Report of Working Group F
Rapport du Groupe de travail F

NOTICING AND ENGAGING THE MATHEMATICIANS IN OUR CLASSROOMS

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Many characteristics describe the work of a mathematician. These characteristics just as readily apply to the work of "professional" mathematicians (e.g. people who "do math" as a career, researching and publishing in the field) as they do to "amateur" mathematicians (e.g. people who "do math" (without funding), be it students, teachers, or teacher educators). The focus of this working group was to explore different ways in which teachers, mathematics educators, and (professional) mathematicians come to appreciate themselves and their students as mathematicians.

Through engagement with mathematical tasks, our working group attempted to establish a sense of what it means to "be a mathematician." This developed from a shared vision of fundamental aspects of "doing math" that were exemplified in the tasks, discussions and experiences of our group members. Specific questions we designed to help shape our discussions included:

> *How is it that teachers/teacher educators/mathematicians come to notice and foster mathematical thinking in primary, secondary, and tertiary classrooms?*

This question is motivated by Wheeler's concern that "*the majority of teachers [do] not encourage their students to 'function like a mathematician'*" (Wheeler, 1982, p. 46).

> *How can we as teachers engage students as mathematicians and what types of tasks model what it is that mathematicians "do"?*

This question is motivated by a recognized disconnect between how students experience mathematics in the classroom and how professional mathematicians experience mathematics in research (e.g. Boaler, 2008; Lockhart, 2009).

INTRODUCTION

The emphasised text in the abstract above exemplifies the underlying themes and foci of our working group. While we continue to explore them (throughout our lives and careers), we present below a snapshot of the working group's engagement, thoughts and reflections that inspired and emerged. In what follows, each of us attempts to give voice to our personal and collective experiences, and we do so in a three-part discussion in which (i) Egan illustrates some of the considerations and intentions that went into selecting tasks that, for us, "model what it is that mathematicians do," (ii) Eva highlights the themes and events that emerged during the three days of the working group, and (iii) Ami reflects on corresponding experiences, *noticing* and *fostering*. Our intention is not merely to give a summary or overview of the experience of our working group, but rather to exemplify poignant issues connected to noticing and engaging that struck each of us in different, though related, ways. As such, each of the sections is written as a first-person narrative.

PINK PIG PONDERING

From the moment we received the invitation to help lead a working group on *noticing and engaging the mathematicians in our classrooms*, I knew I had the perfect opportunity to get (some) members of the CMESG/GCEDM community playing with and talking about little pink pigs, at least for a few days. For those of you not familiar with these little pink pigs – hereafter referred to as *mini pigs* – I'll take a minute to explain.

Mini pigs, for me, represent the modern day equivalent to astragali (i.e., ankle bones), which "*archaeologists have found...among the artefacts of many early civilizations*" (Bennett, 1998, p. 8) and, arguably, can be considered the ancestor of cubical dice. However, there are some fundamental differences between astragali and dice. For example, the six sides of astragali, unlike the six sides of a die, are not (necessarily) equally likely to occur. Further, and as another example, the six sides of the astragali are comprised of two different kinds of "sides" (four are long flat sides and two are shorter rounded sides), unlike a die whose six sides are all identical. My intentions were clear: I wanted the members of the working group to be working with and discussing astragali and not dice.

For me, the problem with using dice is a problem with what Taleb (2007) calls *platonicity*, which "*is our tendency to mistake the map for the territory*" (p. xxv). In other words, the problem with dice is a problem with mistaking the theoretical, perfectly true, cube where each side has an exactly equal chance of landing on any one of the six sides (i.e., the map) with the die in our hand (i.e., the territory). By introducing modern day astragali, and not dice to the working group, my intention was to engage in a variety of discussions on: classical, frequentist, and subjective interpretations of probability, sample space, equiprobability, elementary outcomes, events, and other topics, to establish a sense of what it means to "be a mathematician." What happened next, which admittedly, at first, caught us off guard, we now appreciate as a fundamental component to engaging the mathematicians in our classroom. While the following outlines specifics related to my mini pigs, similar discussions occurred for all the tasks we brought to the working group.

