TEACHING MATH: A COMPARISON OF RECOMMENDATIONS FROM RESEARCH AND THE NOVA SCOTIA CURRICULUM

by

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TABLE OF CONTENTS

List of Tables iv

Abstract v

CHAPTER 1: LEARNING AND TEACHING MATHEMATICS

A REVIEW OF THE LITERATURE 1

Importance Of Math Skills..1

Developmental Progression Of Math Skills 2

Cognitive Processes That Support Math Acquisition 5

Working Memory...5

Attention 7

Phonological Awareness and Processing Speed 8

Cognitive Theories Of Math Acquisition 9

Instructional Approaches For The Acquisition Of Math Skills 12

Individual Learning Styles 15

Summary 18

CHAPTER 2: THE NOVA SCOTIA MATHEMATICS CURRICULUM 20

Introduction to The Curriculum 20

Teaching Philosophy..24

Goals for Mathematics Education...27

Engaging all Learners 28

Curriculum Layout 29

CHAPTER 3: LEARNING AN INTEGRATION OF THEORY AND PRACTICE 34

Introduction to the Curriculum 34

Teaching Philosophy..39

Curriculum Layout..43

Summary 46

CHAPTER 4: IMPLICATIONS AND RECOMMENDATIONS....................................48

Recommendations for the Mathematics Curriculum 48

Scope and Sequence Summary 48

Order of Specific Curriculum Outcomes 49

Unnecessary Repetition 49

Implications for Teacher Training Programs 50

Developing Familiarity with Research 50

Training in Developmental Psychology..51

Implications for School Psychologists 51

Familiarity with Curriculum 51

Instructional Methods 52

References 54

LIST OF TABLES

TABLE 1. Specific Curriculum Outcomes in the Nova Scotia Mathematics Curriculum [Number Strand] 32

ABSTRACT

Strong skills in mathematics are necessary to be successful in today's society. Information on how to structure an evidence-based mathematics curriculum is available within the research literature. The mathematics curriculum documents for grades primary, one, and two in the province of Nova Scotia were reviewed in the context of what is recommended in the cognitive science literature about mathematics instruction. Implications for future curriculum documents, teacher training programs, and for school psychologists are discussed.

Chapter 1: Learning and Teaching Mathematics – A Review of the Literature

Importance of Math Skills

Strong skills in math are necessary in order to be successful in today's society. Fundamental math skills, such as algebra and measurement, are becoming increasingly relevant with the rise in quantitative knowledge that is necessary to function in many jobs. Consequences of poor math skills have been associated with reduced employment opportunities and decreased wages (Geary, 2011a). In a review of national studies in reading and mathematics of children and adults in Great Britain, poorly developed mathematics skills were found to have a number of long-term implications; including lower rates of full-time employment, lower annual income, higher rates of employment in lower-paying manual labour jobs, longer periods of unemployment, and lower rates of job promotion (Every Child a Chance Trust, 2009). Data from longitudinal studies conducted in Canada, the United States, and Great Britain, revealed that difficulty with the acquisition of math skills in the early years of school were predictive of later math difficulty even after controlling for intelligence, reading skill, family background, and social-emotional functioning (Duncan, Dowsett, Claessens, Magnuson, Huston … Klebanov, 2007). Early remediation of math skills is imperative in order to help decrease risk of later innumeracy (Geary, Hoard, Nugent & Bailey, 2013).

The Organization for Economic Co-Operation (OECD) conducts an international assessment of 65 countries every three years. Results from the most recent (2012) OECD Program for International Student Assessment (PISA) indicated a statistically significant drop from the mean in the mathematics performance of Canadian 15-year-olds in the past nine years (Brochu, Deussing, Houme, & Chuy, 2012). Within Canada, scores decreased across all provinces, with the exception of Quebec and Saskatchewan. In fact, students from Quebec were ranked sixth in the world, and tied with Japan and Macao. Interestingly, and in contrast to the discovery-based instructional modalities being used in most other provinces in Canada, students in Quebec receive direct instruction with an emphasis on the memorization of basic math facts (Brochu et al., 2012). According to the British Columbia Ministry of Education (2000), such differences in instruction might explain why students from Quebec outperform students from other jurisdictions (Raptis & Baxter, 2006). To ensure that students are provided with appropriate instruction in mathematics, a more thorough understanding of how children typically develop mathematics skills is required.

Developmental Progression of Math Skills

The acquisition of early numerical competencies typically follows a developmental progression. Before entering school, most children have informal experiences with numerical competencies simply by being exposed to numbers and counting in daily life (Ginsburg, Lee, & Boyd, 2008). These competencies are often referred to as *number sense* and involve the conceptual understanding of whole numbers and the basic operations between numbers (Berch, 2005; Gersten & Chard, 1999; Malofeeva, Day, Saco, Young, & Ciancia, 2004). Through formal and informal instruction with their caregivers, children learn that basic symbols, such as Arabic numerals (e.g., 0, 1, 2), and number words (e.g., one, two), have meaning based on the quantities they represent (Dyson, Jordan, & Glutting, 2013). Arabic numerals are one of the first mathematical concepts that children learn and understanding the quantity represented by these symbols provides the foundation for their conceptual insight into formal mathematics (Chu, vanMarle, & Geary, 2015). In preschool and kindergarten, children learn that numbers have relative magnitude and that counting is guided by principles. According to Gelman and Gallistel (1978), there are five principles that govern and define counting. These include one-to-one correspondence (only one number is assigned to each counted object), fixed order (numbers are arranged in a specific order), cardinality (the value of final stated number represents the quantity in the set), abstraction (counting can be applied to heterogeneous items, such as objects or colours) and order-irrelevance (order of enumeration of a set of objects is irrelevant). Through formal instruction, children acquire more advanced number sense (e.g., how numbers are composed and decomposed, place value), and this knowledge can be generalized across larger numbers and more complex operations (Griffen, 2004; National Mathematics Advisory Panel, 2008). These early numerical competencies all involve understanding of the symbolic representation of numbers, as opposed to knowledge gained simply from verbal output or instruction (Feigenson, Dehaene, & Spelke, 2004). Mastery of these competencies (counting, number knowledge, and number operations) in kindergarten has been linked to proficiency in math calculations and math problem-solving through to at least the third grade (Jordan, Glutting, & Ramineni, 2009; Jordan, Glutting, Ramineni, & Watkins, 2010; Jordan, Kaplan, Ramineni, & Locuniak, 2009; Locuniak & Jordan, 2008).

When children first learn to solve basic number operations (e.g., 4 + 2), they typically begin by using the sum procedure whereby they count the value of both addends (4 and 2) individually and then add them together (Geary et al., 2004). For example, for the problem 4 + 2, the child would count 1, 2, 3, 4, and then 1, 2), and then add them together by counting 1, 2, 3, 4, 5, 6. This process is sometimes executed by the child counting along on their fingers (the finger counting strategy), counting along with another representation (e.g., images, objects), or sometimes the set is simply counted out loud (verbal counting strategy; e.g., Siegler & Shrager, 1984). Another common counting procedure is called counting on. The most efficient means of counting on involves the min procedure (counting the larger addend, here 4, and then adding the smaller one, here 2). Infrequently, children will employ the max procedure (counting the smaller addend first and then adding on the larger one; Geary et al., 2004). Over time, children should show a gradual shift from the sum procedure to the more efficient min procedure due to their increased conceptual understanding of counting (Geary et al., 1992; Siegler, 1987). Given that many newly acquired math skills are built upon preexisting ones, mastery of basic numerical competencies (i.e., math facts) is necessary for a deeper understanding and application of more complex math problems (e.g., Baroody, 2003; Ferrari & Sternberg, 1998). Through practice, children begin to create memory representations of basic facts, which leads to a reduction in the use of counting strategies and an increase in memory-based problem solving (Geary et al., 2004).

Two of the most common forms of memory-based problem solving are direct retrieval and decomposition. Direct retrieval occurs when an answer can be easily recovered from long-term memory, and typically involves combined addends that are both less than 10. Decomposition involves reconstructing a partially retrieved answer; for example, to solve the problem 5 + 6, one would first directly retrieve the solution to 5 + 5 and then add 1 to this partial sum. Direct retrieval places the least amount of demands on memory as the answer is more automatized (Cooney & Hirsch, 1990). These memory-based strategies can be applied to more complex problems; however, with direct instruction children can be taught more efficient procedural strategies to solve these problems. For example, children can be taught columnar strategies to solve multi-digit problems (e.g., 56 + 82), where they sum the integers in the ones column first, and then sum the integers in the tens column (Geary et al., 2004). As the complexity of the task increases, the greater the demand placed upon the student's memory. Efficient use of memory-based processes and procedural strategies leads to reduction in the memory demands.

Cognitive Processes That Support Math Acquisition

In order to better understand how children acquire skills in mathematics, researchers have attempted to delineate specific areas that seem to pose problems for students. This is most often done by comparing the performance of typically achieving students with those who are having more challenges in math. While no consensus has been reached about which cognitive process (or processes) are most important for the acquisition of math skills, researchers have identified a number of cognitive abilities that help support math skills development; such abilities include, but are not limited to, working memory, attention, phonological awareness, and processing speed.

