An Interactive/Collaborative Su Doku Quilt

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Abstract

After introducing Su Doku, a popular number place puzzle, the authors describe a transformation of the puzzle where each number is replaced with a distinct colour. The authors investigate the nature of the experience of solving this transposed version. This, in turn, inspires a design process leading to the creation of an interactive quilt. This process, involving issues of choice of medium, level of interactivity, colour theory and aesthetics, is described. The resulting artefact is a textile diptych accompanied by a collection of coloured buttons, constituting a solvable puzzle and its solution.

1. Introduction

Sometimes called the "Rubik's cube of the 21st century" [1], the mathematical puzzle Su Doku has been sweeping the world. The rules of the puzzle are simple:

> The puzzle is most frequently a 9×9 grid, made up of 3×3 subgrids called "regions" […]. Some cells already contain numbers, known as "givens" or "clues". The goal is to fill in the empty cells, one number in each, so that each column, row, and region contains the numbers 1-9 exactly once. Each number in the solution therefore occurs only once in each of three "directions". [2]

For the rare reader who has not heard of this puzzle, the online encyclopaedia entry at wikipedia [2] can answer many questions both historical and analytical. In this paper, we describe a method of solution that uses colours instead of numbers and show that the solution results in a pleasing template for a quilt. Using this idea, we design a quilted diptych that depicts a specific puzzle [3] in a way to make it interactive. The numerical puzzle on which we based our work, shown in Figure 1, has been classified as "easy".

1			8	3				2
5	7				1			
			5		9		6	4
7		4			8	5	9	
		3		1		4		
	5	1	4			3		6
3	6		7		4			
			6				7	9
8				5	2			3

Figure 1: *The starting point of the puzzle*

2. The strategy versus the technique

Under normal circumstances, solving a Su Doku puzzle involves scanning, marking up and analysing [2] digits in the grid. Digits need to be manipulated mentally and placed, in various combinations in the appropriate cells of the grid, based on logical reasoning. Different people use different strategies to find the solution to a given puzzle, and, as is often the case in problem solving, these strategies appear to be more different than they really are, since the logic that leads different people to the solution is based on the same starting point and the same rules.

We distinguish two components to the solving of a Su Doku puzzle: strategy and technique. The strategy refers to the conceptual processes used to determine the content of each cell, such as, for example, elimination. In contrast, the technique refers to the physical manipulation of instruments, which, in the traditional solving process, are pencil and paper. Our main interest is in the latter: we investigate a technique that emphasises sensory-motor skills over logico-mathematical skills. In the technique we are describing, each number in the puzzle is replaced by a colour.

3. The colour technique

At first glance, it would seem that simply replacing the digits with colours does not change the solving method. In an experiential sense, however, it can. For instance, instead of holding a single pen or pencil in her writing hand, the solver would hold 9 different coloured ones in her other hand. For each blank cell, she puts down the pens corresponding to invalid colours. The pens remaining in her hand represent the possible solutions for that cell. If she is holding a single pen, that colour is the solution and the cell can therefore be coloured in. If she is holding several pens, she needs to record each of these possibilities. In a later stage, the solver determines the final solution for each cell by elimination. The consequence of the change of medium is that when it comes time to eliminate possibilities, this takes place through *separating off* pens (physical, motor), rather than *selecting* numbers (mental).

To illustrate these steps in the colour technique, we describe the solving of the puzzle for the central "region" of our example (number 5 in Figure 1, if counting sequentially left-to-right and top-to-bottom). Borrowing from the quilter's vocabulary, we refer such a 3×3 region as a block. Within the block, as the order in which the cells are investigated is open, we choose ones that highlight possible scenarios.

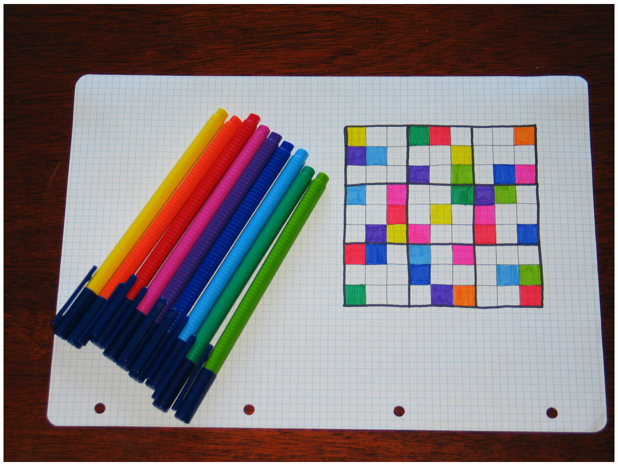

Figure 2: *The starting point in colour*

In Figure 2, the numbers have been replaced by colours (The colour version of these pictures can be found in the CD, and we recommend that you colour the photographs on this and the next two pages yourself to follow the discussion more easily). The chosen colours are: 1-*yellow*, 2-*orange*, 3-*red*, 4-*pink*, 5-*violet*, 6-*dark-blue*, 7-*light-blue*, 8-*dark-green*, 9-*light-green*.

