many letters does the longest name in the class have? Shortest?

SP01.06 A good graph should communicate some overall impressions of the data to a reader. Students should be able to answer questions using the display of the collected data. It is also important that each graph accurately represents the data and includes clear labelling and a title.

SCO SP02 Students will be expected to construct, label, and interpret bar graphs to solve problems. [PS, R, V]			
[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

- SP02.01** Determine the common attributes, title, and axes of bar graphs by comparing bar graphs in a given set.
- SP02.02** Create bar graphs from a given set of data including labelling the title and axes.
- SP02.03** Draw conclusions from a given bar graph to solve problems.
- SP02.04** Solve problems by constructing and interpreting a bar graph.

Performance Indicator Background

SP02.01 Present students with vertical and horizontal bar graphs that represent two different sets of data. Discuss what common attributes the two bar graphs have, such as title, axes, labels for the axes, numerical scale, and bars. Discuss how the two bar graphs are different; for example, the titles of the graphs, labels for the axes, lengths and widths and spacing of the bars, and how some graphs have horizontal bars and others have vertical bars.

SP02.02 Once data has been collected, it should be organized and displayed so that questions can be asked and answered. Students have made line plots that have common attributes and the appearance of a bar graph. Students can discuss how they can change their line plots into bar graphs. As they create their bar graphs, have them check to see if all the attributes of the bar graph are visible.

SP02.03 Although students may be able to create bar graphs, some may experience difficulty with drawing conclusions from them. To develop the skill of interpreting graphs, students should be given bar graphs and be asked to draw conclusions. They should be encouraged to ask or write questions that go beyond simplistic reading of a graph. Both literal questions and inferential questions should be asked, such as, What can you tell about _____ by looking at this graph? How many more/less than ...? Based on the information presented in the graph, what other conclusions can you make? Why do you think _____? Eventually, when trying to solve the problem about what foods should be offered in the cafeteria, students would examine a bar graph that has the title, What Foods Should Be Available on the Cafeteria Menu? and be able to tell what food selections were considered the favourites by noticing which bars are the highest or longest. They may also draw conclusions that more students want healthy food for lunch than not healthy food.

SP02.04 Students should understand that to solve some problems, collecting and organizing graphs can help people to reach conclusions. Data is usually collected to answer questions, to discover something of interest, or most importantly, to solve a problem. Some examples of problems students might be interested in include, What should students be allowed to do during the lunch break? What foods should be available on the cafeteria menu? What foods should be removed from the cafeteria menu? What activities would you like to do in the gym? To answer questions like these, or to solve these kinds of problems, students could collect and display data, then interpret it.

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