Working memory. The relationship between working memory and mathematics ability has been widely examined (e.g., Andersson, 2008; DeStefano & LeFevre, 2004; Fuchs, Geary, Compton, Fuchs, Hamlett & Seethaler, 2010; Swanson & Beebe-Frankenberger, 2004). Working memory can be conceptualized as a short-term working space that can temporarily hold information in mind so that an individual may complete a task (Passolunghi, Vercelloni, & Schadee, 2007). Baddeley and Hitch (1974) proposed a three-component model of working memory comprised of a *central executive*, which is responsible for the attentional control of actions and coordinates the *phonological loop* (short-term maintenance of linguistic information) and the *visuo-spatial sketchpad* (short-term maintenance of visuo-spatial information). Some researchers have examined the relationship between individual components of working memory and mathematics achievement, whereas others have looked at working memory as a unitary construct.

The central executive has been frequently linked with measures of math achievement (e.g., De Smedt, Janssen, Bouwens, Verschaffel, Boets, & Chesquiere, 2009; Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007; Mazzocco & Kover, 2007). Geary and colleagues (2008) found that after controlling for intelligence, improvement in the accuracy with which students in first and second grade placed numbers on a number line was related to their central executive capacity. They attributed this accuracy in number placement to the attentional and inhibitory control of the central executive. Barrouillet and Lepine (2005) found that higher central executive capacity in third and fourth graders was associated with more frequent direct retrieval of addition facts. The authors posited that these children were able to more easily form associations between numbers (e.g., 2 + 5 = 7), and that once these associations were formed in long-term memory, they were able to access their relationship and inhibit other incorrect associations.

Not all research, however, has supported the relationship between the central executive and math skills. For example, Geary and colleagues (2012) found that the importance of the central executive varied across strategies used and grade levels. Specifically, the central executive was more predictive of the use of effective strategies (e.g., direct retrieval and use of the min procedure) in the first grade, than it was of the use of these strategies in later grades. Additionally, the central executive was not predictive of the use of less effective strategies (e.g., decomposition for simple or complex problems) in the first grade, but it was predictive in later grades. This could be in part due to the low overall use of decomposition in earlier grades. Overall, Geary and colleagues (2012) found that individual differences in the central executive were more reflective of strategy use (e.g., direct retrieval and min counting) in earlier grades and more reflective of rate of execution in later grades.

Other researchers have found that the phonological loop has been implicated in mathematics problem solving (e.g., Gathercole & Pickering, 2000) and visuo-spatial working memory has also been associated with early counting skills (Kyttala, Aunio, Lehto, van Luit, & Hautamaki, 2003) and general mathematics ability more broadly (Maybery & Do, 2003). Alternatively, other researchers (e.g., Passolunghi & Siegel, 2001; Swanson & Sachse-Lee, 2001) have found evidence to support a more general working memory deficit.

Attention. The ability to control and sustain attention has been linked with academic achievement broadly and has been shown to predict achievement test scores and marks in preschool and early elementary grades (e.g., Alexander, Entwisle, & Dauber, 1993; Raver, Smith-Donald, Hayes, & Jones, 2005). A meta-analysis of longitudinal studies from the United States, Great Britain, and Canada (Duncan et al., 2007) has linked school-entry attentional skills with academic outcomes, even after controlling for cognitive ability and prior skill level. As it pertains to math, attention has been found to be a strong predictor of achievement across a number of competences including: simple arithmetic, multidigit calculations, word problems (Fuchs, Compton, Fuchs, Paulsen, Bryant, & Hamlett, 2005; Geary, Hoard and Nugent, 2012), algebra (Fuchs et al., 2012), and fractions (Fuchs et al., 2013, 2014, Jordan et al., 2013). In a study of third grade paper-and-pencil arithmetic skills, Fuchs and colleagues (2006) demonstrated that once the effects of in-class attentive behaviour were controlled for, the central executive and level of intelligence no longer remained significant predictors of achievement. This is consistent with their prior intervention study which demonstrated that after controlling for a number of cognitive skills, teacher ratings of attention uniquely predicted math achievement in fact fluency, addition and subtraction calculations, and story problems (Fuchs et al., 2005). Teacher ratings of attention through elementary school have also been associated with rates of high school graduation (Pagani, Japel, Larose, Vitaro, Tremblay, & McDuff, 2007). Related skills such as self-regulation, task perseverance, and the ability to inhibit irrelevant stimuli have been shown to be related to how much time children spend participating in academic activities (Duncan et. al., 2007) and how quickly they learn (Clark, Pritchard, & Woodward, 2010).

Phonological awareness and processing speed. Within the field of reading acquisition, there is now an undeniable scientific consensus that early reading instruction should include direct, explicit and systematic teaching of phonological awareness (the understanding of speech sounds within words) and phonics (a method of instruction that teaches the relationship between letters and sounds; Ehri et al., 2001; Vellutino, Fletcher, Snowling, & Scanlon, 2004). Phonological awareness, along with processing speed (the fluency of processing information; Geary, Hoard, & Bailey, 2012), has been linked with achievement in various mathematics competencies. It has been argued that both of these skills are necessary for math; without them, individuals would not be able to accurately and efficiently access the verbally coded mental representations needed to solve computational problems (Hecht, Torgesen, Wagner, & Rashotte, 2001; Simmons & Singleton, 2008). Fuchs et al. (2012) examined the role of processing speed and phonological processing in pre-algebraic knowledge. Students were administered tests of single-digit calculation and word problems at the beginning of second grade and the end of third grade. Multilevel path analyses, controlling for the variance of in-class instruction, revealed indirect effects of phonological processing on calculation skill. Multiple level effects of calculations and word problems, however, mediated the effects of processing speed. The authors noted that it was possible that correctly solving calculation problems requires efficient processing of information before it is lost from working memory which suggested that lower-level cognitive resources, such as processing speed and phonological processing, exert indirect effects on comprehending text and play a role in the foundational skills required for pre-algebra. Notably, 43% of the variance in pre-algebraic knowledge remained unaccounted for, suggesting that other resources (e.g., other forms of working memory or reasoning) might be responsible for the transition from arithmetic to pre-algebra. Nonetheless, these findings extended Fuchs and colleagues (2005) prior work, which demonstrated that phonological processing (as measured by rapid digit naming and first/last sound matching) predicted math fact fluency.

Cognitive Theories of Math Acquisition

Unlike the area of reading acquisition, where the necessity of skills in phonological processing is well established (e.g., Vellutino et al., 2004), there remains to be a consensus as to what cognitive processes are necessary for acquisition of mathematics skills. Many researchers argue that it is a combination of domain-general cognitive abilities that exert an influence on math achievement (e.g., Fuchs et al., 2006). Domain-general abilities are those thought to influence learning in all areas (e.g., math, language, history), and include constructs such as working memory, executive functions, processing speed, and semantic memory (e.g., Geary, 2004; Geary, Hoard, Nugent, & Bailey, 2012). Geary's (2004) model of mathematics learning proposed that achievement in math arises from a combination of foundational mathematical competencies and domain-general abilities.

Other researchers propose that domain-specific abilities exert an influence on math achievement. Rousselle and Noel (2007), for example, argue that children with mathematics difficulties have deficits accessing magnitude information from numerical symbols. This is known as the *access deficit hypothesis* (see also Wilson & Dehaene, 2007). According to this view, challenges with math are due to an immature development in relating numerical symbols to their underlying meaning (Rouselle & Noel, 2007). This is in contrast to the *defective number module hypothesis*, which assumes the underlying core deficit of mathematics difficulties lies in the processing of numerical information. Landerl and Kolle (2009) found that while children with mathematics difficulties were indeed slower on the symbolic than non-symbolic number comparison tasks (consistent with the access deficit hypothesis), these students were slower on both tasks in comparison to a control group (consistent with the defective number module hypothesis; see also Willburger, Fussenegger, Moll, Wood, & Landerl, 2008; Mussolin, Mejias, & Noel, 2010). More recently, in a longitudinal study of children from grade two to four, efficiency of numerical processing was found to be an indicator of mathematics difficulties (Landerl, 2013).

Still, other researchers (e.g., Jordan, Kaplan, & Hanich, 2002; Mazzocco & Myers, 2003) suggest that there is not one single core deficit that is characteristic of students with mathematics difficulties, but instead, challenges result from multiple causes. In a sample of fifth and six graders from Sweden, students with and without mathematics difficulties were administered a number of cognitive and numerical processing tasks (Andersson & Ostergren, 2012). Students with mathematics difficulties had deficits on a visual-spatial working memory task and on a task that required quick retrieval of information from long-term memory. Students with mathematics difficulties also demonstrated deficits in the *approximate number system* (ANS; the ability to represent approximate quantities on a number line) and in the *object tracking system* (OTS; the ability to mentally keep track of a set of up to four objects). The authors concluded that mathematics difficulties in children are not due to a single deficit but due to a number of deficits and that patterns will present differently across students.

In summary, there are no firm conclusions about the underlying problems for students with challenges in mathematics. Some researchers explain variation in ability in terms of domain-general cognitive skills, while others seek to explain this variation in terms of specific numerical competencies. Still other researchers argue that variation results from a combination of domain-general abilities along with specific numerical competencies (Geary, 2004).

