

Processing Speed Impairments of Children with Arithmetic Disabilities

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ABSTRACT

The processing speed ability of children with arithmetic disabilities (AD; N = 19) and their normally achieving (NA) peers (N = 21) was examined across a series of 4 naming speed tasks (rapid digit naming, rapid letter naming, object naming, and quantity naming) and 4 sequencing speed tasks (making trails numbers, making trails letters, number counting, and alphabet). Children with arithmetic disabilities were found to be significantly slower across 4 tasks: rapid digit naming, object naming, quantity naming, and making trails numbers. No differences between the two groups were found on the rapid letter naming, making trails letters, number counting, and alphabet. Overall, results suggested that children with AD were significantly slower on tasks that contained numerical-based information. That children with AD were significantly slower on the object naming task may indicate weaker visual-spatial memory performance. Results are discussed in terms of the potential utilization of processing speed in the identification of arithmetic disabilities and the importance of processing speed in arithmetic calculation.

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CHAPTER ONE

INTRODUCTION

In the field of learning disabilities, research examining mathematical disabilities is in its infancy. With regard to reading disabilities, the developmental trajectories of reading development have been extensively investigated and empirically documented over the past 20 years (Gersten, Jordan, & Flojo, 2005). Furthermore, a number of valid measures have been identified that enable educators to assess specific cognitive processes that predict which students will likely have difficulty with learning to read (e.g., Comprehensive Test of Phonological Processing, [Wagner, Torgesen, & Rashotte, 1999]). These measures are being used as screening tools within schools to provide support for struggling students and to promote early intervention for children in kindergarten and Grade 1. However, literature examining the field of mathematical disabilities has not yet developed this wealth of screening tools that can assist researchers and educators in the identification and intervention of children who are experiencing difficulty developing expertise in mathematics. Limiting the development of intervention for students with mathematical disabilities is a clear understanding of the cognitive impairments that are characteristic of a mathematical disability. Various disciplines such as cognitive psychology, child development, and curriculum-based assessment have conducted a small number of studies and have found mixed results regarding the screening and development of mathematical disabilities (Gersten et al., 2005).

To date, research has shown that children with mathematical disabilities have deficits in the retrieval of basic combinations of numbers (i.e., number facts) from long-term memory (Geary, 2004; Hanich, Jordan, Kaplan, & Dick, 2001). This deficit seems

to inhibit their ability to understand mathematics and to grasp more complex mathematical concepts (Gersten et al., 2005). In addition, research on mathematical disabilities has also suggested that maturity and efficiency of counting strategies are additional areas of deficit for these children and might be valid predictors of the ability to profit from mathematics instruction (e.g., Geary, 1993, Geary, Hamson, & Hoard, 2000; Siegler & Shrager, 1984). Finally, difficulties in calculation fluency also appear to be a defining characteristic of mathematical disabilities (e.g., Hanich et al., 2001). As a group, children with mathematical disabilities tend to use developmentally immature arithmetical procedures (i.e. finger counting) and make a high frequency of procedural errors (e.g., Geary, 1990; Geary, Brown, & Samaranayake, 1991).

When examining the cognitive mechanisms that might underlie the difficulties that children with mathematical disabilities face, a number of studies have addressed the implications of working memory, however, the underlying cause of this memory deficit is unclear (Bull & Johnston, 1997). In particular, there is a noticeable absence of research exploring the role processing speed plays in working memory and long-term memory and the overall development of children's arithmetic performance. Frequently, research examining processing speed includes only one or two measures and report the presence or absence of a processing speed impairment based upon using this single task (e.g., Bull & Johnston, 1997). In addition, it is possible that tasks intended to measure processing speed are actually tapping into different cognitive process, such as short-term memory or attention.

The purpose of this study was to investigate whether children with arithmetic disabilities are characterized by a processing speed impairment. Research has yet to

thoroughly investigate the specific contributions and the role processing speed makes to children's mathematical performance. Because mathematics takes a place near the top of the curricular pyramid, early identification and intervention is essential to ensure children's mathematical growth. Without intervention children are likely to face substantial problems in their educational development as well as in their daily experiences throughout life (Gersten et al., 2005).

CHAPTER TWO

LITERATURE REVIEW

Research examining the cognitive domains underlying mathematical disabilities is in its infancy. Specifically, research examining the role processing speed plays in mathematics and its potential involvement in mathematical disabilities is unclear. It is well documented that children struggling in mathematics fail to develop proficiency in basic number facts (Geary, 2004). These children are less accurate and tend to provide correct answers more slowly than their normally achieving peers (e.g., Jordan & Montani, 1997). In addition, children with mathematical disabilities rely on inefficient calculation strategies such as finger counting while their normally achieving peers are using more sophisticated strategies such as direct retrieval from long-term memory (Geary, 1990; Geary & Brown, 1991). Due to these findings, processing speed might be a possible cognitive deficit for children with mathematical disabilities, and a cognitive domain that might predict their poor development of basic number facts.

Processing Speed

Research in cognitive psychology, developmental psychology, and neuropsychology have all alluded to the view that speed of information processing is a key element in one's ability to reason, think, and remember (Kail, 2000). Generally, research has shown that differences in processing speed reflect developmental changes in a global mechanism that limits performance on a range of tasks. In addition, the speed at which individuals are able to process information has been linked to general intelligence. Children's ability to process information rapidly increases throughout development. As

processing speed increases, individuals are able to use their working memory more effectively, which in turn, enhances performance on a range of reasoning tasks (Case, 1985; Case Kurland, & Goldberg, 1982). In addition, processing speed can directly affect performance by allowing faster retrieval of task-relevant information from long-term memory (Kail, 2000). Overall, rapid processing speed is an important aspect of general intellectual power.

Developmental Changes in Processing Speed

Research has found that speeded performance develops substantially throughout childhood and moderately throughout adolescence (Kail, 1992). That is, processing speed increases rapidly through early childhood, increases less rapidly during adolescence, reaches its apex during early adulthood, and gradually begins to decline in late adulthood. Overall, processing speed seems to reach functional maturity in early to middle adolescence (Kail, 1986). The same pattern of development can be seen when different speeded tasks are executed. On a wide range of motor, perceptual, and cognitive tasks where participants are required to respond rapidly, a common pattern of age differences has been found. Specifically, 8- to 10-year-olds generally respond at a speed that is 5 to 6 standard deviation units below the average speed for young adults whereas 12- and 13-year-olds respond at speeds more than a full standard deviation below the average for young adults (Kail, 1991). The consistent pattern that is evident across development indicates that a global mechanism limits the speed with which children and adolescents can process information. Said differently, research suggests a common mechanism not specific to particular tasks or domains might be responsible for age-related changes in

speeded performance and in the development of an overall information-processing system (Kail, 2000).

Researchers have looked within cognitive psychology to find basic areas of mental functioning that might correspond to this global information processing mechanism. Kail (2000) gives the example of the mind working as a network of multiple nodes which are interconnected. These networks are analogous to brain processes and neural networks, albeit at a much less complex process than the actual human nervous system. Basically, information is processed at a speed which would correspond to the rate of transmission from one node to another while only a limited number of nodes can be active at any time. Additionally, the age-related changes seen across development might reflect an age-related change in the rate of neural communication. Thus, it is reasonable to view processing speed as a basic parameter of cognitive functioning that changes with development, due, at least in part, to underlying neurological factors (Kail, 2000).

Processing Speed and Intelligence

Processing speed, or the rate at which an individual is able to process various kinds of information, is also an important element of intelligence. In fact, most psychometric theories of intelligence include a factor called perceptual speed or processing speed (Kail, 2000). Studies comparing populations with intellectual impairments, studies of cognitive development, and studies predicting childhood intelligence from infancy all show that speed of information processing is an important element in intelligence. For example, Kail (1992) found that the relationship of reaction times for individuals with and without mental retardation appeared much the same as the

relationship of reaction times for children and adults. Fry and Hale (1996) reported that processing speed, memory, and reasoning are all interrelated. When 7- to 19-year-olds were administered measures of processing speed, working memory, and fluid intelligence, it was found that age-related increases in speed were associated with increased working memory capacity, which was associated with higher scores on tests of fluid intelligence.