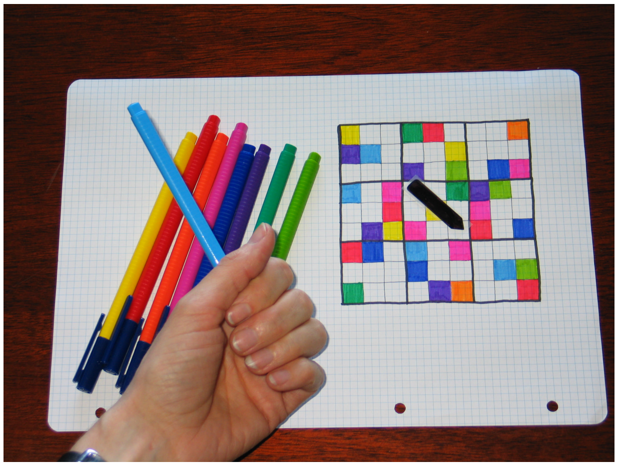

Figure 3: *Step 1, a "sure"*

In Figure 3, to determine the solution for the cell that is indicated with the arrow (block 5, cell 9), the solver is able to eliminate *dark-green, yellow* and *pink* because they occur in the block itself. Additionally, investigating the column eliminates *light-green*, and *orange*. Finally, *violet*, *red* and *dark-blue* are also eliminated because they appear in the same row as cell 5-9. This leaves only *light-blue* in the hand, making it the solution for this cell. Holding a single pen this early in the solution process is not a frequent occurrence, and we call this a "sure". The colour is noted down in the cell (see Figure 4).

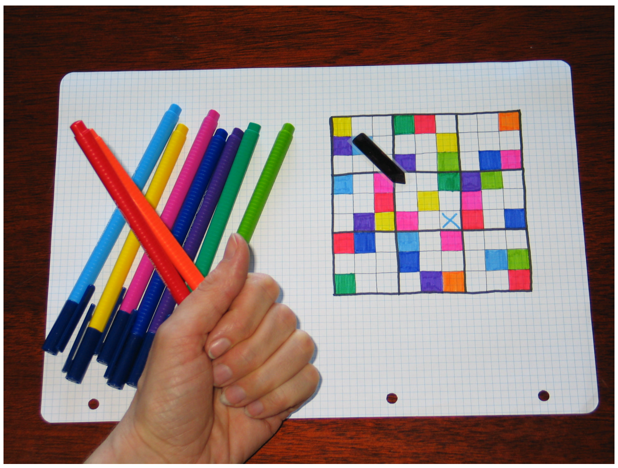

Figure 4: *Step 2, two possibilities*

For cell 5-1, starting with the nine colours again, the block rule eliminates *dark-green*, *yellow*, *pink*, and *light-blue* (because of the "sure" in the previous move), leaving *orange*, *red*, *violet*, *dark-blue* and *light-green*. The row rule allows the solver to remove *violet* and *light-green* as well, leaving *orange*, *red* and *dark-blue*. The column already contains *dark-blue*, and therefore, after all three rounds of elimination, the solver is left with *orange* and *red*. In this case, the final colour for that cell is not yet determined. The solver records these possibilities as little marks in the cell (see Figure 5).

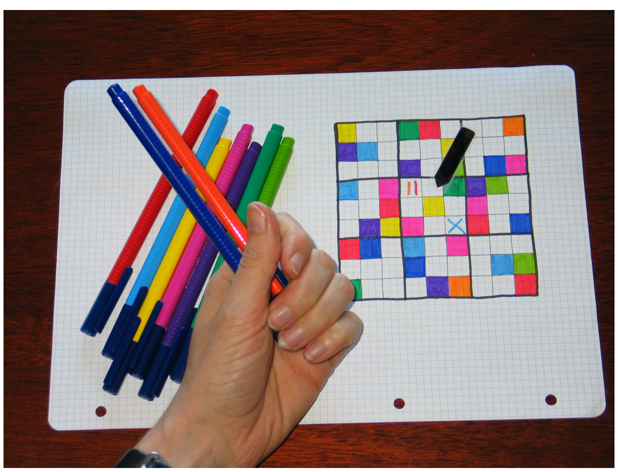

Figure 5: *Examining cell 5-2*

In cell 5-2, because of the block rule, she can again eliminate *dark-green*, *yellow*, *pink* and *light-blue* leaving *orange*, *red*, *violet*, *dark-blue* and *light-green* in the hand. The row rule allows the solver to remove the *violet* and *light-green* pens, and the column rule eliminates *red*. This leaves the solver with *dark-blue* and *orange* in hand, which the solver records in the cell (see Figure 6).