Here is the actual email communication that occurred after the initial posing of the problem to be used in our working group. (Note: the final version of the mini pigs problem is found in the appendix.)

> ***Egan:*** *do not accept any of Ami's rewording of the pig questions*
>
> ***Eva:*** *thanks for the comments. I accepted/rejected as suggested. I think we want to leave out some of the details from the problems because deciding what path to choose regarding such things, or even knowing the conventions, IS part of mathematical thinking,*
>
> ***Ami:*** *I agree with Eva's point, but do think the original problem is ill-posed (perhaps deliberately) and we could prepare to engage in discussion on when ill-posed questions are helpful in fostering math thinking (and why) and when they are not.*
>
> ***Eva:*** *P.S. the French translations of the pig questions are more akin to my revisions than the original phrasings and could probably be changed for consistency.*
>
> ***Egan:*** *Yes, the wording of the questions is deliberate. I don't, necessarily, agree with Ami's answers and, also, don't think if someone came up with the response they would be done with the task...two things I hope to happen during the working group. Nevertheless, how about the following changes...*
>
> ***Ami:*** *I like your wordings and don't think you've lost any of the power of the questions with them. For part c) do you mean that the pig will land both on snout and side at the same time? Or are you hoping they'll compare the probabilities of flipping snout or flipping side?*
>
> ***Eva:*** *the back and forth seems to have quieted down, so please see if the new, attached version is reflective of the feedback. (BTW I had to adjust the phrasing for 2 as it is a single pig being tossed 3 times so it doesn't make sense to say: "the probability of them landing...". Is that ok??)*
>
> ***Egan:*** *think it would sound better if they all said "what is the probability of landing." How about this....*
>
> ***Ami:*** *Egan, I'm still unclear about what you're asking in question 1c, and would like to go over it with you before we do the pig question in WG.*
>
> ***Egan:*** *Hi Ami and Eva: How about this....*
>
> ***Eva:*** *I can see that we want to avoid being too specific with the outcome description so as to avoid closing down avenues of discussion. Conversely, I think we want to respect certain norms so I would be ok with the curly brackets everywhere. Can you please come to an agreement by, say, Friday, so that I can make the copies?*
>
> ***Ami:*** *Egan: I agree, {snout, side} is great. Thanks for clarifying.*
>
> ***Egan:*** *Ok, but I am still looking for an acceptable alternative.*
>
> ***Egan:*** *Ok: How about this!*
>
> ***Eva:*** *Hi, I will go with this.*

As witnessed in the conversation presented above, a large portion of our preconference planning was spent formulating, via negotiation, the mini pigs problem and some of the others. Coming up with a precise formulation of the question was not a phenomenon restricted to the leaders of the working group. Members of the working group, who were orally presented with the mini pigs problem during the conference, also spent the majority of their time working on a precise formulation of the question. For me (whose group worked on this problem), the amount of time spent focusing on the problem was of note because, in many instances in probability, the question is the culprit. "*Outcomes are not uniquely determined from the description of an experiment, and must be agreed upon to avoid ambiguity*" (Weisstein, 2010). This agreement on the question, witnessed in our email communication above and also during the working group (WG) session, drew our attention to an interesting domain where we may be able to notice and foster mathematical thinking in our classroom.

Perhaps in a twist, to notice mathematical thinking in our classrooms, we may need to turn our attention from how our students answer a question to how they engage with the question asked. That being said, the resources we have available, e.g., textbooks, etc., present questions that, for the most part, are largely stripped of any purposeful ambiguity or issues surrounding the wording of the question. However, all is not lost. To engage our students as mathematicians and present them with tasks that model what it is that mathematicians do, i.e. negotiate the problem, we can provide them with tasks that encourage negotiation of the problem before we inevitably turn to our fascination with the solution. While probability is a natural place to find such questions (e.g., the Boy-Girl problem), other areas of mathematics are just as rich, as witnessed with the other questions presented in our working group.

INSTANCES OF THE DIGIT THREE

In the case of "Instances of the Digit Three," for example, although the formulation had also been revised before the conference, ambiguities came up during the WG and the meaning of the question was negotiated, as well.