Processing Speed and Memory

Processing speed is an important cognitive process that has been associated with memory performance. For instance, slow processing speed impairs performance on specific tasks because individuals are unable to complete all components of a task in the time they are provided (Kail, 1992). Specific cognitive components that are related to memory have been found to be sensitive to age-related changes in processing speed. For example, Kail (1992) found that age and processing speed independently contributed to articulation rate, which influenced accurate recall of information. A general model of working memory was also examined which focussed on the central executive and the articulatory loop which is used to store phonological information. Information can be lost quickly in the articulatory loop but can be refreshed through rehearsal. With increasing age, individuals needed less time to execute cognitive tasks. With greater processing speed, information can be refreshed more frequently in the articulatory loop. In addition, age contributed independently to more rapid rehearsal. More rapid rehearsal of words yielded greater recall accuracy (Kail & Park, 1994). Evidence supporting rehearsal rate is also found in studies examining articulation rate. Children with speech disorders, who

articulate words more slowly, are less accurate at remembering spans of digits than children without speech disorders (Raine, Hulme, Chadderton, & Bailey, 1991). Links between processing speed, articulation rate, and memory have also been examined by Kail and colleagues (e.g. Kail, 1992; Kail & Park, 1994). These studies found that age was associated with greater articulation rate and greater memory span. Results suggested that processing speed not only affects speeded tasks, but those which did not require speeded elements.

Processing Speed and Reading

Speed with which children can name familiar objects has been suggested as a predictor of reading skill. For example, Wolf, Bally, and Morris (1986) found a significant correlation ($r = .66$) between kindergarten children's naming speed (letters and digits) and their performance on a word recognition task in Grade 2. In a similar study, which compared digit naming speed and scores on the Reading Recognition subtest of the Peabody Individualized Achievement Test (PIAT), rapid naming of digits predicted word recognition and comprehension (Spring & Davis, 1988). Kail (1994) continued with this line of research and found that processing speed followed a developmental path where age-related changes in processing speed was associated with faster naming speeds, which was related to word recognition and to reading comprehension. Kail found that processing times are determined by a global mechanism that actually limits the speed with which most cognitive processes are executed. Here, naming tasks appeared to predict decoding of words because they measured the speed with which codes for familiar stimuli can be accessed. The faster an individual is able to

access and to process information, the more time they have to integrate other pieces of information before it is lost from working memory.

Processing Speed and Mathematics

Mathematics is a broad domain which encompasses many different branches such as measurement, calculation, and quantification through the use numbers or symbols. At the elementary grade a variety of different strands make up the mathematics curriculum. These include strands such as numeration, measurement, arithmetic, algorithmic computation, and problem solving (Fuchs et al., 2006). A building block of these strands is counting and basic arithmetic. Frequently, when children enter school, they have been already exposed to several mathematical skills such as number recognition and counting. Each of these is related to basic addition abilities, which is one of the foundational skills throughout the school curriculum (National Council of Teachers of Mathematics, 2000, 2006).

When first learning how to solve simple addition problems, children typically count both addends (e.g., $5 + 3$). To do this, many children will count using their fingers (finger counting) and in other instances they are able to count verbally the addends (verbal counting). The two most common strategies children use to count the addends (with or without the use of their fingers) are the counting-all procedure or the counting-on procedure (Geary & Hoard, 2005). When engaged in the counting-all procedure, children count all addends beginning with 1. Children initiate the counting-on procedure by stating the larger valued addend and then counting the number equal to the smaller addend (e.g. stating 5, then counting 6, 7, 8 to solve $5 + 3$). The counting-on procedure

requires the child to understand that stating the cardinal value of the first addend is a shortcut, in that they do not have to begin counting from 1. This procedure also requires the child to understand that the order in which numbers are added together does not affect the accuracy of the answer (Geary, 1994).

Children's counting strategies improve as their conceptual understanding of counting improves. Verbal counting is a common strategy used by children in kindergarten and Grade 1. Kindergarten children mainly engage in the counting-all procedure, while those in Grade 1 typically use the counting-on procedure (Geary, 1994). Important to note is that children typically do not need to be taught the different addition strategies or the progression toward increased efficiency of these strategies; rather, children tend to discover these strategies and progress to more efficient procedures independent of instruction (Siegler & Jenkins, 1989). This improvement in understanding is also accompanied by a gradual shift from using the counting-all strategy to using the counting-on strategy (Geary, Bow-Thomas, & Yoa, 1992).

A more advanced strategy used to solve addition problems involves retrieving the answers directly from long-term memory. The representation of basic math facts in long-term memory is related to the development of children's counting procedures. When a child uses counting to solve a problem, the answers generated by the count become associated in memory. For example, the child counts "5, 6, 7," to solve the problem $3 + 4$, the last number counted, 7, becomes associated with $3 + 4$ (Geary, 1994). For the association to happen, the augend, addend, and generated answer must be held simultaneously in working memory. It is more likely that a child using the counting-on procedure instead of the counting-all procedure will develop an automatized fact because

there is less information to be held in working memory for a shorter amount of time. If a child solves the same problem (i.e., $3 + 4$) using the counting-all procedure by counting “1, 2, 3, 4, 5, 6, 7,” it is possible that they will forget the first number of the problem (i.e., 3) before finishing the count. Thus, the child might be able to generate the correct answer, but forget all or a proportion of the problem in the process, preventing an association in long-term memory to develop (Geary, 1994). In other words, slow counting might hinder the development of associations between problems and answers, and their association in long-term memory.

Once a child has ‘memorized’ math facts (i.e., automatized recall) whereby they can accurately and quickly recall math facts from long-term memory, children are able to use more advanced memory-based problem-solving strategies. When confronted with a math problem, the child is able to retrieve answers automatically from memory instead of engaging in less efficient procedures, such as counting. Decomposition, an advanced strategy using memorized facts, involves reconstructing the answer based on the retrieval of a partial sum (Geary & Hoard, 2005). For example, to answer the question $6 + 7$, a child might retrieve the answer to $6 + 6$ (i.e., 12), and then add 1 to the partial sum.

Overall, as children become more familiar with a range of strategies and improve strategy use, they become more efficient at using memory-based strategies and are able to produce answers more quickly. Slow counting leads to slow development of memory representation of math facts, whereas fast counting leads to development of memory representation with many fewer counting trials (Geary et al., 1992). As the total time taken to answer problems decreases due to better efficiency, the demands on working memory also decrease allowing the child to engage in more complex problem solving.

Mathematical Disabilities

It is estimated that about 6% of school-aged children have some type of learning disability in mathematics (Badian, 1983; Kosc, 1974; Rivera, 1997). Literature has identified potential areas of cognitive deficits for these children; however, differences in cut-off levels on achievement scores seem to be related to which cognitive processes reach significant levels of impairment. That is, a learning disability in mathematics can present itself in the form of deficits in the ability to process information in one domain (e.g., arithmetic) or in one or multiple individual competencies within each domain (e.g., addition) (Geary & Hoard, 2005). The basic cognitive deficits of children with mathematical disabilities have been found to involve five component skills: counting and arithmetical procedures, fact retrieval, conceptual knowledge, working memory, and speed of processing (Geary, 1994). Of these, the procedural and fact-retrieval components are functional skills which are used directly to solve arithmetic problems. The remaining components represent abilities that contribute to the procedural and fact-retrieval components (Geary, 1994).

Arithmetic Impairments of Children with Mathematical Disabilities

Evidence for weak procedural and fact-retrieval skills have been found by Geary (1990). Grade 1 children with mathematical disabilities and normally achieving children in Grade 1 were administered a series of addition tasks. Strategies that children used to solve simple addition problems as well as how quickly they executed each strategy were noted. Overall, it was found that children who showed a persistent mathematical disability were characterized by a high frequency of procedural (i.e., counting

procedures) and fact-retrieval errors, the frequent use of the immature counting-all procedure, and a large variability in the speed of executing both the counting and retrieval strategies. It was also noted that the solution-time patterns for fact retrieval were unsystematic when compared to those of normally achieving children.

A follow-up study was conducted 10 months later, at the end of Grade 2 (Geary et al., 1991). Normally achieving children and those who were previously identified as having a mathematical disability but had improved over time increased their use of fact retrieval and decreased their use of counting to solve addition problems. In addition, these groups made fewer counting and fact-retrieval errors and were also faster at executing both types of strategies. Children with mathematical disabilities made no improvements in the kind of strategies they adopted over the 10 month period. In particular, they showed no change in the number facts they remembered (about 25%), and they also had a very high rate of errors. Specifically, by the end of Grade 2, children with mathematical disabilities were making eight times as many fact-retrieval errors as normally achieving children. This fact-retrieval deficit was reflected in a difficulty retrieving facts from long-term memory. As the facts were retrieved from memory, the associated retrieval speeds were highly unsystematic, thus, causing a high frequency of retrieval errors.

Overall, children with mathematical disabilities seem to catch-up with their normally achieving peers in counting strategies but their ability to retrieve basic facts does not improve over time. When these children do retrieve arithmetic facts from long-term memory, they commit more errors and show error and reaction time patterns that differ from those found in their normally achieving peers (e.g., Geary, 1990; Geary &

Brown, 1991; Rasanen & Ahonen, 1995). Two potential sources of these retrieval difficulties have been found; a deficit in the ability to represent phonetic/semantic information in long-term memory (Geary, 1993) and a deficit in the ability to inhibit irrelevant associations from entering working memory during problem solving (Geary et al., 2000).