Instructional Approaches for the Acquisition of Math Skills

The acquisition of mathematics skills is also influenced by the appropriate context and content of instruction (Geary, 2008). Over the last several decades, there has been a shift from more traditional explicit instruction, to instruction that focuses on exploration, discovery, and invention (i.e., constructivism; Alfieri, Brooks, Aldrich, & Tenenbaum, 2011). Before proceeding, it is prudent that we examine the wide range of instructional conditions that are encompassed within the framework of explicit instruction as compared to instruction that is more in line with constructivist approaches to learning.

Explicit instruction involves the direct and systematic teaching of fundamental skills. Students are provided with information that fully explains concepts and procedures, along with feedback, examples, and teacher support to help them learn (Kirschner, Sweller, & Clark, 2006). New concepts are incrementally introduced when the student has achieved mastery of previously taught information (Carnine, Silbert, Kame'enui, & Tarver, 2004). Mastery is achieved through repetition and practice. Although some educational theorists believe that this approach (sometimes termed *drill and kill*) is detrimental to learning and student well-being, research has established that children require practice in order to develop proficiency in a given area (Imbo & Vandierendonck, 2008). Advocates of this approach have concluded that optimal learning occurs under conditions of direct and explicit instruction for both novice and intermediate learners (e.g., Clark 2009; Klahr, 2009; Mayer 2004; Sweller, Kirschner, & Clark, 2007).

Constructivist (or discovery-based) approaches to learning posit that students learn best when they explore, discover, and invent concepts on their own (Alfieri et al., 2011). There are a number of instructional approaches that are encompassed within the framework of discovery learning (e.g., pattern detection, simulation, working through manuals) and a number of terms are used to refer to what is essentially the same concept (e.g., discovery learning, enquiry learning, constructivist learning). Broadly speaking, these learning approaches require the student to independently conceptualize target information with provided materials (Destrebecqz, 2004; Khlar & Nigam, 2004; Rittle-Johnson, 2006). Manipulatives are used to explore novel concepts, to uncover patterns, and to determine causalities. Discovery/constructivist approaches to instruction may be unassisted, minimally-guided, or highly-guided, depending on the difficulty level of the task and the intended outcome or theoretical perspective of the teacher (Alfieri et al., 2011). Proponents of constructivist approaches to learning agree with Piaget's claim that when a child is prematurely taught something that he could have otherwise discovered himself, he does not understand the concept as completely (Piaget, 1970, p. 715). Advocates of this approach also propose that children who discover things on their own will be better able to apply this knowledge to novel situations than children who are explicitly taught (e.g., Bredderman, 1983; Stohr-Hunt, 1996).

Discovery-based/constructivist approaches to learning, however, are incongruent with what we know from the cognitive science literature. One major flaw inherent within the enquiry-based approach is that the necessary metacognitive skills for this method (e.g., conceptualizing, organizing, and integrating relevant material) strain the learner's working memory, and thus optimal learning cannot occur (e.g., Anderson, Reder, & Simon, 2000; Rittle-Johnson, 2006; Kirschner, Sweller, & Clark, 2006). This could be particularly problematic in children with mathematics difficulties, as they often have associated working memory deficits (e.g, Andersson, 2008; Fuchs et al., 2010); thus, enquiry-based approaches that require use of metacognitive strategies place these children at an even greater, and arguably unnecessary, disadvantage (Timmermans & Van Lieshout, 2003). Further, these metacognitive strategies do not always transfer across problem types. Timmermans and Van Lieshout (2003) found that in a sample of students with learning disabilities, children who received direct instruction showed transfer effects on addition-without-regrouping performance, but children who experienced a constructivist approach to instruction did not. Although the constructivist instruction group (where children were encouraged to develop their own strategies) had a larger repertoire of strategies, the strategies did not result in more efficient problem solving.

Researchers and educators alike continue to dispute the degree of instructional guidance necessary for optimal learning. Advocates of explicit instruction argue that the most efficient learning occurs under conditions of direct and systematic teaching of facts (e.g., Alfieri et al., 2011; Duffy, 2009). Unlike discovery-based learning, direct instructional approaches serve to reduce the likelihood of children encountering inconsistent or misleading feedback and drawing incorrect conclusions (Klahr & Nigam, 2004). And while enquiry-based learning is said to promote more flexible solution strategies (Klein, Beishuizen, & Treffers, 1998), this appears true only for a subset of students. While children who are performing at or above average in math are able to test their own ideas and apply them to new situations, children who are achieving within the lower end of the math distribution have more difficulty with this approach (Geary, Brown, and Samaranayake, 1991).

Individual Learning Styles

The idea that it is important to match instructional approaches to individual learning styles has been discussed in the education literature for upwards of 40 years; however, the definition and usefulness of the concept of learning styles have been controversial from the very beginning (e.g., Dunn, 1983, 1990; Kavale & Forness, 1987; Lovelace, 2005). The original intention behind learning styles was to provide teachers and educators with a framework for discussion about learning and instruction (Jennings, 2012); instead, the learning styles framework has evolved into something that requires classifying students into categories that are not always well defined. The term *learning style* has been defined as "a combination of environmental, emotional, sociological, physical, and psychological elements that permit individuals to receive, store, and use knowledge" (Dunn, 1983; p. 496); or "the way in which students begin to concentrate on, process, internalize, and remember new and difficult information" (Jennings, 2012). Learning style models range from holistic approaches (with an emphasis on environmental, emotional, sociological, physiological, and psychological variables; Dunn et al., 2009) to more specific models that focus on information gathering, processing, and retrieval (Felder & Silverman, 1988). Depending on the model, learning styles are theorized to be influenced by a number of variables, including but not limited to: sociological preferences, emotional preferences, perceptual preferences, cultural preferences, age, gender, achievement, and brain structure. Some propose learning styles to be inherent within the individual (Hawk & Shah, 2007), whereas others propose that they are contextual (Reynolds, 1997).

There are numerous systems that are used to classify different learning styles. A meta-analysis of the learning styles literature from the UK, the US, and Western Europe identified 71 different models of learning styles (Coffield, Moseley, Hall, & Ecclestone, 2004b). Commonly discussed learning styles include the visual, the auditory, and the kinesthetic learner. A visual learner has a preference for information presented in diagrams, charts, maps, and other symbolic representations, rather than verbally presented information. An auditory learner has a preference for information presented through lectures, discussions, and other spoken formats, rather than information presented in a visual form. A kinesthetic learner has a preference for gaining information through physical experiences and interactions, rather than through listening or viewing. Combinations of the three styles can exist (Fleming, 2015). In fact in 1979, Dunn and Dunn proposed that approximately 40 percent of students have some combination of the auditory, visual, and tactile/kinesthetic learning styles.

The idea behind identifying different learning styles was to help students capitalize on their strengths in receiving, assimilating, and retaining information, by providing them with a better understanding of how they learn. It was also intended to provide the teacher with instructional methodologies designed to maximize student strengths (Jennings, 2012). Hadfield (2006) identified three options for differentiating instruction based on learning styles within the classroom: matching teaching style to learning preference; alternating between various styles to accommodate a wider number of students; explicitly teaching strategies so that learners can adapt to other styles. There is no consensus as to which teaching strategy is the most effective for which learning style (Denig, 2004; Honey & Mumford, 1992; Slack & Norwich, 2007). Some researchers (e.g, Petty, 2004) argue that there is benefit for a student to experience a range of styles and strategies; while others (e.g., Halstead & Martin, 2002; Mokhtar, Majid, & Foo, 2008) have found that optimal learning occurs under conditions when the teaching strategy matches the learning style.

There are a variety of published research studies, reviews, and meta-analyses which advocates contend provide support for the learning style hypothesis (that instruction should be delivered in a way that is commensurate with the learner's style). However, others have contended that these studies lack sufficient *scientific* evidence to support the learning style hypothesis. For example, Landrum & McDuffie (2010) noted that the majority (35 out of 36) of the studies that Dunn et al. cited in their 1995 meta-analysis were unpublished dissertations from a single university. In addition to the reliance on unpublished and therefore not peer-reviewed reports, this particular meta-analysis had statistical issues pertaining to the interpretation of effect sizes (Kavale et al., 1998; Stahl, 1999). Additional reviews of the learning style hypothesis (Carbo, 1983; Lovelace, 2005) have failed to account for other theories and explanations of learning processes that are offered in psychology and sociology research (Coffield et al., 2004a, 2004b). For example, Carbo (1988) argued that phonics instruction was not a necessary component of reading acquisition and instead advocated that reading is most improved when reading programs are matched to student learning styles. In response to criticisms (e.g., Stahl, 1999), Carbo later conceded that there is a role for phonics when, "the learners reading style matches the phonics method" (2005, p. 48). Currently, there is insufficient evidence to support the positive effects pertaining to the practical application of learning styles (Allcock & Hulme, 2010; Coffield et al., 2004; Cook, Gelula, Dupras,

& Schwartz, 2007; Kavale & Forness, 1987; Landrum & McDuffie, 2010; Lehmann & Ifenthaler, 2012; Martin, 2010; Muse, 2001; Rohrer & Pashler, 2012).