The possibility of noticing "mathematical thinking" in ourselves or in others is mediated by a number of contextual and affective factors (Jacobs, Lamb, & Philipp, 2010), the first few of which, as discussed with respect to the mini pigs, are connected to the ways in which the initial problem is posed. For example, a problem can be very convergent in that the possible interpretations are few and almost equivalent. Conversely, a mathematical situation can be presented in a divergent enough way that it may not even include a specific question posed.

In the case of the work done on the problem known as "Instances of the Digit Three" (see appendix), I chose to present the problem in a format that was designed to be as specific as possible so as to reduce, wherever possible, possibilities of ambiguities that might generate interpretations of the problem other than the one that I intended. In the case in point, I provided the problem in written form, which gave the solvers a referable artefact, and included generous detail so as to converge on the meaning that was intended. At the same time, to make the problem more engaging, I added a "real life" context. These two factors can interfere with each other: the problem, as was the case in point, was interpreted by the participants through the lens of their previous experience with situations that were either similar to the provided context (is the "ground floor" Level 0 as it is in Europe, or Level 1 as it is in the US?), or mathematically analogous (a version of this problem has circulated previously that involved the sequential numbering of the pages of a book).

From the beginning of the participants' work on the problem, two methods of engaging with the ambiguities of the problem emerged. Some groups decided to work on "disambiguating" the problem as much as possible before initiating the solving phase. For example, the issue raised earlier about whether the ground floor counts as 0 or 1 was verified by me. Other assumptions and ambiguities were resolved internally, through agreement amongst the solvers of the particular group. In the second method, the solvers jumped in and began working on the problem, and negotiated the ambiguities as they arose. The latter method was justified by this group because, as was said, more ambiguities were almost certainly going to appear throughout the process, and so it was not feasible to "clean it all up" in advance.

Criteria for the negotiation of ambiguities fell into two main categories:

1. Efforts were made to keep the problem as "realistic" as possible, that is, ambiguities were resolved by selecting the situation that "could happen this way."

2. Decisions were made that involved accepting (possibly) unrealistic but pragmatic assumptions in order to position the problem at a comfortable location (accessible yet challenging), within the continuum of *trivial – easy – accessible/comfortable – challenging – perceived as impossible*.

During the negotiations that took place, reflections were made that incorporated three points of view. The speaker reflected on herself/himself as: (1) mathematician, (2) student, and (3) teacher. This distinction connects to an additional contextual factor that comes into play, involving the power dynamics and motivations of the participants.

In a conference working group, such as Working Group F of CMESG/GCEDM 2010, the participants were all present of their own volition. Further, they freely chose to participate in this particular set of activities. Although I, as the problem poser, was potentially seen as the owner of the problem, the other participants were not, therefore, under pressure to "perform" a resolution that I had designed and imposed on them. The consequence of this dynamic is that the solvers had the choice to engage with the problem as they saw fit, and to verify with myself, or not, whether their interpretation was as I had anticipated. This is decidedly different from the social context in classrooms, where power differentials abound.

As is typical in a problem solving situation involving a group, there were episodes during which some of the participants felt lost. Upon reflection, these conditions, which were sustained for various lengths of time, yielded descriptions such as:

- Being lost, but still following the language that the group is using
- Being lost completely (having a perspective that is so far off from the group's that it is unrelated)
- Being lost by oneself or with other people

A discussion of the way to get back into the flow, if possible or desirable, included active ways such as asking for explanations or clarifications, and more passive ways whereby things happened around the participant that allowed her/him to get back into the flow: it happened as s/he listened.

When the group was beginning to close in on the solution, comments were made that pertained to the "messiness" of the solution, and the fact that having well-organised data can improve the process of resolution. In response, other participants commented that sometimes it is easier to produce organised notes on a *post-hoc* basis, in retrospect. Indeed, the point was made that organising your thinking is a sign of "mathematical thinking" and that the organisation need not be linear, necessarily. Other signs included:

- Conjecturing
- Disproving
- Looking for patterns
- Looking for shortcuts
- Making *systematic* lists
- Looking for degrees of freedom (and reducing them)
- Thinking about the reverse problem (it was remarked, in fact, that a reversal of the wording of the problem would have made it cleaner, more accessible, perhaps even trivial)

At a later stage, participants also reflected on some of their problem solving experiences within the group. A turn of the table yielded what participants perceived their partners as having done that they could term "thinking like a mathematician." Examples of responses included:

- Pausing
- Verbalising
- Back tracking
- Never giving up
- Attending to details
- Writing down strategy
- Eliminating redundancies
- Finding easy starting points
- Finding a better way, *even in retrospect*
- Thinking of simpler problems and evaluating them
- Asking questions to clarify assumptions or to write down possibilities

A comment was also made concerning the fact that, in trying to ascertain what our partners do that is "mathematical," it can be difficult to tell which partner is responsible for what thought or idea.