Jordan and Montani (1997) found evidence for retrieval deficits when examining problem solving and number-fact skills of Grade 3 children with specific and comorbid mathematical disabilities. Children with both mathematical disabilities and comorbid disabilities relied more on backup strategies than normally achieving children. Conversely, children who were normally achieving used retrieval more often than children with mathematical disabilities, singular and comorbid. Overall, fact retrieval difficulties appear to be a central impairment in mathematical disabilities.

Cognitive Impairments of Children with Mathematical Disabilities

Working Memory

Working memory impairments have also been found to be a defining characteristic of children with mathematical disabilities. Working memory is defined as one's ability to hold information in current consciousness while simultaneously engaging in other mental processes (Just & Carpenter, 1992). Working memory is a triple component system controlled by the central executive, an attention-driven control mechanism that directs information to and from two core slave systems (Baddeley, 1986). These slave systems include a language-based phonological loop and a visuospatial sketchpad. Literature examining working memory function in children with mathematical

disabilities has established that these children do not perform as well as typically achieving children. However, it is not well understood which mechanisms of working memory are responsible for the mathematical impairments of these children (Geary et al., 2007).

For example, McLean and Hitch (1999) found visual spatial working memory deficits as well as executive processing deficits in children with mathematical disabilities. When compared to younger ability-matched children, these children were impaired in their ability to control interactions with long-term memory. Siegel and Ryan (1989) found specific working memory deficits in tasks that involved processing of numerical information. In addition, Hitch and McAuley (1991) confirmed this numerical processing deficit and found no differences when examining verbal working memory tasks. Contrary to these studies, Swanson and Sachse-Lee (2001) found deficits in both general working memory as well as verbal working memory when children with learning disabilities were compared to their normally achieving same-aged peers. Passolunghi and Siegel (2001, 2004) found supporting evidence of a general working memory deficit in children with mathematical disabilities, not restricted to a numerical working memory.

Processing Speed

Numerous studies examining the cognitive processes that underlie mathematical disabilities have implicated short-term memory and working-memory. However, the underlying deficits of these memory impairments remain unclear (Bull & Johnston, 1997). Researchers have implicated processing speed, the maximum rate that cognitive functions are preformed (Kail, 1992), as a potential domain of cognitive functioning

impairment in these children. However, there remains a noticeable lack of research and consistency among studies exploring the specific role of processing speed in children with mathematical disabilities. Studies have found group differences in processing speed (e.g., auditory-based deficits such as rapid naming of speech and non-speech tones; Waber et al., 2001, and visually-based deficits such as parallel and serial filtering tasks; Weiler et al., 2000) when comparing children with and without learning disabilities. Because it is widely held that children with mathematical disabilities frequently provide correct answers more slowly and fail to show a shift from procedural-based problem solving (e.g., counting) to memory-based problem solving (i.e., direct retrieval) (e.g., Geary, 1990; Geary et al., 1991; Geary, Widaman, Little, & Cormier, 1987), processing speed might be a cognitive domain that contributes to calculation difficulties.

Bull and Johnston (1997) found that processing speed was the best predictor of arithmetic competence among 7-year-old children. In fact, processing speed subsumed all of the variance in arithmetic skill and was found to be the best predictor of mathematics performance. More specifically, as mathematics performance increased, time taken to complete the processing speed tasks decreased. In addition, time taken to retrieve arithmetic answers was also correlated with processing speed, suggesting that this measure represents a potential area of impairment for children with mathematical disabilities. Processing speed might facilitate counting speed; as young children gain speed in counting when performing arithmetic calculations, they successfully pair problems with their answers in working memory before decay sets in allowing associations in long-term memory to be established.

Additional evidence for processing speed differences was found by Hanich et al. (2001). Normally achieving children were significantly faster than children with mathematical disabilities when engaged in exact calculation or arithmetic combination tasks. Two lines of reasoning might account for completion time differences between children with and without mathematical disabilities. First, children with mathematical disabilities might use the same types of strategies as normal achieving children to solve arithmetic problems but are slower at overall execution of basic operations (Kirby & Becker, 1988). Second, children with mathematical disabilities and normally achieving children might use a different mix of problem-solving strategies and it is these differences that account for variations in computation times. From here, the issue that needs to be addressed is whether normal achieving children and children with mathematical disabilities differ in processing speed when they use the same strategies to solve problems.

Differences between normally achieving children and children with mathematical disabilities have also been found in retrieval times. Efficient retrieval (quick and accurate) requires strong representations of problems and their answers in long-term memory. If children are able to retrieve efficiently answers from long-term memory there is no need to engage in time consuming procedures such as finger counting or other laborious strategies. When examining children with mathematical disabilities and children with comorbid mathematical and reading disabilities, Jordan, Hanich, and Kaplan (2003) reported that children with mathematical disabilities appeared to have consistent difficulties with rapid fact retrieval and, by extension, calculation fluency. These children performed just as poorly as children with comorbid disabilities on a

forced-retrieval task where they were required to respond to number combinations quickly. Both children with mathematical disabilities and children with comorbid disabilities relied on finger counting, even at the end of Grade 3. Again, these results suggest that children with mathematical disabilities (singular and comorbid) have trouble making the transition from procedure-based to memory-based calculation.

In contrast, when utilizing an age-matched/achievement-matched design, Berg (2008a) found no differences across processing speed tasks between children with severe arithmetic difficulties (SAD) and their chronologically aged-matched (CAM) peers. Specifically, children with SAD were not impaired on digit naming tasks (i.e., retrieval from long-term memory), counting speed tasks (i.e., processing number sequences), and digit articulation (i.e., retaining information within current consciousness).

Sequencing Speed and Naming Speed Deficits

Sequencing Speed

Two areas that are likely to hold particular importance in understanding the cognitive impairments of children with mathematical disabilities are sequencing speed and naming speed. Sequencing speed is the speed at which individuals can identify stimuli that have standard numerical sequences (e.g., digits—1, 2, 3, etc.) or standard non-numerical sequences (e.g., letters—A, B, C, etc.). It has often been reported that as children gain speed in sequencing (i.e., counting) to solve arithmetic problems they are better able to pair efficiently problems with answers, and thus, establish associations in memory. Lemaire and Siegler (1995) reported that improvement in children's speed and accuracy is reflected in four types of specific strategic changes. When faster and more

accurate computation is developed, children will use new strategies, will increasingly use more efficient strategies to solve problems, will use each strategy more effectively, and will be more adaptive when choosing among strategies.

With slow sequencing speed (i.e., counting), representations are likely to decay or be forgotten before counting is completed (Hitch & McAuley, 1991). The main assumption is that if items are articulated faster, then more items can be refreshed in memory before decay. Such a scenario would decrease the possibility of the problem and answer from becoming associated in working-memory: slow counting speed would likely cause the augends to decay allowing associations between the problem and answer impossible. Even if the child were able to generate the correct answer, slow processing speed might decrease the possibility of the problem-answer association from being stored in long-term memory. Thus, the development and mastery of basic arithmetic facts is likely to be related to the speed of executing computational strategies, as well as to computational accuracy (Geary et al., 1991).

Evidence for sequencing speed differences was found by Hitch and McAuley (1991). These researchers found that normally achieving children and children with mathematical disabilities differed in fluency and control of counting operations. Children with mathematical disabilities were found to be slower at simple counting tasks such as reciting the numbers from 1 to 20, thus affecting their ability to perform many arithmetical tasks such as simple calculations involving storing temporary numerical information. Again, this inability to hold and manipulate information in working memory is likely to cause information to decay before associations can be made, thus hindering long-term memory representations.

Studies have found mixed results when assessing verbal counting speed, number encoding, answer production, and naming speed. When examining verbal-counting strategies, Geary (1990) found no difference in counting speed between Grade 1 children who were normal achieving and children with mathematical disabilities. However, children with mathematical disabilities were slower at other processes such as number encoding and answer production. In a follow-up study, results revealed that by Grade 2, children with mathematical disabilities were slower at counting than normal achieving children (Geary et al., 1991). Contrary to these findings, Geary and Brown (1991) found no counting-speed differences between Grade 4 normally achieving children and children with mathematical disabilities. Again, children with mathematical disabilities were found to be slower than their normally achieving peers at other processes such as number encoding.

Naming Speed

A second processing speed factor theoretically associated with mathematical ability is naming speed. Research examining the cognitive deficits of children with reading disabilities has found deficits in the rapid recognition and retrieval of visually presented linguistic stimuli (i.e., naming speed). Wolf and Bowers (1999) proposed that individual differences in the visual processing speed of letters might represent sources of reading dysfunction. They suggested that naming speed contributes to reading disabilities and that slow naming speed impairs the automatized retrieval of word information in memory. A similar process might explain the difficulties children with mathematical disabilities have storing and retrieving arithmetic facts from memory. If children with

mathematical disabilities experience difficulty storing and retrieving numbers in memory due to slow naming speeds, they will subsequently have difficulty storing and retrieving answers to arithmetic calculations.