Summary

Taken together, findings from the reviewed literature have several important implications for educators and other practitioners, especially those who offer consultation or play a role in educational planning and policy development. First, researchers have demonstrated that the acquisition of numerical competencies follows a developmental progression (e.g., Geary, 1993; Jordan et al., 2010; Locuniak & Jordan, 2008). Children learn that basic symbols (e.g., 0, 1, 2, 3) have meaning based on the quantities they represent, that numbers have a relative magnitude, and that counting is guided by certain principles (e.g., Gelman & Gallistel, 1978; Ginsburg, Lee, & Boyd, 2008). Over time, children develop an understanding for how numbers are composed/decomposed. With sufficient practice and repetition, children acquire memory representations of basic facts, which subsequently lead to memory-based problem solving strategies (Geary et al., 2004; Griffen, 2004; Siegler, 1987). Second, there is a large evidence-based body of knowledge that supports instructional methods that are systematic and explicit in nature. In contrast to constructivist approaches to learning, explicit and systematic methodologies are more efficient, promote transfer across tasks, and reduce the metacognitive demands placed upon the student (e.g., Alfieri et al., 2011; Imbo & Vandierendonck, 2008; Kirschner, Sweller, & Clark, 2006). Lastly, there is simply insufficient scientific evidence to advocate for differentiated instruction based upon individual learning styles. Although differentiating instruction based upon learning styles is intuitively and philosophically appealing, researchers have consistently demonstrated that students are not necessarily all heterogeneous with respect to how they learn (e.g., Dunn, 1983, 1990; Kavale & Forness, 1987; Lovelace, 2005).

Chapter 2: The Nova Scotia Mathematics Curriculum

Introduction to Curriculum

All of the information outlined within this chapter was retrieved from the *Mathematics Grade Primary, One, and Two Curriculum* documents (Nova Scotia Department of Education and Early Childhood Development, 2013a, 2013b, 2013c). Aside from the curriculum outcomes, which are specific to each grade, all other information (e.g., introduction, contexts for learning) contained in the mathematics curriculum documents is the same. For this reason, references to *the mathematics curriculum* in this chapter pertain to all three documents unless otherwise specified.

The Nova Scotia mathematics curriculum was adopted from the Western and Northern Canadian Protocol's (WNCP) *The Common Curriculum Framework for K-9 Mathematics* (Alberta Education, 2006). The *Common Curriculum Framework* was developed by the seven ministries of education (Alberta, British Columbia, Manitoba, Northwest Territories, Nunavut, Saskatchewan, and Yukon Territory), in collaboration with teachers and other educators, parents, and business representatives. Included within the mathematics curriculum are statements about beliefs about mathematics, general and specific student outcomes, and performance indicators. As noted in the mathematics curriculum, outcomes and performance indicators in the *Nova Scotia* mathematics curriculum were adapted specifically to Nova Scotia. Included within the mathematics curriculum is an emphasis on key concepts within and across each grade to create a greater depth and understanding of material. In addition, the Nova Scotia mathematics curriculum states that it recognizes the importance of strong foundational skills in numeracy, and thus it emphasizes the importance of number sense and operations in the earlier grades. The mathematics curriculum document states that it was created to provide a common base for defining expectations and maintain consistency of student outcomes within the province of Nova Scotia and to communicate to all education partners the high expectations for mathematics learning within the province. The mathematics curriculum also notes that it was informed by national and international research by the WNCP and the National Council of Teachers of Mathematics (NCTM).

Learning outcomes in Nova Scotia (from Grade Primary through Nine) are organized into strands; these include: Number (N), Patterns and Relations (PR), Measurement (M), Geometry (G), and Statistics and Probability (SP). Each strand has a corresponding general curriculum outcome (GCO) as well as several specific curriculum outcomes (SCO) and performance indicators. GCOs are overarching statements regarding expectations within each strand, while SCOs are statements regarding specific skills and knowledge within each strand. Performance indicators are statements that specify the depth, breadth, and expectation for the outcome and help teachers determine whether students have achieved the corresponding specific curriculum outcome. In addition to specific and general outcomes, the mathematics curriculum also advises teachers on the amount of time they should allocate to mathematics instruction on a daily basis. Specifically, the *Time to Learn Strategy Guidelines for Instructional Time: Grades Primary – 6* (Nova Scotia Department of Education, 2002) recommends that 45 minutes be required daily in Primary to Grade Two, and 60 minutes be required daily for Grades Three to Six. The mathematics curriculum notes that the rationale for this time frame is that it is follows a constructivist approach to teaching through problem solving.

Ongoing assessment is one important component that is outlined in the Nova Scotia mathematics curriculum. Assessment methods within the classroom, as outlined in the mathematics curriculum, include the provision of: specific goals and learning outcomes; examples, rubrics, and models to clarify outcomes and highlight important features; progress monitoring and feedback; self-assessment; and a classroom environment that encourages discussion and fosters a deeper understanding of learned material and concepts. Further, the mathematics curriculum advises that assessment of student learning should be commensurate with curriculum outcomes, clearly define criteria for success and expectations of students' performance, include a wide variety of assessment tools, and produce useful information to inform instruction. In order to obtain a balanced assessment, the mathematics curriculum suggests integration of the following: conversations/conferences/interviews (with individual students, teachers, and groups), products/work samples (e.g., journals, drawings/charts/tables, paper-and-pencil tests, surveys, self-assessments), and observations (planned and unplanned, shared/guided, performance tasks, individual conferences, checklists, interactive activities).

The Nova Scotia mathematics curriculum states that there are seven critical components that students must encounter "in order to achieve the goals of mathematics education and encourage lifelong learning in mathematics" (Nova Scotia Department of Education and Early Childhood Development, 2013b, p. 12). Students are expected to: a) communicate and express their understanding of mathematics; b) apply new mathematical knowledge through problem-solving; c) make connections between mathematical concepts and everyday applications of mathematics; d) become proficient in mental mathematics and estimation; e) identify and use appropriate technology as a tool for learning and problem-solving; f) engage in visualization to facilitate processing of information, making connections, and problem-solving; and e) develop mathematical reasoning.

The mathematics curriculum specifies that learning through problem solving should be the emphasis of mathematics across all grade levels. Students are encouraged to explore a wide variety of problems, are challenged to find multiple solutions to problems, and are encouraged to create their own problems. The mathematics curriculum also emphasizes the importance of connecting mathematical ideas to the real world in order for students to "view mathematics as useful, relevant, and integrated" (Nova Scotia Department of Education and Early Childhood Development, 2013b, p. 13) and states that connections should be made among different representational modes (e.g., contextual, concrete, pictorial, linguistic/verbal, and symbolic). The mathematics curriculum also states that mental mathematics is the cornerstone for estimation and offers students a variety of alternative algorithms and nonstandard techniques for finding answers. It notes that mental mathematics and estimation skills must be learned in context (rather than in isolation) so that students may apply them to solve problems.

The Nova Scotia mathematics curriculum also states that there are seven elements that define mathematics which are interwoven throughout the document; these include: change, constancy, number sense, relationships, patterns, spatial sense, and uncertainty. *Change* refers to the idea that mathematics is dynamic rather than static. Mathematics requires students to search for explanations of that change, to make predictions, look for patterns, and describe and quantify their observations. *Constancy* can be better understood by the following terms: stability, conservation, equilibrium, steady state, and symmetry (Nova Scotia Department of Education and Early Childhood Development, 2013b, p. 16). Many properties in mathematics remain stable while outside conditions change. By recognizing constancy, students are able to solve problems. *Number sense* has been referred to as the most important foundation of numeracy (British Columbia Ministry of Education, 2000, p. 146). The mathematics curriculum states that true understanding of numbers goes beyond the skills of counting, memorizing facts, and the rote use of algorithms. Instead, it states that true understanding of numbers develops as a by-product of learning (rather than as a result of direct instruction), and is enhanced when students are able to connect their number knowledge to real life experiences and when they can use benchmarks and referents. R*elationships* and *patterns*, refer to the concrete, visual, or symbolic association among numbers, sets, shapes, objects, and concepts. The mathematics curriculum notes that students should develop fluency between representations and be able to recognize, extend, create, and identify mathematical patterns and relationships. *Spatial sense* involves visualization, mental imagery, and spatial reasoning. Spatial sense offers one means of interpreting and reflecting upon the physical environment in its 3-D or 2-D representation. Lastly, *uncertainty* refers to how interpretations of data and predictions made from such data lack certainty. This component requires students to assess the reliability and validity of data interpretation, and develop their understanding of probability.

Teaching Philosophy

The Nova Scotia mathematics curriculum outlines four assumptions and beliefs that contribute to mathematics learning. First, it states that mathematics is an active and constructive process. Drawing connections between students' interests, abilities, and experiences is a key component to developing numeracy skills. For example, to practice counting in Grade One, the curriculum suggests having students draw pictures of their favourite toys and then count the total number of items. In Grade Two, teachers are encouraged to allow students to choose a favourite story and then create related addition and subtraction problems which could be shared with the class via dramatizations, pictures, or writing. The mathematics curriculum also advises that teachers use real-world and authentic contexts for developing problems whenever possible. For example, in order to develop number knowledge in Grade Two, children could be asked to identify where they might see the number 26 in their neighbourhood. Additional ways of drawing on students' interests and varying abilities to facilitate learning include the use of role-playing, using verbal/concrete or pictorial representations, and playing games or having discussions (as opposed to using rote memorization) to consolidate basic math facts.