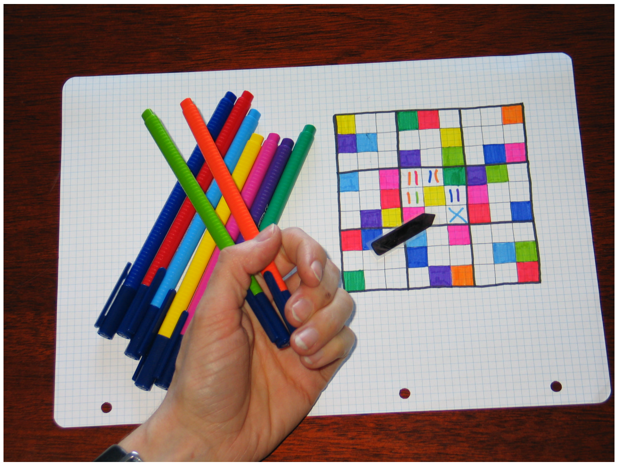

Figure 6: *Examining cell 5-8*

Using the same technique and strategy on cells 5-4 and 5-6 results in *orange* or *light-green* and *violet* or *dark-blue*, respectively.

In the last unexplored cell, 5-8, the solver is again left with *orange*, *red*, *violet*, *dark-blue* and *light-green* after applying the block rule. Applying the row and column rules then leaves the solver with *orange* and *light-green* in hand. The marks are noted in the cell.

The solver has now evaluated all the cells in the block, by considering only the givens and "sures". She can now determine some of the cells further through deductive reasoning and comparing.

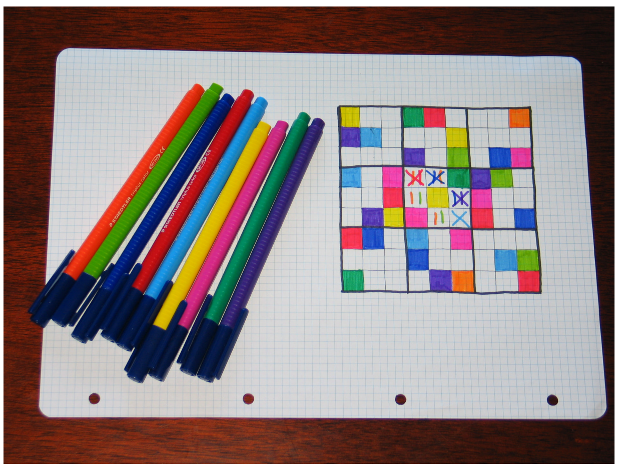

Figure 7: *Solving by elimination*

For example, cell 5-6 is the only one in the block where *violet* is possible. This sets up a domino effect: by elimination, since cell 5-6 is not *dark-blue*, cell 5-2 is the only possible place for it. Further examination reveals that cell 5-1 is the only possible place for *red* (see choices in Figure 6).

Cells 5-4 and 5-8 cannot be determined at this stage: their resolution depends on the elimination of colours in their respective rows and columns outside of block 5.

The above sequence of moves shows the kinds of reasoning and movement that can occur while solving a Su Doku using coloured pens, instead of a single pen and symbols (be they numbers or otherwise). This demonstration is only the beginning of the design journey, however.

4. Harmonising the colours

As a consequence of the rules of the puzzle, each of the colours is distributed, though randomly, evenly across the grid. For this reason, a finished puzzle with all its colours possesses an overall balance, or harmony, depending on the choice of the colours. It is this choosing of colours and the accompanying design issues that we discuss next.

Traditionally, the colour wheel [4] is subdivided into the three primary colours (red, yellow and blue), the three secondary colours, which combine two primary colours (red-yellow for orange, yellow-blue for green and blue-red for violet), and the six tertiary (combining a primary with an adjacent secondary, for example blue and violet giving indigo) as shown in Figure 8, below. This total of twelve, however, does not suit the needs of colouring the Su Doku, if we want to maintain the puzzle's inherent balance. It is possible, however, to shift the colours so they are still regularly distributed along the wheel, but total nine. We have shown this in the outermost ring in Figure 8.