Additional support for a weakness in long-term memory storage (i.e., naming) has been found by Garnett and Fleischner (1983) when assessing performance on rapid automatized naming (RAN) tasks. They found a significant relationship between proficiency in computing basic number facts and performance on the RAN task; basic facts computation was associated with greater speed on RAN, while less proficient computation was associated with slower RAN performance. Overall, children with learning disabilities performed poorer than their normally achieving peers. Again, these children with learning disabilities were suggested to be weak automatizers compared to their normally achieving peers.

As noted previously, a more recent study examining cognitive impairments of children with severe arithmetic disabilities was conducted by Berg (2008a). An age-matched/achievement-matched design was used, where 3 groups of children (i.e., severe arithmetic difficulty, SAD; chronologically age-matched, CAM; arithmetic achievement-matched, AAM) were compared across 2 different naming speed tasks. The first task consisted of a digit naming exercise, in which the child was required to read aloud sets of 9 randomly ordered Arabic digits (1 through 9). The second task consisted of a counting dots exercise, in which the child counted a matrix of black dots presented on a white card. In contrast to previous findings, children with SAD did not show any processing speed (i.e., naming speed) impairments. Notably, significant age-related differences were found. Specifically, the AAM children performed significantly slower than both children

with SAD and CAM children on the naming digits task, while younger children were slower than both groups on the counting dots tasks.

Although significant results were not found, limitations in design may account for this absence. Firstly, counting speed differences likely vanish due to rehearsal effects as children gain experience with counting and get older. The SAD group consisted of children in Grade 3 to 6 who may have become experts at counting, thus, decreasing the gap between the CAM group and AAM group with regards to counting speed. Utilizing a younger sample may have yielded different results. Secondly, the digit naming task only contained 9 digits and may not have been sensitive enough to pick up differences in speed. Finally, the design did not include any sequencing speed tasks, which may tap into alternative kinds of processing speed difficulties.

For children, encoding and efficient retrieval of math facts is essential in order to understand and comprehend dialogue about number concepts and various problem-solving approaches in school (Gersten et al., 2005). Students with mathematical disabilities who use slow immature strategies to perform arithmetic tasks are likely to progress slower and become confused during instruction when teachers assume that they can effortlessly retrieve such information and use this assumption as a basis for explaining more complicated concepts (Gersten et al., 2005). Consistently, research has documented that children with mathematical disabilities generally tend to process mathematical information more slowly than their normally achieving peers. Thus, processing speed is likely to be an important area to consider when investigating the cognitive impairments of children with mathematical disabilities. Investigation of the speed with which children process numerically-based information, specifically counting

speed and naming speed, would provide researchers and educators insight into understanding the difficulty that children experience with mathematics, specifically arithmetic.

The Present Study

The present study was designed to investigate whether children with arithmetic disabilities are characterized by a processing speed impairment. Repeatedly, literature has shown that children who struggle in mathematics fail to show proficiency in calculation, are less accurate, do not develop sophisticated strategies such as rapid retrieval, and provide correct answers more slowly than their normally achieving peers (e.g., Geary, 2004; Jordan & Montani, 1997; Geary 1990, Geary & Brown, 1991). Overall, a child's success in mathematics begins with their ability to develop skills such as counting and basic arithmetic, which are building blocks for mathematical development.

A child's ability to efficiently process information may be the underlying cognitive deficit involved in arithmetic and calculation disabilities. Research to date, has yet to come to an agreement about this conclusion. For example, Bull and Johnston (1997) reported that processing speed was the single best predictor of math achievement. However, Berg (2008a) found no differences in processing speed between normally achieving children and those with severe arithmetic difficulties. Notably, when examining working memory, Siegel & Ryan (1989) found that children with specific arithmetic disabilities had a deficit in working memory for numerically-based information only. The present study was designed to extend these lines of research by utilizing a younger sample size, including multiple measures of naming speed tasks and

sequencing speed tasks, and by including tasks involving numerical and non-numerical information when comparing processing speed ability of children with AD and those who are NA.

CHAPTER THREE

METHOD

Data for this proposed study was part of a larger study previously conducted by Berg. The purpose of the Berg study was twofold. The first purpose was to investigate the role of specific cognitive processes in children's arithmetic calculation. The second purpose was to examine the cognitive processing impairments of children with mathematical disabilities. The present proposed study aligns with the second purpose.

A total of 87 children in Grades 2 and 3 from five schools in Atlantic Canada participated in this study. Children's ages ranged from 94 months to 110 months. The socioeconomic status of individual students was not assessed; however, each of the schools that participated in the study was located in predominately working class and middle-class neighbourhoods. All children spoke English as their first language. No child had been identified as having English language difficulties that would have made it difficult for the child to complete any of the study activities. Children completed a series of tests that assessed children's academic achievement, fluid intelligence, naming speed, sequencing speed, short-term memory, working memory, executive functioning, and mental arithmetic (written addition and mental addition). Only those tests that related to the purpose of the present proposed study were selected for analysis; these are described below.

Participants

From the original study two groups of children were purposely selected. One group was identified as academically normally achieving in arithmetic and one group was identified as having an arithmetic disability. The term arithmetic disability was

purposefully chosen to describe children who were experiencing significant difficulty in mathematics according to the following rationale. First, the term disability refers to a specific cognitive deficit or impairment that is persistent over time. In addition, children in the present study were all classified as having average fluid intelligence, as reflected in their scores within the normal range on the RCPM. Second, arithmetic, as opposed to mathematics, was chosen due to the nature of the standardized achievement test used (i.e., the WRAT3). The WRAT3 includes problems containing single digit and multiple digit addition and subtraction, what is typically referred to as computation. The term mathematics is a broader term that refers to problem solving, algebra, and geometry, skills that were beyond the scope of the present study.

The classification of children into two groups corresponded with standard procedures used in research (e.g., Swanson & Sachse-Lee, 2001). Children were identified as normally achieving in arithmetic if they met each of three criteria: (1) they scored above the 30th percentile (equivalent to a standard score of 92) in arithmetic (WRAT3-A), (2) they scored above the 30th percentile (equivalent to a standard score of 92) in reading (WRAT3-R), and (3) they scored within normal range in fluid intelligence (i.e., > 80 and < 120 on the RCPM). Children were identified as having an arithmetic disability if they met each of three criteria: (1) they scored at or below the 25th percentile (equivalent to a standard score of 90) in arithmetic (WRAT3-A), (2) they scored above the 30th percentile (equivalent to a standard score of 92) in reading (WRAT3-R), and (3) they scored within normal range in fluid intelligence (i.e., > 80 and < 120 on the RCPM).

Instruments

Children's performance on 3 test batteries was selected for analysis in the present study. The first battery focussed upon children's academic achievement in arithmetic and reading. The second battery measured children's fluid intelligence. A third battery assessed children's processing speed ability.

Academic Achievement

The Wide Range Achievement Test-Third Revision (WRAT3; Jastak & Jastak, 1993) was administered to measure children's arithmetic achievement and reading achievement. The arithmetic subtest focuses upon reading numbers, counting, mental arithmetic, and written calculation. The reading subtest focuses upon recognizing and naming letters, and pronouncing words. The WRAT3 has been used extensively to assess children's achievement and to identify children with learning disabilities (e.g., Mabbott & Bisanz, 2003; Wilson & Swanson, 2001). Raw scores and standard scores ($M = 100$, $SD = 15$) based on age-appropriate norms was converted into percentile scores for each child. These scores were used to form the two participant groups.

Fluid Intelligence

The Raven's Colored Progressive Matrices (RCPM; Raven, 1976) was administered to measure children's fluid intelligence. This test has been used by other researchers in studies of children's arithmetic calculation (e.g., Swanson & Sachse-Lee, 2001). In each test item, the child was asked to identify the missing segment required to complete a larger pattern. A child's score (range 0 to 36) was the total number of

problems solved correctly. Raw scores and standard scores ($M = 100$, $SD = 15$) based on age-appropriate norms were converted into percentile scores for each child. These scores were used to form the two participant groups.

Processing Speed

Eight tasks were administered to assess children's processing speed. These tasks can be subdivided into two categories: naming speed and sequencing speed.

Naming Speed

Four tasks assessed children's naming speed for numbers, letters, objects, and quantities. While numbers and quantity naming tasks were each included because of their clear relationship to mathematics, the inclusion of letters and objects naming tasks was to investigate whether any naming speed impairment was specific to mathematical representations or related to a general naming impairment.