A second assumption and belief outlined in the mathematics curriculum is that optimal learning occurs under conditions of clear expectations, ongoing assessment, and feedback. An example of ongoing assessment, across Grades Primary, One, and Two, is to observe and record the way in which students count (e.g., do they demonstrate confidence and are they able to verbally or pictorially explain their reasoning?). Other means of assessment are reflected in the performance indicators (e.g., describe a personal strategy for solving a problem) that correspond to each SCO. The mathematics curriculum states that a number of approaches and contexts should be used for assessment of learning and that these can be determined on an individual level, as a small group, or as a whole class.

A third assumption and belief stated within the mathematics curriculum is that learners comprise a heterogeneous group with varied learning styles, rates of learning, and background knowledge/experiences. The mathematics curriculum notes that students learn by making connections to real-life experiences and by constructing their own meaning of mathematics; given the diversity of learning styles and developmental stages within a classroom, students benefit from working with and translating various representations (e.g., concrete, pictorial, contextual, and symbolic) to optimize their learning. Across the Primary, Grade One, and Grade Two years, students are often asked to demonstrate their knowledge using as many representations as possible. The mathematics curriculum suggests that teachers interview students in order to determine why students use certain strategies noting that this will provide insight into a student's thinking and help teachers to identify groups of students that may benefit from similar types of instructional modalities. Students are also encouraged to share their thinking and problem-solving strategies with other students in the classroom in order to facilitate discussion and shared thinking.

Lastly, the mathematics curriculum outlines that ideal conditions for learning occur in an environment that supports learning, risk-taking, and critical thinking, and fosters a positive attitude and perseverance. For example, the learning environment should be respectful of all student experiences and opinions and should inspire taking risks, posing questions, and generating hypotheses. In order to create more divergent thinking and the development of personal strategies, Grade Two teachers are encouraged to write number sentences horizontally (i.e., as 2 + 3) rather than vertically. According to the mathematics curriculum, students who develop personal strategies to solve problems do not find problems that require regrouping of numbers any more difficult than problems that do not require regrouping. In order to provide students with equitable opportunities to learn, the mathematics curriculum encourages teachers to employ a variety of instructional approaches and assessment activities. Specifically, three key principles are provided: 1) content must be instructed in a manner that is flexible and offers many ways of representation; 2) students must be given the opportunity to demonstrate their learning and understanding in multiple ways; and 3) teachers must provide multiple options for students to engage in learning. The mathematics curriculum states that effective teachers offer a range of mediums from which students can demonstrate their knowledge; they find ways to balance individual, small-group, and whole-class approaches to instruction; and they involve students in the criteria for assessment and evaluation, scaffold instruction and assignments, and provide meaningful feedback and frequent check-ins throughout the learning experience.

Goals for mathematics education. The mathematics curriculum states that in order to experience success, students must be taught to set realistic and attainable goals. Further, they must have a means in which to monitor their progress towards achieving their goals. The process of revisiting and assessing initial goals is reportedly an important step in becoming an autonomous and responsible learner. Additional goals of the mathematics curriculum include the ability to “confidently solve problems, communicate and reason mathematically, appreciate and value mathematics, make connections between mathematics and its applications, and become mathematically literate adults, using mathematics to contribute to society” (Nova Scotia Department of Education and Early Childhood Development, 2013b, p. 22). Students who have achieved these goals will have reportedly gained an appreciation for mathematics as a science, a philosophy, and an art. As a result, these students will not only have a positive attitude towards mathematics, but they will persevere on math-related tasks, contribute to mathematics discussions, take mathematical risks, and exhibit curiosity towards situations involving mathematics.

Engaging all learners. The Nova Scotia mathematics curriculum states that it was designed to provide learning opportunities that are culturally proficient, equitable in assessment and instruction, and reflect the diversity represented in today's classrooms. According to the mathematics curriculum, engagement is fundamental to learning; therefore, the mathematics curriculum advises that students be provided with many opportunities to become invested in their learning. Emphasized throughout the mathematics curriculum is the importance of teachers' demonstrating a genuine belief in each student's potential to learn. The curriculum notes that students are more likely to be motivated, participate, persist in challenging situations, and reflect on their learning when they feel as though their teacher sees them as an individual learner and person.

According to the mathematics curriculum, learning preferences vary widely from student to student and are influenced by both the context/purpose and the type of information being presented. In order to accommodate the greatest number of students, teachers are encouraged to design instruction in a way that is commensurate with multiple learning styles. The mathematics curriculum identifies the following three learning styles (in no particular order) as the most common: 1) *auditory* (e.g., listening to lectures or engaging in discussion); 2) *kinesthetic* (e.g., using manipulatives or taking notes); and 3) *visual* (e.g., interpreting information such as graphs or watching videos).

The mathematics curriculum states that although students are expected to learn using all modalities, they often exhibit a preference for one or two that come more naturally to them.

The Nova Scotia mathematics curriculum states that it is necessary to value the importance of gender equity and inclusion within the classroom and that all students are held to the same academic standard, are given equal opportunity for contributions within the classroom, and receive gender-fair language and respectful listening from their teachers. Teachers are encouraged to respond to the cultural diversity within the classroom by using a range of culturally-proficient instruction and assessment strategies. The mathematics curriculum notes that all classrooms might also have students in them who have English as an Additional Language (EAL), learning challenges, varying levels of development, and/or who are exceptional learners in some other way and suggests that students who have EAL may require adaptations to curriculum outcomes or temporary individualized outcomes until their English language skills become more proficient. The mathematics curriculum requires that teachers adjust the breadth, depth, and/or pace of instruction to meet individual students' needs. In addition, the mathematics curriculum states that students who are gifted flourish when challenged by problem-centred, inquiry-based learning and open-ended activities.

Curriculum Layout

The format of the Nova Scotia mathematics curriculum was designed so that it is possible for teachers to easily see the progression of skills that students are expected to achieve across the academic year. Within the mathematics curriculum, each strand (Number, Patterns and Relations, Measurement, Geometry, Statistics and Probability) is broken down into separate sections. The first page of each section includes the name of the strand and the GCO. The second page of each section provides an overview of the SCOs (approximately 8-12 depending on the grade), including a brief description of each. The order in which the outcomes are presented does not necessarily prescribe the order in which skills should be taught (Nova Scotia Department of Education and Early Childhood Development, 2013b, p. 18); instead, specific outcomes are presented in relation to the overarching general curriculum outcomes.

Following the introduction of each SCO, there is a standard sequence of information that is presented. First, the mathematics curriculum outlines the processes and performance indicators that are specific to that outcome. A scope and sequence text box is then provided in order to relate the SCO to the previous and subsequent grade SCO's. This was created in order to help teachers gain a broader understanding of the SCO skill progression across grades (Nova Scotia Department of Education and Early Childhood Development, 2013b, p. 18). Please refer to Table 1 for a summary of the SCO skill progression, as outlined by the scope and sequence boxes, across Grade Primary, Grade One, and Grade Two. The table is organized so that when looking across the columns the reader can see the progression of an SCO across grades, and when looking down the columns the reader can see all of the SCOs for that grade. Empty cells reflect an SCO from an earlier grade that has been discontinued or an SCO that has yet to be introduced.

Following the scope and sequence table in the mathematics curriculum is a brief section that outlines the relevant background information to provide context for the SCO. Included in this section is information on the developmental progression of skills and various definitions/explanations of terms or concepts. The next section pertains to assessment, teaching, and learning. Across the SCOs, a number of sample tasks are offered as suggestions for formative or summative assessment. Similarly, a number of instructional strategies are offered for planning daily lessons. Various models and manipulatives (e.g., blocks, calendars, coins, counters) are then presented to supplement the assessment and instructional strategies, as is a table of linguistic terms (e.g., numeral, parts, ten-frames) that both the teacher and student should be familiar with. Lastly, the curriculum document offers a number of print resources to the teacher to help support the content outlined by the SCO.

Table 1

Specific Curriculum Outcomes in the Nova Scotia Mathematics Curriculum [Number Strand]

Grade Primary	Grade One	Grade Two
N01: Students will be expected to say the number sequence by: • 1s from 1 to 20 • 1s starting anywhere from 1-10 and from 10-1	N01: Students will be expected to say the number sequence by: • 1s, forward and backward between any two numbers from 0-100 • 2s to 20, forward starting at 0 • 5s to 100, forward starting at 0, using a hundred chart or a number line • 10s to 100, forward starting at 0, using a hundred chart or a number line	N01: Students will be expected to say the number sequence by: • 1s, forward and backward, from any point to 200 • 2s, forward and backward, starting from any point to 100 • 5s and 10s, forward and backward, using starting points that are multiples of 5 and 10 respectively to 100 • 10s, starting from any point to 100
N02: Students will be expected to recognize, at a glance, and name the quantity represented by familiar arrangements of 1-5 objects or dots.	N02: Students will be expected to recognize, at a glance, and name the quantity represented by familiar arrangements of 1-10 objects or dots.	--
--	--	N02: Students will be expected to demonstrate if a number (up to 100) is even or odd.
--	--	N03: Students will be expected to describe order or relative position using ordinal numbers (up to tenth)

Grade Primary	Grade One	Grade Two
N03: Students will be expected to relate a numeral, 1-10, to its respective quantity. N06: Students will be expected to demonstrate an understanding of counting to 10.	N03: Students will be expected to	--
N04: Students will be expected to represent and describe numbers 2-10 in two parts, concretely and pictorially.	N04: Students will be expected to represent and partition numbers to 20.	N04: Students will be expected to represent and partition numbers to 100.
--	N05: Students will be expected to compare sets containing up to 20 objects to solve problems using referent and one-to-one correspondence.	N05: Students will be expected to compare and order numbers up to 100.
--	N06: Students will be expected to estimate quantities to 20 by using referents.	N06: Students will be expected to estimate quantities to 100 by using referents.
--	N07: Students will be expected to demonstrate an understanding of conservation of number for up to 20 objects.	--
--	N08: Students will be expected to identify the number, up to 20, that is one more, two more, one less, and two less than a given number.	N08: Students will be expected to demonstrate and explain the effect of adding zero to or subtracting zero from any number. N10: Students will be expected to apply mental mathematics strategies to quickly recall basic addition facts to 18, and determine related subtraction facts.