REFLECTIONS ON NOTICING AND FOSTERING: A TEACHER, A TEACHER EDUCATOR, AND A MATHEMATICIAN WALK INTO A BAR…

At the very close of our working group (minutes after the end actually), an objection to one of our guiding questions arose; it was in regard to our distinction between teacher, teacher educator, and mathematician. To be clear, this distinction was made deliberately, and while some may have worried that it was divisive, we saw it as both relevant and important to noticing. The word "noticing" is used in a sense similar to (or at least our evolving understanding of) Mason's (2002) *Discipline of Noticing*.

Mason writes: "*the very heart and essence of noticing is being awake in the moment to possibilities*" (2002, p. 144), which stems from Gattegno's idea that only awareness is educable. For me, someone who is not so disciplined in noticing, being awake to possibilities is contingent on the focus of my attention in that moment. It is through this idea of focus of attention that there is relevance in the distinction between teacher, teacher educator, and mathematician. During the three days with our working group I noticed my attention focused on very different matters, depending for instance on which problem was being addressed or with whom I was working. What I noticed (in myself and others), how I interacted or responded, what choices were made, what was emphasised or avoided, and what and how I tried to foster, varied as my attention shifted between teaching, observing as a researcher, and mathematizing. That is, the distinct "roles" in which I found myself – teacher, mathematics education researcher (more so for me during the WG than "teacher educator"), and mathematician – carried with them different goals. Some of the goals were personal (what did I hope to learn from my colleagues) and some were communal or "public" (what did I hope my colleagues would get from the WG). My goals tended to be pretty broad, maybe a bit vague, and, despite varying with respect to the aforementioned roles, always, always, were the goals of a WG facilitator.

Part of being a WG facilitator, or at least one part of what I learned about being a facilitator for *this* WG, is that being able to move flexibly between one role and another is very important. Flexibility requires, in Mason's language, noticing a moment of choice, and then responding in an appropriate way. Mason (2002) writes: "*The real work of noticing is to draw the moment of awakening from the retrospective into the present, closer and closer to the* *point at which a choice can be made*" (p. 76). He is referring to a wide range of moments and choices, many of which are much more subtle than the ones I describe here. Nevertheless, the "roles in which I found myself" were made through choices in the moment, though they were largely defined by others – by their questions, their actions, by my interpretations of their actions and needs. In the following paragraphs, I share some of the moments I noticed and where my attention was focused at the time, as well as some of the associated choices made in relation to fostering.

Right off the bat, day one of our WG, I found myself in a role that many others can probably sympathise with: "math sales-person." I presented a problem (Cutting the Cube) that involved geometry, visualisation, imagination, and I was met with resistance. For a variety of reasons, not all of which were brought to light, my fellow group members really did not want to imagine suspending a cube and slicing it in half. I was left feeling worried, anxious, and even disappointed. Experiencing very much the same emotions many mathematics teachers feel when trying to convince, say, a group of teenagers that solving for *x* is a valuable way to spend their time. So, I reacted. That's probably the best way to describe it – I reacted to my colleagues' resistance by trying to "sell" the problem, by digging for any reason I could find that might convince them to give it a chance. Accordingly, my attention was at first focused on how to make the problem work for this group of people. As they got deeper into the solving, my attention shifted from their reluctance to their strategies, noticing what got them talking and questioning, and eventually (when my anxiety subsided) I noticed their math. *Their* math because each small subgroup did something different from the next, because they focused on different aspects of the problem than expected, because they did a variety of interesting things while I appreciated the math in their actions and in their work. I noticed an iPod being used as a ruler and later being abandoned for bits of paper with ruled markings. I noticed charts and diagrams, movement and gesture, furrowed brows and smiles; I thought "this is good" and chose to encourage people to share their thinking across the different subgroups. When someone did or said something interesting and unexpected, my attention shifted from facilitating the problem-solving to thinking about the math. While I can't speak for the specifics of what others attended to or noticed, I can say that what ended up being fostered in our group – including an exploration that went to a "surprising" level of depth and a discussion around multiple entry points and guidance – was a result of the focus of our attention, of what we were awake to notice.