Rapid Digit Naming. To assess children's ability to rapidly name digits, the rapid digit naming subtest was administered (Wagner et al., 1999). Children were presented with a picture book containing two different pages, each containing four rows and nine columns of six randomly arranged digits. They were instructed to start naming the digits on the top row, from left to right, move to the next row, and so on, until all of the digits were named.

Rapid Letter Naming. To assess children's ability to rapidly name letters, the rapid letter naming subtest was administered (Wagner et al., 1999). Children were presented with a picture book containing two different pages, each containing four rows and nine columns of six randomly arranged letters. They were instructed to start naming the letters on the top row, from left to right, move to the next row, and so on, until all of the letters were named.

Objects Naming. In the Objects Naming task, a series of segments each containing an individual object (i.e., shape) was presented: circle, triangle, square, and diamond. Children were instructed to name the object (i.e., shape) as quickly as possible. For example, when the segment was presented, the correct response was "triangle." Children were instructed to name correctly each object as quickly as possible. There were 8 segments in each of 5 rows for a total of 40 objects. Children were instructed to begin by naming the object at the beginning (left side) of the top row, to continue to the end of the row, and start immediately at the next row after completing the previous row. Instructions to children emphasized the importance of first, to name correctly each object, and second, to name the objects as quickly as possible.

Quantity Naming. In the Quantity Naming task, a series of triangles was presented in individual segments ranging from 1 to 4 triangles. Children were instructed to name the quantity of objects (e.g., triangles). For example, when the segment was presented, the correct response was "3." Children were instructed to name correctly each quantity as quickly as possible. There were 8 quantity segments in each of 5 rows for a total of 40

segments. Children were instructed to begin by naming the quantity at the beginning (left side) of the top row, to continue to the end of the row, and start immediately at the next row after completing the previous row. Instructions to children emphasized the importance of first, to name correctly each quantity, and second, to name the quantity as quickly as possible.

Sequencing Speed

Four tasks assessed children's sequencing speed. Two tasks tapped verbal sequencing speed for sequences: counting 1 to 20 and alphabet A to Z. Two tasks tapped written sequencing speed: making trails 1 to 11 and making trails A to K. While the numerical sequencing tasks were each included because of their clear relationship to mathematics, the inclusion of letters sequencing tasks was to investigate whether any sequencing impairment was specific to numerical representations or related to a generalized sequencing impairment.

Counting 1 – 20. To assess how quickly and accurately children can count from 1 to 20, the counting 1-20 test was administered.

Alphabet A-Z. To assess how quickly and accurately children can recite the alphabet, the alphabet A–Z test was administered.

Making Trails Numbers. The Making Trails Numbers task involved visual scanning and mental and motor speed to connect numbers in sequential order. Children

were presented with a series of 11 circles on a page, with each circle containing a number from 1 to 11. Children were instructed to draw lines as quickly as possible connecting each circle to the next in sequential order beginning at 1 and finishing at 11. Instructions to children emphasized the importance of first, to connect the circles in the correct order, and second, to connect the circles as quickly as possible.

Making Trails Letters. The Making Trails Letters task involved visual scanning and mental and motor speed to connect letters in sequential order. Children were presented with a series of 11 circles on a page, with each circle containing a number from A to K. Children were instructed to draw lines as quickly as possible connecting each circle to the next in sequential order beginning at A and finishing at K. Instructions to children emphasized the importance of first, to connect the circles in the correct order, and second, to connect the circles as quickly as possible.

Data Analysis

The purpose of this study was to investigate whether children with arithmetic disabilities are characterized by a processing speed impairment. Independent samples *t*-tests were conducted to examine whether children academically normally achieving and children with mathematics disabilities differ in the speed to process information. Two separate sets of analyses will be conducted. The first set of *t*-tests will examine naming speed abilities related to numbers, letters, objects, and quantities. The second set of analyses will examine sequencing speed abilities related to mental sequencing (numeric and alphabetic) and written sequencing (numbers and letters).

Significant differences in means scores between groups were indicated using a .05 alpha level due to the exploratory nature of the current study. To determine the magnitude of significant group differences, Cohen's d were calculated. The magnitude of group differences were interpreted using Cohen's (1977) effect size ranges: small effect, around $d = .20$; medium effect, around $d = .50$; large effect, greater than $d = .80$.

CHAPTER FOUR

RESULTS

Participant Classification

The purpose of the present study was to investigate whether children with arithmetic disabilities (AD) are characterized by a processing speed impairment. Participants were classified into two groups: children with arithmetic disabilities (AD; $n = 19$) and children who were academically normally achieving (NA; $n = 21$). Participants were classified according to standard scores in three areas: Wide Range Achievement Test, Third Edition – Arithmetic (WRAT3-A), Wide Range Achievement Test, Third Edition – Reading (WRAT3-R), and Ravens Color Progressive Matrices (RCPM). Means and standard deviations for participant classification measures are reported in Table 1.

Three independent samples t -tests were conducted to validate the classification of participants. Results revealed a significant difference in standard scores between children with AD ($M = 84.32$) and NA children ($M = 103.24$) on the WRAT3-A, $t(38) = 11.58$, $p < .001$, $d = 3.77$. As expected, no significant differences in standard scores were found between children with AD ($M = 99.89$) and NA children ($M = 103.71$) on the WRAT3-R, $t(38) = 1.66$, $p = .106$, $d = .53$. Finally, no significant differences were found between the two groups on fluid intelligence ($M = 103.50$ and $M = 107.67$, for AD and NA participants respectively), $t(38) = 1.84$, $p = .074$, $d = .58$. In sum, participants consist of two distinct groups (i.e., NA and AD) and no differences were apparent with regards to reading ability and fluid intelligence.

Insert Table 1 about here.

Processing Speed Tasks

Processing speed was assessed on the basis of two types of tasks: naming speed and sequencing. Naming speed was examined using four separate tasks: rapid digit naming, rapid letter naming, object naming, and quantity naming. Similarly, four tasks were used to examine sequencing: making trails numbers, making trails letters, number sequencing, and alphabet sequencing.

Naming speed. To explore whether children with AD experienced impairments in naming speed relative to the NA children, independent samples *t*-tests were conducted on the four measures: rapid digit naming, rapid letter naming, object naming, and quantity naming. Means and standard deviations for the naming speed tasks are reported in Table 2. Results revealed a significant difference between the two groups in their ability to name digits (rapid digit naming). Children with AD ($M = 43.53$) were significantly slower than their NA peers ($M = 38.00$) at naming digits, $t(38) = -2.62, p = .013, d = .82$. Conversely, no differences were found between children with AD ($M = 45.79$) and NA children ($M = 43.05$) in their ability to name letters (rapid letter naming), $t(38) = -.92, p = .365, d = .32$. When children were asked to name shapes (rapid object naming), a significant difference was again found between the two groups. Children with AD ($M = 51.11$) were significantly slower at naming shapes than their NA peers ($M = 38.48$), $t(38) = -2.88, p = .008, d = .97$. Finally, when asked to name quantities of triangles (quantity naming), a significant difference was again found between the two groups. Specifically, children with AD ($M = 37.10$) were significantly slower at quantity naming than their NA peers ($M = 31.32$), $t(38) = -3.01, p = .005, d = .96$.

Overall, when exploring naming speed ability, children with AD were significantly impaired in their ability to name digits, objects, and quantities when compared to NA children. No differences were apparent in these groups' ability to name letters.

Insert Table 2 about here.

Sequencing Speed. To explore whether children with AD experienced impairments in sequencing speed relative to NA children, independent samples *t*-tests were conducted on the four measures: making trails numbers, making trails letters, number counting, and alphabet reciting. Means and standard deviations for the sequencing speed tasks are reported in Table 3. Results revealed a significant difference between the two groups in their ability to connect number trails (making trails numbers). Children with AD ($M = 16.05$) were significantly slower than their NA peers ($M = 11.11$) at making number trails, $t(38) = -2.62, p = .016, d = 1.00$. Interestingly, no differences were found between children with AD ($M = 15.24$) and NA children ($M = 13.04$) in their ability to connect letter trails (making trails letters), $t(38) = -1.34, p = .189, d = .44$. When children were asked to count from 1 to 20 as fast as they could (number counting), no significant differences were found between children with AD ($M = 5.53$) and NA children ($M = 5.46$), $t(38) = -.29, p = .771, d = .09$. Finally, when asked to recite the alphabet as fast as they could (alphabet), there were no significant differences between the two groups ($M = 5.78$ and $M = 6.03$, for AD and NA children respectfully), $t(38) = .52, p = .606, d = .17$.

Insert Table 3 about here.

Overall, when exploring sequencing speed ability, children with AD were significantly impaired only in their ability to connect trails consisting of numbers. There were no significant differences found between the two groups when asked to connect trails consisting of letters. Additionally, there were no significant differences between children with AD and NA children with respect to their ability to recite numbers from 1 to 20 and the alphabet.