Note. *Empty cells reflect an SCO from an earlier grade that has been discontinued or an SCO that has yet to be introduced.*

Chapter 3: An Integration of Theory and Practice

Despite the fact that we have a significant, evidence-based body of knowledge that could be used to guide teaching and prevent the development of many mathematics difficulties, a large proportion of students in Nova Scotia continue to struggle with mathematics (Brochu, Deussing, Houme & Chuy, 2012). This raises questions regarding the current mathematics curriculum in Nova Scotia with respect to the foundational beliefs from upon it was based and the suitability of the instructional approaches being recommended. The purpose of the current chapter will be to compare and contrast the information reviewed in chapter two with information obtained from the cognitive science literature (chapter one). In light of this, headings in the current chapter will mirror those of chapter two.

Introduction to the Curriculum

The Nova Scotia mathematics curriculum outlines a number of beliefs about learning, identifies various general and specific curriculum outcomes (SCOs), and provides teachers with assessment and instructional strategies for planning their daily lessons. The mathematics curriculum also advises that Grade Primary, Grade One, and Grade Two teachers spend 45 minutes daily on mathematics instruction in order to support a constructivist approach to learning. There are a number of matters within this statement that warrant comment. First, the mathematics curriculum references the *Time to Learn Strategy Guidelines for Instructional Time: Grades Primary – 6* (Nova Scotia Department of Education, 2002) for this 45-minute time allotment. Upon closer evaluation of the latter document, there is no reference to scientifically (or otherwise) gathered data to support their conclusion that this amount of time per day is optimal for promoting mathematics learning. Instead, the document states that *Time to Learn* initiatives have centred on “maximizing and optimizing the amount of time that teachers spend on delivering the prescribed curriculum” (Nova Scotia Department of Education, 2002, p. 1). Second, the mathematics curriculum does not mention the amount of time in the daily 45 minutes that teachers should be allocating specifically for practice. Automaticity of basic math facts, achieved through repetition and practice, has been frequently linked with proficiency in retrieval and use of more efficient strategies to solve problems (e.g., Fuchs, Seethaler, Powell, Fuchs, Hamlett, & Flether, 2008; Power et al., 2009), so it would make sense to provide teachers with information about how to balance time for instruction and time for practice to ensure that this vital component of mathematics learning is included. Lastly, the mathematics curriculum states that allowing 45 minutes of daily instruction is commensurate with a constructivist approach to teaching through problem solving. It is unclear where the justification for this last statement was derived. A review of the literature reveals no specific connection between time and instructional approach. Constructivist methods of learning are not contingent upon amount of *time* of instruction, rather the *type* of instruction.

The mathematics curriculum emphasizes the importance of ongoing assessment. For the most part, this is commensurate with what is found in the cognitive science literature (e.g., Carnine et al., 2004; Kirschner, Sweller, & Clark, 2006) which states that progress monitoring and feedback are essential to learning, and assessment should be commensurate with the outcomes, have clearly defined criteria, and include a variety of assessment tools (e.g., Alfieri et al., 2011). In theory, the mathematics curriculum outlines a number of assessment and learning tasks that are important in order to monitor and evaluate student progress; however, the specific tasks suggested in the curriculum are not supported by the scientific literature. For example, one means of assessment that is frequently recommended throughout the mathematics curriculum is that students create their own personal strategies to solve problems (rather than being taught the standard algorithm). This type of learning task, which can also be used as a form of assessment, is very much in line with constructivist approaches to learning. Constructivists would argue that such an approach promotes students being proactive participants in their own learning process and develops collaboration and problem solving skills (e.g., Cobb et al., 1991; Klein, 1998). Others would argue that direct instruction in procedural methods is superior because early knowledge in a given domain is limited, and so too is children's ability to construct a way to solve a problem. Rittle-Johnson, Siegler, & Alibali, 2001). Explicit and systematic instruction, where skills are explicitly taught in incrementally difficult steps, allows for frequent monitoring of progress and feedback on incorrect responses to avoid development and practice of ineffective or inefficient strategies. Certainly there will be some children who, with minimal guidance, will be successful in generating their own strategies to solve problems. However, researchers have demonstrated that all children benefit from, and that children's learning is not negatively affected by, direct and systematic instruction (Clark, 2009; Kirschner, Sweller, & Clark, 2006; Klahr & Nigam, 2004; Kroesbergen & Van Luit, 2002, 2003; Sweller, Kirschner, & Clark, 2007).

The Nova Scotia mathematics curriculum outlines seven critical components that students must encounter throughout their education in mathematics. The mathematics curriculum does not reference a source for these seven components but presents them as fact; however no evidence to support their validity is provided. One of the seven components states that there should be an emphasis on learning through problem solving; specifically, students should be exposed to a variety of problem-solving approaches, including finding multiple solutions to the same problem, and students should create their own personal strategy to solve a problem. By actively generating their own strategies and being exposed to multiple methods to solve problems, the mathematics curriculum proposes that students gain a better understanding of the relationship amongst numbers (Nova Scotia Department of Education and Early Childhood Development, 2002b, p. 13). In many ways, there is merit to this idea. Learning is an active process whereby the learner only takes in information to which he or she attends to or engages in (e.g., Duncan et al., 2007; Kroesbergen & Van Luit, 2002). In some cases, learning is optimized under circumstances where individuals actively interact with rather than passively receive information. However, the fact that active learning is beneficial does not mean that active learning only occurs when constructivist approaches are used. Active learning can also occur when information is explicitly taught. In fact, there is an abundance of literature that supports the fact that retention of information is optimized under conditions of explicit teaching rather than when self-direction is required (e.g., Klahr & Nigam, 2004; Mayer, 2004; Slamecka & Katsaiti, 1987; Stern & Bransford, 1979). By requiring students to either generate their own strategies or become proficient in multiple strategies, we tax their cognitive resources and reduce the cognitive energy they have for learning, making tasks unnecessarily difficult.

Two other components that are emphasized by the mathematics curriculum are that a) students make connections between mathematical ideas and the real world, and b) that skills must be learned in context (rather than in isolation) so that students can apply these skills to solve problems. While these statements are perhaps true to some degree, the mathematics curriculum fails to emphasize the importance of practice and the direct instruction of basic skills (e.g., counting, memorizing basic facts), which instead are expected to develop as a by-product of learning rather than through explicit instruction (Nova Scotia Department of Education and Early Childhood Development, 2002b, p. 16). One way the mathematics curriculum suggests making connections between mathematical ideas and the real world is through the emphasis of story problems. For example, in Grade Two the mathematics curriculum suggests that students create their own addition and subtraction problems in the context of their favourite story, or that students solve a problem from a story that is provided (e.g., My dad made 43 chocolate chip cookies and some peanut butter cookies. There were 92 cookies on the cupboard. How many were peanut butter?; Nova Scotia Department of Education and Early Childhood Development, 2002c, p. 71, p. 39). While problems presented in this manner may seem more intuitively appealing and motivating for students and educators, solving math problems that are embedded within a story context is much more difficult (Jitendra, Griffin, Haria, Leh, Adams, & Kaduvettoor, 2007). It requires the integration of a number of cognitive processes and places an undue emphasis on the student's ability to understand numbers as they are embedded within the text (Fuchs, Seethaler, Powell, Fuchs, Hamlett, & Flether, 2008). This is not to say that story problems should not be a part of the mathematics curriculum, rather that it becomes problematic when attempts to make connections with the real world takes precedence over the mastery of basic facts and direct instruction of skills.