It's appropriate to comment that these reflections are not about what I noticed and attended to, but what *was* fostered. While an individual may have intentions for fostering, what ultimately is cultivated is a result of the collective. This seems especially pertinent in a situation such as a CMESG working group where the collective includes a variety of perspectives and expertise. Thus, in my view, part of noticing (and fostering) is being aware of how to navigate your intentions for fostering within the dynamic of the group so that the activity you think is being encouraged is actually (or at least something close to) what is being encouraged.

Returning to noticing, another part of what influenced the focus of my attention was comfort and familiarity with the problem. Two of the problems addressed in the WG were relatively new to me (Cutting the Cube and Multi-facets), and so there were times where I really wanted to think about the math; sometimes it was to try and make sense of a new (to me) approach, other times it was to try and make connections between some tangential lines of reasoning and what I saw as the "heart" of the problem. With both problems, my colleagues ended up forming subgroups and working at their own pace and with their own strategies, as indicated above. With Multi-facets, there were some people who were more familiar with the math in question than others. As a result, some subgroups worked through the problem very quickly and others who explored more hesitantly, asking questions and struggling at times to visualize or represent the problem. My attention during this engagement was very much focused on the latter groups. The newness of this problem had consequences on how and what I could foster: for one, the math still had novelty and could capture my thinking; for another, there were unexpected interpretations of the problem, solving strategies, etc., for which I did not have handy responses. This was in contrast to my experience with the WG exploration of the Ping-Pong Ball Conundrum, which is very familiar, as I've used it previously in research. During this problem, the engagement was quite different: the group stayed as a whole, a timely and probing question asked by a group member instigated controversy and spurred discussion, and consensus on a final answer was not reached.

In this context, I found myself defaulting to a role as "education researcher" – noticing cues that triggered me to pose certain questions at certain times, not really with any intent to "lead" my colleagues to a solution, but rather with the intent to challenge and observe responses. In our group discussion of the problem solving, members reflected on the impact of some of my choices. They commented on their emotional responses to hearing that "there is an answer" though not being told what it is – some found it motivational, others found it intimidating. There was a sense that I "held some knowledge" but not a sense that I was thinking mathematically. A colleague observed that I never said "*hmmmm, I don't know*" during their exploration. Unlike facilitating the other two problems, with the Ping-Pong Ball Conundrum the responses, strategies, interpretations, and problematic issues were familiar and anticipated. What was fostered was a lively debate, an intuitive (rather than formal) address of the problem, and *reflecting-in-action* (Mason, 2002) of the problem-solvers as they compared and debated their points of view. What came to light was a tension between fostering mathematical thinking (or any activity) while not engaging in the same. There are questions about this tension that continue to attract my attention. It is something that needs more thought, but my instinct is that there is an important connection between how an individual may choose to foster mathematical thinking and how he or she perceives the role of "teacher," "teacher educator," or "mathematician" and what is being noticed when the focus of attention is not on mathematics.

CONCLUDING REMARKS

After this working group experience, what do we know for certain? That the first step to noticing, fostering, and engaging the mathematicians in our classrooms is to be given the opportunity to engage with mathematics, whether it be with mathematicians, mathematics educators, or mathematics teachers. In other words, to give an opportunity similar to the one we were given (and, looking back, are extremely thankful for).