Summary of Findings

In sum, children with arithmetic difficulties appeared to experience trouble when processing numerical-based information. Specifically, children with AD were significantly impaired at naming digits, making trails consisting of digits, and naming quantities. These deficits were all pronounced, consisting of large effect sizes, ranging from .82 to .99. Interestingly, significant differences were also found between the two groups on the object naming task. Children with AD were significantly slower at naming shapes. The two groups did not differ in their ability to name letters, recite the alphabet, and connect trails consisting of letters. Again, these differences were not meaningful as reflected by their small effect sizes.

CHAPTER FIVE

DISCUSSION

The present study was designed to examine processing speed impairments of children with arithmetic disabilities (AD). The design accounted for several limitations in previous research by including multiple tasks to measure processing speed, using a sample of children experiencing specific arithmetic disabilities (i.e., matched on IQ and reading ability), and using a younger age criteria between 86 and 105 months than previous studies (e.g., Berg, 2008a). Processing speed was measured using a group of naming speed tasks and a group of sequencing speed tasks. The naming speed group consisted of four individual measures: rapid digit naming, rapid letter naming, object naming, and quantity naming. The sequencing speed group consisted of four individual measures: making trails numbers, making trails letters, number counting, and alphabet reciting. Notably, within each group, tasks involving both verbal and visual ability were included. Overall, results suggested that children with AD are slower at processing numerical-based information compared to normally achieving children (NA).

Naming Speed

Of the four naming speed tasks, children with AD were significantly slower than NA children on the rapid digit naming task, rapid object naming task, and the quantity naming task. Notably, as indicated by medium to large effect sizes, these differences suggest that the magnitude of performance that separates these groups is of practical concern. Children with arithmetic disabilities not only displayed slower processing speed than their normally achieving peers, but these slower speeds were of a large and

significant magnitude and may account for the difficulty these children have with developing skills in mathematics. Interestingly, of the four naming speed tasks, no differences were found between the two groups with respect to rapid letter naming. Children with AD were not impaired in terms of their ability to access phonological-based information. This finding highlights the fact that the impairment found in the present research is specific to a numerical process and independent of a phonological process such as that found in reading. It is likely that this processing speed deficit and significantly slower access of numerical-based information stored in long-term memory is hindering mathematics skill development (e.g., development of automatized math facts). These findings are in line with those of Garnett and Fleischner (1983), who found that slower speeds on rapid automatized naming (RAN) tasks were related to an inability to develop automatized basic math facts. Again, these findings are parallel to reading research, as slow naming speed in readers contribute to reading dysfunction due to an impairment of automatized retrieval of word information in memory (Wolf & Bowers, 1999). Again, the presence of an impairment quickly accessing numerical-based information only highlights the possibility that the cognitive deficits involved in arithmetic disabilities are unique from those found in reading disabilities. The relationship between mathematics disabilities and processing speed of numerical-based information is a notable area that should be pursued in future research. Identifying why some children fail to develop math skills (i.e., the cognitive area of deficit attributing to math disabilities) will allow more accurate identification and intervention of mathematics disabilities.

These findings also coincide with previous research that has implicated the importance of processing speed in arithmetic development. Bull and Johnston (1997) assessed 7-year-olds on measures of processing speed, short-term memory, long-term memory, and sequencing ability. Findings indicated that the strongest predictor of arithmetic calculation was processing speed. In addition, Berg (2008b) examined the contributions of processing speed, short-term memory, working memory, and reading to arithmetic calculation among children in Grades 3 to 6. Again, processing speed emerged as a significant contributor to arithmetic calculation; however, this contribution diminished across ages. These findings highlight the developmental nature of processing speed and indicate that its relationship with calculation is more pronounced during the early states of arithmetic development and decreases as children become more proficient. Notably, research has begun to highlight the important role processing speed plays in the development of arithmetic calculation skills during early elementary school.

Finally, that children with AD were significantly slower on the object naming task may implicate the role of visual-spatial processing. A recent study by Schuchardt, Maehler and Hasselhorn (2008) analyzed the role of the different components of working memory in various children with learning disabilities. Results indicated a significant impairment in the functioning of the visual-spatial sketchpad. The children in this study characterized by arithmetic disabilities scored markedly lower than their peers without these disorders on visual-spatial working memory tasks. It is possible that the naming objects task used in the present study tapped into this kind of visual-spatial processing.

Sequencing Speed

Of the four sequencing speed tasks, children with AD were significantly slower than the NA children on the making trails numbers task only. Again, this result suggests that the magnitude of performance differences is of practical significance as reflected by the large effect size. No differences between the two groups were found with regards to making trails letters, number counting, and reciting the alphabet. Previous research has found counting speed differences in younger children in Grade 2 (e.g., Geary et al., 1991); however, this difference was found to decrease by mid elementary level (e.g., Geary & Brown, 1991). It is likely that by the time children reach Grade 3 or 4, practice effects with counting and reciting the alphabet close the gap between those with mathematics disabilities and their normally achieving peers. Due to the novel nature of the making trails task used in the present study, differences in sequencing speed were able to be measured without the consequences of practice effects. Unlike counting, the making trails task is not a regular everyday activity that children are exposed to in the classroom, thus, the opportunity for repeated practice would not likely affect the results.

Overall, sequencing speed deficits have been found to be related to an inability to develop automatized mathematics facts (Lemaire & Siegler, 1995). Due to slow number sequencing, questions and their answers are not paired in long-term memory. Specifically, when a child is engaged in mental addition, they must pair the question with its answer in working memory (i.e., $2 + 3 = 5$) before the math fact can become automatized in long-term memory. If a child is slow at sequencing, it is possible that decay may set in before the question and answer can become associated, hindering its transfer into long-term memory. For example, when using the counting all strategy (i.e.,

counting $1 + 2 + 3 + 4 + 5$ to answer $2 + 5$), the child may forget what the question was by the time they reach the answer. Results of the present study suggest that children with AD are slower at sequencing numerical information. It is also known that these children frequently fail to develop automatized math facts due to the process noted above. If deficits in sequencing speed are related to a child's failure to develop automatized math facts, and a failure to develop automatized math facts is a hallmark of math disabilities, then sequencing speed should be an area of cognitive development thoroughly tested when investigating possible mathematics disabilities in children. More specifically, the present study found a deficit in sequencing speed related to numerical-based information only. It is possible that these children have a specific deficit around numbers. An assessment battery for investigating math disabilities should include processing speed tasks (specifically those testing sequencing and naming speed) that include both numerical and non numerical tasks.

In sum, results of the present study indicate that children with AD have a deficit in processing numerical-based information across naming speed tasks and sequencing speed tasks compared to their NA peers. These results support and extend previous research which indicated that children with AD have specific working memory deficits related to processing of numerical information (Siegel & Ryan, 1989). Siegel and Ryan (1989) concluded that children with specific AD performed significantly poorer on working memory tasks involving counting and remembering the products of their counts as compared to remembering missing words from sentences. Children with combined arithmetic and reading disabilities showed impairments on both tasks.

The present research extends the Siegel and Ryan (1989) study in two ways. First, by including a wider variety of tasks used to measure processing speed and second, by utilizing a younger sample size. Many studies that measure cognitive functioning of children with learning disabilities only include one or two tasks, and attribute general findings related to impairments by using only these tasks (e.g., Siegel and Ryan, 1989). The present research included eight different tasks all designed to measure processing speed. These tasks were also broken into two types of processing speed: sequencing speed (e.g., making trails) and naming speed (e.g., quantity naming). Again, with the inclusion of a variety of measures, results show that children with AD have a deficit in processing speed related to numerical-based information only.

With regards to the age of the sample size, Siegel and Ryan (1989) used a sample size consisting of children who ranged in age from 9 to 13. The present study extended this line of research by including a sample that ranged in age from 7 to 8.5 years. The results clearly suggest that difficulty processing numerical-based information quickly and rapidly begins at an early age. This finding is noteworthy, as it corresponds with early mathematics instruction in the classroom. For example, mental math and rapid recall of basic number facts is required as part of the mathematics curriculum from Grades Primary to 3 in Nova Scotian schools (Nova Scotia Department of Education and Culture. English Program Services, 1998).

It is also noteworthy to mention the results of Berg (2008a), as the present study is an extension of this research. Berg (2008a) included three tasks to measure the processing speed of children with severe arithmetic difficulties: digit naming, number articulation (where children were asked to repeat pairs of single syllable digits as quickly as

possible), and a counting dots task (where children counted dots that were randomly arranged on a card). Although multiple tasks to measure processing speed were utilized, Berg (2008a) concluded that no significant differences in processing speed were found between children with severe arithmetic difficulties and normally achieving children. It is possible that no differences were found between the two groups of children due to the age of the sample. Berg (2008a) used children between the ages of 102 and 144 months, whereas the present research used a significantly younger sample. Research has consistently shown that elementary school children with AD rely on immature strategies to solve problems beyond their normally achieving peers (e.g., Geary, 1993; Geary et al., 1991). It is likely that the children used in Berg's sample, due to an overreliance on counting as a problem solving strategy, had an increased counting ability. Proficiency with counting would, in turn, increase their performance on the counting tasks used in Berg's (2008a) study. The present study accounted for this by including a younger sample size, as these children would not have had a similar opportunity to increase their counting skill.