Because we tend to think of colours nominally, rather than by their position in the spectrum, we have chosen nine names that will help remember the order: Yellow, Orange, Red, Magenta, Violet, Indigo, Blue, Teal and Green. Conceptually, they are not equally spaced on the circle, but when the specific shades are chosen, they can be shifted to accommodate for this condition.

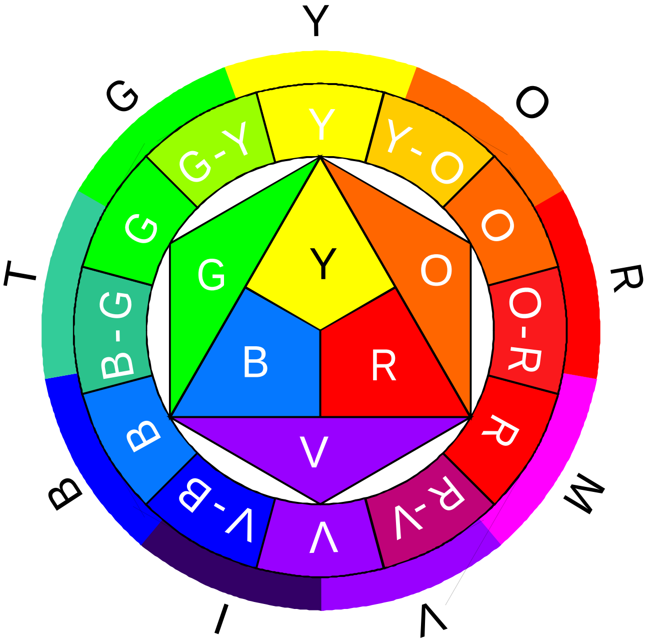

Figure 8: *The traditional colour wheel, and the subdivision into 9 [3]*

Y	M	T	G	R	I	B	V	O
V	B	I	O	M	Y	T	R	G
O	R	G	V	B	T	Y	I	M
B	O	M	R	I	G	V	T	Y
I	G	R	T	Y	V	M	O	B
T	V	Y	M	O	B	R	G	I
R	I	O	B	T	M	G	Y	V
M	Y	V	I	G	R	O	B	T
G	T	B	Y	V	O	I	M	R

Y	O	R	M	V	I	B	T	G

Figure 9: *The solution, using the 9 colours*

Figure 9 shows the solution to the puzzle with the nine colours chosen from the outer ring of the colour wheel of Figure 8. In the photographs depicting the process in section 3, the choice of colours was of course limited by the availability of the pens, and by the quality of the camera.

5. Developing the quilt – some design issues

All the laying out of colours and setting up of 3×3 blocks explained above is immediately recognisable to any quilter that has used this simple block design. Working out how to preserve the challenge of the Su Doku in a quilt, however, drew out our ingenuity.

We wanted the finished product to have a panel depicting the starting point of the puzzle and a panel depicting the finished solution. The puzzle could then be solved by using some additional moveable parts that would replace the written numbers in the conventional technique and the pens in the colour technique. This desire for interactivity is the engine that drove many of the decisions and added another dimension that a quilter normally does not need to work into the design.

The discussion extended from using beads corresponding to the same nine colours as the cells (which proved problematic due to the available materials), to creating patches that would, on one side, have the final colour for each corresponding cell, and on the other, the three, four or sometimes five possible colours for that cell, as illustrated in Figure 10.

Figure 10: *Prototypes of the patches for one possible design*

We pursued this last idea for a while, but later abandoned it because it reduced both the challenge posed by the puzzle and its interactivity. The pre-made patches already depicted predetermined combinations for the cells. In addition, the solver cannot eliminate choices progressively, one at a time, within a single cell. Ultimately, we settled on a design that provided interactivity through the use of buttons covered with the same material as that composing the quilt itself, as illustrated in Figure 11. These buttons can be used in place of coloured pens: they can be held, discarded and positioned to record the possibilities for each. This medium provides no hints to the solver.

Figure 11: *The cloth covered buttons, in the 9 colours of the quilt*

We determined the total number of buttons per colour by calculating how many empty cells could contain each colour, if considering only the givens. This allows a solver to work in any order desired and leaves the choice of strategy open. Table 1 summarises the results of this calculation. Column 1 lists the Su Doku numbers. Column 2 indicates their frequency in the selected puzzle and column 3 the maximum possible number of buttons needed. This tabulation reveals a pattern: number 2, with the least givens (2), requires the most buttons (23). In contrast, numbers 3, 4 and 5, with the most givens (5) require the fewest buttons (4, 7 and 7 respectively).