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APPENDIX: THE PROBLEMS

CUTTING THE CUBE Close your eyes and imagine a cube. Select one of the vertices of your cube, attach a string to it and hang your cube by that string. Now imagine a horizontal plane that will slice your cube exactly in half. What does the cross-section (the intersection of the plane and the cube) look like? How do you know? EXTENSION This next scenario will be hard to imagine, but we can try to extend some of the reasoning from before. Now, instead of a regular cube, "imagine" a 4-D hypercube – that is, a cube with four spatial dimensions. As before, we want to hang this hypercube by a vertex and slice it exactly in half with a 3-D hyperplane. We can't do much to visualize what this will be like, but we can still talk meaningfully about what the cross-section of the hypercube will look like. It will be a 3-D object and one that should have many of the same geometric (and algebraic) properties of the previous scenario. How can we figure out what this cross-section will look like? What shape will it be?	**COUPONS UN CUBE** Fermez les yeux et imaginez un cube. Choisissez un de ses sommets et suspendez le cube par ce sommet. Imaginez qu'un plan horizontal coupe le cube en deux parties égales. De quoi a l'air la coupe, l'intersection entre le plan et le cube? Comment le savez-vous? CONTINUATION Ce nouveau scénario est difficile à visualiser, mais essayons de pousser le raisonnement précédent plus avant. Au lieu d'un cube ordinaire, prenons un hyper-cube 4-D, c'est-à-dire un cube à quatre dimensions spatiales. Comme précédemment, suspendons l'hyper-cube par un sommet et coupons le exactement en deux avec un hyper-plan 3-D. La visualisation de cette configuration est problématique mais il est tout de même possible de parler sensiblement de la forme de la coupe. Ce sera un objet 3-D, que nous pourrions tenir en main et qui devrait avoir des propriétés géométriques (et algébriques) communes à celui du scénario précédent. Comment pourrions-nous déterminer de quoi la coupe a l'air? Quelle forme a-t-elle?
PIGS 1. Two pigs are tossed. a) Determine the possible outcomes. b) Assign a probability to each outcome. *Note: 1c), 2, and 3 can't be given out until 1a) and 1b) are completed by the group. c) What is the probability of landing a snout and a side? 2. A pig is tossed three times. What is the	**LES COCHONS** 1. Deux cochons sont lancés. a) Déterminez les résultats possible. b) Assignez une probabilité à chaque résultat possible. *Note : 1c), 2, et 3 ne doivent pas être posées avant que la réponse à 1a) et 1b) n'aie été établie. c) Quelle est la probabilité qu'ils tombent un naseau et un côté?

probability of landing {2 sides, 1 back}? 3. Three pigs are tossed (all at once into a perfectly circular ring). What is the probability of landing {2 sides, 1 back}?	2. Un cochon est lancé trois fois. Quelle est la probabilité qu'il tombe {2 côtés, 1 dos}? 3. Trois cochons sont lancés (en même temps et dans un anneau parfaitement circulaire). Quelle est la probabilité qu'ils tombent {2 côtés, 1 dos}?
INSTANCES OF THE DIGIT 3 You are working for a big construction company that just finished constructing a very large office building that has 100 offices on each floor. The offices are numbered consecutively and it is your job to install the office numbers. The format of the numbers reflects the position of the office: for example, office 067 is the 68^{th} office on the ground floor and office number 667 is six floors higher. The digits are sorted in boxes of 1000. Unfortunately, the 3's have to be back-ordered and there is only one box of 1000. Assuming you install the office numbers in order, what number will you be installing when you use the last digit '3' of the box? Could you reach the number in the previous question without having opened a second box of any other digit? EXTENSION Can you find the pattern that determines when the second box for each digit is needed?	**INSTANCES DU CHIFFRE 3** Vous travaillez pour une compagnie de construction qui est en train de terminer un édifice à bureaux énorme, avec 100 bureaux par étage. Les bureaux sont numérotés de façon consécutive et vous êtes responsables de l'installation de ces numéros. Leur format indique la position des bureaux : par exemple, le bureau 067 est le $68^{\text{ème}}$ bureau sur le rez-de-chaussée et le bureau 667 est six étages plus haut. Les chiffres à clouer sur les portes sont rangés en ordre dans des boîtes de 1000. Par malheur, une seule boîte de « 3 » est arrivée et les autres sont en rupture de stock. Si vous procédez par ordre, quel numéro serez-vous en train de mettre en place quand vous utiliserez le dernier « 3 »? Pourriez-vous arriver à ce numéro sans ouvrir une des autres boîtes? CONTINUATION Pouvez-vous découvrir la régularité qui détermine quand la deuxième boîte d'un des chiffres est nécessaire?
THE PING-PONG BALL CONUNDRUM Imagine the following scenario… You have an infinite set of ping-pong balls numbered with the natural numbers and a very large barrel. You are about to embark on an experiment, which lasts 60 seconds. In 30 seconds, the task is to place the first 10 balls into the barrel and remove the ball numbered 1. In half of the remaining time, the next 10 balls are placed in the barrel and ball number 2 is removed. Again, in half the remaining time (and working more and more quickly),	**L'AFFAIRE DE LA BALLE DE PING-PONG** Imaginez le scénario suivant… Vous avez un ensemble infini de balles de ping-pong numérotés avec les nombres naturels et un baril énorme. Vous êtes sur le point de commencer une expérience d'une durée de 60 secondes. Dans les premières 30 secondes, la tâche est de placer les premières dix balles de ping-pong dans le baril, et de retirer la balle numéro 1. Dans la moitié du temps restant, les prochaines dix balles sont placées dans le baril et la balle numéro 2 en est retirée. De