That children with AD were found to have a deficit processing numerical-based information only is nicely explained by a theory put forth by Robinson, Menchetti, and Torgesen (2002) who define a two-factor theory of math disabilities. This two-factor theory states that weak phonological processing abilities underlie the learning difficulties of children with combined math and reading disabilities, while weak number sense is an underlying deficit of children who have math disabilities alone. The concept of number sense is gaining importance in the field of mathematics research as a potential parallel to phonological awareness. For example, number sense has been found to be an important

predictor of future mathematics ability (Locuniak & Jordan, 2008). In light of the numerical-based processing speed impairments experienced by children with AD in the current study, these results lend support to use of number sense as a theory to understand mathematical learning disabilities; specifically access to number-based representations from long-term memory (digit naming) and the number line (number sequencing, making trails numbers). Children in the present study were matched on IQ and reading ability and defined by below average ability in arithmetic calculation only. Results are in line with this two-factor theory, as children with AD showed deficits in sequencing and naming tasks of numerical-based information only. They did not differ from the NA children in their ability to complete sequencing and naming tasks of non numerical-based information (e.g., letters).

Although a processing deficit of numerical-based information is explained nicely by difficulty with number sense, it does not yet tap into the cognitive process underlying number sense. Research has found a causal relationship between phonological awareness deficits and reading difficulty (e.g., Wolf & Bowers, 1999); however, number sense is more accurately conceived of as an element of knowledge base that contributes to processing efficiency, as opposed to being the cognitive process itself. Just as phonemic awareness can be considered a skill or type of knowledge that is built on the cognitive construct of phonological processing (Robinson et al., 2002), perhaps naming speed and sequencing speed tasks are skills or a type of knowledge that are built on an as of yet undefined or unnamed cognitive process. Overall, the present research points to a definite relationship between processing speed deficits of numerical-based information and arithmetic disabilities.

Limitations

Results of the present study are limited due to the relatively small sample size used to make up the AD group. However, although a small sample size limits the generalizability of the results of the present study, it is in line with the majority of the research in the field. Although small, significant results consisted of medium to large effect sizes. Additionally, results should be interpreted with caution due to the classification procedures used. Although measures were specifically taken to isolate children experiencing true difficulties (i.e., NA > 30th percentile; AD < 25th percentile), psychoeducational testing was not performed, thus, a diagnosis of a true math disability could not be made. Finally, it should be noted that previous research has shown that children's performance on testing measures is unstable. That is, children's scores can vary significantly from test to test in as little as two years (Jordan et al., 2003). Jordan et al. found that for a subtype of Grade 2 children, their scores varied between being classified as normally achieving or disabled, and vice versa, with regards to reading disabilities, mathematics disabilities, or both. Scores for the present classification of participants may be considered a specific snap shot of ability, as testing was performed only once.

Implications

An important direction for future research would be to extend the present study to include different age groups. In addition, the present study could follow the same group of children across ages. A longitudinal design would allow for the development and comparisons of practice effects as well as developmental and cognitive changes that may

occur over the elementary years. Finally, a significant difference was found between the two groups with regards to object naming. The object naming task measured visual-spatial memory (i.e., the representation of shapes). Further investigation into the differences in visual-spatial memory ability and its relationship to arithmetic disabilities will help inform the field by expanding the knowledge around specific strengths and deficits of these children.

Although the present study did not directly measure the relationship between academic achievement and processing speed deficits of children with AD, results are relevant to educators who work with children experiencing these problems. Specifically, these results can help inform the field of school psychology, and those directly involved in diagnosing arithmetic disabilities, specifically, and mathematics disabilities in general. Presently, it is common practice in school psychology to link deficits in working memory and processing speed to difficulties across all academic subjects (e.g., Flanagan & Kaufman, 2004). Although these deficits are thought to impair skill development across academic subjects, specifically mathematics, the field has yet to find conclusive evidence that these deficits are the cause of such disabilities. The more research investigating the cognitive underpinnings of arithmetic disabilities, the clearer the picture becomes around diagnosis, intervention, and instruction.

Finally, results of the present study have important implications for classroom teachers. The difference in processing speed ability between children with arithmetic disabilities and their normally achieving peers is very significant. It will be important for teachers to understand the magnitude of the impairment that children with arithmetic disabilities experience. These children will require a program of intervention that is

systematic, consistent, and intensive. Programs, like those that focus on developing reading skills (i.e., intervention that is systematic and explicit), will be needed to allow these children to develop basic math skills. Repetition and over-teaching will help insure that these children will develop the skills they need to acquire higher level mathematics skills and experience academic success.

REFERENCES

- Baddeley, A. D. (1986). *Working Memory*. Oxford, UK: Oxford University Press.
- Badian, N. A. (1983). Arithmetic and nonverbal learning. In H. R. Myklebust, (Ed.), *Progress in Learning Disabilities* (Vol. 8, pp. 235-264). New York: Grunt & Stratton.
- Berg, D. H. (2008a). Cognitive impairment of children with severe arithmetic difficulties: Cognitive deficit or developmental lag? *Exceptionality Education Canada*, 18, 59-91.
- Berg, D. H. (2008b). Working memory and arithmetic calculation in children: The contributory roles of processing speed, short-term memory, and reading. *Journal of Experimental Child Psychology*, 99, 288-303.
- Bull, B., & Johnston, S. R. (1997). Children's arithmetical difficulties: Contributions from processing speed, item identification and short-term memory. *Journal of Experimental Psychology*, 65, 1-24.
- Case, R. (1985). *Intellectual development: Birth to adulthood*. San Diego: Academic Press.
- Case, R., Kurland, D. M., & Goldberg, E. (1982). Operational efficiency of short-term memory span. *Journal of Experimental Psychology*, 33, 386-404.
- Fry, A. F., & Hale, S. (1996). Processing speed, working memory, and fluid intelligence: Evidence for a developmental cascade. *Psychological Science*, 7, 237-241.
- Flanagan, D. P. & Kaufman, A. S. (2004). *Essentials of WISC-IV Assessment*. Hoboken, NJ: John Wiley & Sons Inc.

- Fuchs, L. S., Fuchs, D., Compton, D. L., Powel, S. R., Seethaler, P. M., Schatschneider, C., & Fletcher, J. M. (2006). The cognitive correlates of third-grade skill in arithmetic, algorithmic computation, and arithmetic word problems. *Journal of Educational Psychology, 98*, 29-43.
- Garnett, K., & Fleischner, J. E. (1983). Automization and basic fact performance of normal and learning disabled children. *Learning Disabilities Quarterly, 6*, 23-230.
- Geary, D. C. (1990). A componential analysis of an early learning deficit in mathematics. *Journal of Experimental Child Psychology, 49*, 363-383.
- Geary, D. C. (1993). Mathematical disabilities: Cognitive, neuropsychological, and genetic components. *Psychological Bulletin, 114*, 345-362.
- Geary, D. C. (1994). *Children's mathematical Development: Research and practical applications*. Washington, DC: American Psychological Association.
- Geary, D. C. (2004). Mathematics and learning disabilities. *Journal of Learning Disabilities, 37*, 4-15.
- Geary, D. C., Bow-Thomas, C. C., & Yao, Y. (1992). Counting knowledge and skill in arithmetical cognition: A longitudinal study of process and concept deficits in children with learning disability. *Journal of Experimental Child Psychology, 77*, 236-263.
- Geary, D. C., & Brown, S. C. (1991). Cognitive addition: Strategy choice and speed-of-processing differences in gifted, normal, and mathematically disabled children. *Developmental Psychology, 27*, 398-406.
- Geary, D. C., Brown, S. C., & Samaranayake, V. A. (1991). Cognitive addition: A short longitudinal study of strategy choice and speed-of-processing differences in

- normal and mathematically disabled children. *Developmental Psychology*, 27, 787-797.
- Geary, D. C., Hamson, C. O., & Hoard, M. K. (2000). Numerical and arithmetical cognition: A longitudinal study of process and concept deficits in children with learning disability. *Journal of Experimental Child Psychology*, 77, 213-239.
- Geary, D. C., & Hoard, M. K. (2005). Learning disabilities in arithmetic and mathematics: Theoretical and empirical perspectives. In J. I. D. Campbell (Eds.), *Handbook of mathematical cognition* (pp. 253-267). New York: Psychology Press.
- Geary, D. C., Hoard, M. K., Craven, J. B., Nugent, L., & Numtee, C. (2007). Cognitive mechanisms underlying achievement deficits in children with mathematical learning disability. *Child Development*, 78, 1343-1359.
- Geary, D. C., Widaman, K. F., Little, T. D., & Cormier, P. (1987). Cognitive addition: Comparison of learning disabled and academically normal elementary school children. *Cognitive Development*, 2, 149-169.
- Gersten, R., Jordan, N. C., & Flojo, J. R. (2005). Early identification and interventions for students with mathematics difficulties. *Journal of Learning Disabilities*, 38, 293-304.
- Hanich, L. B., Jordan, N. C., Kaplan, D., & Dick, J. (2001). Performance across different areas of mathematical cognition in children with learning difficulties. *Journal of Educational Psychology*, 93, 615-626.
- Hitch, G. J., & MacAuley, E. (1991). Working memory in children with specific arithmetical learning difficulties. *British Journal of Psychology*, 82, 375-386.