Teaching Philosophy

The Nova Scotia mathematics curriculum was designed based on several key assumptions and beliefs about learning. Not all of these assumptions and beliefs are commensurate with what we know from the cognitive science literature. First, the Nova Scotia mathematics curriculum states that, “students learn by attaching meaning to what they do and need to construct their own meaning of mathematics” (Nova Scotia Department of Education and Early Childhood Development, 2013b, p. 21). At a basic level, the literature does support the idea that information is retained in greater detail when it is gathered by learners rather than provided by the instructor (e.g., Alfieri et al., 2011; Slamecka & Graf, 1978). For example, in cultures without formalized education, complex skills are learned through active participation in activities and in the absence of explicit verbal teaching (e.g., Rogoff, 1990). As a result, there is a belief that information obtained through discovery-based approaches is superior to information obtained through direct instruction. This is perhaps why the strong support for using activities that promote making authentic connections between materials and student interests (e.g., drawing pictures of favourite toys and then counting them, identifying numbers within the community) persist. Not to mention that such activities are intuitively appealing and are engaging for students and teachers alike. However, while it is difficult to argue against the assertion that meaningful learning is important, the reliance on having children construct their own meaning using discovery-based approaches may actually be detrimental depending on the nature of the activity and the ability of the learner. Discovery-based learning requires that students select and organize relevant incoming information and then integrate this with their previous knowledge (Mayer, 2003). This requires a high level of problem solving, has the potential to tax the learner's working memory (Rittle-Johnson, 2006; Sweller, 1988), and it requires that the learner have the ability to process and attend to relevant information (Kirschner et al., 2006). Not all students in the early elementary grades have the necessary metacognitive skills to be able to successfully encode and interpret information presented in such a manner. A direct and systematic approach to instruction on the other hand, is beneficial for all types of learners (e.g., Clark 2009; Klahr, 2009; Mayer 2004; Sweller, Kirschner, & Clark, 2007) and can still be accomplished in a manner that is meaningful and enjoyable for students and educators.

The second belief about learning that is outlined in the mathematics curriculum is that learning occurs under conditions of clear expectations, assessment, and feedback. Theoretically, this belief is largely supported by the cognitive science literature (e.g., Kirschner, Sweller, & Clark, 2006; Carnine et al., 2004); however, some of the ways in which this theory is translated into practice within the mathematics curriculum are problematic. Certain approaches and contexts used to evaluate student performance are inefficient and unnecessarily strain their cognitive resources. For example, one of the performance indicators in the Grade Two mathematics curriculum requires students to become proficient in eight different mental mathematics strategies to solve basic addition facts (Nova Scotia Department of Education and Early Childhood Development, 2013c, p. 73). Requiring students to achieve mastery in eight different mental mathematics strategies is problematic as it is an inefficient (e.g., Jitendra et al., 2007) and takes away from time that could be spent practicing and consolidating basic math skills (e.g., Imbo & Vandierendonck, 2008). Learning multiple methods to solve mental mathematics problems not only places an undue demand on cognitive resources (making the task more difficult), but it requires children to attend to the relevant information, search their working memory for the most appropriate method, and then integrate the two (Kirschner et al., 2006; Mayer, 2003; Rittle-Johnson, 2006; Sweller, 1988). Similar examples of inefficient and cognitively demanding assessment methods are seen throughout all three mathematics curriculum documents. Given that mastery of basic numerical competencies is necessarily for a deeper understanding of complex math problems (e.g., Baroody, 2003; Ferrari & Sternberg, 1998), and that mastery of these competencies is achieved through repetition and practice (Geary et al., 2004; Imbo & Vandierendonck, 2008), it is interesting that the assessment tools outlined within the mathematics curriculum primarily focus on exposure to multiple methods to solve problems.

A third belief that is pervasive throughout the Nova Scotia mathematics curriculum is that students have their own unique learning style, rate of learning, and background knowledge and experiences. The mathematics curriculum states that learning styles influence the way in which students "make sense of, receive, and process information, demonstrate learning, and interact with peers and their environment" (Nova Scotia Department of Education and Early Childhood Development, 2013c, p. 25). Specifically, the mathematics curriculum attempts to recognize the following groups of learners: *auditory*, *kinesthetic*, and *visual*. Interestingly, the mathematics curriculum does not reference any literature to support the identification of these groups, nor does it provide any rationale for differentiated instruction for these groups. The mathematics curriculum suggests that students be provided with a number of representations (e.g., concrete, pictorial, symbolic, and contextual) in order to demonstrate their knowledge and optimize their learning experience (e.g., Nova Scotia Department of Education and Early Childhood Development, 2013b, p. 43). The mathematics curriculum also advises teachers to conduct individual student interviews (to gain insight into their thinking), so that teachers can identify groups of students who learn from similar types of instruction (Nova Scotia Department of Education and Early Childhood Development, 2013c, p. 75).

While taking learning styles into account might be helpful for some students or teachers, it becomes problematic when a reliance on learning styles means that other effective teaching and learning methods are deemphasized (Rayner, 2007). Some children will exhibit a preference for a given instructional method; however, by assuming that all students are heterogeneous with respect to how they learn, and thus require differentiated instruction, policy makers are drawing inferences that have yet to be affirmed by the cognitive science literature (e.g., Pashler, McDaniel, Rohrer, & Bjork, 2009). What research does support, is that regardless of learning style, the acquisition of math skills generally follows the same progression (e.g., Elbaum et al., 2000; Geary, 1993; Jordan et al., 2010; Locuniak & Jordan, 2008). Not only does the Nova Scotia mathematics curriculum fail to provide information about evidence-based instruction that would allow for this progression of skills (i.e., direct and systematic instruction), it fails to provide any references to substantiate the emphasis on individual learning styles. In addition, the mathematics curriculum fails to provide information about a consistent, valid, and reliable way to measure learning styles, and does not specify whether such learning styles reflect a student preference or ability. Validity and reliability concerns aside, the mathematics curriculum does not distinguish whether learning styles are a reflection of student *preference* or student *ability*; something that is also absent from the literature on learning styles (Pashler et al., 2009). Aside from the fact that researchers have yet to determine which teaching strategies are most effective for different types of learners (e.g., Denig, 2004; Honey & Mumford, 1992; Slack & Norwich, 2007), and that some researchers argue there is benefit to exposing students to different modalities of instruction (e.g., Petty, 2004), there is simply insufficient evidence to support the practical application of learning styles (Allcock & Hulme, 2010; Coffield et al., 2004; Cook, Gelula, Dupras, & Schwartz, 2007; Kavale & Forness, 1987; Landrum & McDuffie, 2010; Lehmann & Ifenthaler, 2012; Martin, 2010; Muse, 2001; Rohrer & Pashler, 2012).

Curriculum Layout

The Nova Scotia mathematics curriculum states that the format was designed so that teachers could easily see the progression of skills across grades (Nova Scotia Department of Education and Early Childhood Development, 2013a, p. 16). Each strand (e.g, Number, Patterns and Relations, Measurement, Geometry, Statistics and Probability) has a general curriculum outcome (GCO) and several specific curriculum outcomes (SCOs). Although the mathematics curriculum provides a *scope and sequence* box for each SCO there is not always a direct one-to-one correspondence between skill in the previous and subsequent grades. For example, for the SCO *N03* in the Grade One mathematics curriculum, there are two SCOs that match up with it from the Grade Primary mathematics curriculum, but no SCO that matches up with it in the Grade Two mathematics curriculum. Further, the mathematics curriculum does not offer one large table that outlines the SCO integration across grades. This does not facilitate a broad understanding of the scope and progression of the outcomes and would make it more difficult for a teacher to find out about which earlier skills to target for a student who was underachieving.

The Nova Scotia mathematics curriculum does not advise teachers on the preferred order of presentation of the SCOs within the classroom. While perhaps this would allow the teacher greater flexibility with respect to how he/she can structure the curriculum (i.e., introduction of skills/concepts), it is interesting that the order of presentation in the curriculum does not follow a prescribed order. Given that mathematics skills are cumulative and follow a specific developmental trajectory (e.g., Geary, 1993; Jordan et al., 2010; Locuniak & Jordan, 2008), one would think that the mathematics curriculum would follow a specified order within each grade.

In terms of the natural progression of skills across grades, the Nova Scotia mathematics curriculum is generally in line with the recommendations from the literature. The Grade Primary mathematics curriculum does introduce children to the idea that numbers have a relative magnitude and that counting is guided by principles (e.g., Gelman & Gallistel, 1978). The Nova Scotia mathematics curriculum does ensure that students understand that Arabic numerals (e.g., 0, 1, 2), and number words (e.g., one, two), have meaning based on the quantities they represent as recommended in research (e.g., Dyson, Jordan, & Glutting, 2013). For example, in the Grade Primary curriculum, students begin by counting and comparing small sets of numbers from 1-10 and identifying a number with its respective quantity. In the Grade One curriculum, set sizes are increased (e.g., compare sets up to 20 objects, demonstrate conservation of number up until 20), and students are introduced to skip counting (by 2s, 5s, and 10s to 10). Students are also asked to compare sets up to 20 and to add and subtract single digit numbers. In the Grade Two curriculum, set sizes are again increased (e.g., compare and order numbers from 1-100, estimate quantities to 100), and students are introduced to the idea of adding or subtracting zero to a number. This process of increasing difficulty follows what is suggested in the literature and as students develop a more advanced number sense (e.g., how numbers are composed/decomposed), this knowledge is generalized across more complex problems and operations (Griffen, 2004; National Mathematics Advisory Panel, 2008). For example, by Grade Two, the curriculum notes that students are expected to demonstrate an understanding of addition (of 1- and 2-digit numbers) with sums of up to 100, and the corresponding subtraction facts. The mathematics curriculum also highlights the importance of committing basic facts to long-term memory so that in time, students will develop automaticity and no longer need to rely upon mental mathematics strategies (Nova Scotia Department of Education and Early Childhood Development, 2013c, p. 74). This is in line with what is found within the cognitive science literature (e.g., Baroody, 2003; Geary et al., 2004; Ferrari & Sternberg, 1998). In general, the curriculum does an adequate job of presenting information about the progression of skills across grades, but it lacks evidence-based information about approaches to teaching and the means with which students will demonstrate skill acquisition (e.g., provision of personal strategies, becoming proficient in multiple strategies, use of a variety of representations).