balls numbered 21 to 30 are placed in the barrel, and ball number 3 is removed, and so on. After the experiment is over, at the end of the 60 seconds, how many ping-pong balls remain in the barrel? How do you know?	nouveau, dans la moitié du temps restant (et en travaillant de plus en plus vite), les balles 21 à 30 sont placées dans le baril et la balle numéro 3 en est retirée, et cetera. Quand l'expérience prend fin, après les 60 secondes, combine de balles se trouvent dans le baril? Comment le savez-vous?
BOYS AND GIRLS What is the probability that Anne Gull has two boys? *Anne Gull: I have two children.* *Matthew Maddux: Is the older one a boy?* *Anne Gull: Yes.* *Anne Gull: I have two children.* *Matthew Maddux: Is at least one a boy?* *Anne Gull: Yes.* *Anne Gull: I have two children.* *Matthew Maddux: Do you have a boy?* *Anne Gull: Yes. His name is Laurie.*	**DES GARÇONS ET DES FILLES** Quelle est la probabilité qu'Anne Gull a deux garçons? *Anne Gull : J'ai deux enfants.* *Matthew Maddux : Le plus vieux est-il un garçon?* *Anne Gull : Oui.* *Anne Gull : J'ai deux enfants.* *Matthew Maddux : Y a-t-il au moins un garçon?* *Anne Gull : Oui.* *Anne Gull : J'ai deux enfants.* *Matthew Maddux : As-tu un garçon?* *Anne Gull : Oui. Oui, il s'appelle Laurent.*
MULTI-FACETS Picture to yourself a length of rope, lying on a table in front of you. The cross section of the rope is a regular *n*-sided polygon. Slide the ends of the rope towards you so that it almost forms a circle. Now mentally grasp the ends of the rope in your hands. You are going to glue the ends of the rope together but before you do, twist your right wrist so that the polygonal end rotates through one n^{th} of a full revolution. Repeat the twisting a total of *t* times, so that your mental wrist has rotated through *t* n^{ths} of a full revolution. NOW glue the ends together, so that the polygonal ends match with edges glued to edges. When the mental glue has dried, start painting one facet (flat surface) of the rope and keep going until you find yourself painting over an already painted part. Begin again on another facet not yet painted, and	**MULTI-FACETTE** Imaginez un segment de corde, posé sur une table devant vous, dont la coupe transversale est un polygone régulier à *n* côtés. Vous glissez les extrémités de la corde l'une vers l'autre pour les connecter. Mentalement, attrapez les deux extrémités dans vos mains. Vous allez les coller ensemble, mais avant de le faire, vous allez tordre votre poignet droit pour que l'extrémité polygonale soit tournée de 1 *n*-ième de tour. Répétez cette torsion *t*-fois, afin que votre poignet imaginaire ait tourné de *t n*-ième de tour. À présent, collez les deux extrémités pour que les cotés des polygones s'alignent. Quand votre colle mentale aura séché, peignez une facette de la corde (une surface plane) et continuez jusqu'à ce que vous ayez peint toute la facette. Recommencez avec une nouvelle facette et une autre couleur. De combien de