- Jordon, N. C., Hanich, L. B., & Kaplan, D. (2003). A longitudinal study of mathematical competencies in children with specific mathematics difficulties versus children with comorbid mathematics and reading difficulties. *Child Development, 74*, 834-850.
- Jordon, N. C., & Montani, T. O. (1997). Cognitive arithmetic and problem solving: A comparison of children with specific and general mathematics difficulties. *Journal of Learning Disabilities, 30*, 624-634.
- Just, A. M., & Carpenter, P. A. (1992). A capacity theory of comprehension: Individual differences in working memory. *Psychological Review, 99*, 122-149.
- Kail, R. (1986). Sources of Age Difference in speed of Processing. *Child Development, 57*, 969-987.
- Kail, R. (1991). Developmental change in speed of processing during childhood and adolescence. *Psychological Bulletin, 109*, 490-501.
- Kail, R. (1992). Processing speed, speech rate and memory. *Developmental Psychology, 28*, 899-904.
- Kail, R., & Park, Y. (1994). Processing time, articulation time, and memory span. *Journal of Experimental Child Psychology, 57*, 281-291.
- Kail, R. (2000). Speed of information processing: Developmental change and links to intelligence. *Journal of School Psychology, 38*, 51-61.
- Kirby, J. R., & Becker, L. D. (1988). Cognitive components of learning problems in arithmetic. *Remedial and Special Education, 9*, 7-16.
- Kosc, L. (1974). Developmental dyscalculia. *Journal of Learning Disabilities, 7*, 1-17.

- Lemaire, P., & Siegler, R. S. (1995). Four aspects of strategic change: Contributions to children's learning of multiplication. *Journal of Experimental Psychology: General*, *124*, 83-97.
- Locuniak, M. N., & Jordan, N. C. (2008). Using kindergarten number sense to predict calculation fluency in second grade. *Journal of Learning Disabilities*, *41*, 451-459.
- Mabbott, D. J., & Bisanz, J. (2003). Developmental change and individual differences in children's multiplication. *Child Development*, *74*, 1-17.
- McLean, J. F., & Hitch, G. J. (1999). Working memory impairments in children with specific arithmetic learning difficulties. *Journal of Experimental Child Psychology*, *74*, 240-260.
- National Council of Teachers of Mathematics. Commission on Standards for School Mathematics. (2000). *Curriculum and evaluation standards for school mathematics*. Reston, VA: Author.
- National Council of Teachers of Mathematics. Commission on Standards for School Mathematics. (2006). *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics*. Reston, VA: Author.
- Nova Scotia Department of Education and Culture. English Program Services. (1998). *Atlantic Canada mathematics curriculum: Grades primary – 3/Nova Scotia*. Nova Scotia: Author.
- Passolunghi, M. C., & Siegel, L. S. (2001). Short-term memory, working memory, and inhibitory control in children with difficulties in arithmetic problem solving. *Journal of Experimental Child Psychology*, *80*, 44-57.

- Passolunghi, M. C., & Siegel, L. S. (2004). Working memory and access to numerical information in children with disability in mathematics. *Journal of Experimental Child Psychology, 88*, 348-367.
- Raine, A., Hulme, C., Chadderton, H., & Bailey, P. (1991). Verbal short-term memory span in speech-disordered children: Implications for articulatory coding in short-term memory. *Child Development, 62*, 415-423.
- Rasanen, P., & Ahonen, T. (1995). Arithmetic disabilities with and without reading difficulties: A comparison of arithmetic errors. *Developmental Neuropsychology, 11*, 275-295.
- Rivera, D. P. (1997). Mathematics education and students with learning disabilities: Introduction to the special series. *Journal of Learning Disabilities, 30*, 2-19.
- Robinson, C. S., Menchetti, B. M., & Torgesen, J. K. (2002). Toward a two-factor theory of one type of mathematics disabilities. *Learning Disabilities Research, 17*, 81-89.
- Schuchardt, K., Maehler, C., & Hasselhorn, M. (2008). Working memory deficits in children with specific learning disorders. *Journal of Learning Disabilities, 41*, 514-523.
- Siegel, L. S., & Ryan, E. B. (1989). The development of working memory in normally achieving and subtypes of learning disabled children. *Child Development, 60*, 973-980.
- Siegler, R. S., & Jenkins, E. (1989). *How children discover new strategies*. Hillsdale, NJ: Erlbaum.

- Siegler, R. S., & Shrager, J. (1984). Strategy choices in addition and subtraction: How do children know what to do? In C. Sophian (Ed.), *The origins of cognitive skills* (pp. 229-293). Hillsdale, NJ: Erlbaum.
- Spring, C., & Davis, J. M. (1988). Relations of digit naming speed with three components of reading. *Applied Psycholinguistics*, *9*, 315-334.
- Swanson, H. L. & Sachse-Lee, C. (2001). Mathematical problem solving and working memory in children with learning disabilities: Both executive and phonological processes are important. *Journal of Experimental Child Psychology*, *79*, 294-321.
- Waber, D. P., Wolff, P. H., Weiler, M. D., Bellinger, D., Marcus, D. H., Forbes, P., & Wypij, D. (2001). Processing of rapid auditory stimuli in school-age children referred for evaluation of learning disorder. *Child Development*, *72*, 37-49.
- Wagner, R. K., Torgesen, J. K., & Rashotte, C. A. (1999). *Comprehensive test of phonological processing*. Austin, Texas: Pro-Ed.
- Weiler, M.D., Holmes-Bernstein, J., Bellinger, D., & Waber, D.P. (2000). Processing speed in children with attention deficit/hyperactivity disorder, inattentive type. *Child Neuropsychology*, *6*(3), 218-234.
- Wilson, K. M., & Swanson, H. L. (2001). Are mathematics disabilities due to a domain-general or a domain-specific working memory deficit? *Journal of Learning Disabilities*, *34*, 237-248.
- Wolf, M., Bally, H., & Morris, R. (1986). Automaticity, retrieval processes, and reading: A longitudinal study in average and impaired readers. *Child Development*, *57*, 988-1000.

Wolf, M., & Bowers, P. G. (1999). The double-deficit hypothesis for the developmental dyslexias. *Journal of Educational Psychology, 91*, 415-438.

Table 1

Means and Standard Deviations for Chronological Age, Academic Achievement, and Fluid Intelligence

Tasks	AD (<i>n</i> = 19)		NA (<i>n</i> = 21)	
	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>
Chronological Age	97.95	4.44	94.43	4.59
Arithmetic				
Raw	19.32	1.34	23.00	2.32
Standard	84.32	4.03	103.24	6.01
Reading				
Raw	29.05	3.33	29.19	2.40
Standard	99.89	8.87	103.71	5.47
Fluid Intelligence				
Raw	24.42	4.33	25.38	2.91
Standard	103.50	8.21	107.67	6.07

Note. AD = arithmetic disability; NA = normally achieving.

Table 2

Means and Standard Deviations for Naming Speed Tasks

Tasks	AD ($n = 19$)		NA ($n = 21$)	
	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>
Naming Speed				
Rapid Digit Naming	43.53	8.15	38.00	4.97
Rapid Letter Naming	45.79	11.89	43.05	5.43
Object Naming	51.12	17.16	38.48	8.79
Quantity Naming	37.10	5.85	31.32	6.27

Note. AD = arithmetic disability; NA = normally achieving.

Table 3

Means and Standard Deviations for Sequencing Speed Tasks

Tasks	AD ($n = 19$)		NA ($n = 21$)	
	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>
Sequencing Speed				
Making Trails Numbers	16.05	8.02	11.11	1.93
Making Trails Letters	15.24	6.59	13.04	3.47
Number Counting	5.53	0.58	5.46	0.84
Alphabet	5.78	1.52	6.03	1.52

Note. AD = arithmetic disability; NA = normally achieving.