One important component of mathematics acquisition that is not emphasized within the Nova Scotia mathematics curriculum is the matter of practice. Despite the fact that researchers have demonstrated that practice is beneficial for all types of learners (e.g., Fuchs et al., 2010; National Mathematics Advisory Panel, 2008), some researchers (e.g., Coyne et al., 2009; Jitendra et al., 2001) have found that educators and curriculum designers believe that practice is only truly important for students with learning difficulties, and even caution against drill and rote memorization of facts (e.g., National Research Council, 2001). The positive effects of practice have been correlated with proficient and automatic retrieval of math facts and can help to reduce cognitive processing demands (e.g., working memory, processing speed) that are often associated with completing math problems (Geary & Widaman, 1992; Fuchs et al., 2013; Powell, Fuchs, & Fuchs, 2009).

Summary

Information accumulated from the cognitive science literature and the Nova Scotia mathematics curriculum has led to several important conclusions for school psychologists, educators, and policy makers. First, there is a large evidence-based body of research that clearly delineates the developmental trajectory of mathematics skills and the instructional methods that support the acquisition of such skills (e.g., Alfieri et al., 2011; Geary, 1993; Jordan et al., 2010; Kirschner, Sweller, & Clark, 2006; Locuniak & Jordan, 2008). The order in which skills are presented across grades in the Nova Scotia mathematics curriculum is largely commensurate with what is outlined within the literature; however, the Nova Scotia mathematics curriculum deviates from what is recommended in terms of the instructional and assessment strategies being suggested. Specifically, the Nova Scotia mathematics curriculum advises teachers to follow a constructivist approach to learning, whereby discover-based methods of instruction are emphasized over the explicit and systematic teaching of facts. Second, the Nova Scotia mathematics curriculum states that it was founded on the belief that instruction should be differentiated based upon individual learning styles. Researchers, however, have consistently demonstrated that although children will learn at various rates and paces, they are not necessarily heterogeneous with respect to how they learn (e.g., Dunn, 1983, 1990; Kavale & Forness, 1987; Lovelace, 2005). Lastly, the Nova Scotia mathematics curriculum has a number of practical barriers (e.g., ease in viewing the progression of skills across grades, lack of a specific order in which to introduce outcomes, and unnecessary repetition of information) that could compromise the ability of teachers to efficiently and effectively use the document.

Chapter 4: Implications and Recommendations

Recommendations for the Mathematics Curriculum

Scope and sequence summary. The Nova Scotia mathematics curriculum was designed with the intent that teachers would be able to easily see the progression of skills that students are expected to achieve across the academic year (Nova Scotia Department of Education and Early Childhood Development, 2013b, p. 18). A scope and sequence text box was created in order to relate each of the specific curriculum outcomes (SCO) to previous and subsequent grades (Nova Scotia Department of Education and Early Childhood Development, 2013b, p. 18). This is likely useful for teachers who need to reference information one grade below or above the grade they are teaching; however, it would not be useful to inform programming for a child who is more than one year behind or ahead more typically achieving classmates. In this case, the teacher would need to locate another document in order to gain insight into which skills should be addressed. Even if teachers were to reference curriculum documents from other grades, because there is not one scope and sequence table that clearly links the outcomes within and across grades, it may be difficult for a teacher to identify the relevant outcomes from previous or subsequent grades. This could be particularly problematic for teachers who have combined classes, as they would theoretically have an even wider range of student abilities. If the Nova Scotia mathematics curriculum offered a scope and sequence box of the entire arrangement of specific outcomes across grades, it could substantially facilitate the location of skills and teachers' understanding of the expectations of how mathematics skills develop across grades.

Order of specific curriculum outcomes. The Nova Scotia mathematics curriculum does not provide teachers with a specified order in which they are to introduce specific outcomes (Nova Scotia Department of Education and Early Childhood Development, 2013b, p. 18), so teachers must decide about the order in which they will cover the outcomes in relation to the overarching general curriculum outcomes. This assumes that all teachers have a good understanding of mathematics and how the skills and topics are cumulative and build on each other. This could be problematic for a number of reasons. For example, should a student relocate to another school halfway through the academic year, they may be faced with re-doing previously taught information or they may have missed content in a given area. Without a standard order of presenting information, how can the province of Nova Scotia ensure that all children receive the same exposure to grade-level concepts? Further, given that we know the acquisition of math skills follows a developmental trajectory (e.g., Geary, 1993; Jordan et al., 2010; Locuniak & Jordan, 2008), it is interesting that the mathematics curriculum does not have a prescribed order that mirrors this natural progression within grades. Not to mention that without having a specific order in which skills are taught, teachers may be left feeling overwhelmed or uncertain of how to introduce skills; this could be particularly problematic for newer and less experienced teachers. Having an explicit order in which to introduce SCO's would allow for more consistency across schools, be more in line with the literature, and would offer teachers more support in the implementation of the mathematics curriculum.

Unnecessary repetition. Throughout the Nova Scotia mathematics curriculum documents, there is a considerable amount of unnecessary repetition. For example, under the *Assessment Strategies* section of each SCO, the following questions are offered as guidance: “What are the most appropriate methods and activities for assessment student learning? How will I align my assessment strategies with my teaching strategies?” (Nova Scotia Department of Education and Early Childhood Development, 2013a, p. 126). Similarly, under the *Planning for Instruction* section of each SCO, there are five guiding questions for the teacher to consider; for example, “Does the lesson fit into my yearly/unit plan?” (Nova Scotia Department of Education and Early Childhood Development, 2013a, p. 27). All of these questions, and many others, are repeated verbatim following their respective heading in each SCO. This repetition makes it difficult to identify more pertinent information for each SCO, as it requires sifting through previously read information. One potential solution to this repetition would be for the mathematics curriculum to offer a summary page at the beginning of the document that outlined various questions to consider when planning for instruction or assessment. In this way, teachers could have an easy reference page should they have questions, and it would make planning daily lessons more efficient as relevant information would be easier to locate.

Implications for Teacher Training Programs

Developing familiarity with research. Given that there is a large evidence-based body of knowledge that could be used to guide teaching and promote learning, teacher training programs should ensure that a component of their training focus on understanding and interpreting educational research. Specifically, teachers would benefit from learning how to be critical research consumers (e.g., learn the difference between scientific and non-scientific sources of information, different types of logic used in science, difference between theoretical research and applied research). In this way, teachers will be more prepared to differentiate between scientific (e.g., rationalism, empiricism) and non-scientific (e.g., intuition, tenacity, authority) information (Martella, Nelson, Morgan, & Marchand-Martella, 2013). With a greater understanding of how to think critically about research, teachers would be able to critically analyze and make informed decisions about the information about assessment and instructional methods that are outlined in the Nova Scotia mathematics curriculum.

Training in developmental psychology. Teacher training programs would also benefit from having a greater emphasis on developmental child psychology. Learning and development are iterative processes that occur gradually and continuously across the years (Siegler, 1996). As a result, teacher-training programs should emphasize that all learning occurs in the context of a child's developmental level. Knowledge of brain development, including the development of working memory and other cognitive processes, is essential to understanding how children learn and what types of tasks they are capable of at each age. Further, knowledge of the developmental progression of math (and other academic) skills could also help future teachers plan the order in which they introduce specific curriculum outcomes. This knowledge could also help to inform the depth and breadth to which teachers emphasize the concepts outlined in the mathematics curricula.

Implications for School Psychologists

Familiarity with curriculum. Responsibilities of school psychologists in Nova Scotia include prevention, consultation, assessment, intervention, professional development, and research (Nova Scotia Department of Education, 2009). As such, it is imperative that school psychologists be familiar with the curriculums in place so that they can effectively collaborate with teachers and school personnel and make informed recommendations based upon the literature and what is currently being implemented within their schools. Familiarity with the curriculum is also necessary so that school psychologists understand the expectations within and across each grade. This knowledge will allow them to correctly interpret results from standardized tests. For example, familiarity with the curriculum could allow school psychologists to differentiate reasons why a student is unable to answer a given question (i.e., if they do not understand the content or if they have not yet been taught this skill).

Being familiar with the curriculum also includes being familiar with the language that is used. Knowing, for example, that a *number sentence* includes a combination of two addends and a solution, or being familiar with the difference between a GCO and an SCO. Knowledge of this language is important in order for school psychologists to effectively communicate and ask questions to teachers, particularly with respect to the content that is being covered across grades.

Instructional methods. It is also important for the school psychologist to be familiar with the instructional practices (e.g., discovery-learning, multiple methods to solve problems) that are being implemented within Nova Scotia, so that they can make recommendations that consider the context of the classroom. Recommendations made by school psychologists should be informed by research; however, these recommendations are not useful if they are at odds with what is outlined within the curriculum. Part of the role of a school psychologist is to collaborate with teachers regarding evidence-based practices. This includes discussions about research and how certain instructional methods and assessment strategies that are outlined within the Nova Scotia mathematics curriculum are not necessarily beneficial for all children, and how to help teachers translate what is recommended in the research into practice.

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