use another colour. How many colours do you need?	couleurs aurez-vous besoin?
JEOPARDY On Friday March 16, 2007 history was made. For the first time ever there was a three-way tie (for first place) on Jeopardy! People involved with the show decided to contact you, yes you, about the odds, which you claim are....	**JEU TÉLÉVISÉ** Le vendredi 6 mars 2007, un moment historique a eu lieu. Pour la première fois, les trois joueurs furent ex aequo (pour la première place) dans un jeu télévisé. Les organisateurs ont décidé de vous demander, oui, vous, quelles sont les chances que cela se produise… Qu'en pensez-vous?
SU DOKU Can you create a completed Su Doku puzzle without an obvious pattern, starting with a blank grid? What are some useful strategies?	**SU DOKU** Pouvez-vous créer un puzzle Su Doku complet sans régularité apparente à partir d'une grille vide? Quelles seraient des stratégies utiles?
CHROMINO® Chromino® is a modified domino game whereby each tile is a rectangle which consists of three squares each coloured in one of five colours. Players alternate, laying down tiles to create a board. To lay down a new tile, it is required to touch the already-laid tile(s) in such a way that the new tile touches existing ones on at least two contact segments, and that the colours must match across the contacts. If every possible tile exists in the set, can you work out how they were placed to produce the provided image (see Figure 1)? EXTENSION If you had the choice of any tile in the set at each turn, what could be the most compact configuration you could construct?	**CHROMINO®** Chromino® est une version modifiée du jeu classique de domino. Dans cette version, chaque pièce est un rectangle constitué de trois carrés, chacun coloré d'une de cinq couleurs. À chaque tour, les joueurs placent une pièce sur la table. Pour pouvoir poser une pièce dans le jeu, celle-ci doit s'agencer à une pièce déjà posée de même couleur sur au moins deux des huit « arêtes de contact ». Si toutes les pièces possibles font partie du jeu, pouvez-vous déterminer où elles sont placées (figure 1)? CONTINUATION Si vous aviez le choix de poser n'importe quelle pièce à chaque tour, quelle serait la configuration la plus compacte possible?

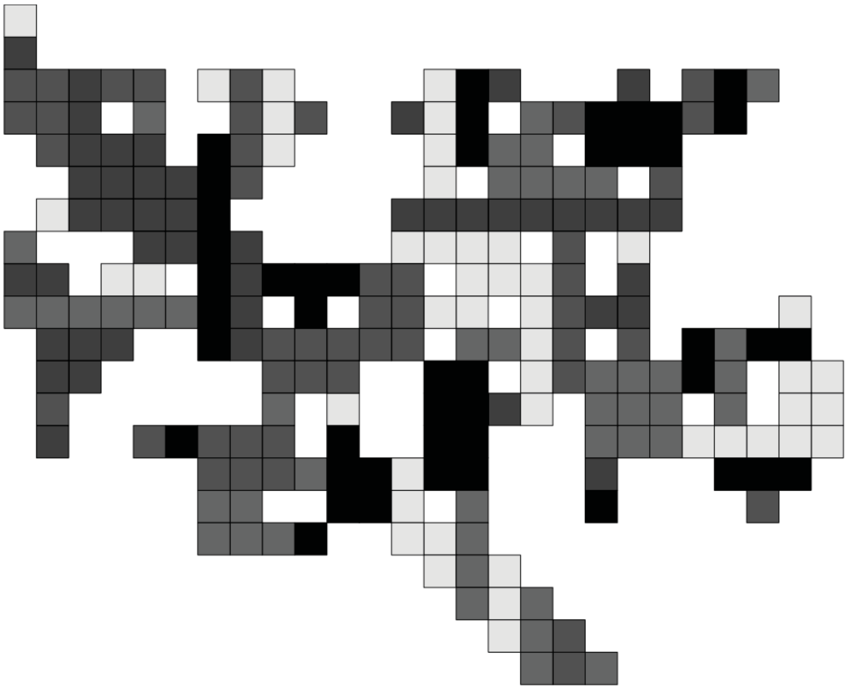

Figure 1. Chromino® configuration