Another important component of the design process involved choosing the colours. We still had to decide what colour was what number. In the case of the pens, we had simply assigned the colours to follow the spectrum (clockwise in Figure 8) starting from 1-yellow.

Table 1, column 5 shows the colours of the textiles, in order. We chose this sequence for two reasons. Firstly, the number of buttons required for colour 2 stands out: if all the possible colours per cell were displayed simultaneously, colour 2 would dominate. We decided that magenta, the most vibrant of the nine colours, would therefore provide the most pleasing energy to the starter quilt if it displayed all the buttons simultaneously.

Secondly, having chosen to follow the spectrum, we decided on the direction to follow around the colour wheel. Of the nine colours, more than half of the materials are in the violet/blue/green section of the spectrum. Consequently, we elected to counter-balance this bias by emphasising, through the initial number of givens, the magenta/red/orange/yellow section. This section then totalled 17 givens, against 18 for violet/blue/green, and meant that we were following the wheel counter-clockwise.

#	Givens	Maximum # of Buttons	Pen colours	Initial colour (first quilt)	Used colour (first quilt)
1	4	12	Yellow	Violet	Violet
2	2	**23**	Orange	Magenta	Magenta
3	5	**4**	Red	Red	Red
4	5	7	Magenta	**Orange**	**Blue**
5	5	7	Violet	Yellow	Yellow
6	4	8	Dark-blue	Green	Green
7	4	10	Light-blue	Teal	Teal
8	3	17	Dark-green	**Blue**	**Orange**
9	3	15	Light-green	Indigo	Indigo
Total	35	103			

Table 1: *Frequency of distribution and colour choices*

Using the colour associations of column 5 results in 62 buttons in the violet/blue/green section against 41 in the magenta/red/yellow/orange. This would produce a preponderance of dark buttons. To remedy this, blue was substituted for orange and vice versa, changing the ratio from 62:41 to 52:51 and giving the distribution of column 5. In this scenario, the sequence of frequencies along the spectrum becomes: Violet (12), Magenta (23), Red (4), Orange (17), Yellow (7), Green (8), Teal (10), Blue (7) and Indigo (15). The impact of this last design decision on the number of givens was minor: from 17:18 to 19:16.

We made two more colour decision. To mimic the original printed puzzle, we selected an off-white material, and for the frame, the colour of number 9, indigo because it was taken from the cells and contrasted well with the blanks.

6. Additional design decisions

There were many other decisions to be made along the way. Scale, for example, was important. The puzzle had to be manageable. The size of the buttons was determined by available supplies and affected the size of the cells. Ultimately, we chose a 2” x 2” square for the single cell, and this determined the overall puzzle dimensions.

At an early stage, the discussion even involved the making of a triptych, with the starting point on one side, an intermediate stage in the middle, and the final solution on the other side. In the end, we settled on a diptych, and the discussion moved on to whether the panels should be connected. Should they be sown back-to-back, allowing them to be hung in space and admired from two sides, or should the solver be able to set them side by side? In the end, we made two separate panels (see Figure 13), which allowed for flexibility both in hanging and playing.

Figure 13: *The finished quilts*

7. Conclusion

When we embarked on this journey from puzzle to quilt, we did not know where it would lead. The initial transition from numbers to colours connected with our mutual love of colour. It also showed results that were reminiscent of works of Constructivist and Swiss Concrete art of the previous century, such as Joseph Albers, Max Bill, Karl Gerstner, Paul Klee and Richard Paul Lohse [5]. In addition, the geometry, as well as the colour distribution underlying the puzzle, invoked thoughts of quilting prompting us to re-interpret the puzzle as a coloured quilt. Using buttons instead of pens preserved the sensory-motor flavour of the puzzle-solving experience.

The design process itself was satisfying and multi-faceted. The variety of the avenues we explored, the complexity of the debates that the issues provoked and the range of media we considered provided us with continuing inspiration. Translating the puzzle into a quilt created a piece of mathematical art.

References

[1] http://www.conceptispuzzles.com/articles/Sudoku/

[2] http://en.wikipedia.org/wiki/Sudoku

[3] Gould, Wayne (Su Doku Grand Master). New York Post Su Doku: The Official Utterly Addictive Number-Placing Puzzle Book 1. New York: Harper Collins.

[4] Itten, Johannes (1985). Der Farbstern. Otto Maier Verlag Ravensburg.

[5] Guderian Dietmar (n.d.). Mathematik in der Kunst der Letzten Dreißig Jahre. Ebringen i. Br.: Bannstein-Verlag.